2

$u(x,0) = \phi(x) = 1$

The solution from last question:

$$u(x,t) = \sum_{n=1}^{\infty} \tilde{A}_n \sin(n\pi x) e^{-(n\pi)^2 t}$$

Plug IC:

$$u(x,0) = \sum_{n=1}^{\infty} \tilde{A}_n \sin(n\pi x) = 1$$

$$\left\langle \sum_{n=1}^{\infty} \tilde{A}_n \sin(n\pi x), \sin(m\pi x) \right\rangle = \langle \phi(x), \sin(m\pi x) \rangle$$

$$\text{LHS} = \sum_{n=1}^{\infty} \tilde{A}_n \int_0^1 \sin(n\pi x)\sin(m\pi x)\mathrm{dx} = \begin{cases} 0 & \text{if } m \neq n \\ \frac{1}{2} & \text{if } m = n \end{cases}$$

$$\text{RHS} = \int_0^1 \phi(x)\sin(m\pi x)\mathrm{dx} = \frac{\tilde{A}_n}{2}$$

$$\tilde{A}_m = 2\int_0^1 \phi(x)\sin(m\pi x)\mathrm{dx}$$

Plug $\phi(x) = 1$

$$\tilde{A}_m = 2\int_0^1 \sin(m\pi x)\mathrm{dx} = -\frac{2}{m\pi}\cos(m\pi x)\Big|_0^1 = -\frac{2}{m\pi}(\cos(m\pi) - \cos(0)) = \frac{2}{m\pi}(1 - \cos(m\pi)) =$$

$$\tilde{A}_1 = \frac{4}{\pi},\ \tilde{A}_2 = 0,\ \tilde{A}_3 = \frac{4}{3\pi},\ \tilde{A}_4 = 0,\ \tilde{A}_5 = \frac{4}{5\pi},$$

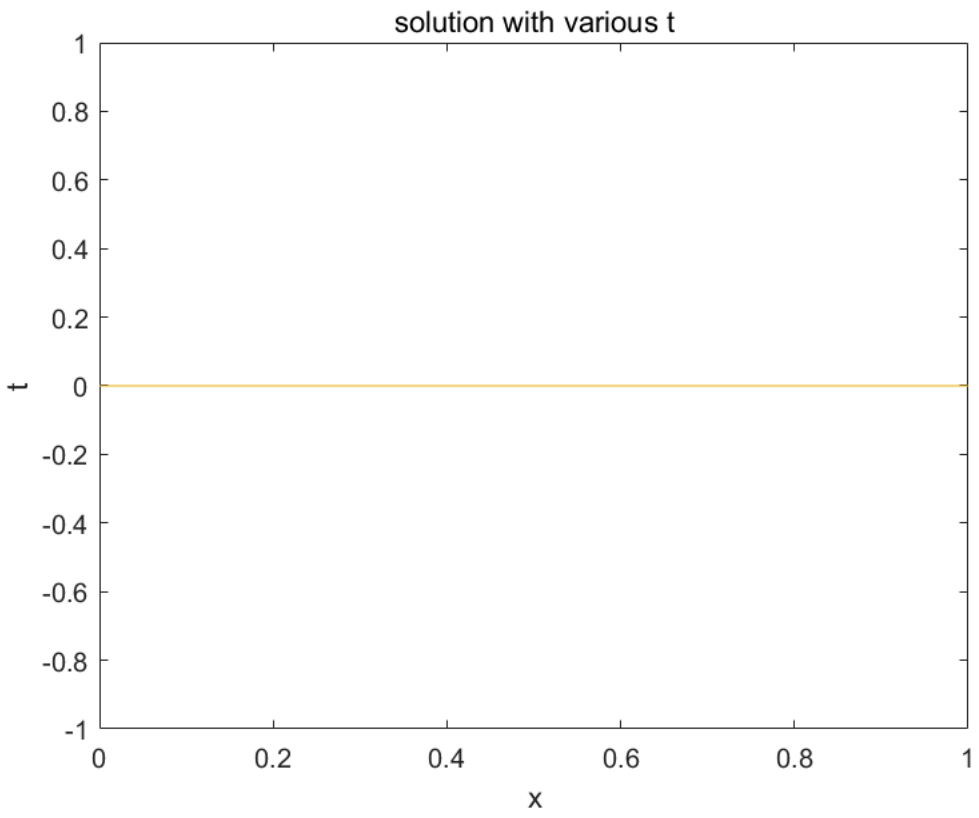
$$u(x,t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}(1 - (-1)^n)\sin(n\pi x)e^{-(n\pi)^2 t} = \frac{4}{\pi}\sin(\pi x)e^{-\pi^2 t} + \frac{4}{3\pi}\sin(3\pi x)e^{-9\pi^2 t} + \frac{4}{5\pi}\sin(5\pi x)e^{-25\pi^2 t} + \ldots$$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi}\sin((2n-1)\pi x)e^{-((2n-1)\pi)^2 t} \qquad (6)$$

```
clear
n=50;%set n=50
x=linspace(0,1,500);%generate x

for t=1:500
    %loop for t
    plot(x,u(x,t,n))%plot with each t
    hold on
end
title("solution with various t")
xlabel("x")
```

