I’m going to talk about the runge kutta method today

First let’s review

For the normal ODE 

with the initial condition 

We can use varies methond to solve it

Like froward Euler method

backward Euler method

Trapezoid

Which the accuracy of froward and backward Euler is O(h) and trapezoido is O(h^3)

Since h is small, so higher order h has the more accurate the method will be.

So I will introduce a new method runge kutta which will has a higher accuracy.

From those method before we can see, all these find of method has a similar form, which is



h is the step size, is a function that contain x,y

In froward Euler method ,in backward Euler method

and in trapezoido

So we can define a equation about the accuracy and equation 1

Which is 

Which means the true value of x+h,is y(x+h) minus the approximate value we predicted, y(x)+hphi, the differentce between them is the O(hp+1)

Then we can take integral of the ODE from xn to xn+1 we will get:



from equation 2

Y(x+h)is y(xn+1) and y(x) is the y(xn),h is the width from xn+1 to xn

So the results of integration is so the area of hphi

So we can derive that



So therefore compute the integration according to hphi is main goal.

we need use Newton-Cotes formula which is



The integration will become

 (4)

Ci is a constant lamdai is small number

The integral are deveide into r pices and the sum of width and eahc parth times the function value

And as i incresing the lamda i h will also increasing which make the x increase a little bit more by the scale of r

The more r we take the more accuracy se we wil get.

For each y point yn need to be predict from all of point above.

So we can derive the whole ruuge kutta eqation like



where

 (5)

And this is only a noncomplete formula since all of the c i s lamda i s and mu ij s is unknown constans

Now I will use r=2 for example



where

 (6)

So in these equations c1 c2, lamda2 and mu21 are the unknowns

So we have 4 unknowns

To figure out these 4 unknowns we need taylor expansion of the 



We don;t want 

Therefore we can write

 (8)

As described in the ODE

And also for second order we will use the take diffriential by part so

 9

And same thing with the third order

 10

And so on

So we can take(8) (9) (10) in to the equation (7)

We will get

 11

I will show only two terms here

The difference of the approximate value and true value is

 12

Also

 13

So if combine equation (12) (13) and (11)

We will get





We want R too be smallest so we need the coefficients of fn and h to be 0

So we will get



Since we have 3 equaitons and 4 unknowns we will get infinite number of solution

So in order to make the compute simple

I will choose some simple value

Let ,

Therefore

and 

So the second order R-K method from equation() will become



where



Which is also called modified Euler mothod

So if let r=3 or r=4 and we do the same process again we will get the third order and fourth order RK method

Which is

RK3



And RK4



Since RK2 has accurate of O(h3)

RK3 will be O(h4)

And RK4 will be O(h5)

So example y’=2y

y(0)=1

The application of RK4 is the cacluation of some long term of ode like the astrounmy

Like orbit the planet and how the gravity effect the comet