Book-Value Wealth Taxation, Capital Income Taxation, and Innovation*

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Abstract

Can capital taxes be designed to incentivize innovation, improve productivity, and ultimately increase welfare? We study this question theoretically in a model of firm dynamics with entrepreneurs that face collateral constraints. The key feature of the model is that the distribution of entrepreneurial productivity is endogenously determined as the outcome of costly innovation and managerial effort. We show that there is a unique equilibrium with positive innovation, heterogeneous returns, and misallocation of capital. Furthermore, because a (book-value) wealth tax is independent of the outcome of innovation, entrepreneurs keep the upside from their investments; therefore, a wealth tax incentivizes innovation, resulting in higher aggregate productivity. The wealth tax also works through a second mechanism, which reallocates capital from low-productivity to high-productivity entrepreneurs, reducing misallocation and further increasing productivity. In contrast, we show that the capital income tax is neutral for innovation, misallocation, and aggregate productivity. Further, shifting from a capital income tax to a wealth tax (keeping the government's budget balanced) increases aggregate capital, output, and wages. As for optimal taxation, we show that the optimal mix depends on the capital intensity of production and shifts towards a higher wealth tax and a lower capital income tax, as the intensity increases. For a range of plausible parameter values, the optimal wealth tax is positive, whereas the capital income tax can be positive (but small) or negative (a subsidy).

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1 Introduction

How should capital taxation be structured when innovation is endogenous? In particular, can a wealth tax or a capital income tax (or a combination of the two) be designed to incentivize innovation, improve productivity, and ultimately increase welfare? While these and related questions underlie many of the current policy debates, these questions have not been studied in the academic literature. However, it is clear that these different forms of taxation shape the distribution of after-tax returns of entrepreneurs differently, thereby affecting their incentives to innovate differently. This question becomes especially prominent in light of the growing body of empirical evidence that documents large and persistent heterogeneity in returns across individuals.[1]

Therefore, in this paper, we study theoretically the optimal mix of capital income and wealth taxes when entrepreneurial productivity and, consequently, the distribution of returns respond endogenously to taxation. We fully characterize how each tax affects the incentives to innovate, which in turn determine the share of high-productivity entrepreneurs in the economy, aggregate productivity, and welfare. We build upon the framework of Moll (2014), which models entrepreneurs with heterogenous productivities subject to collateral constraints (and hand-to-mouth workers) in a tractable fashion that allows aggregation. We modify this framework to introduce endogenous innovation, managerial effort, and a corporate sector, in a discrete-time perpetual-youth setting.

Specifically, entrepreneurs with different productivities produce a final good using a common constant-returns-to-scale technology that combines capital and labor. We model innovation as an upfront investment (modeled as a utility cost) by each newborn entrepreneur that yields a high-productivity project, with a probability of success that increases in the investment level. Each period, entrepreneurs have access to a bond market with zero net supply, where they can borrow subject to a collateral constraint; or lend to other entrepreneurs or to the corporate sector. The corporate sector is not constrained in its borrowing, and it produces with the same technology as entrepreneurs. Finally, workers supply labor inelastically and are hand-to-mouth, so all the wealth is held by entrepreneurs. Each newborn entrepreneur inherits the average wealth of entrepreneurs who die in that period. All agents have log preferences over consumption.

We study a government that taxes the wealth and capital income of entrepreneurs to fund government purchases and transfers to workers. An important feature of our model is that the wealth tax is levied on the *book value* of an entrepreneur's assets, not on the *market*

[1] For empirical evidence on persistent return heterogeneity, see, Campbell, Ramadorai and Ranish (2019), Fagereng, Guiso, Malacrino and Pistaferri (2020), Bach, Calvet and Sodini (2020), and Smith, Zidar and Zwick (2023).

value of the entrepreneurial firm they own. This distinction is crucial for understanding the implications of the wealth tax we study in this paper, as we discuss in a moment.

In principle, capital taxation affects outcomes through two different channels. The first channel works through the *incentives for innovation effort* faced by newborn entrepreneurs, by affecting the distribution of after-tax returns to innovation.[2] In particular, because a *book-value* wealth tax depends only on the wealth of the entrepreneur and not on whether innovation results in a high-productivity and profitable project, the entrepreneur is the residual claimant on the entire right tail of the profit distribution resulting from innovation. This upside potential boosts the incentives for innovation. By contrast, the capital income tax is levied proportionally on the realized returns of the project, eliminating this incentive effect. This channel will play an important role in the determination of the optimal mix of capital income and wealth taxes.

The second channel works through the *use-it-or-lose-it* effect of wealth taxation that results in a more efficient allocation of capital across entrepreneurs — an effect that is absent from capital income taxation. Essentially, a wealth tax puts the same tax burden on entrepreneurs with the same wealth level *regardless of their productivity*, whereas the capital income tax puts a higher tax burden on more productive entrepreneurs (relative to their wealth). Therefore, the wealth tax shifts the tax burden from high-productivity to low-productivity entrepreneurs, improving the allocation of capital.

Both of these channels work most effectively when the wealth tax is levied on the book value rather than the market value of entrepreneurs' capital. This is because two entrepreneurs with the same assets but different productivities have the same book value but (potentially very) different market values of their firms, as the latter incorporates the productivity (through future returns) of the entrepreneur. Therefore, a market-value wealth tax puts a higher tax burden on more productive entrepreneurs (relative to their wealth), looking partly like a capital income tax. This difference is particularly relevant when considering the incentives for innovation into technologies that make capital more productive (and therefore have a higher market value and produce higher returns). This is why we propose a *book-value wealth tax* as a more interesting and potentially more effective policy tool than the standard wealth tax based on market values.[3]

We show five main results. First, we establish that there exists a unique stationary equilibrium in which there is positive innovation effort. This equilibrium exhibits *capital*

[2] In Section 6, we characterize optimal taxes in an extension in which entrepreneurs exert *managerial effort* every period, which affects their productivity.

[3] A book-value wealth tax does not take into account accrued (unrealized) capital gains, facilitating its implementation, and effectively taxes capital gains upon transactions, as suggested by Aguiar, Moll and Scheuer (2024). It also sidesteps the valuation of closely-held assets at market prices and relies on information readily available to tax authorities, reducing the administrative burden of implementing it.

misallocation and *return heterogeneity*. Intuitively, this is because the upside potential provided by return heterogeneity is necessary to incentivize entrepreneurs to pay the cost of innovation. To the extent that one believes that high-productivity projects require costly innovation in real life, this result suggests that return heterogeneity is a natural outcome to expect. Another appealing feature of this model is that the stationary wealth distribution has a Pareto right tail, as in the US data, and the thickness of the tail is determined by the rate of return of high-productivity entrepreneurs.

Capital misallocation comes from the fact that high-productivity entrepreneurs are collateral-constrained and therefore earn higher rates of return on wealth than low-productivity entrepreneurs. However, the equilibrium does not require unrealistically restrictive collateral constraints: We derive an upper bound on the constraint (e.g., the loosest constraint) that sustains this equilibrium in terms of model primitives and show that, for a range of plausible parameter values, it allows for borrowing that (far) exceeds the current level of aggregate debt in the US (e.g., measured by the debt-to-GDP ratio).

Second, we show a *neutrality* result that draws a sharp distinction between the two taxes on capital: After-tax returns in steady state are independent of the capital income tax but do depend on the wealth tax, which increases their dispersion.[4] In particular, by rewarding higher return projects, the wealth tax incentivizes innovation effort, increasing the number of high-productivity entrepreneurs and aggregate productivity. This *extensive margin* benefit of wealth taxation would be missed by models with *exogenous* productivities, which is a commonly made assumption. In those models, while the wealth tax would still increase aggregate productivity, this would occur only through the reallocation of wealth toward high-productivity entrepreneurs (the "use-it-or-lose-it" effect).[5] By contrast, the capital income tax has no effect on—it is neutral with respect to—innovation or the distribution of entrepreneurial productivity in our model, because it has no effect on the dispersion of after-tax returns.

Of course, an independent increase in the wealth tax—holding the capital income tax constant—would lead to lower capital accumulation and output, even as productivity and innovation increase. However, when the government balances its budget, there are further gains from wealth taxation as it allows the government to reduce the capital income tax. This change in the mix of taxes on capital results in an increase in the equilibrium levels

[4] This stark result about the independence of equilibrium returns from the capital income tax emerges in our framework due to the combination of log utility and constant returns to scale in production.

[5] Note that all the allocative effects of the wealth tax in the present model come from the change in after-tax returns as there is no behavioral response—saving rates are (endogenously) constant, thanks to log utility. Guvenen et al. (2019) study a rich quantitative model with a more general CRRA utility function and show that the behavioral savings response strengthens the gains from a wealth tax, which suggests that relaxing the log utility assumption would strengthen the results we establish here.

of output, capital, and wages. This is because the wealth tax is *less distorting* than the capital income tax in the presence of return heterogeneity due to its effectiveness in spurring innovation and mitigating misallocation. The magnitudes of the increase in capital, output, and wages depend critically on how much an increase in aggregate productivity translates into higher output, which is determined by the capital intensity of production — measured as the output elasticity with respect to capital, α. This result will be key in characterizing the optimal combination of taxes.

Third, we study the welfare implications of an increase in the wealth tax (and a corresponding reduction in the capital income tax to balance the budget) and provide necessary and sufficient conditions for welfare gains for each type of agent. Workers always benefit from a higher wealth tax, thanks to higher wages. High-productivity entrepreneurs benefit because they are now wealthier, due to both the higher initial wealth they inherit and the faster wealth growth they experience, thanks to their higher after-tax returns. Although low-productivity entrepreneurs also start out with higher wealth, they experience slower wealth growth (or faster decline) because of their lower after-tax returns. The latter effect dominates, leading to welfare losses, unless the capital intensity of production, α, is unrealistically high. Overall, the expected returns of newborn entrepreneurs also decrease (even as higher returns for ex-post high-productivity entrepreneurs drive wealth accumulation), leading to ex-ante welfare losses, again, unless α is very high.

Putting these pieces together, the welfare change for the entire population (of workers plus entrepreneurs) from a change in the tax mix toward a higher wealth tax depends on the magnitudes of the increase in wages and the wealth level (both of which increase with α) versus the loss from the lower average wealth growth of entrepreneurs — from a lower average expected return. As a result, the condition for average welfare gain amounts to a lower bound on α, which turns out to be around *one-third* for a wide range of parameter values.

Fourth, we characterize the *optimal combination* of capital income and wealth taxes. The key parameter in this characterization is the capital intensity of production, α, which determines the pass-through of productivity gains into wages and capital. The characterization divides the range of α values into three intervals. When α is sufficiently high ($\alpha > \overline{\alpha}$), a given increase in the wealth tax translates (passes through via productivity) into gains in wages and wealth that are large enough that the optimal wealth tax is positive. It is also high enough that the capital income tax turns negative (a subsidy) to balance the government's budget. At the other extreme, when α is sufficiently low ($\alpha < \underline{\alpha}$), the signs of the two taxes flip — a combination of a negative wealth tax and a positive capital income tax. In between the two thresholds ($\underline{\alpha} < \alpha < \overline{\alpha}$), both taxes are positive (but small). This interval turns out to be typically quite narrow—between 0.3 and 0.4 for reasonable parameter values.

Fifth, we introduce ongoing *managerial effort*, which is meant to capture entrepreneurs' involvement in their businesses as active managers. Entrepreneurs choose a costly managerial effort every period, which further increases the productivity of the project. We show that the capital income tax distorts managerial effort by taxing the resulting profits and therefore reducing the marginal return to effort (similar to Jones, 2022). By contrast, we show that the wealth tax does not distort managerial effort at all, because it is independent of profits and leaves entrepreneurs as the residual claimant of the profits resulting from their additional managerial effort. Therefore, increasing the wealth tax and reducing the capital income tax further increases output and wages through the incentives for higher managerial effort.

Related Literature

An important common element in the earlier literature on capital taxation is the assumption of homogenous returns. Because capital income and wealth taxes are equivalent under this assumption, an analysis of the differences between the two taxes is naturally absent from this earlier literature, which focuses on capital income taxation (a short list includes Judd 1985; Chamley 1986; Aiyagari, 1995; Imrohoroglu, 1998; Erosa and Gervais, 2002; Conesa, Kitao and Krueger, 2009; Kitao, 2010; Saez and Stantcheva, 2018; Garriga, 2019; Straub and Werning, 2020).

However, there has been renewed interest in research on wealth taxation in recent years, partly in response to rising wealth concentration at the top, which led to various policy proposals to tax wealth. Some of these recent papers estimate the behavioral savings response to changes in wealth taxes (Seim, 2017; Jakobsen, Jakobsen, Kleven and Zucman, 2019; Londoño-Vélez and Ávila-Mahecha, 2021; Brülhart, Gruber, Krapf and Schmidheiny, 2022; Ring, 2024a), whereas others estimate the migration response of the very rich to a wealth tax (Jakobsen, Kleven, Kolsrud, Landais and Muñoz, 2023; Agrawal, Foremny and Martínez-Toledano, 2024). By contrast, there have been few theoretical studies of wealth taxation, especially when returns are heterogeneous, and to our knowledge, no analysis of the use-it-or-lose it effect of wealth taxes until very recently.[6,7]

There is a burgeoning literature that studies capital taxation in the presence of

[6] Although Allais (1977) and Piketty (2014) verbally described the use-it-or-lose-it mechanism, they did not study it.

[7] Scheuer and Slemrod (2021) is an excellent survey on wealth taxation that also discusses practical issues in implementation.

heterogenous returns. For example, Guvenen, Kambourov, Kuruscu, Ocampo and Chen (2019, 2023), Gaillard and Wangner (2022), Boar and Midrigan (2023), Beare and Toda (2023), and Boar and Knowles (2024) build quantitative macro models with production to study optimal capital income and wealth taxation. These models builds upon a workhorse framework (especially in the firm dynamics, development, and capital misallocation literatures), with firms that face idiosyncratic productivity shocks and collateral constraints (e.g., Quadrini, 2000, Cagetti and De Nardi, 2006, and Buera, Kaboski and Shin, 2011, among many others). We follow Moll (2014) and assume constant-returns-to-scale in production within this framework, which affords significant tractability.

