7. Let $u(x) = \dfrac{1}{x+1}$ on [0,2]. Approximate u on [0,2] using n equally spaced subintervals for n=2,5,10 using.

a) n cubic splines with the polynomial approach

b) n cubic splines with B-spline approach

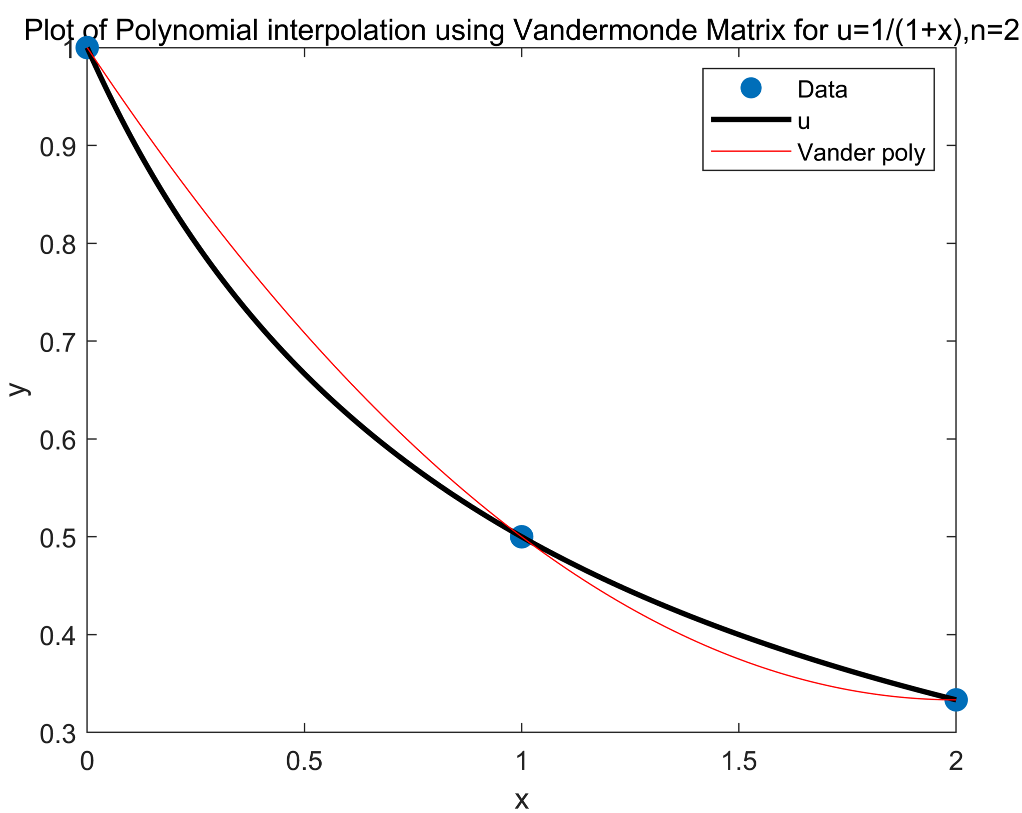
c) n cubic using the spline command in matlab.

Plot your results and compare to those from H1. How do your results change if you use a 'natural' spline or a 'clamped' spline.

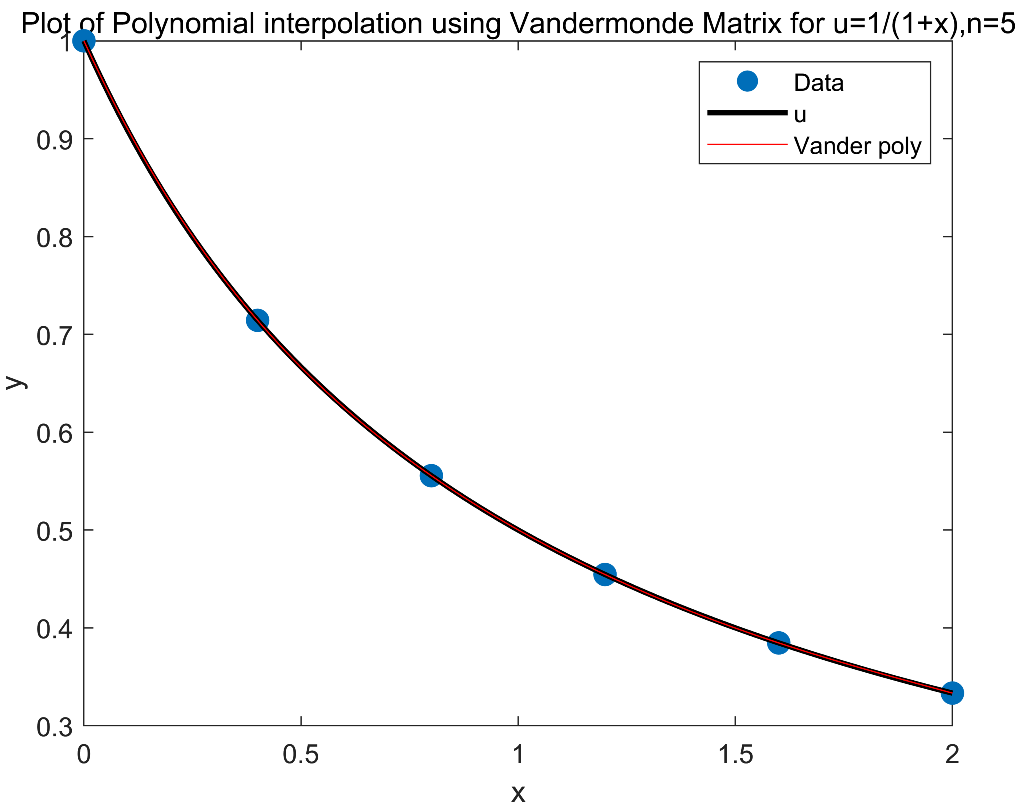
a)

We will use the same method with problem 5.

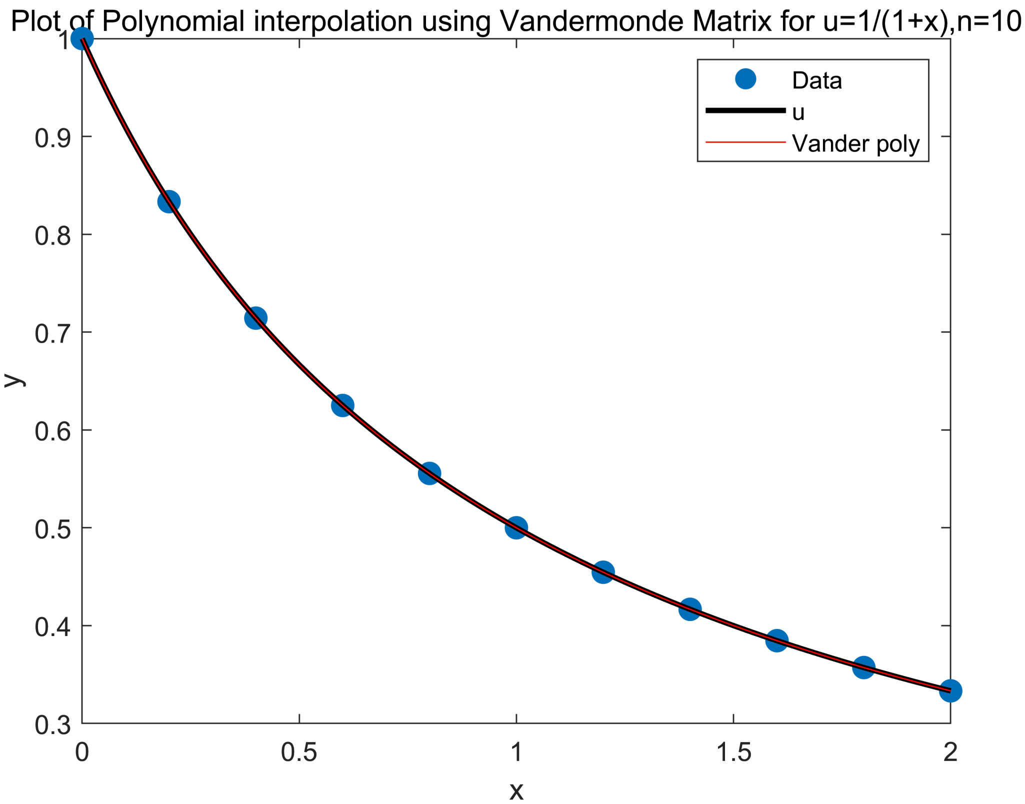
```
clear
a=0;
b=2;
x=a:0.01:b;
ut=u(x);%real

%a)
%Vandermonde matrix
n = 2; % number of intervals
[x2V,y2V,xfine2V,u_Vander_fine2V,a_vec2V]=van(a,b,n);
plotA(x2V,y2V,xfine2V,u_Vander_fine2V,n)
```



