ii)Then, write code in Matlab to solve your problem from (i) to determine the solution u(x). Compare your result to that from (a) at the grid nodes.

h=0.25 and $h = \frac{1}{n+1}$

so$n = \frac{1}{h} - 1 = 3$

From the resulte above $K = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

From equation (1)

$$[U_{i+1} - 2U_i + U_{i-1}] = h^2 f(x)$$
$$[U_{i+1} - 2U_i + U_{i-1}] = h^2 (x_i - 2)$$

To build K

From the resulte above $K = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

if we use

```
n=3;
e = ones(n, 1);
K1 = spdiags([-e, 2*e, -e], -1:1, n, n)
```

```
K1 =
   (1,1)        2
   (2,1)       -1
   (1,2)       -1
   (2,2)        2
   (3,2)       -1
   (2,3)       -1
   (3,3)        2
```

We could findout that the sign is opposite with the function we want

So

```
e = ones(n, 1);
K2 = -spdiags([-e, 2*e, -e], -1:1, n, n)
```

```
K2 =
   (1,1)       -2
   (2,1)        1
   (1,2)        1
   (2,2)       -2
   (3,2)        1
   (2,3)        1
   (3,3)       -2
```

And also, for i=n, from the problem

$u'(1) = 4$

$$\frac{U_{n+1} - U_n}{h} = 4$$
$$U_{n+1} - U_n = 4h$$
$$U_{n+1} = 4h + U_n$$

so the last term of the equation (1) is

$$[4h + U_n - 2U_n + U_{n-1}] = h^2(x_i - 2)$$
$$[-U_n + U_{n-1}] = h^2(x_i - 2) - 4h \qquad (2)$$

So the last term of K is -1.

And for vector b, according to equation (1)

$$b_i = h^2(x_i - 2) \qquad (3)$$

Also from (2), the last term of b is:

$$b_n = h^2(x_n - 2) - 4h \qquad (4)$$

```
clear
h = 0.25;%define h
n = 1/h-1; %compute n
u0 = 0;%boundary condition u(0)=0

x = linspace(1/(n + 1), 1 - 1/(n + 1), n);%define xi
```

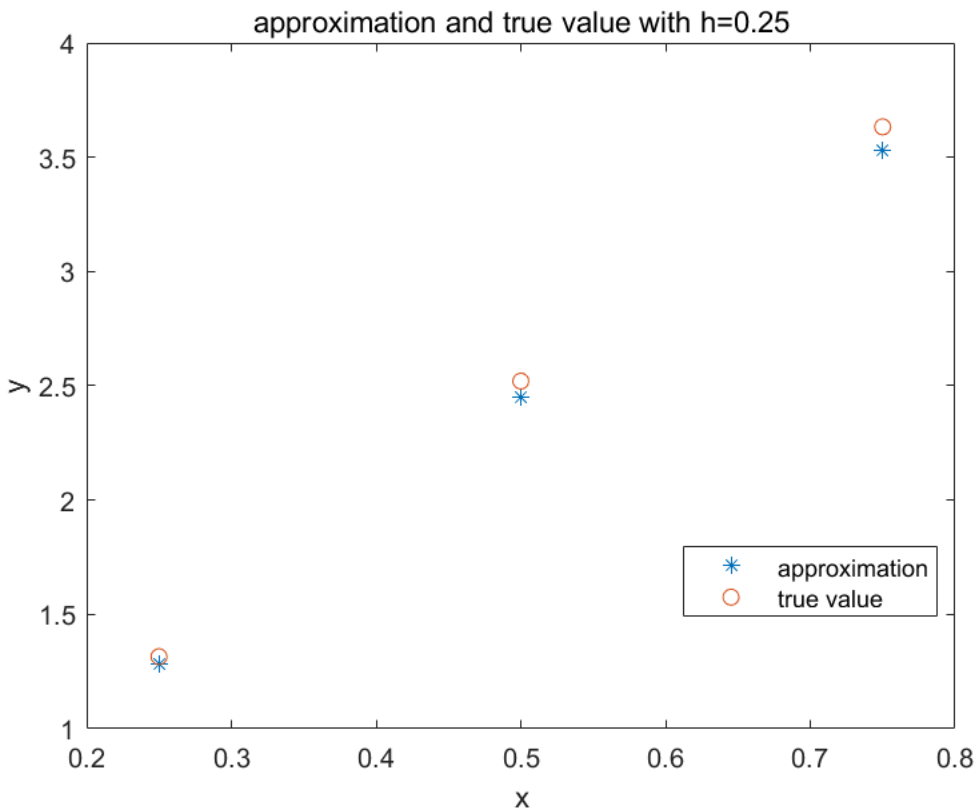
```
x = 1×3
    0.2500    0.5000    0.7500
```

```
r = realf(x); %calculate the ture value
u = finap(n, u0) %calculate the approximation value
```

```
u = 3×1
    1.2812
    2.4531
    3.5312
```

```

%plot the value
plot(x, u, '*')
hold on
plot(x, r, 'o')
xlabel('x')
ylabel('y')
title('approximation and true value with h=0.25')
legend('approximation','true value','Location','best')
hold off
```



