Solved example of PCA:-

S.No.	X	Y
1	2.6	2.4
2	0.5	0.7
3	2.2	2.9
4	1.9	2.2
5	3.1	3.0
6	2.3	2.7
7	2	1.6
8	1	1.1
9	1.5	1.6
10	1.1	0.9

- **Covariance Matrix:**

$$C = \begin{bmatrix} cov(x, x) & cov(x, y) \\ cov(y, x) & cov(y, y) \end{bmatrix}$$

where, $\boxed{cov(x, y) == cov(y, x)}$

- **Covariance Formula:**

$$\boxed{cov(x, y) = \sum_{i=1}^{N} \frac{(x_i - \bar{x})(y_i - \bar{y})}{N-1}}$$

$\boxed{\bar{x} = 1.81}$ $\boxed{\bar{y} = 1.91}$

$$\therefore \; cov(x,x) = \sum_{i=1}^{N} \frac{(x_i-\bar{x})(x_i-\bar{x})}{N-1}$$

$$cov(y,y) = \sum_{i=1}^{N} \frac{(y_i-\bar{y})(y_i-\bar{y})}{N-1}$$

$$cov(x,y) = \sum_{i=1}^{N} \frac{(x_i-\bar{x})(y_i-\bar{y})}{N-1}$$

x	$x-\bar{x}$	$(x-\bar{x})(x-\bar{x})$
2.5	0.69	0.476
0.5	-1.31	1.7161
2.2	0.39	0.1521
1.9	0.09	0.0081
3.1	1.29	1.6641
2.3	0.49	0.2401
2	0.19	0.0361
1	-0.81	0.6561
1.5	-0.31	0.0961
1.1	-0.71	0.5041

y	$y-\bar{y}$	$(y-\bar{y})(y-\bar{y})$
2.4	0.49	0.2401
0.7	-1.31	1.4641
2.9	0.99	0.9801
2.2	0.29	0.0841
3.0	1.09	1.1881
2.7	0.79	0.6241
1.6	-0.31	0.0961
1.1	-0.81	0.6561
1.6	-0.31	0.0961
0.9	-1.01	1.0201

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})(y-\bar{y})$
2.5	2.4	0.69	0.49	0.3381
0.5	0.7	-1.31	-1.31	1.5851
2.2	2.9	0.39	0.99	0.3861
1.9	2.2	0.09	0.29	0.0261
3.1	3.0	1.29	1.09	1.4061
2.3	2.7	0.49	0.79	0.3871
2	1.6	0.19	-0.31	-0.0589
1	1.1	-0.81	-0.81	0.6561
1.5	1.6	-0.31	-0.31	0.0961
1.1	0.9	-0.71	-1.01	0.7171

$$cov(x, x) = \frac{5.5490}{9} = \underline{\underline{0.6165}}$$

$$cov(y, y) = \frac{6.449}{9} = \underline{\underline{0.7165}}$$

$$cov(x, y) = \frac{5.5390}{9} = \underline{\underline{0.6154}}$$

- **Covariance Matrix:**

$$C = \begin{bmatrix} cov(x, x) & cov(x, y) \\ cov(y, x) & cov(y, y) \end{bmatrix}$$

$$\boxed{C = \begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix}}$$

- **Find Eigen Values:**

$$\boxed{C - \lambda I = 0}$$

$\underline{\underline{C}}$ = covariance Matrix

$\underline{\underline{I}}$ = Identity Matrix

$\underline{\underline{\lambda}}$ = Eigen values Maxix.

$$\begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0.6165 - \lambda & 0.6154 \\ 0.6154 & 0.7165 - \lambda \end{bmatrix} = 0$$

$$\lambda^2 - 1333\lambda + 0.0630 = 0$$

$$\therefore \boxed{\begin{aligned} \lambda_1 &= 0.0490 \\ \lambda_2 &= 1.2840 \end{aligned}}$$

↳ Eigen values = num. of features.

- **Find eigen vectors** : For each Eigen values.

$$\boxed{C\,v = \lambda\,v}$$

Eigen values

Eigen vector (Going to find).

① For $\lambda_1 = 0.0490$ –

$$\begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = 0.0490 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$0.6165x_1 + 0.6154y_1 = 0.0490x_1$$

$$0.6154x_1 + 0.7165y_1 = 0.0490y_1$$

$\rightarrow 0.5674\,x_1 = -0.6154\,y_1$

$\rightarrow 0.6154\,x_1 = -0.6674\,y_1$

used to find some relationship betⁿ x & y.

$$x_1 = \frac{-0.6154}{0.5674}\,y_1$$

$$\therefore x_1 = -1.0854\,y_1$$

$$\boxed{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} -1.0854 \\ 1 \end{bmatrix}}$$

$$x_1 = \frac{-0.6674}{0.6154}\,y_1$$

$$\therefore x_1 = -1.0812\,y_1$$

$$\boxed{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1.0812 \\ 1 \end{bmatrix}}$$

↳ These are first 2 eigenvectors, we have found till now.

(2) For $\lambda_2 = 1.2840$ –

$$\begin{bmatrix} 0.6165 & 0.6154 \\ 0.6154 & 0.7165 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = 1.2840 \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$\rightarrow 0.6165\, x_2 + 0.6154\, y_2 = 1.2840\, x_2$

$\rightarrow 0.6154\, x_2 + 0.7165\, y_2 = 1.2840\, y_2$

$\rightarrow -0.6675 x_2 = -0.6154\, y_2$

$\rightarrow 0.6154\, x_2 = 0.5675\, y_2$

$$x_2 = \frac{-0.6154}{-0.6675} y_2 \qquad\qquad x_2 = \frac{0.5675}{0.6154} y_2$$

$\therefore x_2 = 0.9219\, y_2 \qquad\qquad \therefore x_2 = 0.9221\, y_2$

$$\boxed{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0.9219 \\ 1 \end{bmatrix}} \qquad \boxed{\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0.9221 \\ 1 \end{bmatrix}}$$

↳ These are 2 eigenvector for λ_2 eigenvalue.