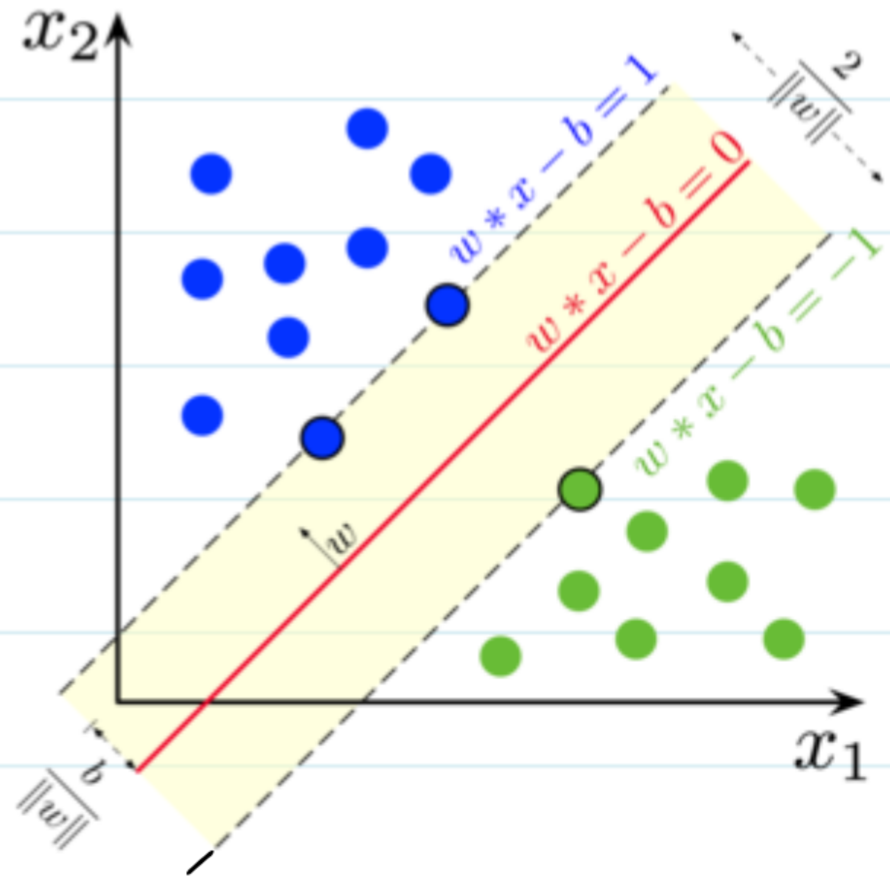


We assumed the equation of hyperplane;

$$\omega . x + b = 0$$

by taking any 2 points at the hyperplane

$$\omega . x_1 + b = 0$$
$$\omega . x_2 + b = 0$$

if we subtract them

$$\omega . (x_1 - x_2) = 0$$

the dot product of a vector on the hyperplane and ω is $= 0$

- this means ω is perpendicular to the hyperplane

$$\bar{\omega} . \bar{x}_i + b \geq 1 \quad \text{for } x^+$$
$$\bar{\omega} . \bar{x}_i + b \leq -1 \quad \text{for } x^-$$