In Guvenen, Kambourov, Kuruscu, Ocampo and Chen (2023), we build an overlapping-generations model that generates key features of the US wealth distribution (e.g., a Pareto right tail and its thickness matching the data), the distribution of individual returns, statistics on entrepreneurs, and others. We show that wealth taxes deliver large gains in output, wages, and welfare, that result not only from higher aggregate productivity but also lower inequality.

The present paper differs in three important ways. *First*, our main focus here is to study taxation in the presence of entrepreneurial innovation and managerial effort, which also implies that the distribution of productivity is endogenous and responds to capital taxes. This feature is absent not only from our earlier paper but also from the literature, both those cited above and below, and is a key contribution of this paper. *Second*, the present paper focuses on an analytically tractable framework and characterizes the precise trade-offs between capital and wealth taxes. *Third,* we consider both taxes together here and characterize their optimal *combination*, whereas our earlier paper considered one of them at a time.[8]

The theoretical focus of the present paper puts it in closer contact with recent theoretical papers by Aguiar, Moll and Scheuer (2024), Gerritsen, Jacobs, Spiritus and Rusu (2024), and Ferey, Lockwood and Taubinsky (2024). In particular, Aguiar, Moll and Scheuer (2024) distinguish between asset price movements that are driven by changes in dividends versus discount rates, and study taxation of realized versus unrealized capital gains. Our paper differs from these contributions by allowing a production structure and modeling innovation and managerial effort, which is absent from these papers which build endowment-economy models and exogenous productivity.

The nature of the optimal policy we find in our paper bears a subtle but interesting

[8] Incidentally, the results of the present paper show that the quantitative conclusions reached in Guvenen et al. (2023) hold more generally—in a standard framework and for a wide range of parameter values—strengthening the conclusions about the advantages of a wealth tax relative to a capital income tax.

resemblance to the one in Itskhoki and Moll (2019), despite the different contexts of the two papers. These authors find that the optimal Ramsey policy in the presence of collateral constraints restricts wages early on in the development process so as to boost the profits of productive entrepreneurs, allowing faster capital accumulation, thereby mitigating the effects of collateral constraints (after which taxes on capital are increased). This is similar to the effects of a wealth tax in our model, which also boosts the after-tax profits of high-productivity entrepreneurs, in turn incentivizing innovation and raising productivity.

Finally, our framework is also related to the literature on power law models that can generate a Pareto tail for the wealth (and income) distribution (see, among others, Champernowne, 1953; Jones, 2015; Gabaix, Lasry, Lions and Moll, 2016; and the review in Benhabib and Bisin, 2018). Especially closely related is Benhabib, Bisin and Zhu (2011) who consider an overlapping-generations model with return heterogeneity and study how the properties of the Pareto tail depend on the model's parameters, including on the estate tax rate. However, they do not study capital income or wealth taxation and returns follow an exogenous process, so they do not analyze the macroeconomic implications of the model, as we do here. Our results for the distribution of wealth are also related to Jones and Kim (2018) and Jones (2022), who study the distribution and taxation of top incomes.

2 Model

Time is discrete. The economy is populated by overlapping generations of homogenous workers (size L) and heterogeneous entrepreneurs (size 1) who differ ex-post in their idiosyncratic entrepreneurial productivity and their wealth. There is also a corporate sector that produces final goods and a government that finances expenditure and transfers to workers taxing wealth and capital income.

Demographics. Workers and entrepreneurs have perpetual-youth life cycles. That is, they face a constant probability of death $1 - \delta$ each period, and, upon death, they are replaced by a cohort of newborns of appropriate size to keep the population constant over time. The wealth of entrepreneurs who die in a period is distributed equally to (i.e., inherited by) all newborn entrepreneurs. Because the population is constant and mortality risk is independent of age, wealth, and productivity, this starting wealth equals the average wealth in the economy, which we denote with $\bar{a}$.[9]

Innovation and Productivity. The *distribution of entrepreneurial productivity* is endogenous and reflects the outcome of a costly and risky innovation process. Specifically,

[9]An alternative assumption would be to assign each newborn to an entrepreneur who dies that period ("parent") and assume that the newborn inherits the wealth of that parent. This case delivers essentially the same results, as we show in Appendix E.

we assume that newborn innovators come up with new ideas for production. The quality of these ideas is captured by the productivity, z, of the technology they describe. Once an idea is generated, the innovator uses it to produce and has access to it for the rest of their lifetime (akin to having a lifetime patent). Entrepreneurial productivity can take on two values: high, z_h, or low, z_ℓ.[10]

Innovation requires costly effort that nevertheless cannot guarantee success for the innovator, as innovation is a risky endeavor. Instead, effort, e, determines the probability that the innovator's idea turns into a high-productivity technology, $\Pr(z = z_h) = p(e)$. The cost of innovation is captured by a strictly increasing, strictly convex, and twice continuously differentiable cost function for effort $\Lambda(e)$, with $\Lambda(0) = 0$ and $\Lambda'(0) = 0$. Thus, there is an endogenously-determined share μ of high-productivity entrepreneurs (which we refer to as "H-type") and a corresponding share $1 - \mu$ of low-productivity entrepreneurs ("L-type") resulting from the entrepreneurs' innovation effort choices.

Preferences. Workers and entrepreneurs share the same preferences, defined over consumption:

$$\mathbb{E}_0 \left(\sum_{t=1}^{\infty} (\beta\delta)^{t-1} \log(c_t) \right), \tag{1}$$

where β is the time discount factor. Workers supply labor inelastically, receive transfers T from the government, and live hand-to-mouth (and therefore hold no wealth). Entrepreneurs are the sole capital/wealth owners in the economy.

The Corporate Sector. The corporate sector produces a homogeneous good combining capital, k, and labor, n, using a constant-returns-to-scale technology with productivity , z_c, so that corporate output is

$$Y_c = (z_c K_c)^{\alpha} N_c^{1-\alpha}. \tag{2}$$

Corporate firms hire labor at wage rate w and can borrow capital in a bond market at interest rate r. Both markets are perfectly competitive. Crucially, corporate firms are not subject to any constraints when borrowing.

The Government. The government taxes capital income at rate τ_k and (beginning-of-period) book-value of wealth, a, at rate τ_a to finance exogenous expenditures, G, and transfers to workers, T.

[10] Allowing more values of z (or even a continuous distribution) is fairly straightforward but comes at the cost of notational complexity without adding new insights, so we do not pursue that approach. We do, however, show that our main results are maintained when productivity varies stochastically over the entrepreneurs' lives as long as entrepreneurial productivity is positively autocorrelated. See Appendix E.

2.1 Entrepreneur's Problem

Entrepreneurial Production. Entrepreneurs differ ex-post in their idiosyncratic productivity, z, and produce a homogeneous good combining capital, k, and labor, n, using a constant-returns-to-scale technology:[11]

$$y = (zk)^{\alpha} n^{1-\alpha}. \tag{3}$$

Entrepreneurs hire labor at wage rate w and can borrow in a bond market at interest rate r to invest in their firm, over and above their own wealth a. The same bonds, which are in zero net supply, can be used as a savings device, which will be optimal for entrepreneurs whose return in equilibrium is lower than the interest rate r. Thus, k can be greater or smaller than a. Entrepreneurs' borrowing is subject to a collateral constraint that depends on their beginning-of-period wealth (a),

$$k \leq \lambda a, \tag{4}$$

where $\lambda \geq 1$. When $\lambda = 1$, an entrepreneur can use only their wealth in production.[12]

Entrepreneurs choose k and n every period to maximize profit,

$$\Pi(a,z) = \max_{\{k \leq \lambda a,\, n \geq 0\}} \left\{ (zk)^{\alpha} n^{1-\alpha} - rk - wn \right\}, \tag{5}$$

which yields their labor demand function:

$$n(a,z) = \left(\frac{1-\alpha}{w}\right)^{1/\alpha} zk(a,z), \tag{6}$$

where $k(a,z)$ is the optimal capital choice, obtained by substituting (6) into (5):

$$k(a,z) = \underset{\{k \leq \lambda a\}}{\operatorname{argmax}} \left[\alpha \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha} z - r \right] k. \tag{7}$$

[11] For convenience, we assume no depreciation, but this is easy to relax. The assumption of constant-returns-to-scale is critical for the tractability of our analytical results. Its main role is to generate constant marginal returns to capital. We consider that the clarity afforded by this assumption is well worth the limitations it imposes over our analysis.

[12] This specification of the collateral constraint is analytically tractable and is widely used in the literature (see, for example, Banerjee and Newman, 2003; Buera and Shin, 2013; and Moll, 2014). It can also be motivated as resulting from an underlying limited commitment problem (see Guvenen et al., 2023 for further discussion). The importance of financial constraints has broad empirical support; see, e.g., Gomes, Yaron and Zhang (2006); Hvide and Møen (2010); Duygan-Bump, Levkov and Montoriol-Garriga (2015); Benmelech, Frydman and Papanikolaou (2019); Ring (2023).

The constant-return-to-scale technology implies that entrepreneurs whose marginal return to capital (first term in equation 7) is greater than r borrow up to their collateral constraint, and set $k(a,z) = \lambda a$, whereas those whose marginal return is below r do not produce and instead lend all their wealth in the bond market to earn return r. Therefore, optimal entrepreneurial income can be written as $\Pi(z,a) = \pi(z) \times a$, where

$$\pi(z) \equiv \begin{cases} \left(\alpha \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} z - r\right)\lambda & \text{if } \alpha \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} z > r \\ 0 & \text{if } \alpha \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} z \leq r, \end{cases} \tag{8}$$

is the *excess return* an entrepreneur earns above r.

Entrepreneurial Savings. Entrepreneurs' consumption-savings problem is separable from their production problem. In anticipation of our focus below on the stationary equilibrium of the model, we write the recursive problem as a stationary Bellman equation:

$$\begin{aligned} V(a,z) &= \max_{\{a',c\}} \log(c) + \beta\delta V(a',z) \\ &\text{s.t. } c + a' = R(z)\, a, \end{aligned} \tag{9}$$

where

$$R(z) \equiv (1-\tau_a) + (1-\tau_k)(r + \pi(z)) \tag{10}$$

is the *after-tax gross return on savings,* and the time-invariant taxes τ_a and τ_k, and prices r and w, are taken as given.[13] Importantly, the wealth tax is levied on the *beginning-of-period* wealth, so only the capital income tax is levied on the income flow generated during the period $(r + \pi(z))$. The optimal savings rule for this problem is

$$a'(a,z) = \beta\delta R(z)\, a, \tag{11}$$

which is linear in wealth, with a net savings rate of $\beta\delta$ that is independent of productivity (thanks to log utility), although the gross savings rate (or the growth rate of their wealth) does depend on z through the rate of return they earn, $R(z)$. Therefore, all the reallocation effects of changes in taxation operate through their effect on returns.[14]

The resulting value of an entrepreneur of type $i \in \{h, \ell\}$ with assets a is:

$$V_i(a) = \frac{1}{1-\beta\delta}\log(a) + \frac{1}{(1-\beta\delta)^2}\left[\log(\beta\delta)^{\beta\delta}(1-\beta\delta)^{1-\beta\delta} + \log R_i\right], \tag{12}$$

[13]Unless when needed for clarity, we suppress the dependence of individual functions on taxes to avoid excessive notation.

[14]As noted in footnote 5, the effects of the wealth tax would likely be stronger with a utility function with a savings response.

which is obtained by substituting the solution of the entrepreneurs' problem into (11). We use $V_i(a)$ to denote $V(a, z_i)$ to simplify notation. The details of these derivations can be found in Appendix A.1.

Entrepreneurial Productivity and Innovation. The distribution of entrepreneurial productivity, captured by μ, depends on the innovation of newborn entrepreneurs who are ex-ante identical. The potential innovators' problem (at birth) is to maximize their ex ante value:

$$\mathbb{V}_0(\overline{a}) \equiv \max_e \; p(e) V_h(\overline{a}) + (1 - p(e)) V_\ell(\overline{a}) - \frac{1}{(1-\beta\delta)^2} \Lambda(e), \tag{13}$$

where $\mathbb{V}_0(\overline{a})$ denotes the maximized value. The resulting value of an idea corresponds to the value of an entrepreneur with productivity z_h or z_ℓ, depending on the outcome of the innovation process (equation 12).

Without loss of generality, we set $p(e) = e$ to simplify the problem. So, in equilibrium, the share of high-productivity entrepreneurs, μ, is given by the optimal effort choice of innovators. That is, by the solution of the following equation:[15]

$$\Lambda'(e) = (1-\beta\delta)^2 (V_h(\overline{a}) - V_\ell(\overline{a})) = \log R_h - \log R_\ell. \tag{14}$$

So, the effort choice (and the share of H-type entrepreneurs) depends on the return gap, $\log R_h - \log R_\ell$. A higher return gap generates higher incentives for effort as it captures the difference in lifetime payoffs between high- and low-productivity entrepreneurs.

The tax system can therefore shape the incentives for innovation through its effects on returns. The key for this is how taxes affect differentially high and low after-tax returns resulting in changes in the return gap.

2.2 Recursive Stationary Competitive Equilibrium

The equilibrium is the solution to a fixed point in the innovation choice of entrepreneurs and the distribution of returns it implies: The level of innovation determines the distribution of entrepreneurial productivity and, with it, the overall productivity in the economy and returns; the distribution of returns determines in turn the incentives to innovate and the innovation effort of entrepreneurs.

Before we go on to discuss how aggregate variables are determined in equilibrium, we make two important remarks. First, entrepreneurial activity is only possible in equilibrium if there are entrepreneurs who are more productive than the corporate sector, $z_h > z_c$.

[15] We assume that the cost function Λ is such that a corner solution is never optimal. This is done by evaluating the equation at $Z \in \{z_\ell, z_h\}$ and ensuring that the solution is interior in both cases.

Otherwise, it is optimal for all entrepreneurs to lend their assets to the corporate sector. This is the only case in which the stationary equilibrium exhibits homogeneous returns, as $R_h = R_\ell = (1-\tau_a) + (1-\tau_k)\, r$.

