c)

(iii) Generalize your code to solve the problem with $h = \frac{1}{20}$

$n = \frac{1}{h} - 1 = 19$

For matrix K, it can be determine by generate (1/h)*Kn matrix

For vector F:

$F = 4v(1) - \int_0^1 \mathrm{fvdx}$ (1)

With $h = \frac{1}{20}$, there will be 19 hat function and 1 half hat function

So $4v(1)$ will be 0 except the last term.

For each hat function, suppose that setting the center of each hat function as nh, n=1,2,...19

Three point are needed:

$((n-1)h, 0)$
$(\mathrm{nh}, 1)$
$((n+1)h, 0)$

For function $y = \mathrm{mx} + b$ at the left side of hat function

$0 = m(n-1)h + b$

$1 = \mathrm{mnh} + b$

$\mathrm{mnh} - m(n-1)h = 1$
$\mathrm{mnh} - \mathrm{mhn} + \mathrm{mh} = 1$
$\mathrm{mh} = 1$
$m = \frac{1}{h}$

$1 = \frac{1}{h}\mathrm{nh} + b$
$1 = n + b$
$b = 1 - n$

$y_1 = \frac{1}{h}x + 1 - n$

For function $y = \mathrm{mx} + b$ at the right side of hat function

$1 = \mathrm{mnh} + b$

$0 = m(n+1)h + b$

$\mathrm{mhn} - m(n+1)h = 1$
$\mathrm{mhn} - \mathrm{mhn} - \mathrm{mh} = 1$
$\mathrm{mh} = -1$
$m = -\frac{1}{h}$

$1 = -\frac{1}{h}\mathrm{hh} + b$
$1 = -n + b$
$b = 1 + n$

$y_2 = -\frac{1}{h}x + 1 + n$

Therefore

$$\int_0^1 \mathrm{fvdx} = \int_{(n-1)h}^{\mathrm{nh}} (x-2)\left(\frac{1}{h}x + 1 - n\right)\mathrm{dx} + \int_{\mathrm{nh}}^{(n+1)h} (x-2)\left(-\frac{1}{h}x + 1 + n\right)\mathrm{dx}$$
$$= \frac{h(3\mathrm{hn} - h - 6)}{6} + \frac{h(3\mathrm{hn} + h - 6)}{6}$$
$$= \frac{h(3\mathrm{hn} - h - 6) + h(3\mathrm{hn} + h - 6)}{6} \quad (2)$$
$$= \frac{h(3\mathrm{hn} - h - 6 + 3\mathrm{hn} + h - 6)}{6}$$
$$= \frac{h(3\mathrm{hn} - 6 + 3\mathrm{hn} - 6)}{6}$$
$$= \frac{h(6\mathrm{hn} - 12)}{6}$$

for n=1,2,...,19

h=1/20

Also for the half hat:

$$b = \int_0^1 \mathrm{fvdx} = \int_{(n-1)h}^{\mathrm{nh}} (x-2)\left(\frac{1}{h}x + 1 - n\right)\mathrm{dx}$$
$$B_{20} = \frac{h(3\mathrm{hn} - h - 6)}{6} \quad (3)$$

So for the hat function

F vector is

$$4v(1) - \int_0^1 \mathrm{fvdx}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} - \begin{bmatrix} \frac{h(6h-12)}{6} \\ \frac{h(6\mathrm{h}2-12)}{6} \\ \frac{h(6\mathrm{h}3-12)}{6} \\ \vdots \\ \frac{h(6\mathrm{h}19-12)}{6} \\ \frac{h(3\mathrm{h}20-h-6)}{6} \end{bmatrix} \qquad (4)$$

```
clear

h=1/20;%define h
n=1/h-1;%compute n
e = ones(n+1, 1);

%generate Kn
K = spdiags([-e, 2*e, -e], -1:1, n+1, n+1);
K(n+1,n+1)=1;
K=K*(n+1);

%generate v
v=zeros(1,n+1);
v(end)=1;

%generate b vector 1 to 19 by equation (2)
b=zeros(1,n+1);
for i=1:n
    b(i)=h*(6*h*i-12)/6;
    %equation (2)
end
bn=n+1;
b(end)=h*(3*h*bn-h-6)/6;%equation (3)

F=4*v-b;%equation (4)
F=F';
coe = K\F;%solve u
coe = [0;coe]
```

```
coe = 21×1
         0
    0.2725
    0.5402
    0.8031
    1.0613
    1.3151
    1.5645
    1.8096
    2.0507
    2.2877
```