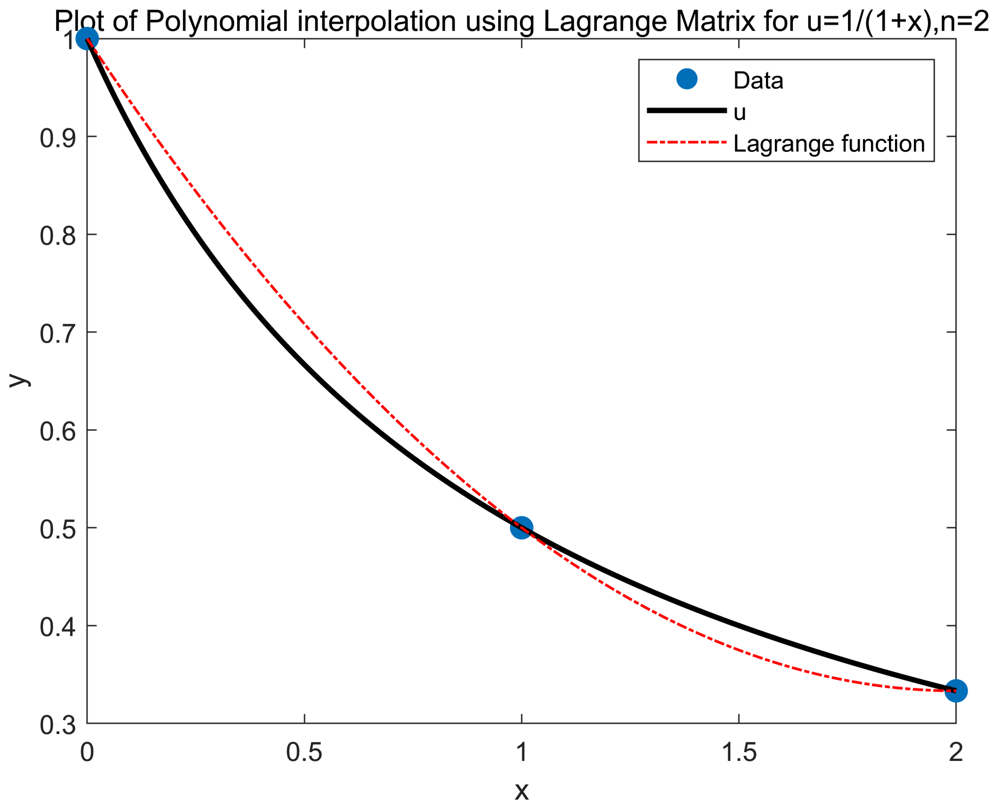
```
n = 5; % number of intervals
[x5V,y5V,xfine5V,u_Vander_fine5V,a_vec5V]=van(a,b,n);
plotA(x5V,y5V,xfine5V,u_Vander_fine5V,n)
```



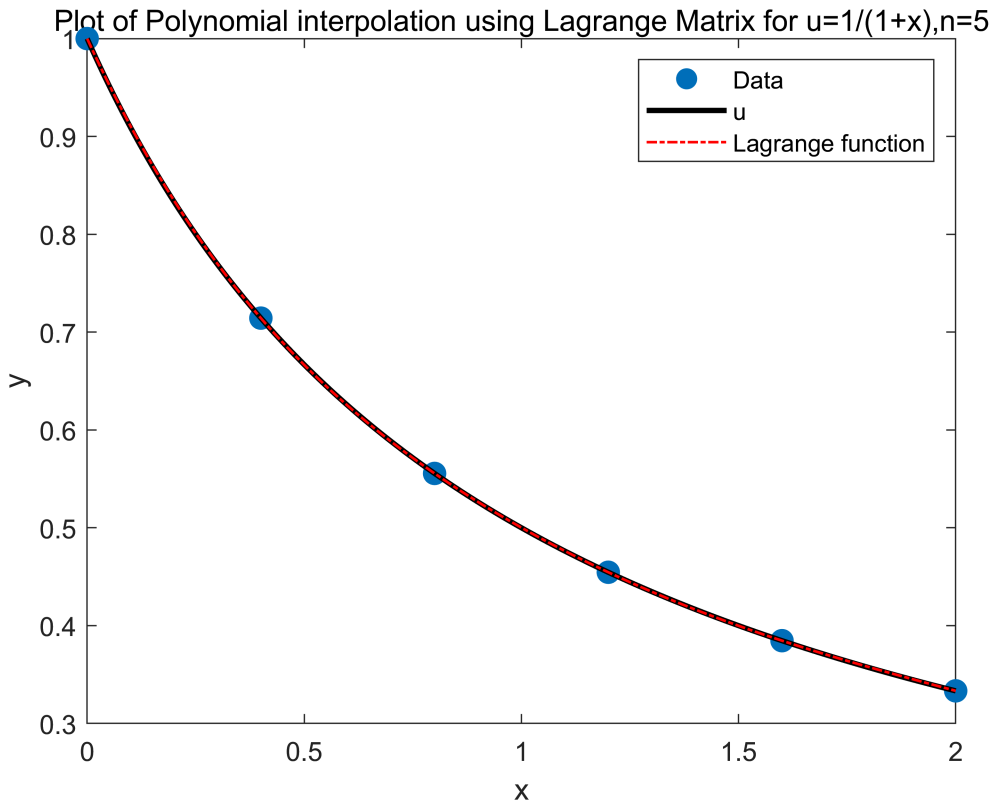

```
n = 10; % number of intervals
[x10V,y10V,xfine10V,u_Vander_fine10V,a_vec10V]=van(a,b,n);
plotA(x10V,y10V,xfine10V,u_Vander_fine10V,n)
```



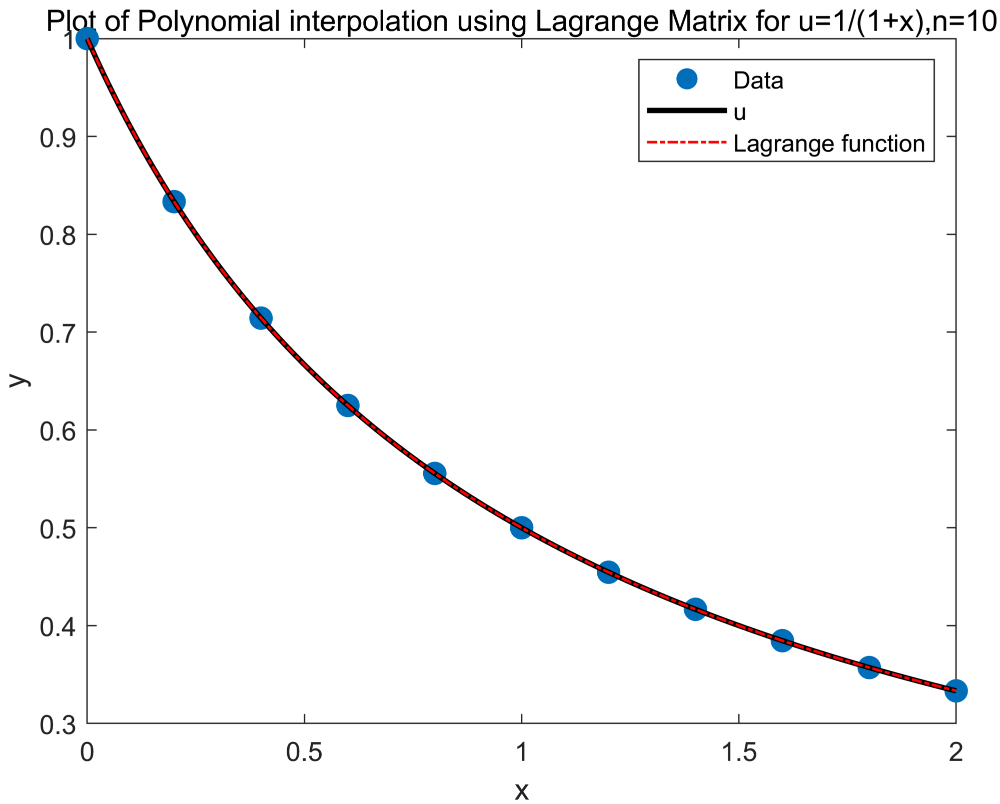

```
%Lagrange fomular
n = 2; % number of intervals
[x2L,y2L,xfine2L,u_Lagrange2L]=lan(a,b,n);
plotAN(x2L,y2L,xfine2L,u_Lagrange2L,n)
```




```
n = 5; % number of intervals
[x5L,y5L,xfine5L,u_Lagrange5L]=lan(a,b,n);
plotAN(x5L,y5L,xfine5L,u_Lagrange5L,n)
```




```
n = 10; % number of intervals
[x10L,y10L,xfine10L,u_Lagrange10L]=lan(a,b,n);
plotAN(x10L,y10L,xfine10L,u_Lagrange10L,n)
```



b)

With B-spline approach

In B-spline method we will use multiple B function

$$B(t)=\begin{cases}\frac{1}{6}t^3 & 0\le t\le 1 \quad (13)\\ \frac{1}{6}[-3(t-1)^3+3(t-1)^2+3(t-1)+1] & 1\le t\le 2 \quad (14)\\ \frac{1}{6}[3(t-2)^3-6(t-2)^2+4] & 2\le t\le 3 \quad (15)\\ \frac{1}{6}[-(t-3)^3+3(t-3)^2-3(t-3)+1] & 3\le t\le 4 \quad (16)\\ 0 & t\le 0 \text{ or } t\ge 4\end{cases}$$