Second, there are no stationary equilibria with innovation in which returns are homogeneous. Without return dispersion there is no innovation effort, $e = 0$ as implied by equation (14), and no H-type entrepreneurs. Therefore, in an equilibrium with innovation there is capital misallocation, return heterogeneity ($R_h > R_\ell$), and a non-degenerate wealth distribution. Moreover, only equilibria with misallocation and return heterogeneity provide an interesting setting for analyzing wealth and capital income taxation. *Therefore, we focus on this type of heterogeneous-return equilibrium with innovation in the rest of the paper.*[16] We impose the following assumption throughout.

Assumption 1. *Corporate and entrepreneurial productivity satisfy $z_\ell \leq z_c < z_h$.*

Under this assumption only the H-type entrepreneurs and the corporate sector produce in equilibrium, as L-type entrepreneurs would rather save all of their wealth in bonds instead of using it in less productive activities.

From this point on, we proceed in two steps. We first define the key variables and conditions that hold in equilibrium. In particular, we give an aggregation result in Lemma 1 that will be useful in subsequent results. We then provide an intuitive discussion of how the labor and bond markets work and how the equilibrium distribution of productivity is determined. We establish the existence and uniqueness of equilibrium in the next section.

Defining some key variables. An important feature of our model is that aggregate variables can be expressed in closed form as functions of aggregate capital

$$K \equiv \mu A_h + (1-\mu)\, A_\ell, \tag{15}$$

(where A_h and A_ℓ are the aggregate wealth of the H-type and L-type, respectively) and aggregate productivity Z (as in Moll, 2014). Productivity is endogenous and equal to the wealth-weighted average of the *effective productivity* of H-types and the corporate sector:

$$Z \equiv s_h z_\lambda + (1-s_h)\, z_c, \tag{16}$$

where

$$s_h \equiv \frac{\mu A_h}{\mu A_h + (1-\mu)\, A_\ell} \tag{17}$$

[16] This would also be the unique equilibrium if the distribution of productivity were continuous. Then, the equilibrium would be characterized by a threshold value of productivity above which entrepreneurs borrow, as in Moll (2014).

is the wealth share of the H-types, and

$$z_\lambda \equiv z_h + (\lambda - 1)(z_h - z_c) \tag{18}$$

is the *effective productivity of their wealth*, that is, the return they earn from their own wealth, captured by z_h, plus the excess return from borrowed capital, $(\lambda - 1)(z_h - z_c)$.

Armed with these definitions, we can now state Lemma 1, which shows that aggregate output can be written solely as a function of aggregate variables (aggregation) and gives expressions for all equilibrium prices.

Lemma 1. ***(Aggregate Variables in Equilibrium)*** *In equilibrium, aggregate output, the wage rate, the interest rate, and gross returns satisfy:*

$$Y = (ZK)^\alpha L^{1-\alpha} \tag{19}$$

$$w = (1-\alpha)(ZK/L)^\alpha \tag{20}$$

$$r = \alpha (ZK/L)^{\alpha-1} z_c \tag{21}$$

$$R_\ell = (1-\tau_a) + (1-\tau_k)\alpha (ZK/L)^{\alpha-1} z_c \tag{22}$$

$$R_h = (1-\tau_a) + (1-\tau_k)\alpha (ZK/L)^{\alpha-1} z_\lambda. \tag{23}$$

The proofs of all lemmas and propositions can be found in Appendix B. However, we briefly discuss key equilibrium conditions leading to the results in this lemma for instructional purposes.

Labor market equilibrium. The labor demand function in (6) is linear in capital, so it can be aggregated to express the labor market clearing condition as

$$n(K_h, z_h) + n(K_\ell, z_\ell) + n(K_c, z_c) = L. \tag{24}$$

Substituting in $n(K, z)$ from (6) gives the expression for the equilibrium wage in Lemma 1.

Bond market equilibrium. Under Assumption 1, both the corporate sector and the H-type produce in equilibrium, demanding funds, while the L-type lend all their wealth and do not produce. In this, the corporate sector plays a central role, as it faces no constraints and is thus a constant source of demand for capital. Therefore, the marginal return of capital in the corporate sector imposes a lower bound on the equilibrium interest rate: $r \geq \alpha z_c (ZK/L)^{\alpha-1}$. The upper bound is given by the marginal product of capital of the H-type entrepreneurs. That is,

$$\alpha \left(\frac{ZK}{L}\right)^{\alpha-1} z_c \leq r \leq \alpha \left(\frac{ZK}{L}\right)^{\alpha-1} z_h, \tag{25}$$

which is obtained by substituting the equilibrium wage (20) into (8).

The supply of capital comes from L-types who lend all their wealth, $(1-\mu)A_\ell$, while the collateral constraint ensures that the H-type can borrow at most $(\lambda-1)\mu A_h$. The heterogeneous-return equilibrium corresponds to the case when the L-type have more wealth to lend than what the H-type are able to borrow, with the corporate sector absorbing the difference:

$$(\lambda-1)\,\mu A_h \;<\; (1-\mu)\,A_\ell. \tag{26}$$

Clearly, this happens when the H-type are not "too rich" relative to the L-type or when the collateral constraint is "not too loose," or both. Indeed, the inequality in (26) simplifies to $s_h < 1/\lambda$, which combines these two conditions. In this case, the H-type borrow up to the collateral constraint—hence $K_h = \lambda A_h$. Corporate firms compete with each other to borrow, bidding the equilibrium interest rate to their marginal product (giving $r = \alpha\,(ZK/L)^{\alpha-1}\,z_c$ in Lemma 1).[17] So, the capital used by the corporate sector is

$$K_c \;=\; \frac{(1-\mu)\,A_\ell - (\lambda-1)\,\mu A_h}{1-\mu} \;>\; 0. \tag{27}$$

Two properties of this equilibrium we mentioned earlier follow from this discussion. First, the fact that $K_h < K$ implies that there is capital misallocation. Second, from equations (22) and (23) in Lemma 1, $R_\ell < R_h$, so the equilibrium features return heterogeneity.

Innovation equilibrium. As mentioned above, the level of innovation effort in equilibrium is determined by the distribution of returns, which is taken as given by individual entrepreneurs. Equilibrium returns are in turn determined by the level of aggregate productivity for any given level of innovation, μ. A stationary recursive competitive equilibrium with innovation therefore requires that *(i)* innovators' effort choice solves problem (13) given returns R_h and R_ℓ, and *(ii)* the aggregate share of H-type is consistent with the optimal effort choice. Because all innovators are identical (at birth), they make the same choices, so this second condition becomes: $\mu \equiv p\,(e) = e\,(R_h, R_\ell)$.

The stationary equilibrium can therefore be stated as a fixed point in μ:

$$\mu^\star \;=\; e\left(R_h\left(\mu^\star\right), R_\ell\left(\mu^\star\right)\right), \tag{28}$$

Given $\mu^\star$, the equilibrium returns are obtained from the steady state marginal return on capital. Then, individual innovators take $\mu^\star$ and the returns R_h and R_ℓ as given when

[17] If instead $z_c < z_\ell$, the corporate sector would not operate in equilibrium and the L-types would bid down the interest rate to set $r = \alpha\,(ZK/L)^{\alpha-1}\,z_\ell$.

making their innovation effort choice, $e(R_h, R_\ell)$. This choice, in turn, implies $\mu^\star$. We provide a complete definition of the equilibrium in Appendix A.2.

3 Characterization of Stationary Equilibrium

We now characterize the stationary equilibrium of the economy with heterogeneous returns and innovation. We first derive two equations that determine the steady-state levels of K and Z for a given level of μ and show how they depend on the wealth and capital income taxes. Then, we prove the existence of a unique equilibrium (a fixed point for μ) and discuss the conditions under which it arises. We end with a short discussion of the stationary wealth distribution among entrepreneurs.

3.1 Steady State Levels of K and Z for a Given μ

Stationary level of K. The assets of the L-type and H-type evolve according to:

$$A_i' = \delta^2 \beta R_i A_i + (1-\delta)\,\overline{a}, \tag{29}$$

that is, a δ fraction of type-i entrepreneurs survive to the next period, each saving $\delta\beta R_i A_i$. The remaining fraction $1-\delta$ die and are replaced with newborn entrepreneurs with initial wealth $\overline{a} \equiv (1-\mu)\,A_\ell + \mu A_h = K$, equal to the average wealth in the economy. Adding up these equations for $i = \{h, \ell\}$ and substituting $\bar{a} = K$, we obtain the law of motion for aggregate capital:

$$\frac{K'}{K} = \delta^2 \beta \underbrace{\left(s_h R_h + (1-s_h)\,R_\ell\right)}_{\text{wealth-weighted avg. return}} + (1-\delta)\,, \tag{30}$$

which shows that the growth rate of K depends on the wealth-weighted average return. Substituting in the expressions for R_ℓ and R_h from Lemma 1 and setting $K' = K$ yields the following equation—that is analogous to the steady-state condition in the neoclassical growth model—where the (after-tax) marginal product of capital is equal to the inverse of the effective discount factor:

$$(1-\tau_a) + (1-\tau_k)\underbrace{\alpha Z^\alpha \left(\frac{K}{L}\right)^{\alpha-1}}_{\text{marginal product of capital}} = \frac{1}{\beta\delta}. \tag{31}$$

Neutrality of the capital income tax. Equation (31) has far-reaching implications and plays a critical role in our subsequent results. In particular, it reveals *a neutrality result* that draws a sharp distinction between the two forms of capital taxation: the after-tax marginal product of capital is *independent of the capital income tax but does depend on the* *wealth tax.* Rearranging (31) makes this easier to see:

$$\underbrace{(1-\tau_k)\,\alpha Z^{\alpha}\left(\frac{K}{L}\right)^{\alpha-1}}_{\text{after-tax marginal product of capital}} = \frac{1}{\beta\delta} - (1-\tau_a)\,. \tag{32}$$

Changing the wealth tax rate τ_a changes the after-tax marginal product of capital on the left hand side of the equation—same as changing the effective discount factor—whereas, changing τ_k causes only K to adjust so as to keep the after-tax marginal product constant and equal to $\frac{1}{\beta\delta} - (1-\tau_a)$. This reveals the different roles of capital income taxes—that are proportional to returns—and book-value wealth taxes—that are independent of returns. Capital accumulation (or de-accumulation) can undo the changes to returns induced by the capital income tax, say by having a lower capital stock and therefore a higher marginal product of capital in response to a higher capital income tax. This is not the case with the wealth tax because it has a differential effect on high- and low-productivity entrepreneurs. Specifically, because entrepreneurs' rates of return, R_h and R_ℓ depend on the after-tax marginal product (eqs. 23 and 22), τ_a increases the levels and dispersion of returns, whereas the τ_k has no effect. The following proposition formalizes these results.

Proposition 1. ***(Capital Income Tax is Neutral for Returns. Wealth Tax is Not)*** *In the stationary heterogeneous-return equilibrium, the after-tax returns of the H-type and L-type are independent of the capital income tax rate but do depend on the wealth tax rate:*

$$R_\ell = 1-\tau_a + \left(\frac{1}{\beta\delta} - (1-\tau_a)\right)\frac{z_c}{Z} \quad \textit{and} \quad R_h = 1-\tau_a + \left(\frac{1}{\beta\delta} - (1-\tau_a)\right)\frac{z_\lambda}{Z}. \tag{33}$$

In particular, the wealth tax has a "use-it-or-lose-it" effect that changes the dispersion of returns and therefore the level of wealth inequality, whereas the capital income tax has no distributional effects.

This neutrality result also affects the incentives that innovators face. Changes in capital income taxes do not affect after-tax returns in equilibrium and therefore do not change the incentives to innovate. By contrast, wealth taxes can (and do) shape innovation through their effect on returns. At its core, this result reflects the fact that individuals with different returns are differentially affected by wealth taxes, while they are homogeneously affected by capital income taxes. Intuitively, τ_k affects the marginal return of capital for both types proportionally and is therefore neutral from a distributional standpoint, as capital adjusts in the steady state, while τ_a affects gross returns additively and therefore has a disproportionate (negative) effect on the returns of the L-type.

Moreover, R_h and R_ℓ do not depend on the level of capital, K, but only in the level of productivity, Z. So, the optimal innovation effort choice is also a function of Z: $e(Z)$. This allows us to express the equilibrium condition for innovation in (28) directly in terms of the relationship between equilibrium productivity and innovation effort: $\mu^\star = e(Z(\mu^\star))$.

