6. For the fundemental B-spline discussed in the lecture, check that $B(t)$, $B'(t)$ and $B''(t)$ are continous across the nodes(knots).

$$B(t) = \begin{cases} \frac{1}{6}t^3 & 0 \le t \le 1 & (9) \\ \frac{1}{6}[-3(t-1)^3 + 3(t-1)^2 + 3(t-1) + 1] & 1 \le t \le 2 & (10) \\ \frac{1}{6}[3(t-2)^3 - 6(t-2)^2 + 4] & 2 \le t \le 3 & (11) \\ \frac{1}{6}[-(t-3)^3 + 3(t-3)^2 - 3(t-3) + 1] & 3 \le t \le 4 & (12) \\ 0 & t \le 0 \text{ or } t \ge 4 \end{cases}$$

PLot $B(t)$

$$B_1(1) = \frac{1}{6}$$

$$B_2(1) = \frac{1}{6}$$

$$B_2(2) = \frac{1}{6}[-3(1)^3 + 3(1)^2 + 3(1) + 1] = \frac{1}{6}[-3 + 3 + 3 + 1] = \frac{1}{6}[4] = \frac{4}{6}$$

$$B_3(2) = \frac{1}{6}[3(t-2)^3 - 6(t-2)^2 + 4] = \frac{4}{6}$$

$$B_3(3) = \frac{1}{6}[3(1)^3 - 6(1)^2 + 4] = \frac{1}{6}[3 - 6 + 4] = \frac{1}{6}[1] = \frac{1}{6}$$

$$B_4(3) = \frac{1}{6}[-(t-3)^3 + 3(t-3)^2 - 3(t-3) + 1] = \frac{1}{6}$$

$$B_4(4) = \frac{1}{6}[-(1)^3 + 3(1)^2 - 3(1) + 1] = \frac{1}{6}[-1 + 3 - 3 + 1] = \frac{1}{6}[0] = 0$$

$$B_5(4) = 0$$

$$B_5(0) = 0$$

$$B_1(0) = 0$$

So $B(t)$ is continous across the nodes

$$B'(t) = \begin{cases} \frac{1}{2}t^2 & 0 \le t \le 1 \\ \frac{1}{6}[-9(t-1)^2 + 6(t-1) + 3] & 1 \le t \le 2 \\ \frac{1}{6}[9(t-2)^2 - 12(t-2)] & 2 \le t \le 3 \\ \frac{1}{6}[-3(t-3)^2 + 6(t-3) - 3] & 3 \le t \le 4 \\ 0 & t \le 0 \text{ or } t \ge 4 \end{cases}$$

$$B_1'(1) = \frac{1}{2}$$

$$B_2'(1) = \frac{1}{3}$$

$$B_2'(2) = \frac{1}{6}[-9 + 6 + 3] = \frac{1}{6}[0] = 0$$
$$B_3'(2) = 0$$

$$B_3'(3) = \frac{1}{6}[9 - 12] = -\frac{1}{2}$$

$$B_4'(3) = -\frac{1}{2}$$

$$B_4'(4) = \frac{1}{6}[-3 + 6 - 3] = 0$$
$$B_5'(4) = 0$$

$$B_5'(0) = 0$$
$$B_1'(0) = 0$$

So $B'(t)$ is continous across the nodes

$$B''(t) = \begin{cases} t & 0 \le t \le 1 \\ \frac{1}{6}[-18(t-1) + 6] & 1 \le t \le 2 \\ \frac{1}{6}[18(t-2) - 12] & 2 \le t \le 3 \\ \frac{1}{6}[-6(t-3) + 6] & 3 \le t \le 4 \\ 0 & t \le 0 \text{ or } t \ge 4 \end{cases}$$

