problem 1

N=4

$$\mathrm{DFT}: \widehat{f_k} = \sum_{j=0}^{N-1} f_j e^{-i\alpha}$$

$$\alpha = \frac{2\pi \mathrm{kj}}{N}$$

let $\omega_N = e^{-\frac{2\pi i}{N}}$, $\widehat{f_k} = \sum_{j=0}^{N-1} f_j \omega_N^{\mathrm{jk}}$

When k=1

$$\widehat{f_0} = \sum_{j=0}^{4-1} f_j \omega_4^{\mathrm{j}0} = [1 \quad 1 \quad 1 \quad 1]$$

When k=2

$$\widehat{f_1} = \sum_{j=0}^{4-1} f_j \omega_4^{\mathrm{j}1} = [1 \quad \omega_4 \quad \omega_4^2 \quad \omega_4^3]$$

When k=3

$$\widehat{f_2} = \sum_{j=0}^{4-1} f_j \omega_4^{\mathrm{j}2} = [1 \quad \omega_4^2 \quad \omega_4^4 \quad \omega_4^6]$$

When k=4

$$\widehat{f_3} = \sum_{j=0}^{4-1} f_j \omega_4^{\mathrm{j}3} = [1 \quad \omega_4^3 \quad \omega_4^6 \quad \omega_4^9]$$

$$\mathrm{DFT} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-\frac{2\pi i}{4}} & \left(e^{-\frac{2\pi i}{4}}\right)^2 & \left(e^{-\frac{2\pi i}{4}}\right)^3 \\ 1 & \left(e^{-\frac{2\pi i}{4}}\right)^2 & \left(e^{-\frac{2\pi i}{4}}\right)^4 & \left(e^{-\frac{2\pi i}{4}}\right)^6 \\ 1 & \left(e^{-\frac{2\pi i}{4}}\right)^3 & \left(e^{-\frac{2\pi i}{4}}\right)^6 & \left(e^{-\frac{2\pi i}{4}}\right)^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-\frac{2\pi i}{4}} & e^{-\frac{4\pi i}{4}} & e^{-\frac{6\pi i}{4}} \\ 1 & e^{-\frac{4\pi i}{4}} & e^{-\frac{8\pi i}{4}} & e^{-\frac{12\pi i}{4}} \\ 1 & e^{-\frac{6\pi i}{4}} & e^{-\frac{12\pi i}{4}} & e^{-\frac{18\pi i}{4}} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-\frac{\pi i}{2}} & e^{-\pi i} & e^{-\frac{3\pi i}{2}} \\ 1 & e^{-\pi i} & e^{-2\pi i} & e^{-3\pi i} \\ 1 & e^{-\frac{3\pi i}{2}} & e^{-3\pi i} & e^{-\frac{9\pi i}{2}} \end{bmatrix}$$

iDFT: $F_N^{-1} = \frac{1}{N}\overline{F_N}$

$$F_N^{-1} = \frac{1}{4}\left\| \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-\frac{\pi i}{2}} & e^{-\pi i} & e^{-\frac{3\pi i}{2}} \\ 1 & e^{-\pi i} & e^{-2\pi i} & e^{-3\pi i} \\ 1 & e^{-\frac{3\pi i}{2}} & e^{-3\pi i} & e^{-\frac{9\pi i}{2}} \end{bmatrix} \right\|$$

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N=4
```

```
N = 4
```

```
F_N=zeros(N,N)
```

```
F_N = 4×4
     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0
```

```
for j=1:N
    for k=1:N
        F_N(j,k)=(exp(-2*pi*i/N))^((j-1)*(k-1));
    end
end
F_N
```

```
F_N = 4×4 complex
   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i
   1.0000 + 0.0000i   0.0000 - 1.0000i  -1.0000 - 0.0000i  -0.0000 + 1.0000i
   1.0000 + 0.0000i  -1.0000 - 0.0000i   1.0000 + 0.0000i  -1.0000 - 0.0000i
   1.0000 + 0.0000i  -0.0000 + 1.0000i  -1.0000 - 0.0000i   0.0000 - 1.0000i
```

```
iF_N=1/4*conj(F_N)
```

```
iF_N = 4×4 complex
   0.2500 + 0.0000i   0.2500 + 0.0000i   0.2500 + 0.0000i   0.2500 + 0.0000i
   0.2500 + 0.0000i   0.0000 + 0.2500i  -0.2500 + 0.0000i  -0.0000 - 0.2500i
   0.2500 + 0.0000i  -0.2500 + 0.0000i   0.2500 - 0.0000i  -0.2500 + 0.0000i
   0.2500 + 0.0000i  -0.0000 - 0.2500i  -0.2500 + 0.0000i   0.0000 + 0.2500i
```

```
iF_N*F_N
```

```
ans = 4×4 complex
   1.0000 + 0.0000i  -0.0000 - 0.000
  -0.0000 + 0.0000i   1.0000 + 0.000
   0.0000 + 0.0000i  -0.0000 + 0.000
   0.0000 + 0.0000i   0.0000 + 0.000
```