Research Assistant Crypto Response

Haile Lagi

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1 Finance

Yara Inc is listed on the NYSE with a stock price of 40 - the company is not known to pay dividends. We need to price a call option with a strike of $45 maturing in 4 months. The continuously-compounded risk-free rate is 3%/year, the mean return on the stock is 7%/year, and the standard deviation of the stock return is 40%/year. What is the Black-Scholes call price?

Parameters given are (assuming the stock doesn't pay dividend it is assumed an American call option is equivalent to an European call option):

Converting all parameters w.r.t. to 1 year:

Given that:

- Black-Scholes call price = (C)
- stock price(S) = \$40
- call option strike(X) = \$45
- maturity(T) = 4 months / 12 = 1/3
- continuously-compounded risk-free rate(r_f) = 3/100 = 0.03
- stock mean return(μ) = 7/100 = 0.07
- standard deviation(σ) = 40/100 = 0.4

the linear black-scholes general form for an European style call option is:

$$C(S,T) = SN(x_1) - BN(x_2) \tag{1}$$

Where:

$$Bondprice(B) = Xe^{-}r_f * T \tag{2}$$

and x_1 is given by:

$$x_1 = \frac{log(S/B)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \tag{3}$$

and to find x_2 from the value of x_1:

$$x_2 = x_1 - \sigma * \sqrt{T} \tag{4}$$

To find the value of $N(x)$ which is the cumulative normal distribution we use the integral form:

$$N(x) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{x} e^{\frac{-u^2}{2}} du \tag{5}$$

First we find $B = Xe^{-}r_f * T = 40e^{-}0.03 * \frac{1}{3}$ $B = 44.552243$

Using this we find x_1

$$x_1 = \frac{log(40/44.552243)}{0.4\sqrt{1/3}} + \frac{1}{2} * 0.4 * \sqrt{1/3} \tag{6}$$

$$= \frac{-0.10778304645914898}{0.2309401076758503} + (0.5 * 0.4 * 0.2309401076758503) \tag{7}$$

$$x_1 \approx -0.35124393624020780465722\!42 \approx -0.3512439 \tag{8}$$

$$x_2 = -0.3512439362402078046572242 - 0.4 * \sqrt{1/3} \approx -0.5821840439160581104608837 \approx -0.5821840 \tag{9}$$

Now we must find the Gaussian probability distribution functions of $N(x_1)$ and $N(x_2)$

$$N(x_1) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{-0.3512439} e^{\frac{-u^2}{2}} du \tag{10}$$

$$N(x_1) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{-0.5821840} e^{\frac{-u^2}{2}} du \tag{11}$$

Sadly the cumulative difference function of a normal distribution does not have a closed form and CANNOT BE SOLVED analytically. The accuracy of the values produced by each estimation methods varies in accuracy. I'll be using a $Z - table$ or $StandardNormalTalbe$

$$N(x_1) = 0.3527026N(x_2) = 0.2802213 \tag{12}$$

Then finally substitution into the general form:

$$C(S, T) = 40 * 0.3627026 - 44.55224 * 0.2802213 \tag{13}$$

The Black-Scholes call price is $\approx$ \$2.02361738929