Stationary level of Z. Equation (31) provided the first condition for the steady state levels of K and Z, focusing on the evolution of aggregate capital. We now impose the second condition that ensures a stationary equilibrium, namely, that the wealth share of each type are constant. Evaluating the law of motion for each type in (29) at $A_i' = A_i$ for $i \in \{h, \ell\}$ and recalling that $\bar{a} = K = (1-\mu) A_\ell + \mu A_h$ implies

$$s_h = \frac{\mu A_h}{\bar{a}} = \frac{(1-\delta)\mu}{1-\delta^2 \beta R_h} \qquad \text{and} \qquad 1 - s_h = \frac{(1-\mu) A_\ell}{\bar{a}} = \frac{(1-\delta)(1-\mu)}{1-\delta^2 \beta R_\ell}. \tag{34}$$

These conditions show the intimate link between returns, R_h and R_ℓ, and the wealth distribution, captured by the wealth share of H-types, s_h. Differences in wealth accumulation come from differences in returns. Proposition 1 further implies that this link works through the equilibrium level of productivity Z, which simultaneously determines returns (as in 33) and reflects the wealth distribution by definition (see 16).[18]

Figure 1 shows how the equilibrium level of productivity, $Z^\star$, is determined as the intersection of the two curves for s_h implied by the wealth accumulation of H- and L-types described in (34). We show in the appendix that these curves necessarily intersect, with the value of $Z^\star \in (z_c, z_h)$ satisfying the following quadratic equation:

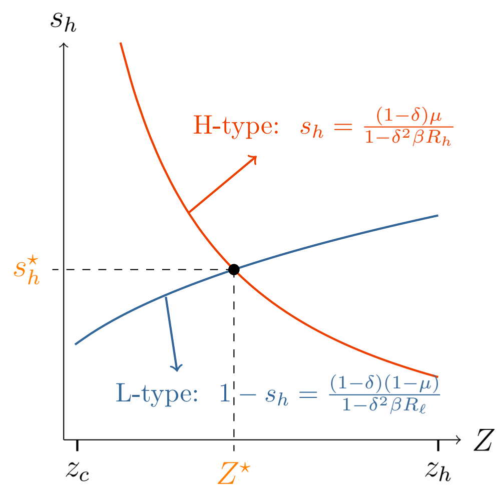
$$\begin{aligned}\left(1-\delta^2\beta(1-\tau_a)\right) Z^2 - \left[(1-\delta)(\mu z_\lambda + (1-\mu) z_c) + \delta(1-\delta\beta(1-\tau_a))(z_\lambda + z_c)\right] Z \\ + \delta(1-\delta\beta(1-\tau_a)) z_c z_\lambda = 0.\end{aligned} \tag{35}$$

This equation reveals a few key properties of the stationary equilibrium. First, we show that only the larger root of this quadratic equation satisfies $z_c < Z < z_\lambda$ (see the appendix). Therefore, if the stationary heterogenous-return equilibrium exists (as we assumed so far), it is also unique. Plugging this value of Z into (31) then determines the steady state level of K. Second, and more important, while τ_a appears in (35), τ_k does not, which means that the capital income tax rate has no effect on equilibrium productivity level—only τ_a does. This result sharpens the neutrality results in Proposition 1 by adding Z to the list of variables that τ_k has no effect on.[19]

[18] In this way, the determination of the equilibrium level of productivity is itself a fixed point problem, as we can map Z onto itself through its relationship to returns, their relationship to wealth shares, and the definition of Z in terms of s_h. Recall that $Z \equiv s_h z_\lambda + (1-s_h) z_\ell$, so a higher Z must follow from an increase in s_h—that is, from the reallocation of wealth towards the H-type, which means higher wealth inequality.

[19] We should note that this stark conclusion of *complete* neutrality follows from the combination of the

Figure 1: Determination of the Equilibrium level of Productivity given μ



Note: The figure shows the steady state levels of the wealth share of the H-types implied by the conditions in (34) as a function of the level of productivity Z, taking μ as given. Productivity affects returns as in equation (33) from Proposition 1.

Long-Run Elasticity of Capital with Respect to Taxes. Before moving forward, we highlight the implications of equation (31) for the response of aggregate capital to taxes as this will be important for our results later. Increasing either tax would (on its own) imply a decrease in the long-run level of capital. But, crucially, the wealth tax affects the level of capital through two channels—directly, by changing the right hand side of (31) and reducing the capital level, as well as indirectly, through its effect on productivity (which increases the capital level)—whereas the capital income tax only has the direct effect. This asymmetry implies different long-run elasticities of capital with respect to each tax:

$$\xi^K_{\tau_a} \equiv \frac{d \log K}{d\tau_a} = -\frac{1}{(1-\alpha)\left(\frac{1}{\beta\delta} - (1-\tau_a)\right)} + \frac{\alpha}{1-\alpha}\frac{d \log Z}{d\tau_a}; \tag{36}$$

$$\xi^K_{\tau_k} \equiv \frac{d \log K}{d\tau_k} = -\frac{1}{(1-\alpha)(1-\tau_k)}. \tag{37}$$

For standard parameter values these elasticities are well within the range reported in Scheuer and Slemrod (2021) in their review of the literature and are consistent with the values reported by Jakobsen, Jakobsen, Kleven and Zucman (2019) (see Figure F.4 in the Appendix). We will have more to say about these formulas in the tax analysis.

constant-returns-to-scale and log utility assumptions. That said, this result suggests that, even in a more general model, the wealth tax is likely to have a stronger impact on distributional outcomes and productivity than the capital income tax. Our quantitative results in Guvenen et al. (2023) confirm this conjecture.

3.2 Existence and Uniqueness of Stationary Equilibrium

We establish the existence of a unique fixed point for $\mu^{\star}$ that characterizes the stationary equilibrium of the economy — that is, a unique solution to (28). Existence of the fixed point follows from standard fixed point arguments relying on Cellina's and Brouwer's fixed point theorems (Border, 1985, Thms. 15.1, 16.1). Uniqueness follows from standard comparative statics results for fixed points after showing that the mapping of $\mu^{\star}$ into itself defined in (28) is monotonically decreasing. Specifically, we show that the equilibrium productivity, Z, is increasing in the share of H-type entrepreneurs, $\mu^{\star}$, using equation (35) and that the optimal effort is decreasing in Z because of its effect on return dispersion using (33). These two results imply the monotonicity of the equilibrium mapping for $\mu^{\star}$. We can now state the main result of this section.

Proposition 2. ***(Existence of a Unique Stationary Equilibrium with Innovation)*** *There exists an upper bound for the wealth tax such that, for $\tau_a < \overline{\tau}_a$, there is a unique stationary equilibrium that features heterogeneous returns ($R_h > R_\ell$). That is, there is a unique value of the share of H-type entrepreneurs, $\mu^{\star}$, such that the optimal level of effort exerted by innovators satisfies $\mu^{\star} = e\left(Z\left(\mu^{\star}\right)\right)$, and $Z\left(\mu^{\star}\right) \in \left(z_c, z_h\right)$ satisfies equation (35). The upper bound for the wealth tax satisfies*

$$\overline{\tau}_a = 1 - \frac{1}{\beta\delta}\left(1 - \frac{1-\delta}{\delta}\frac{1-\lambda\mu^{\star}\left(\overline{\tau}_a\right)}{\left(\lambda-1\right)\left(1-\frac{z_c}{z_h}\right)}\right), \tag{38}$$

where we make the dependence of $\mu^{\star}$ on τ_a explicit.

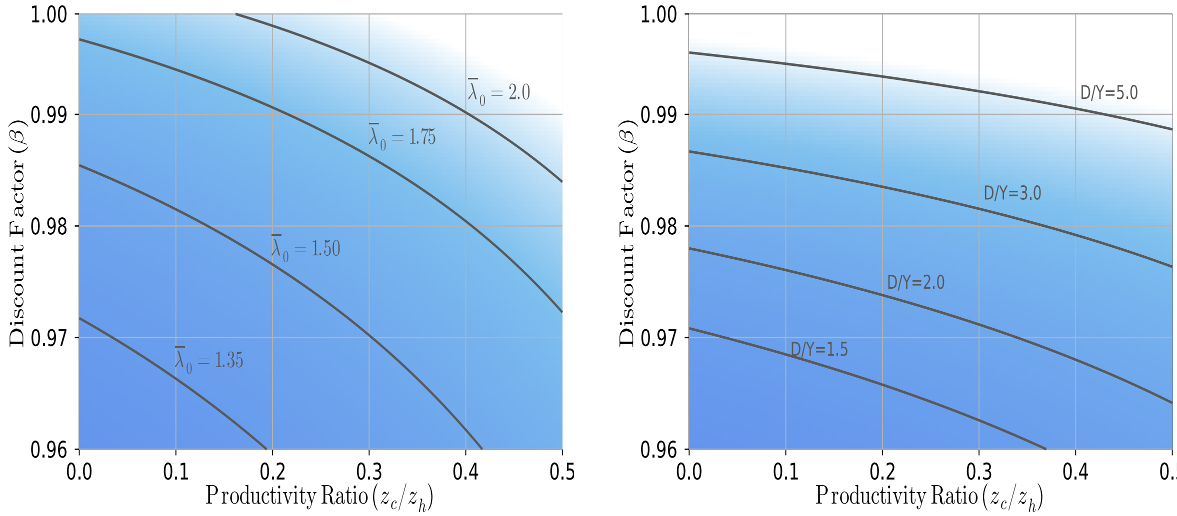
Two comments are in order with respect to condition (38) on the level of wealth taxes. First, τ_k does not appear in (38) as a direct consequence of the neutrality result from Proposition 1. Second, condition (38) guarantees that equilibrium exhibits return heterogeneity by ensuring that condition (26) holds, that is, that $s_h < 1/\lambda$. This role will become obvious as we explore the effects of wealth taxes on returns and inequality.

To understand condition (38) better, recall that condition (26) is equivalent to imposing a mutual bound on wealth inequality, s_h, and the looseness of the entrepreneurs' collateral constraint, λ. It is then convenient to restate it as an upper bound on the collateral constraint of entrepreneurs $\lambda < \bar{\lambda}$, holding fixed the value of wealth taxes and the innovation choice of entrepreneurs. The condition then becomes:

$$\lambda < \overline{\lambda} \equiv 1 + \frac{\left(1-\delta\right)\left(1-\mu\right)}{\left(1-\delta\right)\mu + \delta\left(1-\delta\beta\left(1-\tau_a\right)\right)\left(1-\frac{z_c}{z_h}\right)}. \tag{39}$$

Figure 2: Conditions for Stationary Equilibrium with Innovation

(a) Threshold $\overline{\lambda}_0$ (b) Debt-to-Output Ratio $\left(\lambda = \overline{\lambda}_0\right)$



Note: The left panel plots $\overline{\lambda}_0$ (condition 39 with $\tau_a = 0$) for combinations of β and z_c/z_h. The right panel plots the corresponding debt-to-output ratio, $\left(\overline{\lambda}_0 - 1\right) A_h/Y$. Other parameters are: $\delta = {}^{49}/_{50}$, $z_h = 1$, $\theta = 025$, and $\alpha = 0.4$.

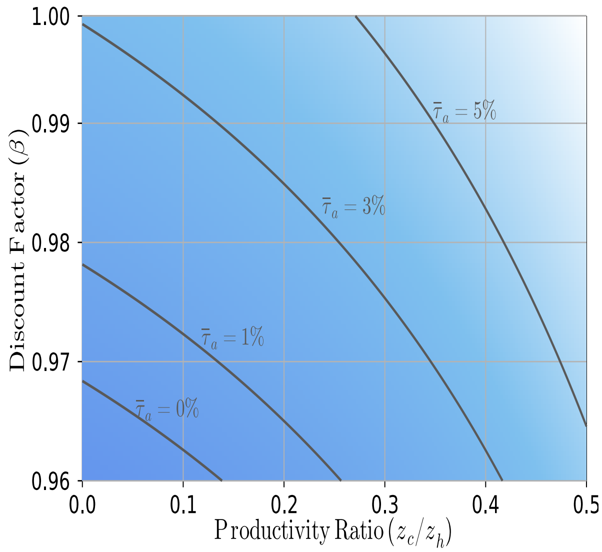
This implied upper bound $\overline{\lambda}$ is decreasing in the wealth tax rate. The reason is that a higher τ_a increases wealth inequality through its effect on returns (this can already be seen in Proposition 1 and we formalize it below in Lemma 2), thereby shifting wealth from the L-type to the H-type ($s_h \uparrow$), which makes it harder for the excess supply condition in (26) to hold, unless λ is reduced. Therefore, $\bar{\lambda}$ must get tighter to disallow high values of λ.[20]

This intuition extends when we take into account that the bounds depend also on the share of H-type entrepreneurs, μ, which is endogenous and responds in equilibrium to tax rates. We show formally in the next section that, as τ_a increases, so does $\mu^\star$ because wealth taxes increase the return gap between H- and L-types (Lemma 2 and Proposition 3). But, as $\mu^\star$ increases, so does s_h, making it harder to guarantee that the demand for funds from H-type entrepreneurs is met by the wealth held by the L-types. This results in the upper bound for wealth taxes, $\overline{\tau}_a$, defined by equation (38).

So, how much borrowing does condition (39) allow for empirically reasonable parameter values? To get a sense about this, we plot the values of $\bar{\lambda}$ (Figure 2a) and the debt-to-GDP ratio (Figure 2b) for different values of β and z_c/z_h and setting $\tau_a = 0$. This threshold, which we denote with $\bar{\lambda}_0 \equiv \bar{\lambda}|_{\tau_a=0}$, characterizes the "loosest" collateral constraint that sustains

[20] It is also easy to see from (39) that $\bar{\lambda}$ is decreasing in μ and z_h/z_c because, again, higher values of both make it easier $s_h < 1/\lambda$ to be violated. $\bar{\lambda}$ is decreasing in δ because longer life-spans benefit the accumulation of assets by H-type entrepreneurs, who have higher returns than the L-types. By contrast, $\bar{\lambda}$ is increasing in β (for a fixed δ) because, as both types become more patient, the aggregate savings in the economy increase, lowering wealth inequality through the redistribution of wealth among newborns ($s_h \downarrow$).

Figure 3: Upper Bound on the Wealth Tax ($\bar{\tau}_a$)



Note: The figure reports the upper bound on the wealth tax from Proposition 2 when innovation is endogenous for combinations of the discount factor (β) and productivity dispersion (z_ℓ/z_h). Other parameters are: $\delta = 49/50$, $z_h = 1$, $\theta = 025$, and $\alpha = 0.4$. λ is such that the debt-to-output ratio is 1.5 when $\tau_a = 0$.

the heterogenous-return (or innovation) equilibrium in the absence of a wealth tax.[21] For a plausible value of $\beta = 0.98$ and $z_c/z_h = 0.3$, the implied $\bar{\lambda}_0$ is above 1.5, which corresponds to a debt-to-GDP ratio $\left(\left(\bar{\lambda}_0 - 1\right)\mu A_h/Y\right)$ well above 2. This is well above the debt-to-GDP ratio in the US in recent years (1.52) reported in Guvenen et al. (2023), confirming that the innovation equilibrium allows substantial amounts of borrowing in the model.

Figure 3 shows the upper bound on wealth taxes defined by condition (38) in Proposition 2 when we set λ to generate a debt-to-GDP ratio of 1.5 under $\tau_a = 0$, so as to match the US economy. The upper bound on wealth taxes is well above practical values for most parameter combinations. For instance, with $\beta = 0.98$ and $z_c/z_h = 0.3$, a wealth tax of over 3% can be sustained.

These comparisons show that the model we analyze allows for substantial borrowing in equilibrium under a wide range of plausible parameter values. This leads us to conclude that the conditions in (38) and (39), ensuring returns are heterogeneous in equilibrium, are not unduly binding and do not impose unreasonable constraints on the model.