For each B function $B_0, B_1, ..B_n$,

Each of B_j are shifted by h with other function

The B_0 ends at a+h

And B_n starts at b-h

B function has length=4

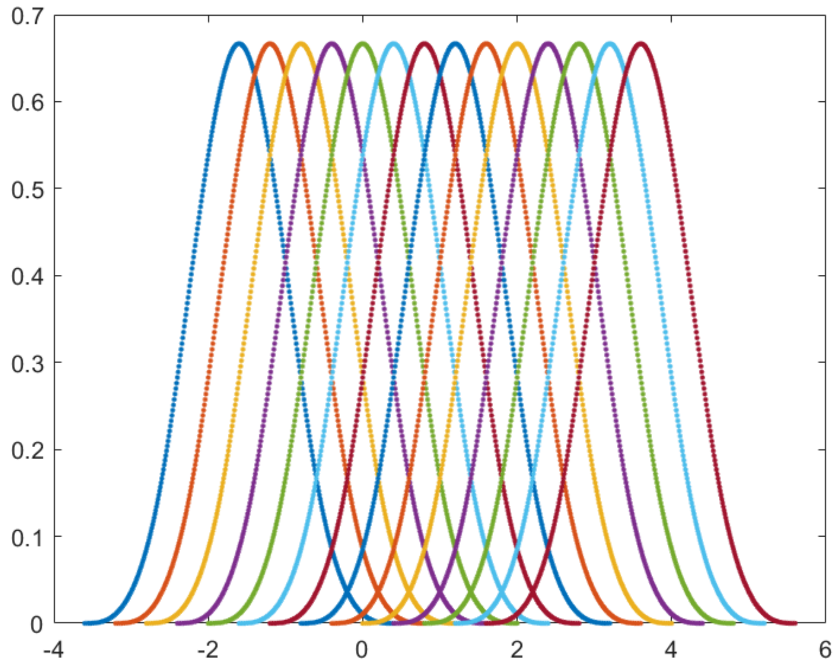
So B_0 starts at a+h-4

And B_n end at b-h+4

So if we graph it

For example n=5,a=0 and b=2

$$h = \frac{(b-a)}{n} = \frac{2}{5} = 0.4$$



So in total, the number of B funtion will be

$$n_b = \frac{(b-h)-(a+h-4)}{h} + 1 = \frac{2-0.4-0-0.4+4}{0.4} + 1 = \frac{5.2}{0.4} + 1 = 14$$

$$n_b = \frac{(b-h)-(a+h-4)}{h} + 1 \qquad (17)$$

The linear combination of B functions are:

$$s(x) = \sum_j^{n_b} \alpha_j B_j \qquad (18)$$

So for $j = 0, 1, 2, \ldots, n_b$

$$u(0 + \mathrm{ih}) = \sum_j^{n_b} \alpha_j B_j (0 + \mathrm{ih}) \qquad (19)$$

for i=0,1,2,...n+1

And we also have

$$u'(a) = \sum_j^{n_b} \alpha_j B'_j(a) \qquad (20)$$

and

$$u'(b) = \sum_j^{n_b} \alpha_j B'_j(b) \qquad (21)$$

we could compute the B'_j first

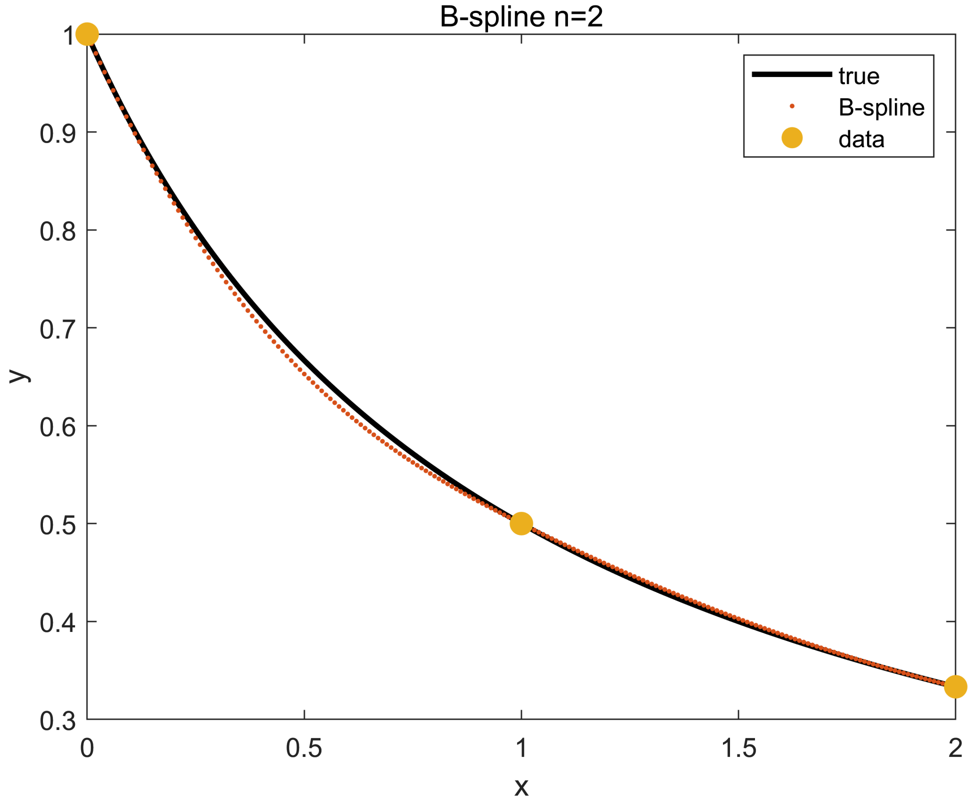
$$B'(t) = \begin{cases} \frac{1}{2}t^2 & 0 \le t \le 1 & (22) \\ \frac{1}{6}[-9(t-1)^2 + 6(t-1) + 3] & 1 \le t \le 2 & (23) \\ \frac{1}{6}[9(t-2)^2 - 12(t-2)] & 2 \le t \le 3 & (24) \\ \frac{1}{6}[-3(t-3)^2 + 6(t-3) - 3] & 3 \le t \le 4 & (25) \\ 0 & t \le 0 \text{ or } t \ge 4 \end{cases}$$

therefore we can write it as matrix

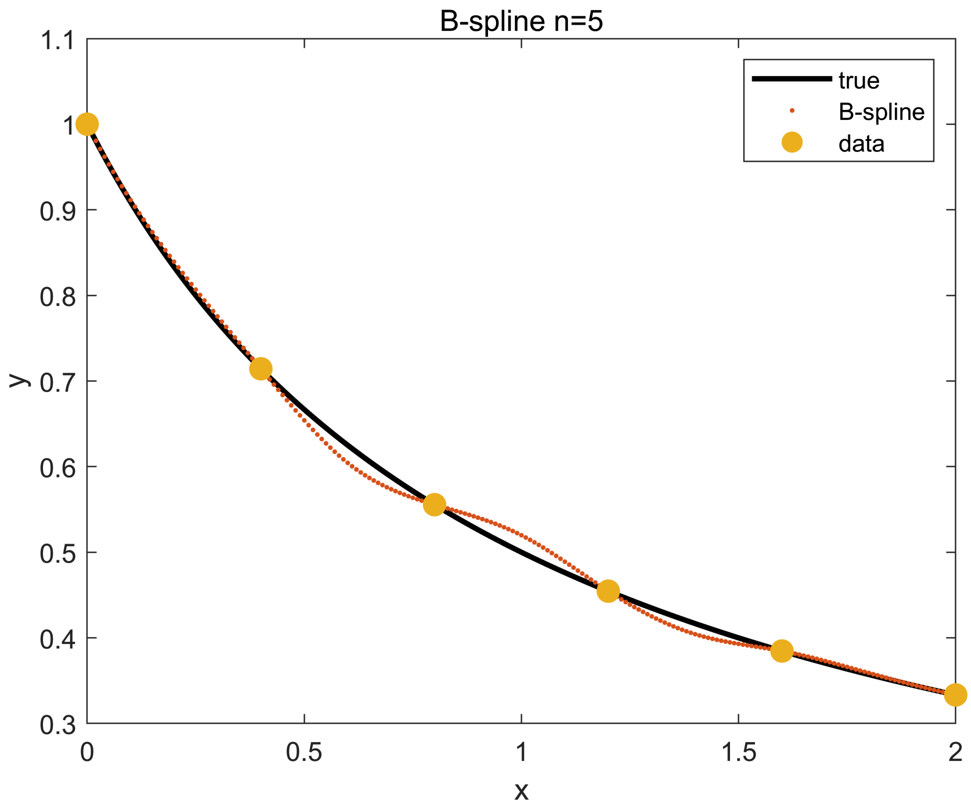
$$\begin{bmatrix} B_0(0+0\mathrm{h}) & B_1(0+0h) & \cdots & B_{n_b}(0+0h) \\ B_0(0+1h) & B_1(0+1h) & \cdots & B_{n_b}(0+1h) \\ \vdots & \vdots & \ddots & \vdots \\ B_j(0+(n+1)h) & B_j(0+(n+1)h) & \cdots & B_j(0+(n+1)h) \\ B'_0(a) & B'_1(a) & \cdots & B'_{n_b}(a) \\ B'_0(b) & B'_1(b) & \cdots & B'_{n_b}(b) \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{n_b} \end{bmatrix} = \begin{bmatrix} u(0+0h) \\ u(0+1h) \\ \vdots \\ u(0+(n+1)h) \\ u'(a) \\ u'(b) \end{bmatrix}$$

