2. Use MC methods to compute the following

a) the AREA trapped by $f(x) = x^2 + 2$ using the rectangular dartboard described in class

b) the AREA trapped by $f(x) = x^2 + 2$ using a trapezoid instead of a rectangle for your dartboard

c) Compare your results from a) and b) to the results you get with Calculus

a)

Let the range of the area trapped from x=0 to x=4

So the maximum of f(x) is $4^2 + 18$, and minimum of f(x) is 0.

step 1: intitialize count=0

step 2: loop from 1 to n

step 3: generate random num x:$0 \le x \le 4$,

y:$0 \le y \le 18$

step 4: calculate $\mathrm{fx} = x^2 + 2$

step 5: if $y \le \mathrm{fx}$increase count by 1. Otherwise goes to step 3

step 6: area=4*18*count/n

```
clear
%step 1 initial count=0
count = 0;
num = 10000;%set n=10000

for j = 1:num
    %step 2 loop from 1 to n
    x = 4*rand;%step 3 generate random number of x from 0 to 4
    y = 18*rand;%generate random number of y from 0 to 18
    fx = x^2+2;%step 4: calculate fx
    if y <= fx
        %test if y is less than fx
        count = count + 1;%count increase by 1
    end
end
%step 6
AreaFx=18*4*(count/num)
```

```
AreaFx = 29.4552
```

So as result the area under $f(x) = x^2 + 2$ by using MC method is around 29

b)

Since $\mathrm{fx} = x^2 + 2$ is curve up so the trapezoid created by (0,0),(4,0),(0,2),(4,18)

The line of (0,2),(4,18) is

$y = 4x + 2$

So in step 3 generate random number of y $0 \le y \le 4x + 2$

The area of trapezoid is (2+18)*4/2=40

And in step 6: area=40*count/n

```
numt = 10000;
countt = 0;
for jt = 1:num
    xt = 4*rand;
    yt = (4*xt + 2)*rand;%modified step 3
    fxt = xt^2+2;
    if yt <= fxt
        countt = countt+1;
    end
end

AreaFxt=40*(countt/numt)
```

```
AreaFxt = 27.4240
```

So as result the area under $f(x) = x^2 + 2$ by using MC method with trapezoid is around 27

c)

$$\int_0^4 x^2 + 2 = \left(\frac{1}{3}x^3 + 2x\right)_0^4 = \frac{1}{3}4^3 + 8 = 29.33$$

So the true value of the area is 29.33, which means that the normal MC has more accuracy.