3.3 Stationary Wealth Distribution

The distribution of wealth among entrepreneurs reflects the differences in their after-tax returns. In equilibrium, not only does it hold that the H-types have higher

[21]Other parameter values are set as follows: Average life expectancy is 50 years ($\delta = 49/50$); the capital intensity is $\alpha = 0.4$; and the capital income tax is $\tau_k = 25\%$.

returns, but their returns are high enough to ensure that they accumulate wealth as they age ($\beta\delta R_h > 1$) while the returns of the L-types lead them to dissave ($\beta\delta R_\ell < 1$). See Lemma 6 in Appendix C. Since both types start life with $\bar{a}$, each group's wealth distribution lies in two non-overlapping (except at $\bar{a}$) intervals: $(0,\overline{a}]$ and $[\overline{a},\infty)$, with (endogenously-determined) discrete mass points: $\left\{\ldots,(\beta\delta R_\ell)^2\,\overline{a},\beta\delta R_\ell\overline{a},\overline{a}\right\}$ and $\left\{\overline{a},\beta\delta R_h\overline{a},(\beta\delta R_h)^2\,\overline{a},\ldots\right\}$. The population share at wealth level $(\beta\delta R_i)^t\,\overline{a}$ is equal to the fraction of each type who has lived exactly t years. So, the wealth distribution has a geometric distribution with parameter δ.[22] An important property of this distribution is that it exhibits the pattern of *fractal inequality* highlighted in Jones and Kim (2018) in the context of Pareto distributions. We provide a formal discussion of the stationary distribution and its response to changes in the environment in Appendix C.

4 Effects of Wealth and Capital Income Taxation

We now consider the effects of increasing the wealth tax (and later reducing the capital income tax to balance the government budget) on equilibrium outcomes. The results here are global in nature—they hold for any starting level of $\tau_a < \overline{\tau}_a$. We abstract from other taxes to focus on the trade-offs between these two forms of capital taxation. In Section 4.1, we do not impose a government budget constraint; in Section 4.2, we do. In Section 5, we turn our attention to the optimal combination of capital income and wealth taxes that maximizes average newborn welfare.

4.1 Effects on Aggregate Returns, Innovation, and Productivity

The main result of this section establishes that an increase in the wealth tax increases innovation, μ, and aggregate productivity, Z. This result showcases the core mechanisms behind the effect of wealth taxation. In a nutshell, higher wealth taxes affect the economy through their effect on returns by two separate and complementary channels. First, wealth taxes increase the after-tax returns of more productive entrepreneurs and decrease those of less productive ones. We can see how these effects are triggered going back to the expression for returns in equation (33) of Proposition 1. Holding all other variables constant, we have

$$\left.\frac{dR_\ell}{d\tau_a}\right|_{Z,\mu} = -\frac{Z - z_c}{Z} < 0 \quad \text{and} \quad \left.\frac{dR_h}{d\tau_a}\right|_{Z,\mu} = \frac{z_h - Z}{Z} > 0. \tag{40}$$

[22] This characterization follows Jones (2015), adapted to the discrete time setting.

This change in returns is reflected in the optimal innovation effort choice of entrepreneurs, who now have incentives for higher innovation, see equation (14).[23] This, in turn, increases aggregate productivity as the distribution of entrepreneurial productivity shifts, as we see in the wealth accumulation equations in (34). Second, the increase in return dispersion also triggers a reallocation of wealth that increase productivity by concentrating wealth in the hands of H-types—the "use-it-or-lose- it" effect.[24]

In equilibrium, both μ and Z necessarily increase in response to an increase in the wealth tax. We provide a formal result below in Proposition 3. The proof builds on standard comparative static results for fixed points found in Villas-Boas (1997), as we have to account for the simultaneous responses of μ and Z to a change in the wealth tax, and the interactions between them. Because τ_k does not affect Z, we can study the effect of τ_a on Z without needing to specify the government budget.

Proposition 3. ***(Innovation and Productivity Gains from Wealth Taxation)*** *For all $\tau_a < \overline{\tau}_a$, an increase in the wealth tax (τ_a) increases the equilibrium share of high-productivity entrepreneurs, $\mu^\star$, and the equilibrium level of productivity $Z^\star$. Capital income taxes do not affect innovation or productivity.*

This result implies that the wealth share of H-type entrepreneurs increases with the wealth tax. We state this as a corollary given its substantive importance for later results.

Corollary 1. ***(Wealth Taxation Increases Wealth Inequality)*** *For all $\tau_a < \overline{\tau}_a$, a higher τ_a reallocates wealth towards the H-type, $ds_h/d\tau_a > 0$.*

Effects of wealth taxes on returns. Because of its importance for the mechanisms at play, we also state an additional result on the response of returns to taxes in equilibrium. The changes in returns must be consistent with the changes in innovation and productivity outlined above, and so we can use equation (34) once more to establish how changes in wealth taxes are reflected in returns. The objective is to determine the effect of wealth taxes on returns through the changes in equilibrium Z and μ:

$$\frac{d\log R_i}{d\tau_a} = \frac{d\log R_i}{d\log Z}\frac{d\log Z}{d\tau_a} + \frac{d\log R_i}{d\mu}\frac{d\mu}{d\tau_a}, \tag{41}$$

[23]It is easier to see the effect o wealth taxes on return dispersion by noting that $R_h - R_\ell = \left(\frac{1}{\beta\delta} - (1-\tau_a)\right)\frac{(z_\lambda - z_c)}{Z}$.

[24]Both effects can be seen graphically in Figure 1 where, for any level of productivity Z, the return of the H-types is higher, which shifts their wealth accumulation curve upwards, similarly, the curve of the L-types also moves up. We establish formally in the appendix that these shifts necessarily result is a higher level of wealth concentration $s_h \uparrow$ and therefore a higher equilibrium productivity, $Z \uparrow$. The result is also immediate from the analysis of the quadratic equation in (35).

where $d \log R_i / d \log Z$ captures the equilibrium change in returns as Z adjusts, holding μ constant, and $d \log R_i / d\mu$ the change as μ adjusts, for $i \in \{\ell, h\}$. Higher productivity must reflect *increases* in R_h and *reductions in* R_ℓ. Moreover, we show that as the dispersion of returns increases, the population-weighted returns decrease both in levels and in logs. An increase in innovation has (holding the wealth shares constant) the opposite implications for returns.

Lemma 2. ***(Returns, Productivity, and Innovation)*** *An increase in productivity, Z, increases the rate of return of the H-type and reduces that of the L-type, holding μ constant,*

$$\xi_Z^{R_h} \equiv \frac{d \log R_h}{d \log Z} > 0 \qquad \textit{and} \qquad \xi_Z^{R_\ell} \equiv \frac{d \log R_\ell}{d \log Z} < 0. \tag{42}$$

Furthermore, the population-weighted average of returns and log returns decline:

$$\frac{d \left(\mu \log R_h + (1-\mu) \log R_\ell\right)}{d \log Z} = \mu \xi_Z^{R_h} + (1-\mu)\, \xi_Z^{R_\ell} < 0. \tag{43}$$

An increase in innovation, μ, holding Z constant, has the opposite effects, so that,

$$\xi_\mu^{R_h} \equiv \frac{d \log R_h}{d\mu} < 0, \qquad \xi_\mu^{R_\ell} \equiv \frac{d \log R_\ell}{d\mu} > 0, \qquad \textit{and} \qquad \mu \xi_\mu^{R_h} + (1-\mu)\, \xi_\mu^{R_\ell} > 0. \tag{44}$$

These results will prove crucial when determining the welfare effects of taxation. For instance, equation (43) means that an increase in the productivity is accompanied by a reduction in the lifetime growth of wealth expected by a newborn entrepreneur (holding μ fixed) because the average elasticity of returns (which determines the growth rate of savings, eq. 11) is negative.

4.2 Effects on Aggregate Variables

We now turn to the response of aggregate capital, output, and wages to changes in the wealth and capital income taxes. To do this, we need to specify the government budget and how it is balanced as K adjusts (according to equation 31) in response to changes in the combination of taxes.

Government budget. The government uses the revenues collected from the capital income and wealth taxes to finance (unproductive) government expenditures G and lump-sum transfers to workers T:

$$G + T \;=\; \tau_k \alpha Y + \tau_a K \;=\; \left(\tau_k + \tau_a \frac{\beta\delta\left(1-\tau_k\right)}{1-\beta\delta\left(1-\tau_a\right)}\right) \alpha Y. \tag{45}$$

What should we assume about total government spending, $G+T$? We consider two assumptions. The first one has empirical appeal and greatly simplifies the analysis: we assume that both G and T are fixed fractions of output. The second one is to assume that $G+T$ is constant—independent of the size of the economy. The first assumption implies that spending will rise/fall linearly with output if different taxes affect the output level, whereas the second assumption implies that the tax experiments we consider are revenue neutral. We start with the first assumption.

Assumption 2. ***(Constant Government Spending Share)*** *Assume that* $G=\theta_G \alpha Y$ *and* $T=\theta_T \alpha Y$*, so total tax revenue is* $G+T=\theta\alpha Y$*, where* $\theta \equiv \theta_G+\theta_T$*.*

This assumption, along with equation (45), implies a tight link between τ_k and τ_a:[25]

$$\frac{1-\theta}{1-\beta\delta}=\frac{1-\tau_k}{1-\beta\delta\left(1-\tau_a\right)}. \tag{46}$$

A special case worth highlighting is when $\theta=0$: there are no revenue requirements, so taxation only serves to redistribute *among entrepreneurs* or to increase productivity. In this case, it must be that either $\tau_k \geq 0$ and $\tau_a \leq 0$ or $\tau_k \leq 0$ and $\tau_a \geq 0$, with no taxation also being feasible, $\tau_k=\tau_a=0$.

We first show that under Assumption 2, all aggregate quantities—capital, output, and wages—increase when the wealth tax is raised and the capital income tax is reduced accordingly to keep the government budget balanced. Since Proposition 3 established that Z rises with τ_a, all we need to show is that aggregates increase with Z. Notice that this increase in productivity and aggregates happens even as the total tax revenue collected increases, as per Assumption 2, and regardless of how the revenue is spent on G versus T. (As we will see in a moment, aggregates would increase even more if we instead assume revenue neutrality.)[26] These results are summarized in the following lemma.

Lemma 3. *Under Assumption 2, the steady-state level of capital is*

$$K=\left(\alpha\frac{\beta\delta\left(1-\theta\right)}{1-\beta\delta}\right)^{\frac{1}{1-\alpha}} Z^{\frac{\alpha}{1-\alpha}} L. \tag{47}$$

The long-run elasticities of aggregate variables with respect to productivity, $\xi_Z^x \equiv \frac{d\log x}{d\log Z}$*, are*

$$\xi_Z^K=\xi_Z^Y=\xi_Z^w=\frac{\alpha}{1-\alpha}. \tag{48}$$

[25] $\tau_k=\theta$ without a wealth tax ($\tau_a=0$) and $\tau_a=\frac{\theta(1-\beta\delta)}{\beta\delta(1-\theta)}$ without a capital income tax ($\tau_k=0$).

[26] If τ_a were raised without changing τ_k, aggregate capital would have decreased, because of the negative (and empirically probably large) elasticity of capital with respect to τ_a in equation (36), see Figure F.4a.

One remark is in order. Equation (47) implies that the steady-state levels of capital, output, and wages respond to the tax mix between wealth and capital income taxes *only* through the effect of the wealth tax on aggregate productivity. Thus, the (semi-)elasticities of aggregates with respect to τ_a are[27]

$$\xi^K_{\tau_a} = \xi^Y_{\tau_a} = \xi^w_{\tau_a} = \frac{\alpha}{1-\alpha} \times \frac{d \log Z}{d\tau_a}. \tag{49}$$

Let us now consider what happens if we assume that the total tax revenue is constant: $G + T = \overline{\theta}$. In this case, because tax revenue does not rise with output, the same rise in τ_a is matched with a larger decline in τ_k than under Assumption 2, implying a stronger positive response of aggregates to τ_a. The following lemma states this result.

Lemma 4. *Assume that total government is fixed, $G+T = \overline{\theta}$. Then, the (semi-)elasticities of capital, output, and wages to a change in the wealth tax satisfy*

$$\xi^K_{\tau_a}, \xi^Y_{\tau_a}, \xi^w_{\tau_a} > \frac{\alpha}{1-\alpha} \frac{d \log Z}{d\tau_a}. \tag{50}$$

In sum, under a balanced budget (as Assumption 2), capital, output, and wages increase in response to an increase in τ_a (and a corresponding reduction in τ_k) with an elasticity of $\frac{\alpha}{1-\alpha} \times \frac{d \log Z}{d\tau_a}$.

4.3 Effects on Individual Welfare

We now focus on the welfare implications of changes in the mix of wealth and capital income taxes on entrepreneurs and workers. The value of entrepreneurs is given in equation (12) and the value of workers is

$$V_w = \frac{1}{1-\beta\delta} \log (w + T). \tag{51}$$

These values, together with the results above, give rise to the following result characterizing the conditions for welfare changes after an increase in the wealth tax.

Lemma 5. ***(Welfare by Agent Type)*** *For all $\tau_a < \overline{\tau}_a$, under Assumption 2, the values*

[27] Notice that the elasticity of capital with respect to wealth taxes under a balanced budget corresponds to the first term in equation (36) as equation (46) implies that the negative second term is exactly offset by the decrease in capital income needed to balance the budget.

of workers and the ex-post and ex-ante values of entrepreneurs satisfy, respectively,

$$\frac{dV_w}{d\tau_a} = \frac{1}{1-\beta\delta}\xi_{\tau_a}^{w+T} > 0; \tag{52}$$

$$\frac{dV_i(\overline{a})}{d\tau_a} = \frac{1}{1-\beta\delta}\xi_{\tau_a}^{K} + \frac{1}{(1-\beta\delta)^2}\xi_{\tau_a}^{R_i}; \tag{53}$$

$$\frac{d\mathbb{V}_0(\overline{a})}{d\tau_a} = \frac{1}{1-\beta\delta}\xi_{\tau_a}^{K} + \frac{1}{(1-\beta\delta)^2}\left(\mu\xi_{\tau_a}^{R_h} + (1-\mu)\,\xi_{\tau_a}^{R_\ell}\right); \tag{54}$$

for $i \in \{h, \ell\}$ and $\xi_{\tau_a}^{x}$ is the (semi-)elasticity of x with respect to τ_a.