Then we could compute α vector and approch the true value by using equation (18)

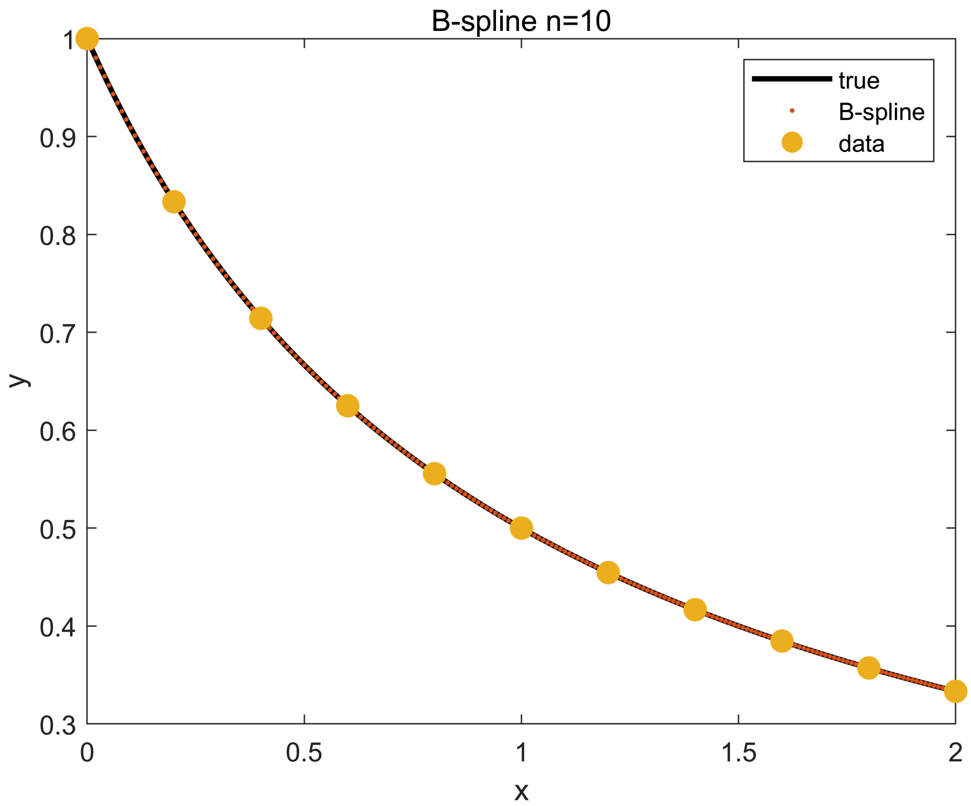
```
n=2;
[bc2,vv2,v2,t2]=b_clam(a,b,n);
[Y2B]=plotB(bc2,vv2,v2,x,ut,n,t2);
```




```
n=5;
[bc5,vv5,v5,t5]=b_clam(a,b,n);
[Y5B]=plotB(bc5,vv5,v5,x,ut,n,t5);
```




```
n=10;
[bc10,vv10,v10,t10]=b_clam(a,b,n);
[Y10B]=plotB(bc10,vv10,v10,x,ut,n,t10);
```



c)

Using clamped-spline

We will cut the function according to the numbers of data

So we define the funtion for n given data:

$$s = s(x) = \begin{cases} P_1(x) & x \in [x_0, x_1] \\ P_2(x) & x \in [x_1, x_2] \\ \vdots & \vdots \\ P_n(x) & x \in [x_{n-1}, x_n] \end{cases}$$

And each one could be written as

$$P_j(x) = a_j + b_j x + c_j x^2 + d_j x^3 \qquad (26)$$

Therefore we will get the equation below

$$\begin{aligned} &P_1(x_0) = u(x_0) \\ &P_2(x_1) = u(x_1) \\ &\vdots \\ &P_n(x_{n-1}) = u(x_{n-1}) \end{aligned} \qquad \text{(27a) and}$$

$$
\begin{aligned}
&P_1(x_1) = u(x_1)\\
&P_2(x_2) = u(x_2)\\
&\vdots\\
&P_n(x_n) = u(x_n)
\end{aligned}
\qquad (27b)
$$

Since we also need the first derivative to be equall

$$
\begin{aligned}
&P'_1(x_1) = P'_2(x_1)\\
&P_2'(x_2) = P_3'(x_2)\\
&\vdots\\
&P_{n-1}'(x_{n-1}) = P_n'(x_{n-1})
\end{aligned}
\qquad (28)
$$

Since we also need the second derivative to be equall

$$
\begin{aligned}
&P''_1(x_1) = P''_2(x_1)\\
&P_2''(x_2) = P_3''(x_2)\\
&\vdots\\
&P_{n-1}''(x_{n-1}) = P_n''(x_{n-1})
\end{aligned}
\qquad (29)
$$

So in total there are $4n$ of unknowns and only $n + n + n - 1 + n - 1 = 4n - 2$ equations

So we need clamped splines:

$$u'(a) = P_1'(a) = P_1'(x_0) \qquad (30)$$

$$u'(b) = P_n'(b) = P_n'(x_n) \qquad (31)$$

if we plug the (26) into (28) and (29)

$$P_j'(x) = 0 + b_j + 2c_j x + 3d_j x^2$$

$$P_j''(x) = 0 + 0 + 2c_j + 6d_j x$$

So for (28)

$$
\begin{aligned}
&P'_1(x_1) = P'_2(x_1) && 0 + b_1 + 2c_1x_1 + 3d_1x_1^2 = 0 + b_2 + 2c_2x_1 + 3d_2x_1^2\\
&P_2'(x_2) = P_3'(x_2) && 0 + b_2 + 2c_2x_2 + 3d_2x_2^2 = 0 + b_3 + 2c_3x_2 + 3d_3x_2^2\\
&\vdots && \vdots\\
&P_{n-1}'(x_{n-1}) = P_n'(x_{n-1}) \Rightarrow && 0 + b_{n-1} + 2c_{n-1}x_{n-1} + 3d_{n-1}x_{n-1}^2 = 0 + b_n + 2c_nx_{n-1} + 3d_nx_{n-1}^2
\end{aligned}
$$

$$
\begin{aligned}
&0 + b_1 + 2c_1x_1 + 3d_1x_1^2 - 0 - b_2 - 2c_2x_1 - 3d_2x_1^2 = 0\\
&0 + b_2 + 2c_2x_2 + 3d_2x_2^2 - 0 - b_3 - 2c_3x_2 - 3d_3x_2^2 = 0\\
&\vdots\\
\Rightarrow\ &0 + b_{n-1} + 2c_{n-1}x_{n-1} + 3d_{n-1}x_{n-1}^2 - 0 - b_n - 2c_nx_{n-1} - 3d_nx_{n-1}^2 = 0
\end{aligned}
\qquad (32)
$$

and (29)

$$
\begin{aligned}
&P''_1(x_1) = P''_2(x_1) && 0 + 0 + 2c_1 + 6d_1x_1 = 0 + 0 + 2c_2 + 6d_2x_1 \\
&P_2''(x_2) = P_3''(x_2) && 0 + 0 + 2c_2 + 6d_2x_2 = 0 + 0 + 2c_3 + 6d_3x_2 \\
&\vdots && \vdots \\
&P_{n-1}''(x_{n-1}) = P_n''(x_{n-1}) \Rightarrow && 0 + 0 + 2c_{n-1} + 6d_{n-1}x_{n-1} = 0 + 0 + 2c_n + 6d_nx_{n-1}
\end{aligned}
$$

$$
\Rightarrow
\begin{aligned}
&0 + 0 + 2c_1 + 6d_1x_1 - 0 - 0 - 2c_2 - 6d_2x_1 = 0 \\
&0 + 0 + 2c_2 + 6d_2x_2 = 0 - 0 - 2c_3 - 6d_3x_2 = 0 \\
&\vdots \\
&0 + 0 + 2c_{n-1} + 6d_{n-1}x_{n-1} - 0 - 0 - 2c_n - 6d_nx_{n-1} = 0
\end{aligned}
\Rightarrow \qquad (33)
$$

so if we write (27a) (27b) (32) (33) into a marix:

$$
\begin{aligned}
&P_1(x_0) = u(x_0) \\
&P_2(x_1) = u(x_1) \\
&\vdots \\
&P_n(x_{n-1}) = u(x_{n-1})
\end{aligned}
\qquad \text{(27a) and}
$$

$$
\begin{aligned}
&P_1(x_1) = u(x_1) \\
&P_2(x_2) = u(x_2) \\
&\vdots \\
&P_n(x_n) = u(x_n)
\end{aligned}
\qquad \text{(27b)}
$$

