Homework 5 - MSSC 6030: Spring 2020

Directions. All work is to be done in *complete sentences*. Assignments must be stapled with a printout of the assignment serving as the first page. Each problem must be on a *separate* sheet of paper. You are welcome to recycle paper, where one side is crossed out to avoid wasting paper, but your work MUST have **no more than one problem per page**. Each problem write-up must begin with the **full statement of the problem**. While you are encouraged to work through confusion with your classmates, your work must be written in your own words. **The assignment is due in dropbox on Wednesday, April 29, 2020 by 3:15pm.**

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1. Using $N = 4$, write out the matrices for the DFT F_4 and the iDFT $\frac{1}{4}\overline{F_4}$ as described in class. Compute each entry of the matrix.

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2. For your matrices above, compute each term to show that F_4 times $\frac{1}{4}\overline{F_4}$ gives you the identity matrix.

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3. Use the data file posted on D2L entitled `DFT_example_data.mat` to determine the original signal from the given frequency data. You are given the sampling points x_j as well as the Discrete Fourier transform F of the data points f. You are tasked with identifying the main frequencies in the signal as well as recovering the signal at the data points i.e. recovering f_j's.

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4. Write out the factorization of F_8 and multiply out all the matrices to get a hand on what is happening. (Note: Do not just use the matlab built in commands fft, actually go through the process. You can perform the matrix multiplication in matlab, but write out the matrices yourself.)

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5. Load in a picture of your choice. Perform FFT filtering as described in the lecture to determine an acceptable % of the Fourier basis that can be zeroed out and still retain image quality. What are you using as a measure of image quality? Why?

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6. Approximate functions using Haar wavelets. Approximate the function $f(t) = 4t^3 - 3t + sin(t)$, for t in $[0, 1]$ using the Haar wavelet basis. How many wavelets do you 'need' to capture the essential behavior? Try just using the first 8 we wrote down on the board together in class (orthonormal version or just amplitude 1 version, either is fine).

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7. Run the Cascade algorithm (posted on D2L) for the following filters h.
   (a) Daubechies 4, i.e. $h = \left(1 + \sqrt{3}, 3 + \sqrt{3}, 3 - \sqrt{3}, 1 - \sqrt{3}\right)/8$
   (b) The Cubic B-spline: $h = (1, 4, 6, 4, 1)/16$
   (c) #12 of Section 4.7: $h = (-1, 2, 6, 2, -1)/8$
   (d) Can you find another one that works?
