Comparing Electoral Systems

Analytical and Experimental Methods

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Electoral System

An electoral system is the mechanism used by a population to select one candidate from a group of a candidates.

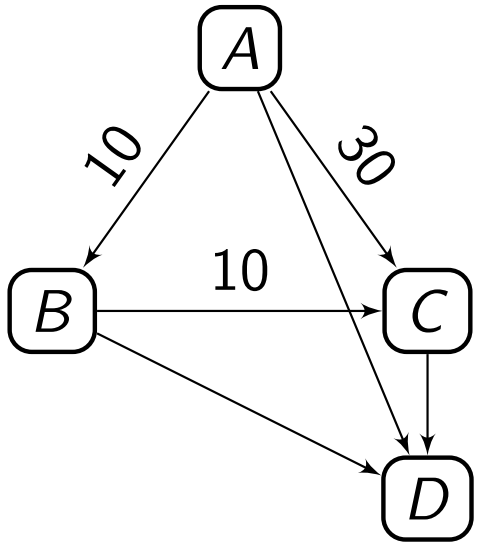
- Plurality
- Vote splitting

Criteria

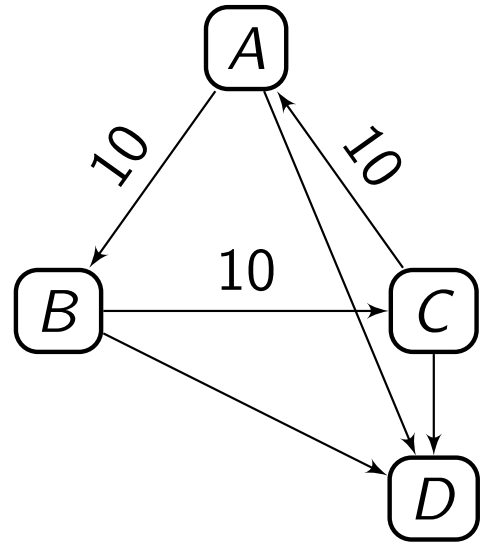
- Condorcet Criterion
- Smith Criterion
- Monotonicity Criterion
- Majority Criterion
- Consistency Criterion
- Independence of Clones Criterion

Condorcet and Smith Criteria

Smith Set: $\{A, B, C\}$



Condorcet winner :)



No Condorcet winner :(

Some systems to consider

- Plurality (Common)
- Instant-Runoff Voting (Maine)
- GT-Method (Rivest, Shen)

Comparing Plurality, IRV, and GT by these Criteria

Criteria	Plurality	IRV	GT
Condorcet Criterion	No	No	Yes
Smith Criterion	No	No	Yes
Monotonicity Criterion	Yes	No	No
Majority Criterion	Yes	Yes	No
Consistency Criterion	Yes	No	No
Independence of Clones Criterion	No	Yes	No

Arrow's Impossibility Theorem; Gibbard-Satterthwaite Theorem

Theorem

For ranked voting with at least three candidates, no electoral system can meet all of the following Criteria:

- *Non-dictatorship*
- *Unanimity*
- *Independence of irrelevant alternatives*

Theorem

No non-trivial electoral system is strategy-proof

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[0] Nisan, Noam, et al. Algorithmic Game Theory. Cambridge University Press, 2008.

[0] http://www.eecs.harvard.edu/cs286r/courses/fall11/papers/Gibbard73.pdf

Simulations: Motivation

- All these criteria are annoying
- It doesn't matter that an electoral system fits various natural-sounding criteria so much as it just picks the best candidate
- New problem: best candidate, model

Pause!