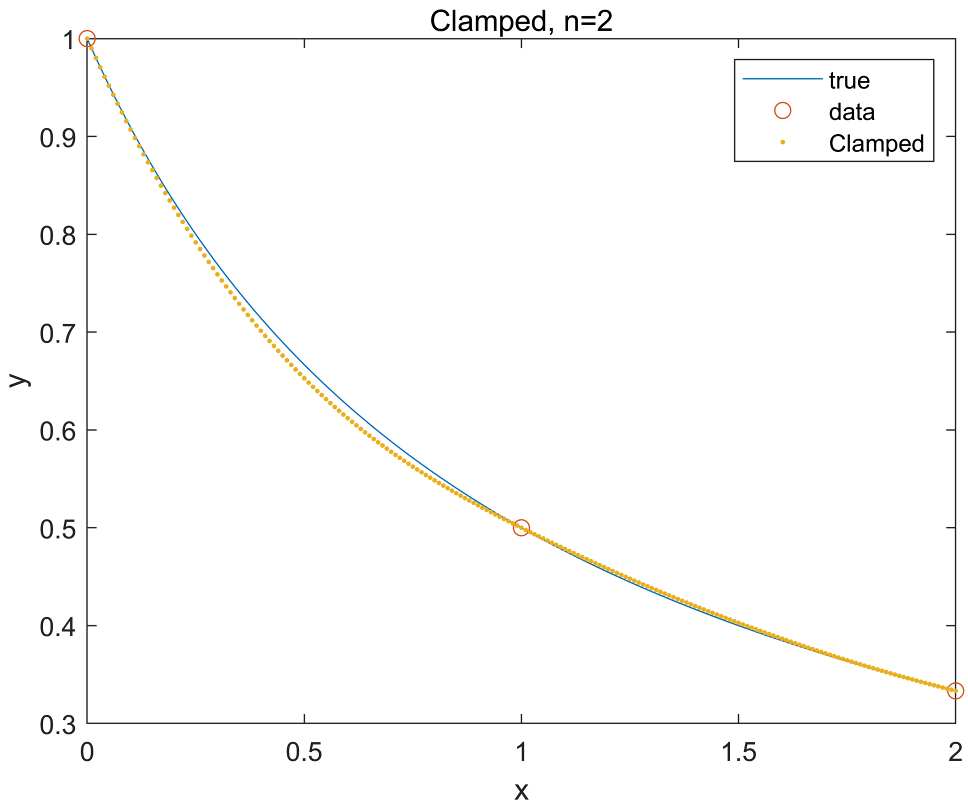
and (30) and (31)

$$
V = \begin{bmatrix}
1 & x_0 & x_0^2 & x_0^3 & 0 & & & & & & & & \cdots & & & & & & & & 0 \\
0 & & & & 1 & x_1 & x_1^2 & x_1^3 & & & & & \cdots & & & & & & & & 0 \\
0 & & & & & & & & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & & & 0 \\
0 & & & & & & & & & & & & \cdots & & & & & 1 & x_{n-1} & x_{n-1}^2 & x_{n-1}^3 \\
1 & x_1 & x_1^2 & x_1^3 & & & & & & & & & \cdots & & & & & & & & 0 \\
0 & & & & 1 & x_2 & x_2^2 & x_2^3 & & & & & \cdots & & & & & & & & 0 \\
0 & & & & & & & & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & & & 0 \\
0 & & & & & & & & & & & & \cdots & & & & & 1 & x_n & x_n^2 & x_n^3 \\
0 & 1 & 2x_1 & 3x_1^2 & 0 & -1 & -2x_1 & -3x_1^2 & & & & & \cdots & & & & & & & & 0 \\
0 & & & & 0 & 1 & 2x_2 & 3x_2^2 & 0 & -1 & -2x_2 & -3x_2^2 & \cdots & & & & & & & & 0 \\
0 & & & & & & & & & & & & \cdots & & & & & & & & 0 \\
0 & & & & & & & & & & & & \cdots & 0 & 1 & 2x_{n-1} & 3x_{n-1}^2 & 0 & -1 & -2x_{n-1} & -3x_{n-1}^2 \\
0 & 0 & 2 & 6x_1 & 0 & 0 & -2 & -6x_1 & & & & & \cdots & & & & & & & & 0 \\
0 & & & & 0 & 0 & 2 & 6x_2 & 0 & 0 & -2 & -6x_2 & \cdots & & & & & & & & 0 \\
0 & & & & & & & & & & & & \cdots & & & & & & & & 0 \\
0 & & & & & & & & & & & & \cdots & 0 & 0 & 2 & 6x_{n-1} & 0 & 0 & -2 & -6x_{n-1} \\
0 & 1 & 2x_0 & 3x_0^2 & & & & & & & & & \cdots & & & & & & & & 0 \\
0 & & & & & & & & & & & & \cdots & & & & & 0 & 1 & 2x_n & 3x_n^2
\end{bmatrix}
$$

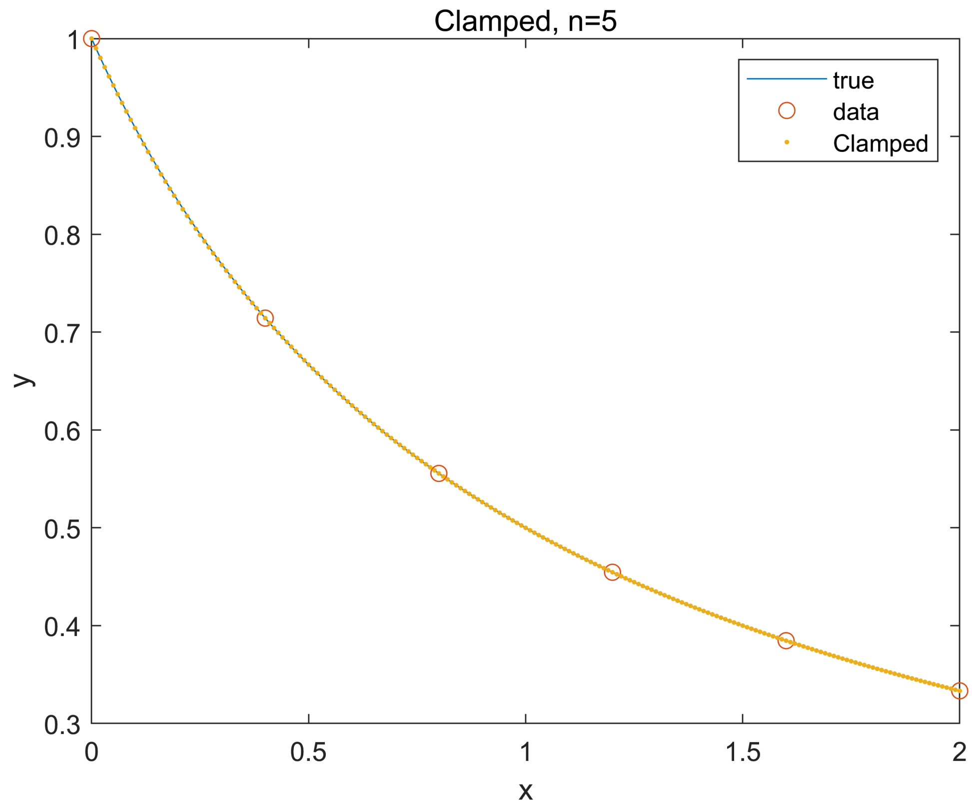
$a = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \\ a_2 \\ b_2 \\ c_2 \\ d_2 \\ \vdots \\ a_n \\ b_n \\ c_n \\ d_n \end{bmatrix}$ and $u = \begin{bmatrix} u(x_0) \\ u(x_1) \\ \vdots \\ u(x_{n-1}) \\ u(x_1) \\ u(x_2) \\ \vdots \\ u(x_n) \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ u'(a) \\ u'(b) \end{bmatrix}$,where $\mathrm{Va} = u$

Then we can compute a from V and u.

