1. Use MC methods to compute the following

a) the AREA trapped by $\sin(x)$ for in $[0, 2\pi)$

b) the integral of $\sin(x)$ from $x = 0$ to 2π

c) (not using MC methods) Compare your results from a) and b) to the results you get with Calculus

a)

Using MC method calculus the area.

Since the area cannot be negative, we need to use the MC on the whole range $[0, 2\pi)$

We will drop darts on the interval of 0 to 2π and maximum, minimum of $\sin(x)$ which is 1 and -1.

Then test if the dart we drop is bounded by the sin(x) and x-axis.

Then we could count the number of darts that in the area and total darts we dropped.

Therefore we could calculate the area trapped by the line.

The specific step is following

step 1: intitialize count=0

step 2: loop from 1 to n

step 3: generate random num x: $0 \le x \le 2\pi$,

y: $-1 \le y \le 1$.

step 4: calculate $\text{fx} = \sin(x)$

step 5: if $0 \le y \le \text{fx}$ or $\text{fx} \le y \le 0$ increase count by 1. Otherwise goes to step 3

step 6: area= $2 \times 2\pi \times$ count/n

```
clear
%step 1 initial count=0
count = 0;
num = 10000;%set n=10000

for j = 1:num
    %step 2 loop from 1 to n
    x = 2*pi*rand;%step 3 generate random number of x from 0 to 2*pi
    y = -1+(1-(-1))*rand;%step 3 generate random number of y from -1 to 1
    fx = sin(x);%step 4 calculate fx
    if (0<=y) && (y<=fx) || (fx<=y) && (y<=0)
        %step 5 test if y is bounded by fx and x-axis
        count=count+1;%count increase by 1
    end
end
%step 6 calculate the total area.
```

```
Areafx=2*pi*2*count/num
```

```
Areafx = 4.0677
```

So the area trapped by the line is about 4

b)

$$\int_0^{2\pi} \sin(x)\mathrm{dx} = 0$$

c)

From the integral we coud see that, the integral of sin(x) from 0 to 2π is 0. But if we calculate

$$\left|\int_0^{\pi} \sin(x)\mathrm{dx}\right| + \left|\int_{\pi}^{2\pi} \sin(x)\mathrm{dx}\right|$$

$$\left|\int_0^{\pi} \sin(x)\mathrm{dx}\right| + \left|\int_{\pi}^{2\pi} \sin(x)\mathrm{dx}\right| = 2 + 2 = 4$$

Which the result from MC method is similar with the true value.