Final Exam - MSSC 6030: Spring 2020

INSTRUCTIONS: This exam is open book, notes, and MATLAB but CLOSED to the internet. Work that is erased or crossed out will not be graded. You must show your work to earn full and partial credit. You must write legibly to earn credit. **You may use your calculator or MATLAB to check your answers but you must show all work unless stated otherwise. As with homework, begin each problem on a new page and write the full problem statement. The exam is due on D2L dropbox by 12:30am on Wednesday, May 6, 2020.**

PROBLEM 1: Consider the boundary value problem $u''(x) = x - 2$, where $u(0) = 0$, and $u'(1) = 4$ for $0 \leq x \leq 1$.

(a) (5 pts) Solve the IVP by hand to find the true solution $u(x)$.

(b) (25 pts) Solve the IVP using the method of *Finite Differences*.

(i) Using $h = 0.25$, write out all steps of the solution method by hand and justifying all entries in the resulting matrix equation.

(ii) Then, write code in *Matlab* to solve your problem from (i) to determine the solution $u(x)$. Compare your result to that from (a) at the grid nodes.

(iii) Generalize your code to solve the problem with $h = \frac{1}{20}$.

(c) (25 pts) Solve the IVP using the *Finite Element Method*.

(i) Using $h = 0.25$, write out all steps of the solution method by hand and justifying all entries in the resulting matrix equation.

(ii) Then, write code in *Matlab* to solve your problem from (i) to determine the solution $u(x)$. Compare your result to that from (a) at the grid nodes.

(iii) Generalize your code to solve the problem with $h = \frac{1}{20}$.

(d) (10 pts) Compare your results from your FD and FEM to the true solution in (a). Which solution is most accurate? How are you classifying 'most accurate'? Additionally, plot the errors of each solution method compared to the true solution.

PROBLEM 2: This problem will use Fourier Sine series (using terms of the form $sin(n\pi x)$) and Haar wavelets (at most 8 terms).

(a) (20 pts) Approximate the following functions on $[0, 1]$. You may use the computer to verify your computations but you must also show the work by hand.

(i) $f(x) = x^2$

(ii) $g(x) = \begin{cases} 0 & 0 \leq x \leq \frac{1}{2} \\ 1 & \frac{1}{2} \leq x \leq 1 \end{cases}$

(b) (5 pts) Plot the function $f(x)$ and its Fourier sine series and Haar wavelet approximations on the same plot. Repeat this process for $g(x)$ in a new figure.

(c) (5 pts) Which expansion does a better job? How many terms are needed to capture the behavior? How are you classifying 'better'?

(d) (5 pts) Compare and contrast the different expansion methods Fourier sine series and Haar wavelets. Which one is most appropriate when, etc? Give strengths and weaknesses of both.

PROBLEM 3: (20 pts) Explain how the FFT is used in de-noising. Illustrate your point via a specific example (different than code I have provided for you).

PROBLEM 4: (10 pts) Explain how Taylor series can be used to determine the order of the error in numerical methods.

PROBLEM 5: (20 pts) What is *regularization* and why is it needed? Give two examples of regularization methods explaining the basics of how they work and for what type of problems they are appropriate.

PROBLEM 6: (20 pts) We have studied various solution methods for solving differential and partial differential equations. Compare and contrast *Finite Differences*, the *Finite Element Method*, *Separation of Variables*, *Monte Carlo* methods, and *Fourier Transform* methods. What types of problems can be solved by each method?

PROBLEM 7: (10 pts) Discuss how you can determine the 'accuracy' of a numerical method or solution.