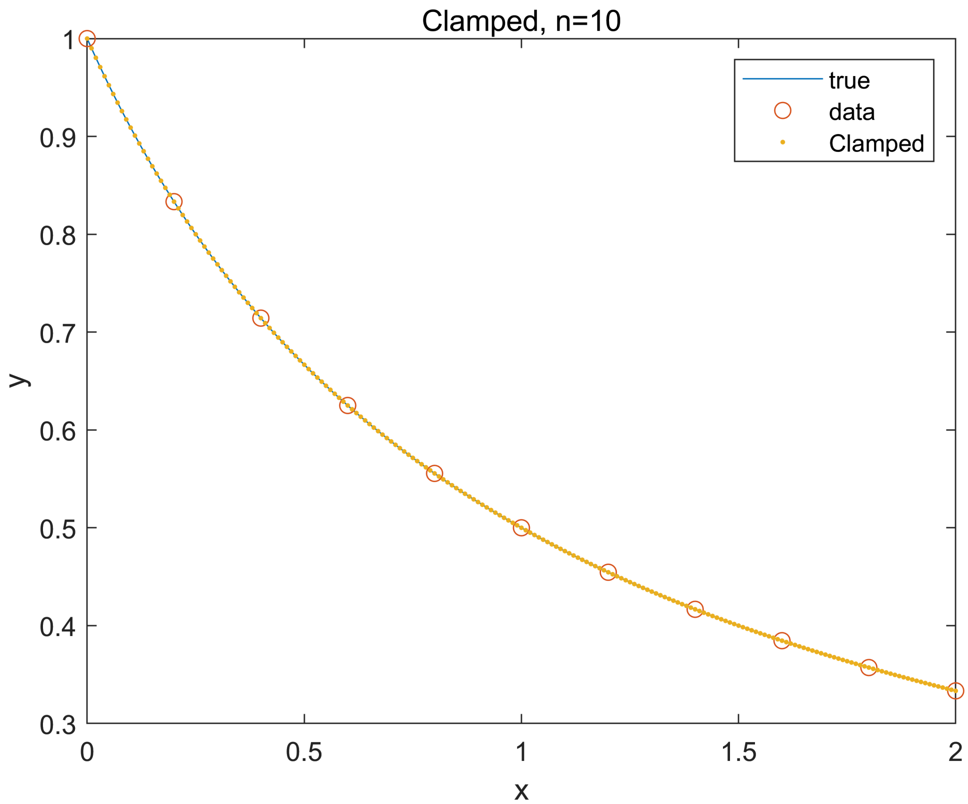
```
n=2;
[X_coeC2]=clam(a,b,n);
[Y2C]=plotCC(x,ut,a,b,X_coeC2,n);
```




```
n=5;
[X_coeC5]=clam(a,b,n);
[Y5C]=plotCC(x,ut,a,b,X_coeC5,n);
```




```
n=10;
[X_coeC10]=clam(a,b,n);
[Y10C]=plotCC(x,ut,a,b,X_coeC10,n);
```



from the plot we could conclude that , the more n we take the more closer two curves will be.

For natrual spline

instand (30) and (31) we will use

$P_1''(a)=P_1''(x_0) = 0$ (34)

$P_n''(a)=P_n''(x_n) = 0$ (35)

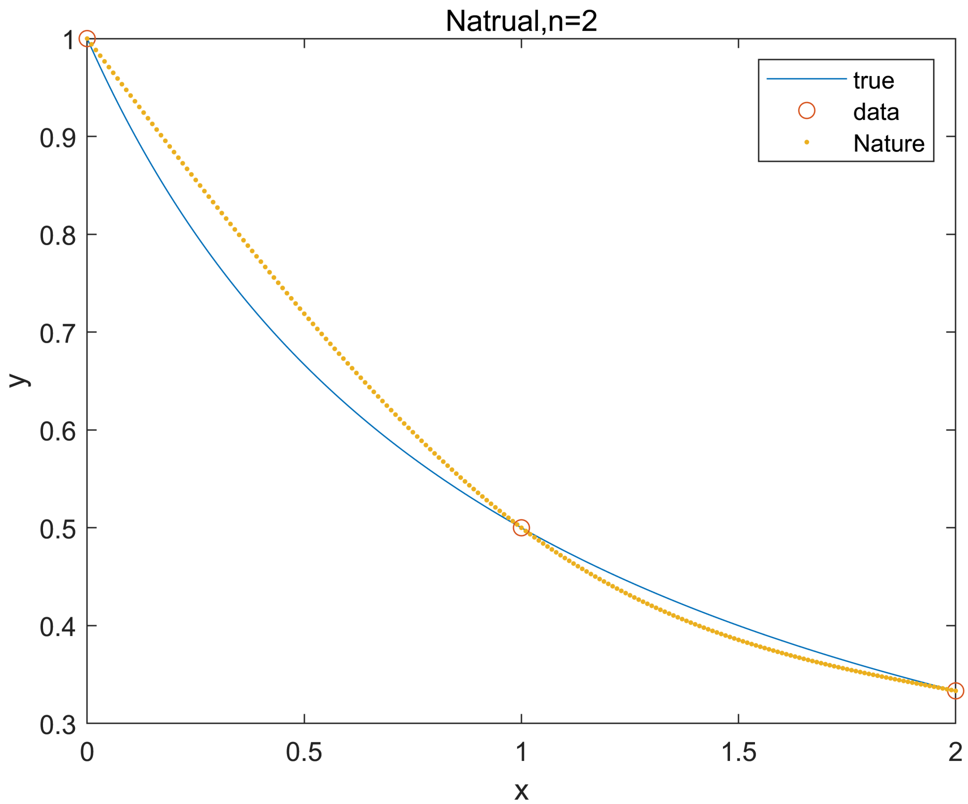
The only things that need to be changed is

last 2 row of the matrix V

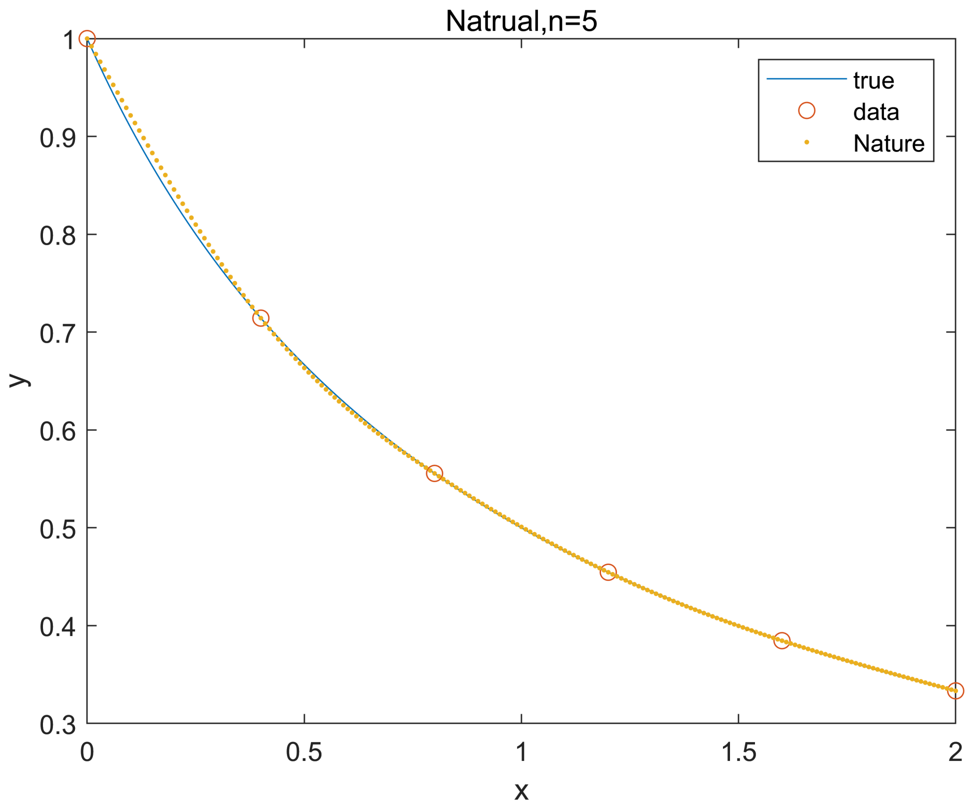
$$\begin{bmatrix} 0 & 0 & 2 & 6a & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 2 & 6b \end{bmatrix} \quad (36)$$

and last 2 terms of u becomes 0.

