

Problem 7

Compute the Fourier transform of $f(x)$ provide that $a > 0$)

$$f(x) = \begin{cases} e^{-ax} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad (30)$$

In order to take Fourier transform we need the formula:

$$F[f] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\xi x} dx \quad (31)$$

Then we can plug function (30) in to (31), becomes

$$\begin{aligned} F[f] &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\xi x} dx = \frac{1}{\sqrt{2\pi}} \left(\int_{-\infty}^0 0 \cdot e^{-i\xi x} dx + \int_0^{\infty} e^{-ax} e^{-i\xi x} dx \right) \\ &= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-(a+i\xi)x} dx \\ &= \frac{1}{\sqrt{2\pi}} \left[\frac{1}{a+i\xi} e^{-(a+i\xi)x} \right]_0^{\infty} \\ &= \frac{1}{\sqrt{2\pi}} \left(\frac{-1}{a+i\xi} \cdot 0 + \frac{1}{a+i\xi} \right) \\ &= \frac{1}{\sqrt{2\pi}} \frac{1}{a+i\xi} \end{aligned}$$