An increase in the wealth tax, coupled with a decrease in the capital income tax, increases the welfare of workers because it increases wages and transfers.[28] As for entrepreneurs, even though their initial wealth increases, the total change in their welfare is, in principle, ambiguous because it depends on the equilibrium response of returns. As we established in Lemma 2, there are opposing forces coming from the increase in productivity and innovation that follow an increase in the wealth tax.

We can nevertheless establish conditions for welfare gains (and losses) among entrepreneurs by exploiting the fact that the changes in wealth and returns operate through different channels. The key is, once again, the neutrality result established in Proposition 1. Only the wealth tax is relevant for the response of equilibrium returns so that this can be determined separate from the response of aggregate variables. The response of initial wealth, $\overline{a}$, to changes in the tax mix depends only on the change in productivity and the elasticity of capital to productivity, which is constant and depends only on the capital intensity, α. Thus, we have,

$$\frac{dV_i(\overline{a})}{d\tau_a} > 0 \quad \text{if} \quad \frac{\alpha}{1-\alpha} > \frac{-1}{1-\beta\delta}\frac{\xi_{\tau_a}^{R_i}}{\xi_{\tau_a}^{Z}}; \tag{55}$$

$$\frac{d\mathbb{V}_0(\overline{a})}{d\tau_a} > 0 \quad \text{if} \quad \frac{\alpha}{1-\alpha} > \frac{-1}{1-\beta\delta}\frac{\mu\xi_{\tau_a}^{R_h} + (1-\mu)\,\xi_{\tau_a}^{R_\ell}}{\xi_{\tau_a}^{Z}}. \tag{56}$$

If the pass-through of productivity to capital is sufficiently strong, that is, if α is sufficiently high, entrepreneurs benefit from the increase in the wealth tax. This is because, even if returns go down, the increase in their initial wealth can compensate this change. However, a decrease in returns proves to be quite costly for entrepreneurs, as it implies a lower lifetime (discounted) wealth growth. This means that only high-productivity entrepreneurs benefit from a change in the tax mix toward a higher

[28]Under Assumption 2, a worker's the total income is $w+T = ((1-\alpha)+\theta_T\alpha)\,Y/L$ with (semi-)elasticity with respect to the wealth tax given by $\xi_{\tau_a}^{w+T} \equiv d\log(w+T)/d\tau_a = \frac{\alpha}{1-\alpha} \times d\log Z/d\tau_a$.

wealth tax as their returns are actually likely to increase for any standard value of α. In Figure F.5a in Appendix F we show that H-types benefit as long as α is above 0.1. By contrast, the threshold values for α implied by the conditions above for L-types and entrepreneurs as a whole turn out to be too high for a range of plausible parameter values, with α having to be above 0.6. See Figures F.5b and F.5c in Appendix F.

5 Optimal Taxation

The government's objective is to maximize the equilibrium utilitarian welfare of newborns, $\mathcal{W}$, by choosing the optimal combination of capital income and wealth taxes, subject to its budget constraint. Let $n_w \equiv {}^{L}\!/\!{}_{(1+L)}$ represent the fraction of workers in the population. The government's objective is $\mathcal{W} = n_w V_w + (1 - n_w) \mathbb{V}_0 (\overline{a})$. We can make the trade-off faced by the government clearer by substituting in the value functions of workers and entrepreneurs from (51) and (12):

$$\mathcal{W} = \frac{1}{1-\beta\delta} \left(n_w \log (w+T) + (1-n_w) \log \overline{a} \right) + \frac{1-n_w}{(1-\beta\delta)^2} \left(\mu \log R_h + (1-\mu) \log R_\ell - \Lambda(\mu) \right) + v, \tag{57}$$

where $v \equiv \frac{1-n_w}{(1-\beta\delta)^2} \log (\beta\delta)^{\beta\delta} (1-\beta\delta)^{1-\beta\delta}$ is a constant. This objective includes the cost of innovation effort and the fact that $\mu = e$ in equilibrium. The government's problem is

$$\max_{\tau_k, \tau_a} \mathcal{W} \quad \text{s.t. } (45). \tag{58}$$

An interior solution balances the effects on the level of aggregates and the changes in returns, and satisfies $d\mathcal{W}/d\tau_a = 0$. Crucially, there is no direct effect on welfare of the change in the innovation in response to changes in the tax combination because μ is being chosen optimally by entrepreneurs, so the effect of innovation only comes through changes in equilibrium returns and productivity. See Lemma 5.

Figure 4 illustrates the forces at play. Increasing the wealth tax (while simultaneously reducing the capital income tax as in 46) increases the levels of initial wealth, wages, and transfers, through its effect on aggregate productivity (Lemma 3). We call this the *level effect* of wealth taxation. Formally, this positive level effect on welfare depends on the elasticities of workers' income and initial wealth with respect to productivity $\left(n_w \xi_Z^{w+T} + (1-n_w) \xi_Z^K \right)$ that give the (percentage) gain in workers' and entrepreneurs' welfare as the wealth tax increases (in turn, raising productivity). These elasticities are constant in our economy (Lemma 3, equation 48) and are both equal to $\alpha / (1-\alpha)$.

However, increasing the wealth tax also results in changes in lifetime wealth growth. This *growth effect* is driven by two forces (Lemma 2). First, holding μ constant, higher wealth taxes induce a decrease in average log returns, decreasing the lifetime wealth growth of entrepreneurs. This effect works through the change in productivity triggered by wealth taxes and is captured by the average elasticity of returns,

$$\xi_Z^g \equiv \frac{1}{1-\beta\delta}\left(\mu\xi_Z^{R_h} + (1-\mu)\,\xi_Z^{R_\ell}\right) < 0, \tag{59}$$

where the superscript g stands for "growth," and the scaling term comes from the discounted sum over entrepreneurs' life times. This effect is decreasing in τ_a, becomes more negative, reflecting the widening gap between low and high returns as the wealth tax increases productivity. At the same time, an increase in wealth taxes incentivizes innovation (Proposition 3) resulting in higher returns and lifetime wealth growth.[29] This second force constitutes the *innovation effect* and captures the average elasticity of returns to innovation,

$$\xi_\mu^g \equiv \frac{1}{1-\beta\delta}\left(\mu\xi_\mu^{R_h} + (1-\mu)\,\xi_\mu^{R_\ell}\right) > 0. \tag{60}$$

The net effect on returns, and therefore the lifetime wealth growth, is negative (as shown in Figure 4). This follows from the same logic as that in Acemoglu and Jensen (2024). The equilibrium change in returns must be consistent with the initial (partial-equilibrium) change that sustains the increase in innovation (see Lemma 2).

The choice of optimal tax combination $(\tau_a^\star, \tau_k^\star)$ balances these gains and losses and is determined by the intersection of the two lines in Figure 4. We formalize this next.

Proposition 4. ***(Optimal Taxes)*** *Under Assumption 2, there is a unique combination of taxes $(\tau_a^\star, \tau_k^\star)$ that maximizes utilitarian welfare $\mathcal{W}$. An interior solution $\tau_a^\star < \overline{\tau}_a$ solves:*

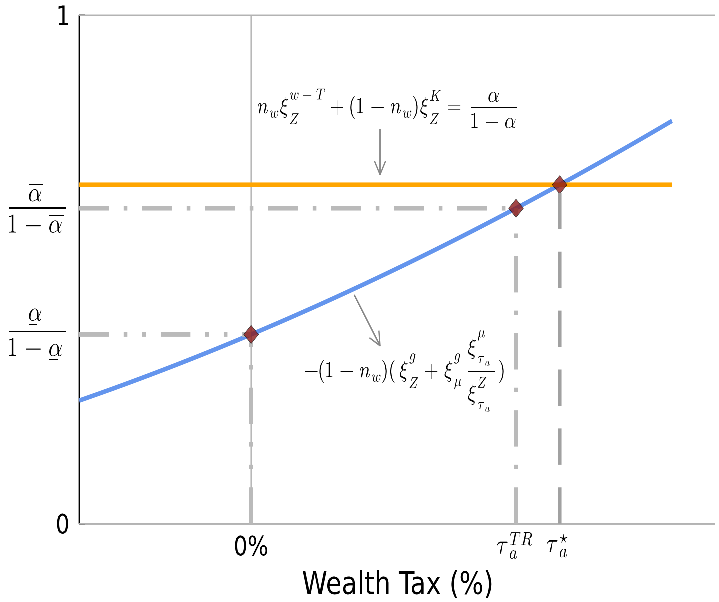
$$0 = \Bigg(\underbrace{n_w\xi_Z^{w+T} + (1-n_w)\,\xi_Z^K}_{\textit{Level Effect}=\frac{\alpha}{1-\alpha}(+)} + (1-n_w)\underbrace{\xi_Z^g}_{\textit{Growth Effect}\,(-)}\Bigg)\frac{d\log Z}{d\tau_a} + (1-n_w)\underbrace{\xi_\mu^g}_{\textit{Innovation Effect}\,(+)}\frac{d\mu}{d\tau_a}, \tag{61}$$

where $\xi_Z^x \equiv \frac{\partial \log x}{\partial \log Z}$ is the elasticity of x with respect to Z and $\xi_\mu^x \equiv \frac{\partial \log x}{\partial \mu}$ is the semi-elasticity with respect to μ. From Lemma 3, $\xi_Z^{w+T} = \xi_Z^K = \frac{\alpha}{1-\alpha}$, so that this condition can be restated as

$$\frac{\alpha}{1-\alpha} = -\,(1-n_w)\left[\xi_Z^g + \xi_\mu^g \times \frac{d\mu}{d\tau_a}\Big/\frac{dlogZ}{d\tau_a}\right]. \tag{62}$$

[29] An important insight is that the level and growth effects can coexist even without the countervailing effect of innovation on lifetime wealth growth. This is because the negative growth effect associated to changes in productivity is different ex-post for the H-type and the L-type, allowing for higher aggregate wealth even as entrepreneurs expect lower wealth growth over their lifetimes. The increase in returns that comes from increased innovation reinforces this result.

Figure 4: Determination of the Optimal Combination of Wealth and Capital Income Taxes



Note: The figure shows the conditions satisfied by the optimal wealth tax solving (61). The horizontal line is the (population) average of the elasticity of workers' income and capital with respect to productivity, ξ_Z^{w+T} and ξ_Z^K, respectively. The increasing line is proportional to the negative of the average elasticity of returns with respect to τ_a $\left(\xi_{\tau_a}^g = \xi_Z^g + \xi_\mu^g \cdot \xi_{\tau_a}^\mu / \xi_Z^\mu\right)$. The optimal wealth tax is denoted by $\tau_a^\star$, the tax reform tax level is $\tau_a^{TR} = \frac{\theta(1-\beta\delta)}{\beta\delta(1-\theta)}$, the level at which $\tau_k = 0$. Other parameters are: $\delta = 49/50$, $\beta\delta = 0.96$, $z_h = 1$, $\theta = 025$, and $\alpha = 0.4$. λ is such that the debt-to-output ratio is 1.5 when $\tau_a = 0$.

This implies two cutoff values for α, $\underline{\alpha}$ and $\overline{\alpha}$, such that $(\tau_a^\star, \tau_k^\star)$ satisfies:

$$\tau_a^\star \in \left[1 - \frac{1}{\beta\delta}, 0\right) \text{ and } \tau_k^\star > \theta \qquad \text{if } \alpha < \underline{\alpha}$$

$$\tau_a^\star \in \left[0, \frac{\theta(1-\beta\delta)}{\beta\delta(1-\theta)}\right] \text{ and } \tau_k^\star \in [0, \theta] \qquad \text{if } \underline{\alpha} \leq \alpha \leq \bar{\alpha}$$

$$\tau_a^\star \in \left(\frac{\theta(1-\beta\delta)}{\beta\delta(1-\theta)}, \tau_a^{\max}\right) \text{ and } \tau_k^\star < 0, \qquad \text{if } \alpha > \bar{\alpha}$$

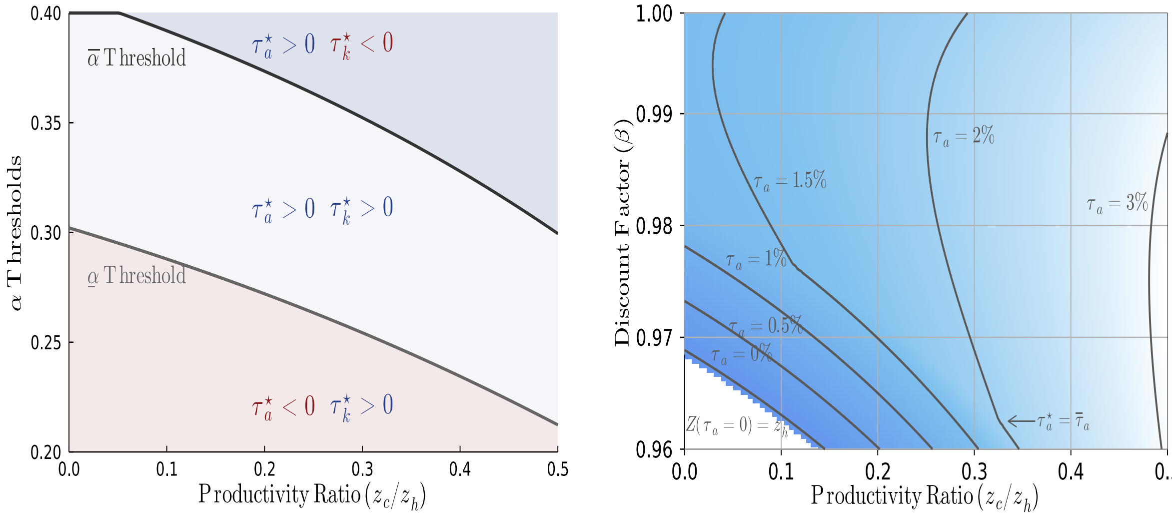
where $\tau_a^{\max} \geq 1$, $\underline{\alpha}$ and $\overline{\alpha}$ are the solutions to equation (61) with $\tau_a = 0$ and $\tau_a = \frac{\theta(1-\beta\delta)}{\beta\delta(1-\theta)}$, respectively. When $\theta = 0$, and there are no revenue needs, so $\underline{\alpha} = \overline{\alpha}$.

