Observational Techniques

2: Signal to Noise

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February 21, 2017

Notes

Recap

- In order to talk about astronomical observations and instrumentation, you need to know the meaning of *magnitude (Vega)*, f_λ, f_ν and be able to convert between units. You also need to understand *nominal* solar units and conversions.
- EM Radiation can be detected by counting photons. These events are described by *Poisson statistics*, where variance is proportional to N and standard deviation proportional to $\sqrt{N}$.
- Detecting faint objects is made difficult by astrophysical backgrounds. Key backgrounds are Galactic synchrotron, the Cosmic Microwave Background, and thermal/scattered Zodiacal light.

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Outline

Notes

The Effect of Backgrounds

- When only photon statistics of the target are important, the signal-to-noise is (yesterday):

$$S/N = \frac{N_t}{\sqrt{N_t}} = \sqrt{N_t} \quad (1)$$

- Where the telescope *primary beam* or *point spread function* contains both the target of interest and background (or foreground) emission, signal-to-noise is limited by the background.
- For photon-counting applications, the signal-to-noise becomes:

$$S/N = \frac{N_t}{\sqrt{N_t + N_b}} \quad (2)$$

- We say observations are *background-limited* if we can ignore N_t in the denominator.

Notes

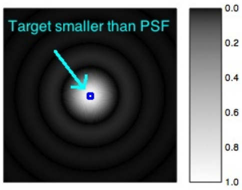
Detecting Resolved Objects

- The key metric for detecting resolved objects is the (surface) *brightness*/specific intensity of a source, particularly in relation to the background. The surface brightness of a source is typically measured in *magnitudes per square arcsec* (visible) or *Jy per square arcsec* (radio)... or even *MJy per sr*. What is a *sr*?.
- For *background-limited* observations, the signal-to-noise in each 1 square arcsec patch is just the number of target object photons collected per square arcsec, divided by the square root of the total number of photons collected per square arcsec (Eqn. 2)
- What about for *n* square arcsec?

Notes

Detecting Point Sources

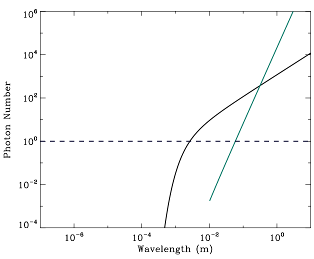
- We can make astrophysical backgrounds smaller by making our *beam* or *point-spread function* (PSF) smaller.
- This only applies to sources that are *unresolved* by our instruments.
- Diffraction-limited telescopes have a PSF of angular extent $\sim \lambda/D$ radians, for a telescope diameter D. Signal-to-noise is then proportional to D^2. Derive This.



Notes

Photon Bunching

- The brightness (i.e. specific intensity) in number of photons per unit time (s) per unit bandwidth (Hz) per *beam* is dimensionless. This is the *average photon number* in quantum mechanics.
- When there are many photons per quantum state we use generally us classical electromagnetism to describe observations.



CMB (black) and Synchrotron (green) backgrounds in terms of photon number.
What is the photon-number of a blackbody radiation field near its peak?

Notes

Photon Bunching (cont)

- We can also use Bose-Einstein statistics to describe the statistical properties of light.
- A perfect laser follows a Poisson distribution of photon number (per s per Hz per beam), but both background and astronomical sources in general follow a thermal, or Bose-Einstein distribution:

$$P(n) = \frac{1}{\langle n \rangle + 1}\left(\frac{\langle n \rangle}{\langle n \rangle + 1}\right)^2 \quad (3)$$

- For large photon number, this approximates an exponential distribution, just like the square of a voltage measured in 2 phases (sine and cosine):

$$\mathrm{Var}(n) = \langle n \rangle + \langle n \rangle^2 \quad (4)$$

- Go over python poisson statistics example again, with revised statistics.

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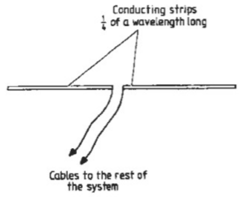
Radio Dishes and Antennae

- A Radio antenna does not detect photons directly... instead, it measures the electric field as an instantaneous voltage across the antenna feed.

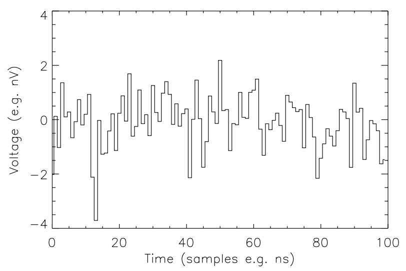


A half-wave dipole.

- This is fundamentally different to detecting photons, because the quantity being measured has a mean of zero (and is Gaussian). The mean-squared voltage is proportional to the power.

Notes

Self-Noise in Radio Observations



- There is a detail as to whether the plot above is voltage, or demodulated voltage (like the voltage going to your car speaker after tuning in to a radio station), but the principle is the same.

Notes

Self-Noise in Radio Observations

- The signal-to-noise in detecting the mean-squared voltage is $1/\sqrt{2}$. See a python example
- So, the signal-to-noise of M samples is $\sqrt{M/2}$
- The number of samples is equal to the bandwidth $\Delta\nu$ multiplied by the integration time Δt, so the (maximum) signal-to-noise of any radio observation is $\sqrt{\Delta t \Delta\nu}$ - this is *independent of source brightness*.
- In practice, radio observations are almost always background limited, so signal-to-noise is still proportional to source flux density, but with a different constant. Naomi will cover this.

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Other Noise Sources

Later, you will learn a little about the physics of detector-specific noise sources. These fall in 2 categories:

1. *Dark Current*: This is the most common detector noise source, which is just like adding another background due to the detector. The term comes from detectors where you get 1 electron per detected photon.
2. *Readout Noise*: This is the kind of noise you'll learn about for CCD detectors. Irrespective of how long you integrate for, you get noise equivalent to $\sigma_{\rm ro}^2$ additional background photons detected.

Many other noise sources (e.g. noise in an intensifier) can be written in terms of the concepts in this lecture.

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Overall Signal-to-Noise

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All that we're now missing do describe signal-to-noise for photon-counting applications is the instrumental efficiency, often called η. This is the probability that a photon the gets to the telescope aperture (or sometimes the top of the atmosphere) is actually detected. Putting all this together, we arrive at the signal-to-noise:

$$S/N = \frac{\eta N_t}{\sqrt{\eta N_t + \eta N_b + N_d + \sigma_{\rm ro}^2}} \quad (5)$$

Summary

Notes

- Backgrounds add to the variance of the noise but not to the signal.
- Dark current and readout noise also add to the variance of the noise.
- The background in terms of photons per quantum state means:
  - Radio observations are treated clasically.
  - Optical observations are treated as if each photon (of a particular wavelength) is an indistinguishable particle.
- EM Radiation can also be detected by measuring an electric field. In the absence of background noise, this has a signal to noise of $\sqrt{\Delta t \Delta \nu}$

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