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# Comparing Electoral Systems

For this report, an electoral system is a social choice function that chooses one winning candidate from a set of alternatives. Such systems can be compared by both analytical and experimental methods. I discuss each briefly, including my experimental process and results for comparing three specific methods: plurality, instant-runoff, and the GT-method.

**Plurality**

In the plurality method, all voters submit their top choice among the candidates, and the candidate with the most votes wins. This is a very common system in democracies all over the world, but it has some issues like vote-splitting in which similar candidates who are each popular among voters actually do poorly because they divide the voting bloc that favors them.

**Instant-Runoff Voting**

Instant-runoff voting is a popular alternative to the plurality method. It is newly used for statewide elections in my home state of Maine, among few others in the US. Voters submit rankings of the candidates that show their preferences between the candidates. Then if a candidate has the majority of first place votes, they win. Otherwise, the candidate with the fewest first place votes is removed from all rankings, and in ballots where they were the first choice, the second choice is moved up into the first place. Now if some candidate has the majority of votes, they win. This process repeats and terminates for finitely many candidates.

**GT-Method**

The GT-Method, as presented in Rivest and Shen's paper, views an electoral system as a player in a two player, zero sum game. The payoff for choosing a particular candidate is the margin between that candidate and the candidate chosen by the opposing electoral system. Iterating this game allows for an optimal mixed strategy: a distribution over the candidates used for selecting a candidate in each game which guarantees the best possible payoff over time. This optimal mixed strategy can be found by linear programming.

**Ranked Preferences**

It is useful to note that while the plurality method requires little information (just the top choice of each voter), many other electoral systems use more information, consisting often of rankings (preferential lists) or scores/approval ratings. Other frequently discussed electoral systems include approval, Borda count, Minimax, score, Schulze, and STAR, though many others exist.

**Criteria for Electoral Systems**

Many desirable criteria have been developed for analytically comparing electoral systems. For example, the majority criterion insists if one candidate is the top choice of a majority of the population, this candidate must win. The monotonicity criterion requires that increasing support for a candidate by raising that candidate in some ranking does not prevent that candidate from winning. To achieve independence of clones, adding identical copies of a candidate to the group of alternatives can not prevent the original candidate from winning. These are just a few examples from a class of many such criteria. These criteria all describe traits that are appealing in a voting system, perhaps feeling natural, but two theorems show us that no voting system can possibly achieve all desirable characteristics.

**Condorcet Winners**

In an election there is always a set of candidates, the Smith set, each of whom is preferred by a majority of voters to all other candidates. If this set has only one candidate, this is the Condorcet winner of the election. A Condorcet winner would win a majority of votes in a contest with any other candidate among the alternatives. I am tempted to believe that in large elections Condorcet winners almost always exist. I did some reading on this topic, but a lack of accurate data makes it difficult to answer this question confidently.

**Arrow's Impossibility Theorem**

This theorem says that there are no electoral systems that simultaneously fulfill the following three criteria:

I. Non dictatorship (a single vote does not determine the outcome)
II. Unanimity (if all voters prefer one candidate, this candidate wins)
III. Independence of irrelevant alternatives (new candidates don't not change a system's evaluation of initial candidates)

**Gibbard-Satterthwaite Theorem**

This theorem, published independently by each academic, states that there are no nontrivial voting mechanisms which are strategy-proof. Here, a nontrivial mechanism is one which either (a) is a dictatorship or (b) limits possible winners to two alternatives only.

**Implication of the Theorems**

One interpretation of these theorems is a realization that analytical criteria can always be used to show the deficiencies of a system, since all systems have them. In this spirit, it is useful to turn to simulations to experimentally analyze how various electoral systems behave. The meaning of simulations depends greatly on the model of voting being used and how results are interpreted.

**Existing Simulations**

Among existing simulations of electoral systems, I enjoyed the visualizations given by Ka-Ping Yee on [their website](). These simulations have each candidate as a point on a plane. Voters then are normally distributed on the plane where a single voter's preferences correspond to the Euclidean distance from that voter's point to each candidate's point (closer is better). In each graphic, trials of 200,000 votes are generated at each point and a winner is computed for a given system and displayed. A main result here is that the plurality method performs poorly (as per my explanation of vote-splitting above), while other systems tend to do better.

**My Simulations**

In my simulations, a given voter is given by nature a utility for each of the candidates. This utility is then used to determine preference rankings (higher is better). By having the utility for each candidate normally distributed with differing means, Condorcet winners rarely occur. If the means among candidates are the same, Condorcet winners sometimes occur. In all simulations I ran, instant-runoff voting selected the same candidate as the GT-Method, but these two sometimes differed from the plurality winner who did not have a majority of votes.

**Future Work**

A more robust model of voter preferences is necessary to make any more substantive claims about the differences between these systems. Work can be done to better understand how to accurately model voter preferences by gathering and analyzing data from real elections or other social choices. For my simulations, I can also consider next correlated utility distributions.

**Here is a link to my Github Repository which includes both the source code for my simulations and the slide show for my presentation of this project. The slideshow includes citations for material I used in this project.**