The most natural models (to me anyways) have Condorcet winners

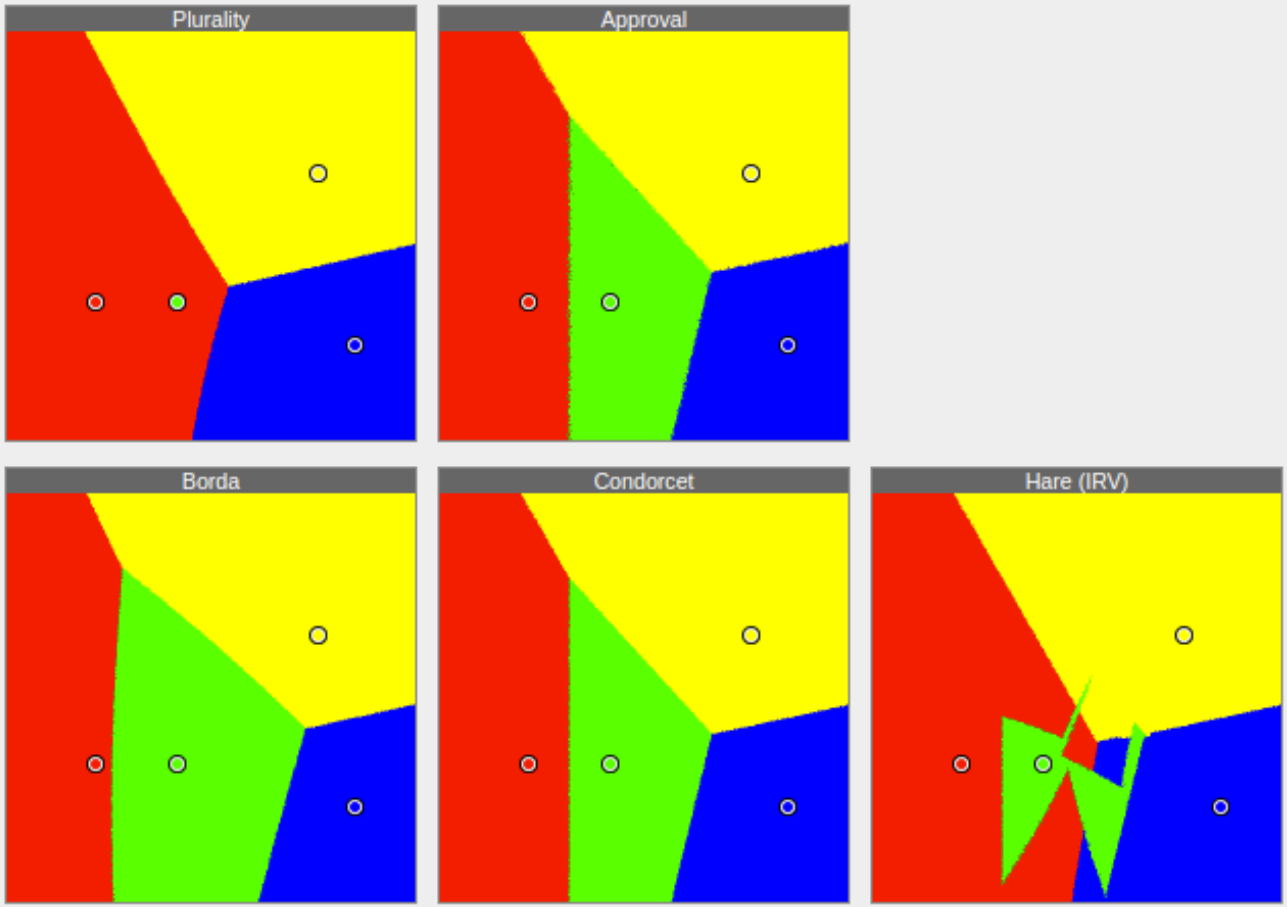
- How to model voter beliefs?
- Condorcet method is the answer? (Princeton agrees with me!)

[0] https://www.princeton.edu/ cuff/voting/theory.html

Existing Simulations: Ka-Ping Yee

Shattered

Here we have red at (0.12, 0.28), yellow at (0.85, 0.70), green at (0.39, 0.28), and blue at (0.97, 0.14). Plurality severely penalizes green, the moderate. The Approval and Condorcet methods yield boundaries exactly halfway between candidates, and once again, the centrist gets the largest winning region with Borda. The Hare method yields a very strange shape for green's winning region.



[0] http://zesty.ca/voting/sim/

Simulations: -.01, 0, .01

Normal distributions
100,000 voters
Always a Condorcet winner, always same result from all three electoral systems

Simulations: -.01, 0, 0

Can we force plurality to fail?

100,000 voters
Still always a Condorcet winner
Now in 5 of 20 trials, plurality failed to choose the Condorcet winner
But the other two systems still always agreed

Simulations: -.01, 0, 1 with a twist

What if liking 1 means disliking 2 and also liking 0?

100,000 voters
Still always a Condorcet winner
Now in 2 of 20 trials, plurality failed to choose the Condorcet winner
But the other two systems still always agreed

Future work!

- Condorcet winners?
- Estimate preferences given rankings

Sources

http://timroughgarden.org/f16/l/l3.pdf
http://timroughgarden.org/f16/l/l3.pdf
Nisan, Noam, et al. Algorithmic Game Theory. 2008.
http://www.eecs.harvard.edu/cs286r/courses/fall11/papers/Gibbard73.pd
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