The approximate value is a little bit off the true value

```
error_h_0_25=sum(abs(u(:)-r(:)))
```

```
error_h_0_25 = 0.2031
```

The total error of n=3 is 0.2031

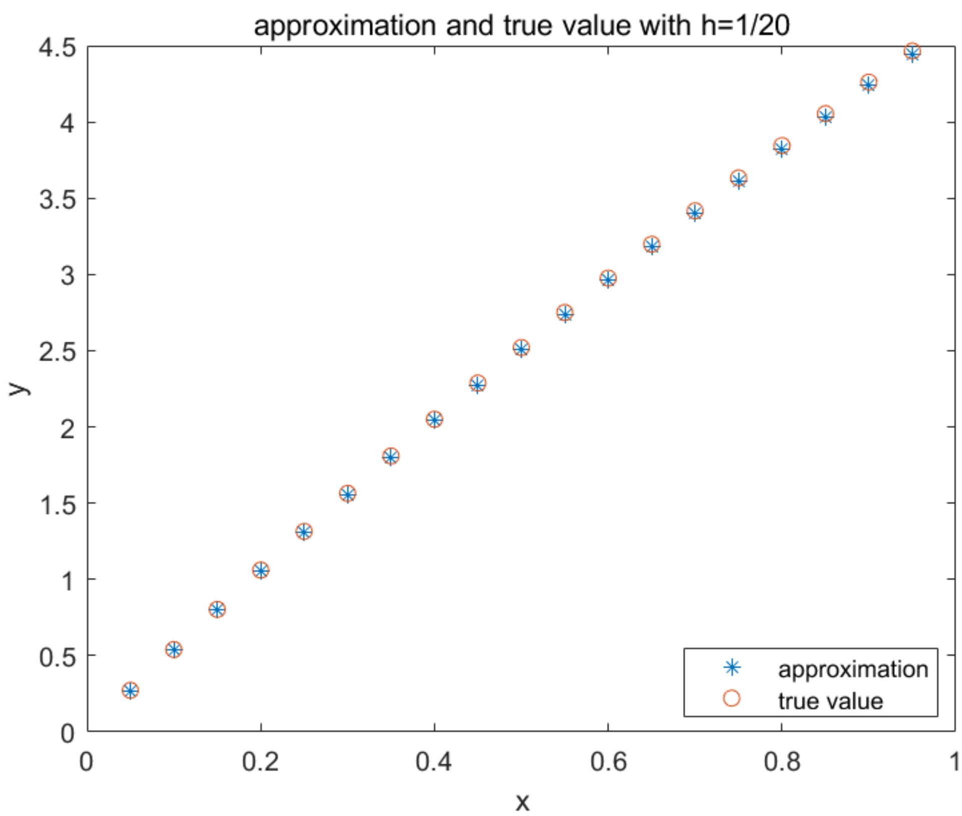
(iii) Generalize your code to solve the problem with $h = \frac{1}{20}$.

```
clear
h2 = 1/20;%define h
n2 = 1/h2-1; %compute n
u0 = 0;%boundary condition u(0)=0

x2 = linspace(1/(n2 + 1), 1 - 1/(n2 + 1), n2);%define xi
r2 = realf(x2); %calculate the ture value
u2 = finap(n2, u0); %calculate the approximation value

%plot the value
plot(x2, u2, '*')
hold on
plot(x2, r2, 'o')
xlabel('x')
ylabel('y')
title('approximation and true value with h=1/20')
legend('approximation','true value','Location','best')
```

```
hold off
```



From the plot we could find out that, at the begining of the plot, the approximate value are almost the same as the true value, but with x increasing the error become larger.

```
error_h_0_005=sum(abs(u2(:)-r2(:)))
```

```
error_h_0_005 = 0.2415
```

The total error of h=1/20 is 0.2415

```
function [y] = f(xi)
%Function of compute f(x) from du^2/d^2x=f(x)
    y = xi-2;
end

function [r]=realf(x)
%Function of compute the true value for comparison. from (a)
    r = 1/6*x.^3 - x.^2+11/2*x;
end

function [u] = finap(n, u0)
%Function of finite difference approximation
    h = 1/(n + 1); %compute h
    xi = linspace(0, 1, n + 2); %define xi
```

```
    xi = xi(2:n + 1); %only need x1 to xn
    b = f(xi)*h*h; %equation (3)
    b(end) = b(end) - 4*h; %equation (4)
    e = ones(n, 1);
    K = -spdiags([-e, 2*e, -e], -1:1, n, n);%form Kn matrix of n by n
    K(n, n) = -1;%modify Kn by changing the (n,n) element to 1
    u = K\b';%calculating

end
```