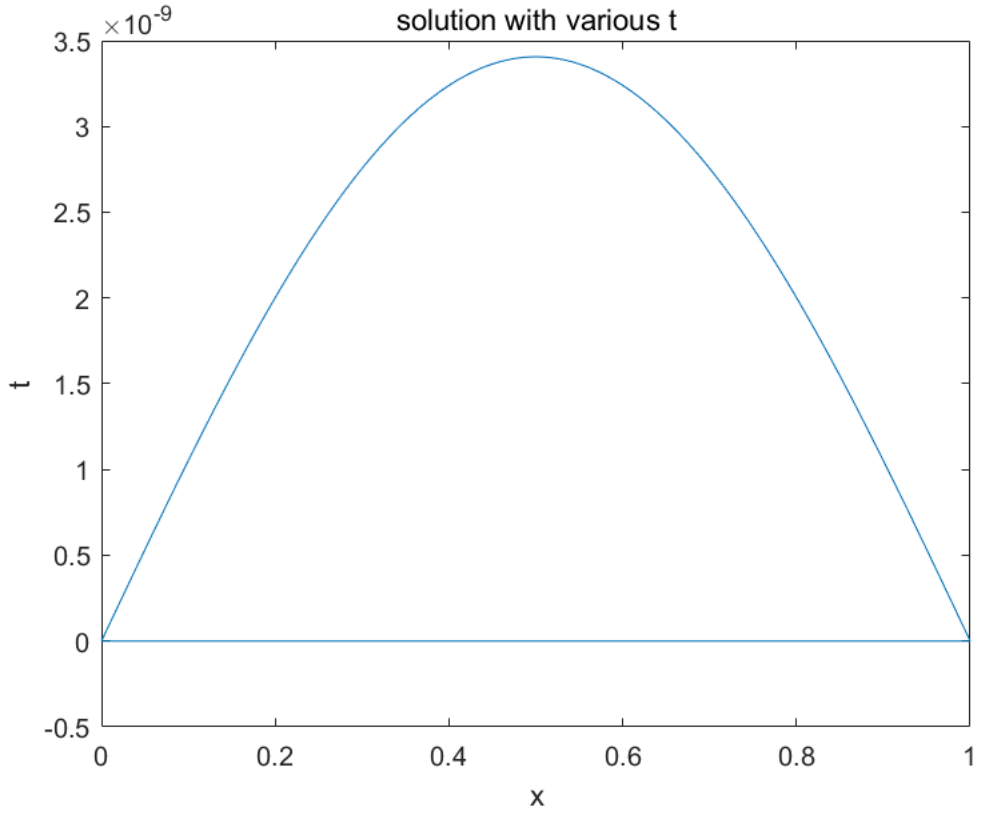
```
ylabel("u(x,t)")
hold off
```



From the graph we could see the solution will not change with t so t can be any number.

So we set t=2

```
t=2;%set t=2
for n=1:50
    %loop for n
    plot(x,u(x,t,n))%plot with each n
    hold on
end
title("solution with various n")
xlabel("x")
ylabel("u(x,t)")
```




```
function [s]=u(x,t,n)
%equation (6)
    s=0;%initialize the sum
    for j = 1:n
        %loop of n
        s=s+4/((2*n-1)*pi)*sin((2*n-1)*x*pi).*exp(-((2*n-1)*pi)^2*t);
        %add sum together
    end
end
```