Figure 4 also clarifies the roles of the thresholds $\underline{\alpha}$ and $\overline{\alpha}$.[30] The lower threshold $\underline{\alpha}$ marks the level of $n_w\xi_Z^{w+T} + (1-n_w)\,\xi_Z^K$ for which $\tau_a = 0$ is optimal. Any $\alpha > \underline{\alpha}$ implies a higher scope for workers' income and capital to rise with the wealth tax and thus a positive optimal wealth tax. The upper threshold $\overline{\alpha}$ is similarly defined by the level of $n_w\xi_Z^{w+T} + (1-n_w)\,\xi_Z^K$

[30] The value of the thresholds depend on Z, μ, and returns, which are endogenous but independent of α (Proposition 1, and equations 14 and 35).

Figure 5: Optimal Taxes

(a) α Thresholds for Optimal Tax Combination (b) Optimal Wealth Tax



Note: The left panel plots threshold values of α for the optimal combination of wealth and capital income taxes for different levels of productivity dispersion (z_c/z_h). The right panel plots the optimal wealth tax rate for combinations of β and z_c/z_h. Other parameters are (when not being varied): $\delta = 49/50$, $\beta\delta = 0.96$, $z_h = 1$, $\theta = 025$, and $\alpha = 0.4$. λ is such that the debt-to-output ratio is 1.5 when $\tau_a = 0$.

for which $\tau_a = \tau_a^{TR} \equiv \theta\,(1-\beta\delta)\,/\beta\delta\,(1-\theta)$ is optimal. At that level, the wealth tax finances all government spending, so $\tau_k = 0$. Consequently, any $\alpha > \overline{\alpha}$ implies that the optimal tax combination is one of a positive wealth tax and a capital income subsidy. Finally, the upper bound on the wealth tax ($\tau_a^{\max}$) ensures that R_ℓ remains positive.

This result highlights the crucial role that the pass-through of productivity to wages and wealth plays in determining the optimal mix of taxes on capital. Shifting the tax mix towards wealth taxes is only desirable in as much as the productivity gains it generates translate into higher incomes for workers and higher aggregate wealth.

Figure 5a shows how the thresholds for α vary with the dispersion in productivity between the H-type and corporate firms when we set $\theta = 0.25$, implying a capital income tax rate of 25% in the absence of wealth taxes. Both thresholds decline as the dispersion of productivity decreases and maintain a gap of about 0.1 that includes the typical values of α used in the literature, between 0.3 and 0.4. For example, a value of α of $1/3$ is always in the intermediate range, implying positive values for the optimal levels of capital income and wealth taxes, while a value of α of 0.4 implies a positive wealth tax and a capital income subsidy if $z_c/z_h \geq 0.1$.

Figure 5b shows the levels of the optimal wealth tax for different combination of parameters, holding α fixed at 0.4. The optimal wealth tax is positive except for corner cases where $\overline{\tau}_a < 0$ (condition 38 in Proposition 2). As the dispersion of productivity decreases, or entrepreneurs are more patient, there is more misallocation and a higher optimal wealth tax rate in the range of 0 to 2 percent for most parameter combinations. The flip side of this pattern is the decrease in the optimal capital income tax eventually becomes a subsidy as the optimal wealth tax increases.

6 Managerial Effort

We now endogenize the *level* of productivity through *managerial effort* choice, m, every period, complementing the endogenous response of the *distribution* of productivity through innovation choice at the beginning of life. Managerial effort augments entrepreneurial productivity z each period but does not affect the share of H-types, μ. We show that capital income and wealth taxes have different effects on the managerial effort choice of entrepreneurs. Only the capital income tax decreases managerial effort directly because it reduces the marginal benefit from exerting effort, while the wealth tax does not distort this margin. This introduces an additional channel through which shifting the tax mix away from capital income taxes increases output and welfare.

We introduce managerial effort, m, in a tractable manner that allows us to identify its core implications for wealth and capital income taxation and solve the model analytically. Managerial effort affects production according to

$$y = (zk)^{\alpha} m^{\gamma} n^{1-\alpha-\gamma}; \quad 0 \leq \gamma < 1 - \alpha. \tag{63}$$

Exerting effort has a utility cost that we capture by modifying the utility function to

$$u(c, m) = \log(c - h(m)), \tag{64}$$

where $h(m) = \psi m$ and $\psi > 0$.[31] Tractability depends on preserving the constant-returns-to-scale in production and abstracting from income effects in the effort choice as in Greenwood, Hercowitz and Huffman (1988).[32] We provide detailed derivations in Appendix D.

The solution of the problem with entrepreneurial effort inherits the properties of our benchmark model in Section 2 after suitable change of variables. We define consumption

[31] More generally, we can let effort affect production according to an increasing function $g(m)$, and we only require that the ratio $h'(m)/g'(m)$ is constant. See Appendix D.

[32] See Ring (2024b) for recent evidence that households respond to wealth taxation by exerting more labor effort and increasing savings, highlighting the role of the income effect.

net of effort costs as $\hat{c} = c - h(m)$ and write the problem as

$$V(a,z) = \max_{\{a', \hat{c}, m \geq 0;\, k \geq \lambda a\}} \log(\hat{c}) + \beta\delta V(a', z) \tag{65}$$

$$\text{s.t. } \hat{c} + a' = \underbrace{\left((1-\tau_a) + (1-\tau_k)\left(r + \frac{\hat{\pi}(z,k,m)}{a} \right) \right)}_{\hat{R}(z)} a,$$

where $\hat{\pi}$ stands for profits net of effort costs:

$$\hat{\pi}(z,k,m) = \max_{\{n\}} \left\{ \quad y - wn - rk - \underbrace{\frac{1}{1-\tau_k}}_{\text{Effective Effort Cost}} h(m) \quad \right\}. \tag{66}$$

Notice how $1/(1-\tau_k)$ scales the disutility of effort, so that a higher τ_k effectively raises the marginal cost of effort (that is, of course, not tax deductible). The wealth tax does not affect this margin as it operates over assets, a, independently of entrepreneurial profits. These facts are behind our key results below.

Interestingly, while the role of capital income taxes in the managerial effort choice of entrepreneurs changes the relationship between capital income taxes and aggregate output, capital, and wages, it does not change the equilibrium behavior of aggregate productivity (equation 35). This is because the capital level adjusts in steady state so that the after-tax return net of effort costs satisfies

$$\hat{R}(z) = (1-\tau_a) + \left(\frac{1}{\beta\delta} - (1-\tau_a) \right) \frac{z}{Z}, \tag{67}$$

just as in Lemma 1, preserving the neutrality of τ_k for returns and productivity. Consequently, the results of our benchmark model regarding the existence of a stationary competitive equilibrium and the innovation and efficiency gains from wealth taxation (Propositions 2 and 3) remain unchanged, as does the behavior of after tax returns (Lemma 2).

Turning to the effects of taxes on aggregate variables, we obtain closed-form expressions for equilibrium quantities as a function of aggregate capital, K, and productivity, Z, paralleling the results of Lemma 1. Our first main result can be seen by inspecting the expression for aggregate managerial effort:

$$M = \left(\frac{(1-\tau_k)\gamma}{\psi} \right)^{\frac{1}{1-\gamma}} (ZK)^{\frac{\alpha}{1-\gamma}} L^{\frac{1-\alpha-\gamma}{1-\gamma}}. \tag{68}$$

The appearance of $(1-\tau_k)$ in the numerator shows that capital income taxation disincentivizes effort in equilibrium because it reduces the after-tax marginal product of effort. Consequently, the capital income tax also reduces aggregate output and wages,

$$Y = \left(\frac{(1-\tau_k)\,\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}} (ZK)^{\frac{\alpha}{1-\gamma}} L^{\frac{1-\alpha-\gamma}{1-\gamma}} \quad \text{and} \quad w = (1-\alpha-\gamma)\left(\frac{(1-\tau_k)\,\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{ZK}{L}\right)^{\frac{\alpha}{1-\gamma}}. \tag{69}$$

By contrast, wealth taxes do not directly affect the effort choice because they do not affect the fraction of profits retained by the entrepreneur and instead affect production and wages only through their effect on aggregate productivity.

The effect of capital income taxes on entrepreneurial effort introduces a new channel affecting the optimal tax combination: Shifting the tax mix away from capital income taxes incentivizes entrepreneurial effort and, through it, increases aggregate output, capital, and wages, as equations (68) to (69) make clear. This mechanism, that we term the *managerial-productivity effect*, works on top of the increase in capital, output, and wages already coming from increased productivity, described in Lemma 3, which is now modified as well by the role of effort in production, γ. Put in terms of the trade-off described in Section 5, entrepreneurial effort *strengthens the (positive) level effect* of a wealth tax without changing the growth effect (because of the neutrality of the capital income tax for after-tax returns). The result is an optimal tax combination that now involves a higher wealth tax and a lower capital income tax.

Proposition 5. *Under Assumption 2, there exists a unique tax combination $\left(\tau^\star_{a,m}, \tau^\star_{k,m}\right)$ that maximizes $\mathcal{W}$. An interior solution $\tau^\star_{a,m} < \overline{\tau}_a$ solves*

$$0 = \Bigg(\underbrace{\frac{\alpha}{1-\alpha-\gamma}}_{\textit{Modified Level Effect }(+)} + (1-n_w)\underbrace{\xi^g_Z}_{\textit{Growth Effect }(-)} \Bigg)\frac{d\log Z}{d\tau_a} + (1-n_w)\underbrace{\xi^g_\mu}_{\textit{Innovation Effect }(+)}\frac{d\mu}{d\tau_a} + \underbrace{\frac{\gamma}{1-\alpha-\gamma}\beta\delta}_{\textit{Managerial-Productivity Effect }(+)} \tag{70}$$

where $\xi^x_Z \equiv \frac{\partial \log x}{\partial \log Z}$ is the elasticity of x with respect to Z and $\xi^x_\mu \equiv \frac{\partial \log x}{\partial \mu}$ is the (semi-) elasticity with respect to μ. Moreover, $\tau^\star_{a,m} > \tau^\star_a$, with $\tau^\star_a$ defined in (61).

7 Conclusions

In this paper, we have studied book-value wealth taxation and capital income taxation in an infinite-horizon economy with heterogeneous entrepreneurial productivity that responds endogenously to the mix of capital income and wealth taxes. We showed an important neutrality result that distinguishes the two forms of taxation: capital income taxation has no effect on the steady state after-tax marginal product of capital and returns, whereas the wealth tax does. In particular, the wealth tax increases the dispersion of after-tax returns. It is through this channel that changing the mix of capital taxation toward the wealth tax affects the distribution of entrepreneurial productivity: The powerful incentives provided by increasing return dispersion incentivize entrepreneurs' innovation and managerial effort. This, in turn, increases productivity, output, wages, and welfare.

We then characterized the optimal combination of capital income and wealth taxes that balances the gains from higher incomes for workers and an increase in wealth with the losses from higher return dispersion. We showed that it features a positive wealth tax if the increase in productivity that the wealth tax generates has a strong-enough pass-through into higher wages, capital, and innovation, something that happens when the capital intensity in the economy, captured by the capital share α, is above a threshold (around 0.3 for a wide range of parameters).

One feature we left out from our analysis is fluctuations in productivity during the life time of entrepreneurs. This could be of interest because such fluctuations would increase misallocation as some of the capital stock will be owned by previously productive entrepreneurs who have lost their productivity. To understand these and other ramifications, we study an infinite-horizon version of our model (without death) with productivity that evolves as a first-order Markov process. In Appendix E, we present this model and show that our main results (e.g., efficiency gains from wealth taxation; the increases in capital, output, and wages with the wealth tax; and the trade-offs that determine optimal taxation) continue to hold as long as entrepreneurial productivity is positively autocorrelated.

Before concluding, we want to discuss an alternative way to understand the importance of book-value wealth taxation and how it differs from levying the wealth tax on the market value. *First*, taxes on the book value of wealth operate very differently from taxes on the market value because they do not tax current or future returns. We can see this in the context of our baseline model where the market value of wealth of an entrepreneur with productivity z and a units of assets is given by the book value of their assets *and* the discounted value of their future returns (which depend on their productivity), with the market interest rate used for discounting:[33]

$$a + \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \delta^t \Pi\left(z, a_t\right) = \underbrace{a}_{\text{Book Value}} + \overbrace{\pi\left(z\right) a}^{\text{Current Capital Income}} + \underbrace{\frac{\beta\delta^2 \frac{R(z)}{1+r}}{1 - \beta\delta^2 \frac{R(z)}{1+r}} \pi\left(z\right) a}_{\text{Future (unrealized) Capital Income}} . \quad (71)$$

Therefore, a tax on market value wealth is a tax on the book value of assets, *plus* a tax on current returns (profits), *and* a tax on future (unrealized) returns. So, conceptually, a tax on the market value of wealth mixes the properties of book-value wealth taxes we have studied with those of a tax on (excess) returns, like the capital income tax.[34]

Second, the taxation of book value wealth helps to address many of the practical implementation issues raised by wealth taxes. While valuing the market value of infrequently-traded and closely-held assets is intrinsically hard, most tax agencies already have access to measures of the book value of private firms and other forms of wealth from standard accounting practices. This makes book-value wealth taxation a viable and theoretically grounded alternative to proposals of wealth taxation based on market values and to the more commonly used capital income tax based on realized returns.

[33] This corresponds to the value of a "growing" Lucas tree whose fruit grows depending on the initial "investment" in the tree, here given by a. The tree gives fruit as long as the entrepreneur is alive, hence the discounting by δ, and its fruit reflects the entrepreneur's savings, hence its future fruit depending on a_t capturing the savings rate. The market value of the entrepreneur's wealth is the value of their assets plus the value of their "tree," given by the present discounted value of its fruit.

