Homework 1 - MSSC 6030: Spring 2020

Directions. All work is to be done in *complete sentences*. Assignments must be stapled with a printout of the assignment serving as the first page. Your name is to be written on the *back* of the final page of the assignment. Each problem must be on a *separate* sheet of paper. You are welcome to recycle paper, where one side is crossed out to avoid wasting paper, but your work MUST have **no more than one problem per page**. Each problem write-up must begin with the **full statement of the problem**. While you are encouraged to work through confusion with your classmates, your work must be written in your own words. The assignment is due in class on **Wednesday, February 26, 2020**.

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1. §3.1 - Problem 10 of Strang:

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2. §3.1 - Problem 11 of Strang:

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3. Graph the frequency spectrum of the following periodic functions:

   (a) $f(x) = \sin(x)$

   (b) $f(x) = \sin(x) + \cos(2x)$

   (c) $f(x) = \sin(x) + \cos(x) + 0.5\sin(3x)$

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4. Let $f(x)$ be defined by

$$f(x) = \begin{cases} 0, & -3 < x \leq -1 \\ 1, & -1 < x \leq 1 \\ 0 & 1 < x \leq 3 \end{cases}$$

   and suppose that $f(x+6) = f(x)$, i.e. f has period 6. Find Fourier Series representation for $f(x)$. Plot the original function $f(x)$ as well as its 5, 10, and 20th partial sums

$$s_m(x) = \frac{a_0}{2} + \sum_{n=1}^{m} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right]$$

   of the Fourier Series for $f(x)$. Use different colors and markers as well as a legend in your plot. Additionally, plot the frequency spectrum of $f(x)$ for $n = 1, \ldots, 20$. Finally, plot the **absolute value** of the 'error' $e_m(x) = f(x) - s_m(x)$ (versus x) for the same values of $m = 5, 10$, and 20. Where does the largest error in the approximation come, i.e. for which values of x is the approximation poorest?

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5. What is Gibb's Phenomenon? Research this and come up with an example where you can both demonstrate Gibb's phenomenon as well as compute the error predicted by Gibb's phenomenon precisely for your example. (Show that your FS representation can never improve at points of discontinuity to be better than BLANK.)

6. Let $f(x)$ be defined by the sawtooth example $f(x) = x$ for $-L < x < L$ with $f(x + 2L) = f(x)$ as examined in class and notes online. Compute the term-by-term derivative of your FS representation for $f(x)$ and compare it to the actual derivative $f'(x) = 1$. Show all work and plot your results.

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7. Compute the Fourier transform of $f(x)$ (provided that $a > 0$) $f(x) = \begin{cases} e^{-ax} & x \geq 0 \\ 0 & x < 0 \end{cases}$

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8. Solve the problem

$$\begin{array}{rll} \text{PDE} & u_t = \alpha^2 u_{xx}, & -\infty < x < \infty \\ \text{IC} & u(x, 0) = e^{-x^2} & -\infty < x < \infty \end{array}$$

by using the Fourier transform.
