PROBLEM 4: (10 pts) Explain how Taylor series can be used to determine the order of the error in numerical methods.

4. with step h.

the taylor expansion of $y(x+h)$ is

$$y(x) + hy'(x) + \frac{h^2}{2!} y''(x) + \frac{h^3}{3!} y'''(x) + \cdots$$

For any numerical method.

the prediction of point x_{n+1} is y_{n+1}

the accuracy is $y(x+h) - y_{n+1}$

Since the prediction of y_{n+1} will need $y(x)$ (which $y(x) = y_0$ is a provided po

$$y(x+h) - y_{n+1} = hy'(x) + \frac{h^2}{2!} y''(x) + \frac{h^3}{3!} y''(x) + \cdots$$

$$- h\phi(x, y, h)$$

which $h\phi$ is function that predict the $[y_{n+1} - y(x)]$

~~So~~ the how the order of (h^n) ~~left~~ means the accurate of the method

~~has been~~

Since y', y'' .... can be determine from $f(x)$

So. the term left from the substraction

has order $(h)^{n+1}$ means the method has nth order accurate.

PROBLEM 5: (20 pts) What is *regularization* and why is it needed? Give two examples of regularization methods explaining the basics of how they work and for what type of problems they are appropriate.

Regularization is the method that helping people choosing data from the noise data. With better Regularization method we could de-noise the data and get better result, reduce the over fit.

Method 1: TSVD

Truncked SVD:

for the problem $m = Af + \varepsilon$

F is the things we want

M is the data we have and sigma is the noise

In order to solve m we need $A^{-1}m \approx f$

The singular values of a picture or matrix are separated to wide range. The condition number will be huge and it will be hard to do the inverse computation

Set a limitation and ignore the singular value smaller than the limitation, doing the pseudo-inverse instant of inverse, set the inverse of smaller singular value to 0.

Then using the least square method find vector $\ell(\vec{m})$ that:

$$\|A\ell(\vec{m}) - \vec{m}\| = \min\|A\vec{z} - \vec{m}\|$$

Then vector $\ell(\vec{m})$ will be the solution

Method: Tikhonov

for the problem $m = Af + \varepsilon$

Tikhonov minimize the expression

$\|AT_\alpha(m) - m\|_2^2 + \alpha\|T_\alpha(m)\|_2^2$ with $T_\alpha(m) \in \Re^n$

write $T_\alpha(m) = \arg\min(\|A\vec{z} - \vec{m}\|_2^2 + \alpha\|\vec{z}\|_2^2)$

$$T_\alpha(m) = VS_\alpha^+ U^T \vec{m}$$

$$S_\alpha^+ = diag(\frac{\sigma_1}{\sigma_1{}^2 + \alpha}, \frac{\sigma_2}{\sigma_2{}^2 + \alpha}, \ldots, \frac{\sigma_{\min}}{\sigma_{\min}{}^2 + \alpha})$$

PROBLEM 6: (20 pts) We have studied various solution methods for solving differential and partial differential equations. Compare and contrast *Finite Differences*, the *Finite Element Method*, *Separation of Variables*, *Monte Carlo* methods, and *Fourier Transform* methods. What types of problems can be solved by each method?

6. F D.

① has larger error, $o(h)$

② Can be solve 1st and 2nd order of ODE with a point given, for ~~two~~ 2nd order ODE need two point or one point and one free end

Numerical method.

FEM

① will be used on solving 2nd order of ODE and PDE

② Has larger computation ③ numerical method.

Separation of Variable.

① will be used on solving PDE with IC ~~and~~ or BC

② True solution will be fund, ③ with some kind of IC, BC will be hard to find the solution.

Monte Carlo:

① Integration: Definite integral

② PDE with IC or BC

③ numerical method ④ take a lot computation by computer.

Fourier Transform.

① Analytical solution will fund

② PDE with 1st or 2nd order.

③ difficult to use if the step of Inverse of Furier Transform cannot be fund

④ Take lots of word to do the convolution.

PROBLEM 7: (10 pts) Discuss how you can determine the 'accuracy' of a numerical method or solution.

7. The accuracy of numerical method can be determined by subtract the approximate value with the taylor expansion of the true value.

And for solution, compute the total absolute error ~~betee~~ between the solution and discretized true solution.