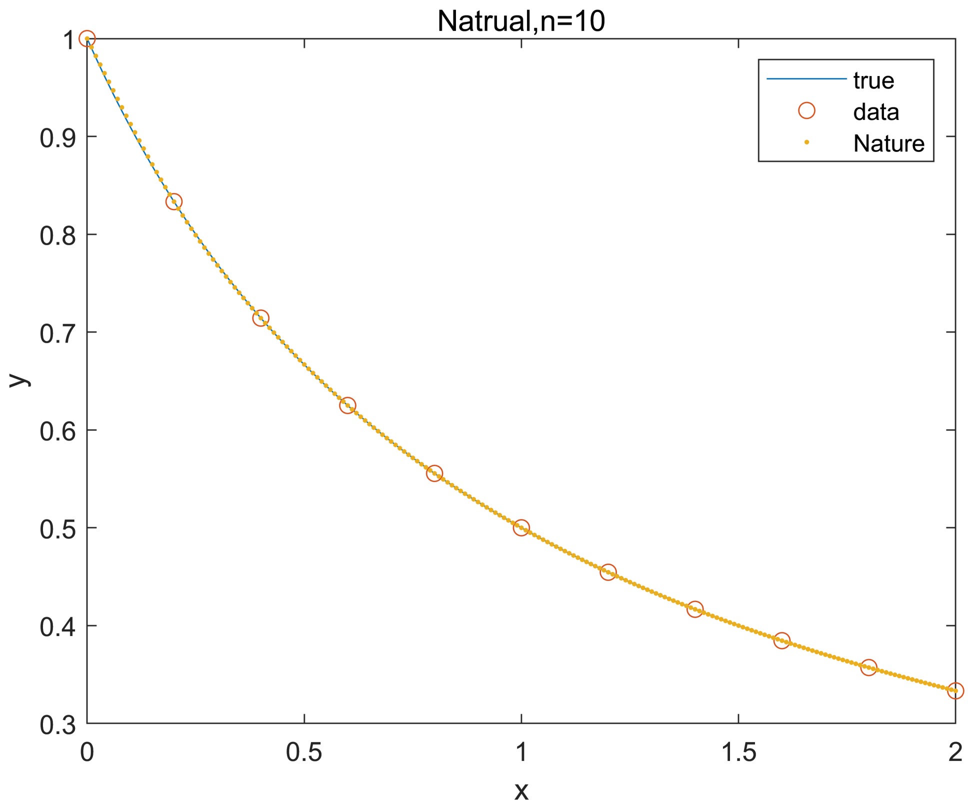
```
n=2;
[X_coeN2]=nat(a,b,n);
[Y2N]=plotCN(x,ut,a,b,X_coeN2,n);
```




```
n=5;
[X_coeN5]=nat(a,b,n);
[Y5N]=plotCN(x,ut,a,b,X_coeN5,n);
```




```
n=10;
[X_coeN10]=nat(a,b,n);
[Y10N]=plotCN(x,ut,a,b,X_coeN10,n);
```



we could find that when n is small the nature spline work bad compare with the clamped.

To specific compute

```
err_Vandermonde_n_2 = sum(abs(u_Vander_fine2V-u(xfine2V)),'all')
```

```
err_Vandermonde_n_2 = 4.5646
```

```
err_Vandermonde_n_5 = sum(abs(u_Vander_fine5V-u(xfine5V)),'all')
```

```
err_Vandermonde_n_5 = 0.0643
```

```
err_Vandermonde_n_10 = sum(abs(u_Vander_fine10V-u(xfine10V)),'all')
```

```
err_Vandermonde_n_10 = 1.3101e-04
```

```

err_Lagrange_n_2 = sum(abs(u_Lagrange2L-u(xfine2L)),'all')
```

```
err_Lagrange_n_2 = 4.5646
```

```
err_Lagrange_n_5 = sum(abs(u_Lagrange5L-u(xfine5L)),'all')
```

```
err_Lagrange_n_5 = 0.0643
```

```
err_Lagrange_n_10 = sum(abs(u_Lagrange10L-u(xfine10L)),'all')
```

```
err_Lagrange_n_10 = 1.3101e-04
```

```
err_B_spline_2=sum(abs(Y2B(:)-ut(:)),"all")
```

```
err_B_spline_2 = 0.9904
```

```
err_B_spline_5=sum(abs(Y5B(:)-ut(:)),"all")
```

```
err_B_spline_5 = 1.5040
```

```
err_B_spline_10=sum(abs(Y10B(:)-ut(:)),"all")
```

```
err_B_spline_10 = 0.0013
```

```
err_Cliamped_2=sum(abs(Y2C(:)-ut(:)),'all')
```

```
err_Cliamped_2 = 0.9904
```

```
err_Cliamped_5=sum(abs(Y5C(:)-ut(:)),'all')
```

```
err_Cliamped_5 = 0.0247
```

```
err_Cliamped_10=sum(abs(Y10C(:)-ut(:)),'all')
```

```
err_Cliamped_10 = 0.0013
```

```
err_Nature_2=sum(abs(Y2N(:)-ut(:)),'all')
```

```
err_Nature_2 = 4.5646
```

```
err_Nature_5=sum(abs(Y5N(:)-ut(:)),'all')
```

```
err_Nature_5 = 0.4788
```

```
err_Nature_10=sum(abs(Y10N(:)-ut(:)),'all')
```

```
err_Nature_10 = 0.0660
```

