6. Find the finite-difference solution of the heat-conduction problem

PDE $u_{tt} = u_{xx} \qquad 0 < x < 1, \quad t > 0$

BCs $\begin{cases} u(0,t) = 0 & t > 0 \\ u(1,t) = 0 & t > 0 \end{cases}$

IC $u(x,0) = \sin(\pi x) \quad 0 \le x \le 1$

for t=0.005, 0.010, 0.015 by the explicit method. Let $h = \delta x = 0.1$. Plot the solution at x=0, 0.1, 0.2, 0.3, ..., 0.9, 1 for t=0.015.

Solving the problem according to the following step

step 1: Input H=0.1,K=0.005 and end point of t,

step 2: calculate number of grid point on x-direction N and t-direction M.

N=1/H+1;

M=1/K+1;

step 3: compute the ratio $R = \frac{K}{H^2}$

step 4: for i=1:M-1

for j=2:N-1

compute u(i + 1, j) = u(i, j) + R*[u(i, j + 1) - 2*u(i, j) + u(1, j - 1)]

end

u(i + 1, N) = [u(i + 1, N - 1) + HG(i + 1)]/(H + 1)

since in this problem the boundary on the left and right side is 0

u(i + 1, N) = 0

end

```
clear
%step 1 input H, K and ending point p
H = 0.1;
K = 0.005;
p = 0.015;

%step 2, calculate N, M
N = 1/H + 1;
M = p/K + 1;

%step 3, calculate R
R = K/(H*H);

x = linspace(0, 1, N);%generate x = 0, 0.1, 0.2, ... ,0.9, 1
y = linspace(K, p, M);%generate t = 0.005, 0.010, 0.015
```

```
%initial condition
u = zeros(M, N);%initial the matrix
u(1, :) = sin(pi*x);%set initial condition u(x, 0) = sin(pi*x)

%step 4
for i = 1:M - 1
    %loop for i:M-1
    for j = 2:N - 1
        %loop for j=2:N-1
        u(i + 1, j) = u(i, j) + R*(u(i, j + 1) - 2*u(i, j) + u(i, j - 1));
    end
    u(i + 1, N) = 0;
end

u = flipud(u);%since the matrix is upside down with the problem we need to flip it up down
result_t_015 = u(1, :)%the result of PDE with t=0.015;
```

```
result_t_015 = 1×11
         0    0.2658    0.5056    0.6959    0.8181    0.8602    0.8181    0.6959 ···
```

```
plot(x, u(1, :))%only plot the row fot t = 0.015
title('solution of PDE with t=0.015')
xlabel('x')
ylabel('t')
```