Problem 3

Graph the frequency spectrum of the following periodic functions:

(a) $f(x) = \sin(x)$

(b) $f(x) = \sin(x) + \cos(2x)$

(c) $f(x) = \sin(x) + \cos(2x) + 0.5\sin(3x)$

Solve:

(a) $f(x) = \sin(x)$ (12)

Periodic function could be written as Fourier Series equation:

$$f(x) = \sum_{n=0}^{\infty} \left[A_n \cos(nx) + B_n \sin(nx) \right] \quad (13)$$

By insert equation (12) to (13). We could find:

$A_1 = 0$, $B_1 = 1$,

Since the equation of spectrum C_n is:

$$C_n = \sqrt{A_n^{\ 2} + B_n^{\ 2}} \quad (14)$$

So if take result of A and B to equation (14)

$C_1 = \sqrt{B_1^{\ 2}} = \sqrt{1^2} = 1$

The plot is:

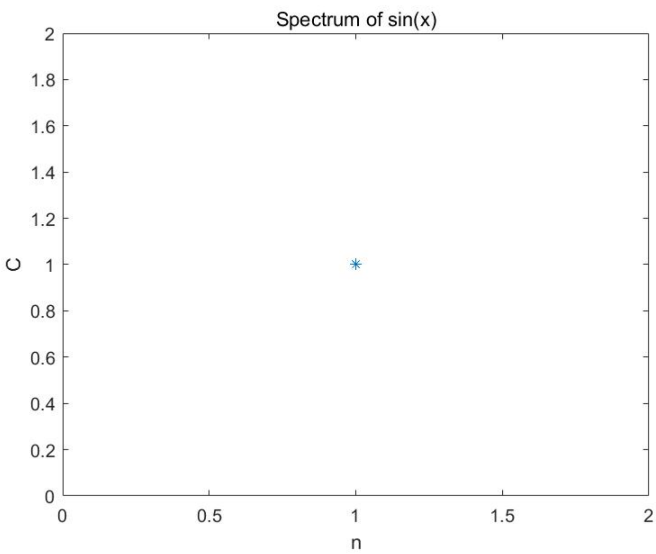

Fig 3.1 The plot of spectrum of sin(x) on the point n=1 is 1

(b) $f(x) = \sin(x) + \cos(2x)$ (15)

From the equation (13) we could get

$A_1 = 0$, $B_1 = 1$,

$A_2 = 1$, $B_1 = 0$,

So we could plug this two conditions to (14):

$C_1 = \sqrt{{A_1}^2 + {B_1}^2} = \sqrt{1^2} = 1$

$C_2 = \sqrt{{A_2}^2 + {B_2}^2} = \sqrt{1^2} = 1$

So the graph of the spectrum is:

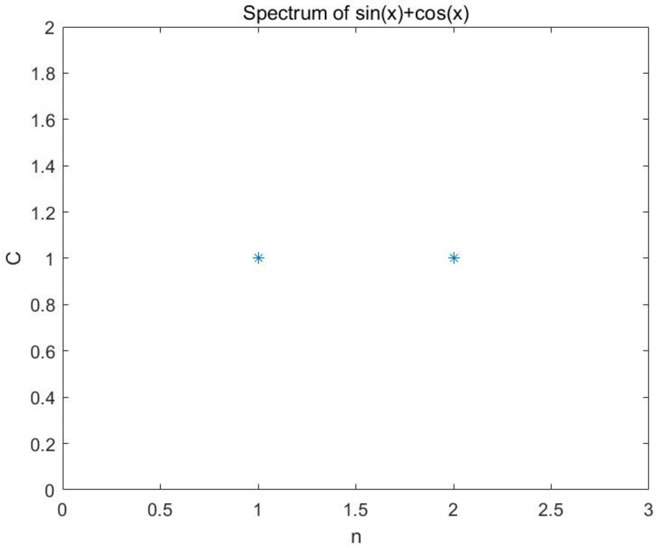

Fig3.2 The plot of spectrum of sin(x)+cos(x) on the point n=1 is 1, on the point n=2 is 1

(c) $f(x) = \sin(x) + \cos(2x) + 0.5\sin(3x)$ (15)

From the equation (13) we could get

$A_1 = 1$, $B_1 = 1$,

$A_2 = 0$, $B_1 = 0$,

$A_3 = 0$, $B_3 = 0.5$,

So we could plug this two conditions to (14):

$C_1 = \sqrt{{A_1}^2 + {B_1}^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$C_3 = \sqrt{{A_3}^2 + {B_3}^2} = \sqrt{(0.5)^2} = \frac{1}{2}$

So the graph of the spectrum is:

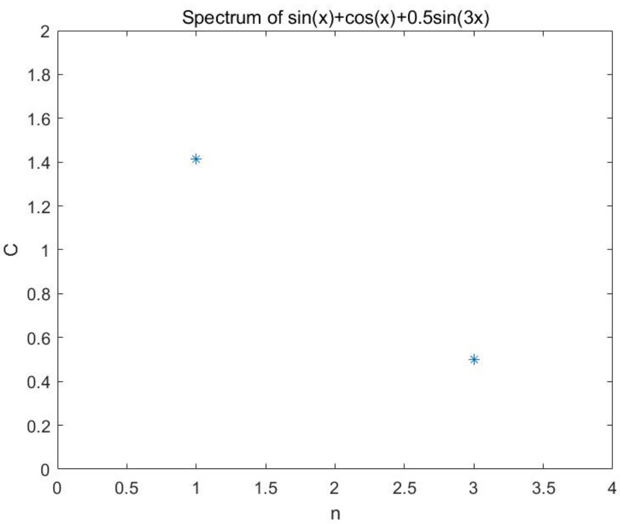

Fig 3.2 The plot of spectrum of sin(x)+cos(x) on the point n=1 is sqrt(2), on the point n=3 is 0.5