So as result, when n is high, vanermode and lagrange will has better result,

and when n is smal, B-spline and clamped will have better result.

```
function [X_coe]=clam(a,b,n)
%function of Clamped spline
%out put the coefficient a,b,c,d
    A=zeros(4*n,4*n);%initial Matrix V
    xx=linspace(a,b,n+1);%general grid
    x=xx(2:end);
    for i=1:n-1
        %general row 1 to n-1 of equation (27a) (32) (33)
        %general row 2 to n of equaiton (27b)
        A(i,(1+(i-1)*4:(i+1)*4))=[1 x(i) x(i)^2 x(i)^3 0 0 0 0];%(27a)
        A(i+n,(1+(i-1)*4:(i+1)*4))=[0 0 0 0 1 x(i) x(i)^2 x(i)^3];%(27b)
```

```
        A(i+n+n-1,(1+(i-1)*4:(i+1)*4))=[0 1 2*x(i) 3*x(i)^2 0 -1 -2*x(i) -3*x(i)^2];%(32)
        A(i+n+n-1+n-1,(1+(i-1)*4:(i+1)*4))=[0 0 2 6*x(i) 0 0 -2 -6*x(i)];%(33)
    end
    A(n,(1+(n-1)*4:n*4))=[1 x(n) x(n)^2 x(n)^3];%the n th row of (27a)
    A(n+n-1+n-1+n-1+1,1)=1;%the first row of (27b)
    A(n+n-1+n-1+n-1+1+1,1:4)=[0 1 2*a 3*a^2];%(30)
    A(n+n-1+n-1+n-1+1+1+1,(4*(n-1)+1:4*(n-1)+4))=[0 1 2*b 3*b^2];%(31)
    rhs=zeros(1,4*n);%initial the u vector
    s=u(x);
    rhs(1:n)=s;%(27a)
    rhs(n+1:n+n-1)=s(1:n-1);%(27b)
    rhs(n+n-1+n-1+n-1+1)=u(a);
    rhs(4*n-1)=uprim(a);%(30)
    rhs(4*n)=uprim(b);%(31)
    X_coe=A\rhs';
end

function [X_coe]=nat(a,b,n)
%function of Clamped spline
%out put the coefficient a,b,c,d
    A=zeros(4*n,4*n);
    xx=linspace(a,b,n+1);
    x=[xx(2:end)];
    for i=1:n-1
        A(i,(1+(i-1)*4:(i+1)*4))=[1 x(i) x(i)^2 x(i)^3 0 0 0 0];
        A(i+n,(1+(i-1)*4:(i+1)*4))=[0 0 0 0 1 x(i) x(i)^2 x(i)^3];
        A(i+n+n-1,(1+(i-1)*4:(i+1)*4))=[0 1 2*x(i) 3*x(i)^2 0 -1 -2*x(i) -3*x(i)^2];
        A(i+n+n-1+n-1,(1+(i-1)*4:(i+1)*4))=[0 0 2 6*x(i) 0 0 -2 -6*x(i)];
    end
    A(n,(1+(n-1)*4:n*4))=[1 x(n) x(n)^2 x(n)^3];
    A(n+n-1+n-1+n-1+1,1)=1;
    A(n+n-1+n-1+n-1+1+1,1:4)=[0 0 2 6*a];%(34)
    A(n+n-1+n-1+n-1+1+1+1,(4*(n-1)+1:4*(n-1)+4))=[0 0 2 6*b];%(35)
    rhs=zeros(1,4*n);
    s=u(x);
    rhs(1:n)=s;
    rhs(n+1:n+n-1)=s(1:n-1);
    rhs(n+n-1+n-1+n-1+1)=u(a);
    X_coe=A\rhs';
end

function [X,Y]=hua(a,b,X_coe,n)
%funtion of processing the data
%input:
%X_coe: the coefficient computed from above
%ouutput:
%X,Y: x coordinate and Y coordinate of the approximate funtion.
    h=(b-a)/n;
    dx=0.01;
    Y=[];
    X=[];
    for i=1:n
        %loop of n
        x=a+(i-1)*h:dx:a+i*h-dx;%compute each part of x coodinate
```

```
        %compute each part of y coodinate according to the coefficient
        y=X_coe(1+(i-1)*4)+ ...
        X_coe(2+(i-1)*4)*x+ ...
        X_coe(3+(i-1)*4)*x.^2+ ...
        X_coe(4+(i-1)*4)*x.^3;
        Y=[Y y];
        X=[X x];
    end
    %last point of Y
    Y=[Y X_coe(4*n-3)+X_coe(4*n-2)*b+X_coe(4*n-1)*b^2+X_coe(4*n)*b^3];
    %last point of X
    X=[X b];
end

function [datax,datay]=dian(a,b,n)
%funtion of compute the data
    datax=linspace(a,b,n+1);
    datay=u(datax);
end

function [ut]=u(x)
%function real function
    ut=1./(x+1);
end

function [up]=uprim(x)
%function of uprime
    up=-(x+1).^(-2);
end

function [bc,vv,v,t]=b_clam(a,b,n)
%function of forming the matrix, u vector and compute the alpha vector
%output
%bc:alpha vector
%vv:index of B function
%v:number of B funciton
    h=(b-a)/n;
    vv=(-4+h:h:2-h);%compute the index of B function equation(17)
    v=length(vv);%compute the number of B function
    Ab=zeros(n+3);%initial the matrix
    t=a:h:b;%define t
    for i=1:n+1
        %loop of i
        for j=1:v
            %loop of j
            Ab(i,j)=BBj(t(i),vv(j));%fill the matrix with equation(19)
        end
    end
    for j=1:v
        %fill the matrix with equation (20) and (21)
        Ab(n+2,j)=BBjprime(a,vv(j));%euqaiton(20)
        Ab(n+3,j)=BBjprime(b,vv(j));%euqaiton(21)
    end
    %general the u vector
```

```
    yb=zeros(n+3,1);
    yb(1:n+1)=u(t);%fill the first n+1 terms with u(t)
    yb(n+2)=uprim(a);%euqaiton(20)
    yb(n+3)=uprim(b);%euqaiton(21)
    bc=Ab\yb;%compute the alpha vector
end

function [Bt1]=B1(t)
%equation (13)
    Bt1=1/6*t.^3;
end

function [Bt2]=B2(t)
%equation (14)
    Bt2=1/6*(-3*(t-1).^3+3*(t-1).^2+3*(t-1)+1);
end

function [Bt3]=B3(t)
%equation (15)
    Bt3=1/6*(3*(t-2).^3-6*(t-2).^2+4);
end

function [Bt4]=B4(t)
%equation (16)
    Bt4=1/6*(-(t-3).^3+3*(t-3).^2-3*(t-3)+1);
end

function [Bt1p]=B1prime(t)
%equation (22)
    Bt1p=1/2*t.^2;
end

function [Bt2p]=B2prime(t)
%equation (23)
    Bt2p=1/6*(-9*(t-1).^2+6*(t-1)+3);
end

function [Bt3p]=B3prime(t)
%equation (24)
    Bt3p=1/6*(9*(t-2).^2-12*(t-2));
end

function [Bt4p]=B4prime(t)
%equation (25)
    Bt4p=1/6*(-3*(t-3).^2+6*(t-3)-3);
end

function [k]=BBj(t,j)
%function of combina B1, B2, B3, B4 together
%j means the shifts take placed on B
    if j<=t && t<=j+1
        k=B1(t-j);
    elseif j+1<=t && t<=j+2
        k=B2(t-j);
```

```
    elseif j+2<=t && t<=j+3
        k=B3(t-j);
    elseif j+3<=t && t<=j+4
        k=B4(t-j);
    elseif t>=j+4 || t<=j
        k=0;
    end
end

function [k]=BBjprime(t,j)
%function of combina B1prime, B2prime, B3prime, B4prime together
%j means the shifts take placed on Bprime
    if j<=t && t<=j+1
        k=B1prime(t-j);
    elseif j+1<=t && t<=j+2
        k=B2prime(t-j);
    elseif j+2<=t && t<=j+3
        k=B3prime(t-j);
    elseif j+3<=t && t<=j+4
        k=B4prime(t-j);
    elseif t>=j+4 || t<=j
        k=0;
    end
end

function [Y]=plotCC(x,ut,a,b,X_coe,n)
%funtion of plot clamped spline
%out put the approximate y value
    [X,Y]=hua(a,b,X_coe,n);%compute the y coordinate
    [datax,datay]=dian(a,b,n);%input the data
    plot(x,ut)%real value
    hold on
    plot(datax,datay,'o')%data
    plot(X,Y,'.')%Clamped
    xlabel('x');ylabel('y')
    legend('true','data','Clamped')
    title(['Clamped, n=',num2str(n)])
    hold off
end

function [Y]=plotCN(x,ut,a,b,X_coe,n)
%funtion of plot nature spline
%out put the approximate y value
    [X,Y]=hua(a,b,X_coe,n);
    [datax,datay]=dian(a,b,n);
    plot(x,ut)
    hold on
    plot(datax,datay,'o')
    plot(X,Y,'.')
    xlabel('x');ylabel('y')
    legend('true','data','Nature')
    title(['Natrual,n=',num2str(n)])
    hold off
end
```

```
function [Y]=plotB(bc,vv,v,x,ut,n,t)
%funtion of ploting the B-spline
%input:
%bc:alpha vector
%vv:index of B function
%v:number of B funciton
%x:grid
%ut:real value
%yb:u vector
%n
    num=length(x);
    Y=zeros(num,1);
    for i=1:v
        for j=1:num
            %compute the linear compbination from equation (18)
            Y(j)=Y(j)+bc(i)*BBj(x(j),vv(i));
        end
    end
    plot(x,ut,'k', 'LineWidth',2)%true value
    hold on
    plot(x,Y,'.')%approximate value
    plot(t,u(t),'.','MarkerSize', 30)%data
    xlabel('x');ylabel('y')
    legend('true','B-spline','data')
    title(['B-spline n=',num2str(n)])
    hold off
end

function [x,y,xfine,u_Vander_fine,a_vec]=van(a,b,n)
%function of computing the Vandermonde martix and correspounding
%coefficients
%take input a,b,and n
%out put x: the x coodinate of data point according to n
%out put y: the y coodinate of data point that compute from x
%xfine: a finer grid of x
%u_Vander_fine: approximate the polynomial using the coefficient compute from V and y
%a_vec: vector a
    h = (b-a)/n;
    x = a + (0:n)*h; % define the x values
    y = u(x); % define the corresponding y values as y_j = u(x_j)
    Vmat = zeros(length(x));
    for ii=1:length(x)
        % loop over the data points
        for jj=1:length(x)
            % loop over the columns of the Vandermonde martix
            Vmat(ii,jj) = x(ii)^(jj-1);
            % since matlab indices starts at 1, so we need to subtract 1
        end
    end
    rr_form     = rref([Vmat,y(:)]);
    a_vec       = rr_form(:,end); % peeling off the last column
    xfine         = a:0.01:b;%define a finer grid x
    u_Vander_fine = a_vec(1)*ones(size(xfine));
```

```
    for jj=1:n
        %compute the polynomial from vector a
        u_Vander_fine = u_Vander_fine + a_vec(jj+1)*xfine.^(jj);
    end
end

function [x,y,xfine,u_Lagrange]=lan(a,b,n)
%function of compute the Lagrange fomular
%input a, b and n
%outpout
%out put x: the x coodinate of data point according to n
%out put y: the y coodinate of data point that compute from x
%xfine: a finer grid of x
%u_Lagrange:  approximate the polynomial
% using the coefficient compute Lagrange fomular
    h = (b-a)/n;
    x = a + (0:n)*h; % define the x values
    y = u(x); % define the corresponding y values as y_j = u(x_j)
    N           = length(x);
    xfine       = a:0.01:b; % define the finer x values
    u_Lagrange  = zeros(size(xfine));
    for k = 1:N
        %loop each k from equation (6)
        w = ones(size(xfine));
        for j = [1:k-1 k+1:N] % This loop the index from equation (8)
            w = (xfine-x(j))./(x(k)-x(j)).*w;
        end
        u_Lagrange = u_Lagrange + w*y(k);
        % This adds up each y_k*l_k(x) term, equation (7)
    end
end

function []=plotA(x,y,xfine,u_Vander_fine,n)
%function of plot the polynomial from Vandermonde martix
%take input:
%x: the x coodinate of data point according to n
%y: the y coodinate of data point that compute from x
%xfine: a finer grid of x
%u_Vander_fine: approximate the polynomial using the coefficient compute from V and y
%n
    plot(x,y,'.','MarkerSize', 30)%plot the data
    hold on;
    plot(xfine,u(xfine),'k', 'LineWidth',2)%plot the true value
    plot(xfine, u_Vander_fine, 'r')%plot the polynomial
    xlabel('x');ylabel('y');
    legend('Data','u', 'Vander poly')
    title(['Plot of Polynomial interpolation using ' ...
        'Vandermonde Matrix for u=1/(1+x),n=',num2str(n)])
    hold off
end

function []=plotAN(x,y,xfine,u_Lagrange,n)
%function of plot the polynomial from Lagrange fomular
%take input:
```

```
%x: the x coodinate of data point according to n
%y: the y coodinate of data point that compute from x
%xfine: a finer grid of x
%u_Lagrange: approximate the polynomial using the coefficient compute Lagrange
%n
    plot(x,y,'.','MarkerSize',30);%plot the data
    hold on;
    plot(xfine,u(xfine),'k', 'LineWidth',2);%plot the true value
    plot(xfine,u_Lagrange,'-.r','LineWidth',1)%plot the polynomial
    xlabel('x');ylabel('y');
    legend('Data','u','Lagrange function')
    title(['Plot of Polynomial interpolation using' ...
        ' Lagrange Matrix for u=1/(1+x),n=',num2str(n)])
    hold off
end
```