$$B_1''(1) = 1$$

$$B_2''(1) = \frac{1}{6}[6] = 1$$

$$B_2''(2) = \frac{1}{6}[-18 + 6] = \frac{1}{6}[-12] = -2$$

$$B_3''(2) = \frac{1}{6}[-12] = -2$$

$$B_3''(3) = \frac{1}{6}[18 - 12] = \frac{1}{6}[6] = 1$$

$$B_4''(3) = \frac{1}{6}[6] = 1$$

$$B_4''(4) = \frac{1}{6}[-6 + 6] = 0$$

$$B_5''(4) = 0$$

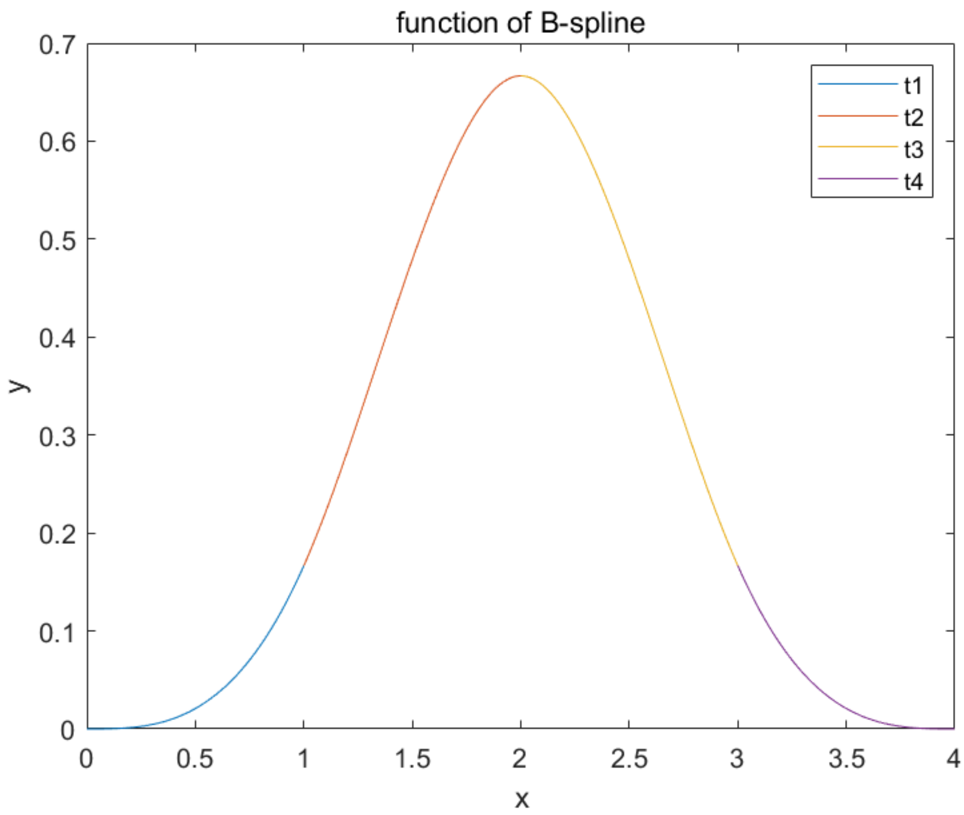
$$B_5''(0) = 0$$

$$B_1''(0) = 0$$

So $B''(t)$ is continous across the nodes

```
clear
%define n
n=0.01;
%define t1,t2,t3 and t4
t1=0:n:1;
t2=1:n:2;
t3=2:n:3;
t4=3:n:4;

%plot
plot(t1,B1(t1))
hold on
plot(t2,B2(t2))
plot(t3,B3(t3))
plot(t4,B4(t4))
xlabel('x')
ylabel('y')
title('function of B-spline')
legend('t1','t2','t3','t4')
hold off
```




```
function [Bt1]=B1(t)
%function of equation (9)
    Bt1=1/6*t.^3;
end

function [Bt2]=B2(t)
%function of equation (10)
    Bt2=1/6*(-3*(t-1).^3+3*(t-1).^2+3*(t-1)+1);
end

function [Bt3]=B3(t)
%function of equation (11)
    Bt3=1/6*(3*(t-2).^3-6*(t-2).^2+4);
end

function [Bt4]=B4(t)
%function of equation (12)
    Bt4=1/6*(-(t-3).^3+3*(t-3).^2-3*(t-3)+1);
end
```