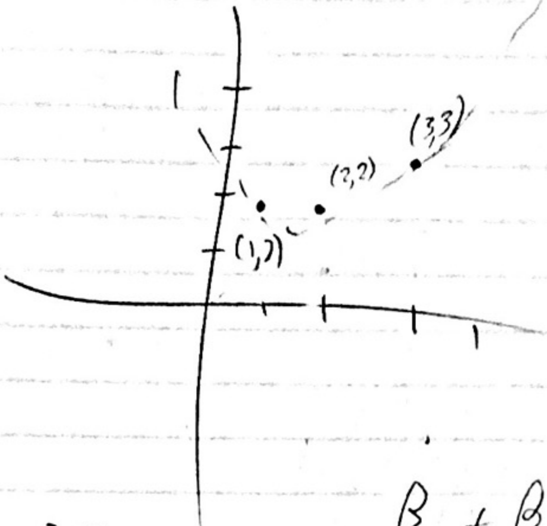
$$\begin{bmatrix}1 & t_0 & t_0^2\end{bmatrix}\beta = \begin{bmatrix}p(t_0)\end{bmatrix} \quad p(t) = \beta_0 + \beta_1 \cdot t + \beta_2 \cdot t^2$$

$$\begin{bmatrix}1 & t_0 & t_0^2\end{bmatrix}\begin{bmatrix}\beta_0 \\ \beta_1 \\ \beta_2\end{bmatrix}$$

$(1,2) \quad (2,2) \quad (3,3)$



$a_0 + a_1 t + a_2 \cdot t^2$

$a_0 + a_1 \cdot t$

$$\begin{bmatrix}1 & t_0 & t_0^2 \\ 1 & t_1 & t_1^2\end{bmatrix}\beta = \begin{bmatrix}p(t_0) \\ p(t_1)\end{bmatrix}$$

$\beta_0 + \beta_1(1) + \beta_2(1) = 2$

$\beta_0 + \beta_1(2) + \beta_2(4) = 2$

$\beta_0 + \beta_1(3) + \beta_2(9) = 3$

$$\begin{bmatrix}1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9\end{bmatrix}\beta = \begin{bmatrix}2 \\ 2 \\ 3\end{bmatrix}$$

$$\begin{bmatrix}1 & t_0 & t_0^2 \\ 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2\end{bmatrix}\beta = \begin{bmatrix}p(t_0) \\ p(t_1) \\ p(t_2)\end{bmatrix}$$

$$\begin{bmatrix}1 & 1 & 1 & 2 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 9 & 3\end{bmatrix} \sim \begin{bmatrix}1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 0 \\ 0 & 2 & 8 & 1\end{bmatrix} \sim \begin{bmatrix}1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 2 & 1\end{bmatrix}$$

$$\sim \begin{bmatrix}1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 1/2\end{bmatrix} \sim \begin{bmatrix}1 & 1 & 0 & 3/2 \\ 0 & 1 & 0 & -3/2 \\ 0 & 0 & 1 & 1/2\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -3/2 \\ 0 & 0 & 1 & 1/2\end{bmatrix}$$

$\beta_0 = 3 \quad \beta_1 = -3/2, \quad \beta_2 = 1/2$

$p(t) = 3 - 3/2\, t + 1/2\, t^2$

$\beta\ \beta\ \beta\ \beta\ \beta\ \beta\ \beta\ \beta\ \beta\ \beta\ \beta\ \beta\ \beta$

$$\begin{bmatrix} 1 & t_0 & t_0^2 \\ 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \end{bmatrix} \beta = \begin{bmatrix} p(t_0) \\ p(t_1) \\ p(t_2) \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & t_0 & t_0^2 \\ 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \end{bmatrix}$ and $x = \beta$ and $b = \begin{bmatrix} p(t_0) \\ p(t_1) \\ p(t_2) \end{bmatrix}$

Is $Ax = b$ true for any $b \in \mathbb{R}^3$

$$\begin{bmatrix} 1 & t_0 & t_0^2 & p(t_0) \\ 1 & t_1 & t_1^2 & p(t_1) \\ 1 & t_2 & t_2^2 & p(t_2) \end{bmatrix} \sim \begin{bmatrix} 1 & t_0 & t_0^2 & p(t_0) \\ 0 & t_1 - t_0 & t_1^2 - t_0^2 & p(t_1) - p(t_0) \\ 0 & t_2 - t_0 & t_2^2 - t_0^2 & p(t_2) - p(t_0) \end{bmatrix} - \frac{t_2 - t_0}{t_1 - t_0} (\text{elem})$$

$$\sim \begin{bmatrix} 1 & t_0 & t_0^2 & p(t_0) \\ 0 & t_1 - t_0 & t_1^2 - t_0^2 & p(t_1) - p(t_0) \\ 0 & 0 & t_2^2 - t_0^2 - \frac{t_2 - t_0}{t_1 - t_0}(t_1^2 - t_0^2) & p(t_2) - p(t_0) - \frac{t_2 - t_0}{t_1 - t_0}(p(t_1) - p(t_0)) \end{bmatrix} \quad \begin{matrix} 5 \\ 4 - \frac{4}{5}(5) \end{matrix}$$

$$\sim \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ 0 & \blacksquare & \cdot & \cdot \\ 0 & 0 & \blacksquare & \cdot \end{bmatrix}$$

