(c) (25 pts) Solve the IVP using the *Finite Element Method.*

(i) Using $h = 0.25$, write out all steps of the solution method by hand and justifying all entries in the resulting matrix equation.

(ii) Then, write code in *Matlab* to solve your problem from (i) to determine the solution $u(x)$. Compare your result to that from (a) at the grid nodes.

(iii) Generalize your code to solve the problem with $h = \frac{1}{20}$.

c). $u''(x) = x - 2 \qquad u(0) = 0 \qquad u'(1) = 4$

1/4 2/4 3/4 $\qquad Lu = u'' \quad f(x) = x - 2$

$$\langle LU_2, \phi_j \rangle = \langle d, \phi_j \rangle$$

$$\int_0^1 u'' v \, dx = \int_0^1 f v \, dx$$

$$\int_0^1 \frac{d}{dx} u' v \, dx - \int_0^1 f v \, dx$$

$$\left[u'v \Big|_{x=0}^{x=1} - \int_0^1 u'v' \, dx \right] = \int_0^1 f v \, dx$$

$$u'(1)v(1) - u'(0)v(0) - \int_0^1 u'v' \, dx = \int_0^1 f v \, dx$$

$$4v(1) - \int_0^1 u'v' \, dx = \int_0^1 f v \, dx$$

$$\int_0^1 u'v' \, dx = 4v(1) - \int_0^1 f v \, dx$$