⋮

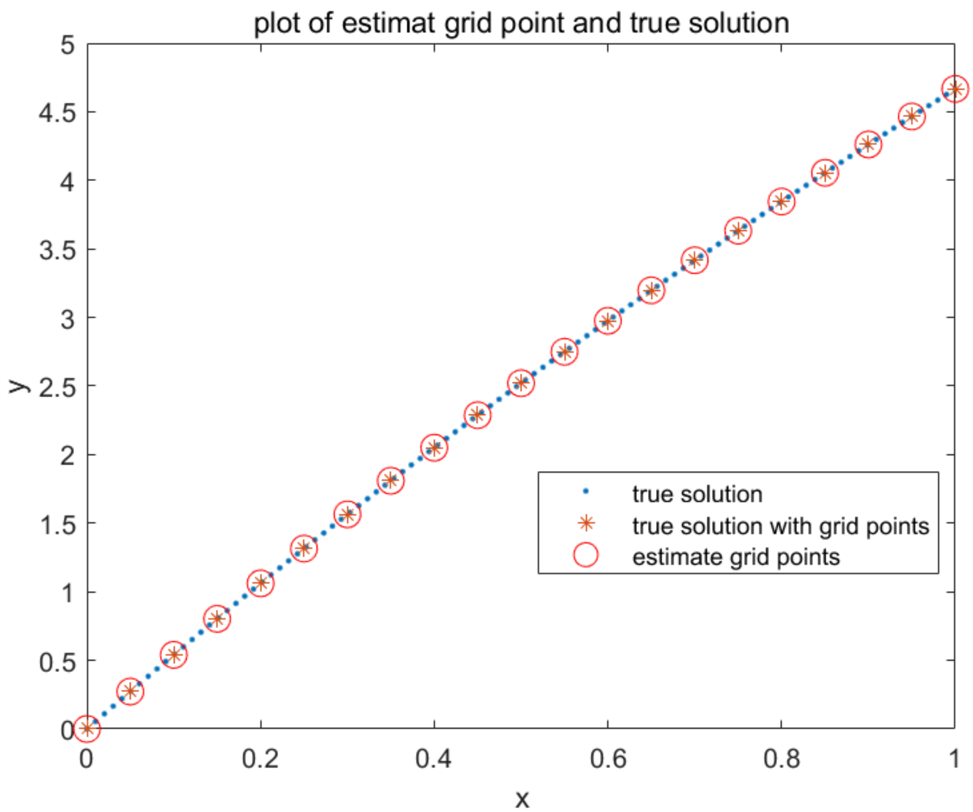
```
x = linspace(0, 1, 100);%define x
xg = linspace(0,1,n + 2);%true x with n+1 points
up = realf(xg);%true value of the grid
u = realf(x);%true solution
```

```
error=abs(coe(:) - up(:))%compute the error
```

```
error = 21×1
10^-14 ×
        0
   0.0278
   0.0555
   0.0666
   0.0888
   0.1110
   0.1332
   0.1554
   0.0888
   0.1332
     ⋮
```

error is still to the order of -14

```
%plot
plot(x, u, '.')
hold on
plot(xg, up, '*')
plot(xg, coe, 'o', 'Markersize', 10, 'Color','r')
legend('true solution', 'true solution with grid points', 'estimate grid points','Location','be
xlabel('x')
ylabel('y')
title('plot of estimat grid point and true solution')
hold off
```




```
function [r]=realf(x)
%Function of compute the true value for comparison. from (a)
    r = 1/6*x.^3 - x.^2+11/2*x;
end
```