Research Assistant Crypto Response

Haile Lagi

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1 Math

Over all real numbers, find the minimum value of a positive real number, y such that

$$y = \sqrt{(x+6)^2 + 25)} + \sqrt{(x-6)^2 + 121}\,. \tag{1}$$

To find a stationary point(in this case mininum) for 1 the procedure is as follows:

1. Given y = f(x) find f'(x)
2. Let $\frac{dy}{dx} = 0$ and solve for the x value(s).
3. find the corresponding y value(s).
4. Determine the nature of the equation using:
   - Second derivative and substitute x if x is positive, negative or zero
   - Alternatively, find gradient before and after the stationary + to -, - to - or + to +

First we simplify 1 by expanding the polynomials of the form $(x + c)^n$ and we have

$$y = \sqrt{(x+6)(x+6) + 25)} + \sqrt{(x-6)(x-6) + 121} = \sqrt{x^2 + 12x + 61} + \sqrt{x^2 - 12x + 157}\,. \tag{2}$$

To find f'(x) using chain rule, in an equation of the form $\frac{dy}{dx} = u^n$ where u is function of x and n is some exponent this holds $\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$. Divide equation 2 into two parts $y = y_1 + y_2$ where

$$y_1 = \sqrt{x^2 + 12x + 61}\,. \tag{3}$$

$$y_2 = \sqrt{x^2 - 12x + 157}\,. \tag{4}$$

Applying the chain rule in 3 and 4

$$\frac{dy_1}{dx} = u^{\frac{1}{2}}\,. \tag{5}$$

where $u_1 = x^2 + 12x + 61$ and $\frac{du_1}{dx} = 2x + 12$

$$\frac{dy_2}{dx} = u^{\frac{1}{2}}\,. \tag{6}$$

where $u_2 = x^2 - 12x - 12x + 157$ and $\frac{du_2}{dx} = 2x - 12$

Combining 5 and 6 we have 7

$$\frac{dy}{dx} = \frac{2x+12}{2 * \sqrt{(}x^2+12x+61)} + \frac{2x-12}{2 * \sqrt{(}x^2-12x+157)}\,. \tag{7}$$

By simple factorization of 2 and Let $\frac{dy}{dx} = 0$

$$0 = \frac{x+6}{(x^2+12x+61)^{\frac{1}{2}}} + \frac{x-6}{(x^2-12x+157)^{\frac{1}{2}}}\,. \tag{8}$$

At this point it becomes impossible to solve for x because the values of x are complex(or imaginary) so, the root(s) of x are **unknown**.

$$y = (x^2+12x+61)^{\frac{1}{2}} + (x^2-12x+157)^{\frac{1}{2}}\,. \tag{9}$$

The minimum value of a real x is assumed to be 0

Substituting $x = 0$ in 9 we have the minimum value of y as:

$$y = \sqrt{61} + \sqrt{157}\,. \tag{10}$$

Finally we have $y_{min} = (0, \sqrt{61} + \sqrt{157})$