we assume y_i such that $y_i = 1$ for + samples

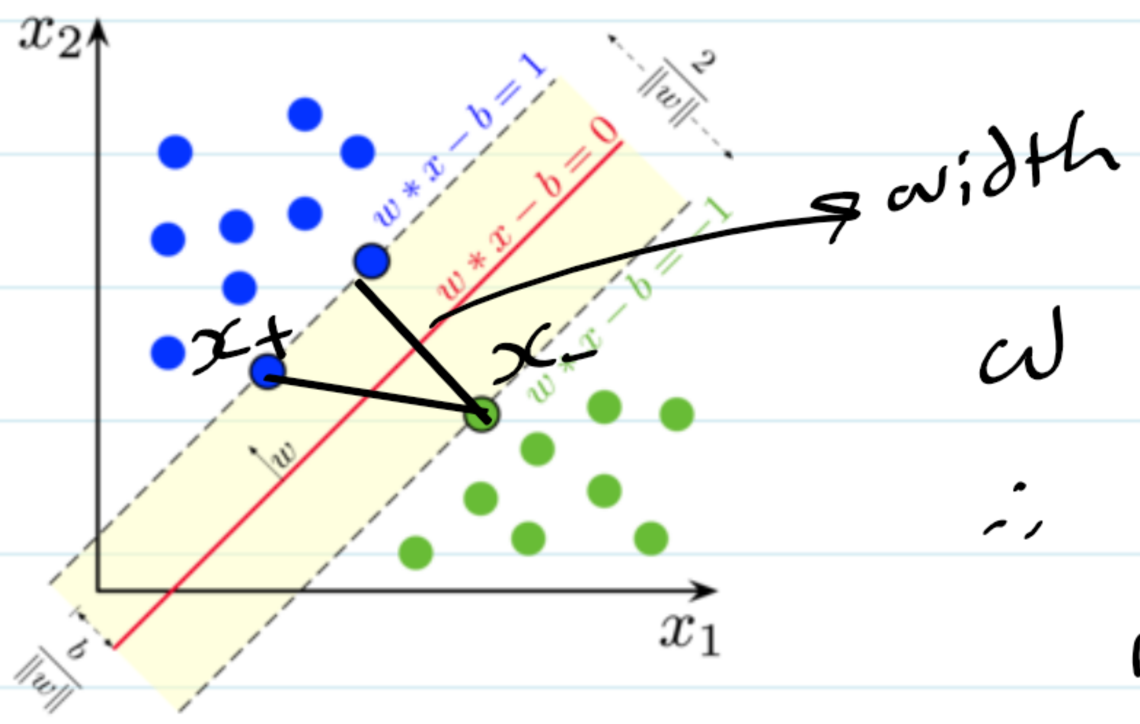
, $y_i = -1$ for - samples

$$y_i (\bar{\omega} . \bar{x}_i + b) \geq 1$$
$$y_i (\bar{\omega} . \bar{x}_i + b) \geq 1$$

same

$$\therefore \quad y_i (\bar{\omega} . \bar{x}_i + b) - 1 = 0 \quad \boxed{1}$$

$$-\bar{\omega} . \bar{x}_i - b - 1 = 0$$
$$-\bar{\omega} . \bar{x}_i = 1 + b$$



ω is perpendicular to the hyperplane

$\therefore$

$$\text{width} = (x_+ - x_-) . \text{unit vector of } \omega$$

$$= (\bar{x}_+ - \bar{x}_-) . \frac{\bar{\omega}}{\|\omega\|}$$

$$= \frac{(\bar{\omega} . \bar{x}_+ - \bar{\omega} \bar{x}_-)}{\|\omega\|}$$

from $\boxed{1}$; $\bar{\omega} . \bar{x}_+ = 1 - b$

$\bar{\omega} . \bar{x}_- = -(1 + b)$

$$\text{width} = \frac{1 - b + 1 + b}{\|\omega\|} = \frac{2}{\|\omega\|} \quad \boxed{2}$$

the goal is to maximize the width

$$\max \frac{2}{\|\omega\|}$$
$$\min \|\omega\|$$
$$\min \frac{1}{2}\|\omega\|^2 \rightarrow \text{تسهيل للحسابات (هي)}$$

using lagrange multipliers to form equation that takes in counter the constraints and then try to minimize it

$$L = \frac{1}{2}\|w\|^2 - \alpha\,[\text{constraint}]$$

↳ equation I is our constraint

$$L = \frac{1}{2}\|w\|^2 - \alpha\,[y_i(\bar{x}_i \cdot \bar{w} + b) - 1 \quad \text{for all } i]$$

$$= \frac{1}{2}\|w\|^2 - \sum_{i=1}^{L} \alpha_i\,[y_i(\bar{x}_i \cdot \bar{w} + b) - 1]$$

$$= \frac{1}{2}\|w\|^2 - \sum_{i=1}^{L} \alpha_i y_i (\bar{x}_i \cdot \bar{w} + b) + \sum_{i=1}^{L} \alpha_i$$

now minimize the equation to get w and b values

$$\frac{\partial L}{\partial w}\left(\frac{1}{2}\|w\|^2 - \sum_{i=1}^{L} \alpha_i y_i (\bar{x}_i \cdot \bar{w} + \underbrace{\cancel{b}}_{0}) + \underbrace{\cancel{\sum_{i=1}^{L} \alpha_i}}_{0}\right) = 0$$

$$\frac{\partial L}{\partial w} = w - \sum_{i=1}^{L} \alpha_i y_i x_i = 0$$

$$\therefore \quad w = \sum_{i=1}^{L} \alpha_i y_i x_i \qquad (3)$$

$$\frac{\partial L}{\partial b}\left(\underbrace{\cancel{\frac{1}{2}\|w\|^2}}_{0} - \sum_{i=1}^{L} \alpha_i y_i (\underbrace{\cancel{\bar{x}_i \cdot \bar{w}}}_{0} + b) + \underbrace{\cancel{\sum_{i=1}^{L} \alpha_i}}_{0}\right) = 0$$

$$\frac{\partial L}{\partial b} = \sum_{i=1}^{L} \alpha_i y_i = 0$$

$$\therefore \quad \sum_{i=1}^{L} \alpha_i y_i = 0 \qquad (4)$$