Inconsistent when

$$t_2^2 - t_0^2 - \left(\frac{t_2 - t_0}{t_1 - t_0}\right)(t_1^2 - t_0^2) = 0$$

$$z^2 - x^2 - \left(\frac{z - x}{y - x}\right)(y^2 - x^2) = 0$$

$$p(t_2) - p(t_0) - \left(\frac{t_2 - t_0}{t_1 - t_0}\right)(p(t_1) - p(t_0)) \neq 0 \text{ and}$$

$$z^2 - x^2 = \left(\frac{z - x}{y - x}\right)(y^2 - x^2)$$

$A = \begin{bmatrix} 1 & t_0 \\ 1 & t_1 \end{bmatrix} \quad x = \beta \quad b = \begin{bmatrix} p(t_0) \\ p(t_1) \end{bmatrix}$

Does $Ax = b$ have a solution for every $b \in \mathbb{R}^2$

$\begin{bmatrix} 1 & t_0 & p(t_0) \\ 1 & t_1 & p(t_1) \end{bmatrix}$

$\sim \begin{bmatrix} 1 & t_0 & p(t_0) \\ 0 & t_1 - t_0 & p(t_1) - p(t_0) \end{bmatrix}$

Inconsistent when

$t_1 - t_0 = 0$ and $p(t_1) - p(t_0) \neq 0$

$t_1 = t_0$ and $p(t_1) \neq p(t_0)$

$(t_0, p(t_0)) \quad (t_1, p(t_1))$

$$p(t_2)-p(t_0) = \left(\frac{t_2-t_0}{t_1-t_0}\right)(p(t_1)-p(t_0))$$

and $$t_2^2-t_0^2 = \left(\frac{t_2-t_0}{t_1-t_0}\right)(t_1^2-t_0^2)$$

$$\Leftrightarrow$$

$$\left(\frac{t_2-t_0}{t_1-t_0}\right) = \frac{p(t_2)-p(t_0)}{p(t_1)-p(t_0)} \text{ and } \left(\frac{t_2-t_0}{t_1-t_0}\right) = \frac{t_2^2-t_0^2}{t_1^2-t_0^2}$$

$$\Leftrightarrow$$

$$\frac{p(t_2)-p(t_0)}{p(t_1)-p(t_0)} = \frac{t_2^2-t_0^2}{t_1^2-t_0^2}$$

$(t_0, p(t_0))$
$(t_1, p(t_1))$
$(t_2, p(t_2))$

$$\Leftrightarrow$$

$$\frac{p(t_2)-p(t_0)}{t_2^2-t_0^2} = \frac{p(t_1)-p(t_0)}{t_1^2-t_0^2}$$

$(1, 1)$
$(2, 2)$
$(3, 2)$

$$(p(t_2)-p(t_0))(t_1^2-t_0^2) = (p(t_1)-p(t_0))(t_2^2-t_0^2)$$

$$p(t_2)\cdot(t_1^2-t_0^2) - p(t_0)t_1^2 = p(t_1)(t_2^2-t_0^2) - p(t_0)t_2^2$$

Overdetermined:

$$\begin{bmatrix} \boxed{} & * & * \\ 0 & \boxed{} & * \end{bmatrix}$$

$$\begin{bmatrix} 1 & t_0 \\ 1 & t_1 \\ 1 & t_2 \end{bmatrix} \beta = \begin{bmatrix} p(t_0) \\ p(t_1) \\ p(t_2) \end{bmatrix}$$

$$\begin{bmatrix} \boxed{} & * & * \\ 0 & \boxed{} & * \\ 0 & 0 & \boxed{} \end{bmatrix}$$

$$\begin{bmatrix} 1 & t_0 & p(t_0) \\ 1 & t_1 & p(t_1) \\ 1 & t_2 & p(t_2) \end{bmatrix} \sim \begin{bmatrix} 1 & t_0 & p(t_0) \\ 0 & t_1 - t_0 & p(t_1) - p(t_0) \\ 0 & t_2 - t_0 & p(t_2) - p(t_0) \end{bmatrix}$$

$(1, 2)$

$(3, 3)$

$(5, 2)$

$$\sim \begin{bmatrix} 1 & t_0 & p(t_0) \\ 0 & t_1 - t_0 & p(t_1) - p(t_0) \\ 0 & 0 & \left(\frac{t_2 - t_0}{t_1 - t_0}\right)(p(t_1) - p(t_0)) \end{bmatrix}$$

Consistent when $p(t_2) = p(t_0)$

or when $t_2 = t_0$

Under:

$$\begin{bmatrix} \boxed{} & * & * & * \\ 0 & \boxed{} & * & * \end{bmatrix}$$