[34] The returns of the L-type are given by r. Profits capture excess returns above this rate (see eq. 8) and are zero for these agents. Therefore, a book value wealth tax is the same as a market value wealth tax for individuals that have market returns.

References

Acemoglu, Daron and Martin Kaae Jensen, "Equilibrium Analysis in Behavioural One-Sector Growth Models," *The Review of Economic Studies*, 04 2024, *91* (2), 599–640.

Agrawal, David, Dirk Foremny, and Clara Martínez-Toledano, "Wealth Tax Mobility and Tax Coordination," *American Economic Journal: Applied Economics (Forthcoming)*, 2024.

Aguiar, Mark A, Benjamin Moll, and Florian Scheuer, "Putting the "Finance" into "Public Finance": A Theory of Capital Gains Taxation," Working Paper 32951, National Bureau of Economic Research September 2024.

Aiyagari, S. Rao, "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting," *Journal of Political Economy*, 1995, *103* (6), 1158–1175.

Allais, Maurice, *L'impôt Sur Le Capital et la Réforme Monétaire*, Hermann, 1977.

Bach, Laurent, Laurent E. Calvet, and Paolo Sodini, "Rich Pickings? Risk, Return, and Skill in Household Wealth," *American Economic Review*, 2020, *110* (9), 2703–2747.

Banerjee, Abhijit and Andrew F. Newman, "Inequality, Growth, and Trade Policy," 2003. http://piketty.pse.ens.fr/files/BanerjeeNewman2003.pdf.

Beare, Brendan K. and Alexis Akira Toda, "Optimal taxation and the Domar-Musgrave effect," 2023.

Benhabib, Jess, Alberto Bisin, and Shenghao Zhu, "The Distribution of Wealth and Fiscal Policy in Economies With Finitely Lived Agents," *Econometrica*, 2011, *79* (1), 123–157.

_ **and** _ , "Skewed Wealth Distributions: Theory and Empirics," *Journal of Economic Literature*, 2018, *56* (4), 1261–1291.

Benmelech, Efraim, Carola Frydman, and Dimitris Papanikolaou, "Financial frictions and employment during the Great Depression," *Journal of Financial Economics*, 2019, *133* (3), 541–563. JFE Special Issue on Labor and Finance.

Boar, Corina and Matthew Knowles, "Optimal taxation of risky entrepreneurial capital," *Journal of Public Economics*, 2024, *234*, 105100.

— **and Virgiliu Midrigan**, "Should We Tax Capital Income or Wealth?," *American Economic Review: Insights*, 2023, *5* (2), 259–74.

Border, Kim C., *Fixed Point Theorems with Applications to Economics and Game Theory*, Cambridge University Press, 1985.

Brülhart, Marius, Jonathan Gruber, Matthias Krapf, and Kurt Schmidheiny, "Behavioral Responses to Wealth Taxes: Evidence from Switzerland," *American Economic Journal: Economic Policy*, 2022, *14* (4), 111–50.

Buera, Francisco J. and Yongseok Shin, "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 2013, *121* (2), 221–272.

— **, Joseph P. Kaboski, and Yongseok Shin**, "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 2011, *101* (5), 1964–2002.

Cagetti, Marco and Mariacristina De Nardi, "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, 2006, *114* (5), 835–870.

Campbell, John Y., Tarun Ramadorai, and Benjamin Ranish, "Do the Rich Get Richer in the Stock Market? Evidence from India," *American Economic Review: Insights*, 2019, *1* (2), 225–240.

Chamley, Christophe, "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 1986, *54* (3), 607–622.

Champernowne, D. G., "A Model of Income Distribution," *The Economic Journal*, 1953, *63* (250), 318–351.

Conesa, Juan Carlos, Sagiri Kitao, and Dirk Krueger, "Taxing Capital? Not a Bad Idea After All!," *American Economic Review*, 2009, *99* (1), 25–48.

Duygan-Bump, Burcu, Alexey Levkov, and Judit Montoriol-Garriga, "Financing constraints and unemployment: Evidence from the Great Recession," *Journal of Monetary Economics*, 2015, *75*, 89–105.

Erosa, Andrés and Martin Gervais, "Optimal Taxation in Life-Cycle Economies," *Journal of Economic Theory*, 2002, *105* (2), 338–369.

Fagereng, Andreas, Luigi Guiso, Davide Malacrino, and Luigi Pistaferri, "Heterogeneity and Persistence in Returns to Wealth," *Econometrica*, 2020, *88* (1), 115–170.

Ferey, Antoine, Benjamin B. Lockwood, and Dmitry Taubinsky, "Sufficient Statistics for Nonlinear Tax Systems with General Across-Income Heterogeneity," *American Economic Review*, October 2024, *114* (10), 3206–49.

Gabaix, Xavier, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll, "The Dynamics of Inequality," *Econometrica*, 2016, *84* (6), 2071–2111.

Gaillard, Alexandre and Philipp Wangner, "Wealth, Returns, and Taxation: A Tale of Two Dependencies," Technical Report, SSRN 2022. https://ssrn.com/abstract=3966130.

Garriga, Carlos, "Optimal Fiscal Policy in Overlapping Generations Models," *Public Finance Review*, 2019, *47* (1), 3–31.

Gerritsen, Aart, Bas Jacobs, Kevin Spiritus, and Alexandra V Rusu, "Optimal Taxation of Capital Income with Heterogeneous Rates of Return," *The Economic Journal*, 09 2024, p. ueae083.

Gomes, João F., Amir Yaron, and Lu Zhang, "Asset Pricing Implications of Firms' Financing Constraints," *The Review of Financial Studies*, 03 2006, *19* (4), 1321–1356.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman, "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, June 1988, *78* (3), 402–17.

Guvenen, Fatih, Gueorgui Kambourov, Burhan Kuruscu, Sergio Ocampo, and Daphne Chen, "Use It or Lose It: Efficiency and Redistributional Effects of Wealth Taxation," *The Quarterly Journal of Economics*, 01 2023, *138* (2), 835–894.

_ , _ , Burhanettin Kuruscu, Sergio Ocampo, and Daphne Chen, "Use It or Lose It: Efficiency Gains from Wealth Taxation," Working Paper 26284, National Bureau of Economic Research September 2019.

Hvide, Hans K. and Jarle Møen, "Lean and Hungry or Fat and Content? Entrepreneurs' Wealth and Start-Up Performance," *Management Science*, 2010, *56* (8), 1242–1258.

Imrohoroglu, Selahattin, "A Quantitative Analysis of Capital Income Taxation," *International Economic Review*, 1998, *39*, 307–328.

Itskhoki, Oleg and Benjamin Moll, "Optimal Development Policies With Financial Frictions," *Econometrica*, 2019, *87* (1), 139–173.

Jakobsen, Katrine, Henrik Kleven, Jonas Kolsrud, Camille Landais, and Mathilde Muñoz, "Wealth Taxation and Migration Patterns of the Very Wealthy," Working Paper, Princeton University 2023.

_ **, Kristian Jakobsen, Henrik Kleven, and Gabriel Zucman**, "Wealth Taxation and Wealth Accumulation: Theory and Evidence From Denmark," *The Quarterly Journal of Economics*, 10 2019, *135* (1), 329–388.

Jones, Charles I., "Pareto and Piketty: The Macroeconomics of Top Income and Wealth Inequality," *Journal of Economic Perspectives*, 2015, *29* (1), 29–46.

_ , "Taxing Top Incomes in a World of Ideas," *Journal of Political Economy*, 2022, *130* (9), 2227–2274.

_ **and Jihee Kim**, "A Schumpeterian Model of Top Income Inequality," *Journal of Political Economy*, 2018, *126* (5), 1785–1826.

Judd, Kenneth L., "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 1985, *28* (1), 59–83.

Kitao, Sagiri, "Labor-Dependent Capital Income Taxation," *Journal of Monetary Economics*, 2010, *57* (8), 959–974.

Londoño-Vélez, Juliana and Javier Ávila-Mahecha, "Enforcing Wealth Taxes in the Developing World: Quasi-experimental Evidence from Colombia," *American Economic Review: Insights*, 2021, *3* (2), 131–48.

Moll, Benjamin, "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?," *American Economic Review*, 2014, *104* (10), 3186–3221.

Piketty, Thomas, *Capital in the Twenty-First Century*, Cambridge: Belknap of Harvard University Press, 2014.

Quadrini, Vincenzo, "Entrepreneurship, Saving, and Social Mobility," *Review of Economic Dynamics*, 2000, *3* (1), 1–40.

Ring, Marius, "Entrepreneurial Wealth and Employment: Tracing Out the Effects of a Stock Market Crash," *The Journal of Finance*, 2023, *78* (6), 3343–3386.

Ring, Marius A K, "Wealth Taxation and Household Saving: Evidence from Assessment Discontinuities in Norway," *The Review of Economic Studies*, 10 2024, pp. 1–30.

— , “Wealth Taxation and Household Saving: Evidence from Assessment Discontinuities in Norway,” *The Review of Economic Studies*, 10 2024, p. rdae100.

Saez, Emmanuel and Stefanie Stantcheva, “A simpler theory of optimal capital taxation,” *Journal of Public Economics*, 2018, *162*, 120–142. In Honor of Sir Tony Atkinson (1944-2017).

Scheuer, Florian and Joel Slemrod, “Taxing Our Wealth,” *Journal of Economic Perspectives*, 2021, *35* (1), 207–30.

Seim, David, “Behavioral Responses to Wealth Taxes: Evidence from Sweden,” *American Economic Journal: Economic Policy*, 2017, *9* (4), 395–421.

Smith, Matthew, Owen Zidar, and Eric Zwick, “Top Wealth in America: New Estimates under Heterogeneous Returns,” *Quarterly Journal of Economics*, 2023, *138* (1), 515–573.

Straub, Ludwig and Iván Werning, “Positive long-run capital taxation: Chamley-Judd revisited,” *American Economic Review*, 2020, *110* (1), 86–119.

Vermeulen, Philip, “How Fat is the Top Tail of the Wealth Distribution?,” *Review of Income and Wealth*, 2018, *64* (2), 357–387.

Villas-Boas, J.Miguel, “Comparative Statics of Fixed Points,” *Journal of Economic Theory*, 1997, *73* (1), 183–198.

ONLINE APPENDIX

Not for Publication

A Further Equations for the Benchmark Model

A.1 Entrepreneur's Problem

Entrepreneurial Production. An entrepreneur's labor demand given a level of capital solves:

$$\pi(z,k) = \max_{n} \left\{ (zk)^{\alpha} n^{1-\alpha} - wn \right\},$$

which yields the labor demand function in equation (6). Substituting (6) into (5) allows us to solve for the entrepreneur's capital choice in equation (7) that implies an optimal capital choice:

$$k(z,a) \quad = \quad \begin{cases} \lambda a & \text{if } \alpha \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha} z > r \\ [0, \lambda a] & \text{if } \alpha \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha} z = r \\ 0 & \text{if } \alpha \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha} z < r. \end{cases}$$

Replacing the capital choice and (6) into (5) yields the optimal entrepreneurial income in (8).

Entrepreneurial Savings. Given constant taxes and prices, the savings problem is

$$V_i(a) \quad = \quad \max_{a'} \left\{ \log\left(R_i a - a'\right) + \beta\delta V\left(a'\right) \right\},$$

where $R_i = R(z_i)$ is defined as in (10) for $i \in \{\ell, h\}$.

We solve the entrepreneur's saving problem via guess and verify. To this end, we guess that the value function of an entrepreneur with productivity z_i, $i \in \{\ell, h\}$, has the form

$$V_i(a) = m_i + n \log(a),$$

where $m_\ell, m_h, n \in \mathbb{R}$ are coefficients. Under this guess, the optimal savings choice is

$$a_i^{'} = \frac{\beta\delta n}{1+\beta\delta n} R_i a.$$

Replacing the savings rule into the value function gives

$$V_i(a) = \log\left(R_i a - a_i^{'}\right) + \beta\delta V_i\left(a_i^{'}\right)$$
$$m_i + n\log(a) = \beta\delta n \log(\beta\delta n) + (1+\beta\delta n)\log\left(\frac{R_i}{1+\beta\delta n}\right) + \beta\delta m_i + (1+\beta\delta n)\log(a)$$

Matching coefficients we solve for $n = \frac{1}{1-\beta\delta}$. This in turn delivers the optimal saving decision of the entrepreneur in equation (11) with constant saving rate $\beta\delta$. We solve for the remaining coefficients from the system of linear equations:

$$m_i = \frac{1}{(1-\beta\delta)^2} \left[\log(\beta\delta)^{\beta\delta}(1-\beta\delta)^{1-\beta\delta} + \log R_i\right]$$

The value of an entrepreneur with productivity z_i, $i \in \{\ell, h\}$, is then

$$V_i(a) = \frac{\log(\beta\delta)^{\beta\delta}(1-\beta\delta)^{1-\beta\delta}}{(1-\beta\delta)^2} + \frac{1}{(1-\beta\delta)^2}\log R_i + \frac{1}{1-\beta\delta}\log(a).$$

A.2 Stationary Recursive Competitive Equilibrium

Definition. A stationary recursive competitive equilibrium *with innovation*, given a government tax policy $\mathcal{T} \equiv (\tau_a, \tau_k)$ and transfer T, consists of a value function for workers V_w, entrepreneurial value functions $\mathbb{V}_0$, V_h, and V_ℓ, entrepreneurial innovation effort choice at age zero, $e(Z)$, entrepreneurial policy functions $a'(a,z)$ and $c(a,z)$, entrepreneurial operating value and policy functions, $\Pi(a,z)$, $k(a,z)$, and $n(a,z)$, prices r and w, an endogenous share of H-type entrepreneurs $\mu^\star$, and a distribution of entrepreneurs $\Gamma(a,z)$ such that

i. Effort choice, $e(Z)$, solves the optimal innovation problem (13) as in (14), and $\mathbb{V}_0$ is the associated ex ante value function;

ii. V_i satisfies the Bellman equation (9) for type-i entrepreneurs' consumption-saving problem, and a' and c are the corresponding policy functions, given r and w; and $V_w = \frac{1}{1-\beta\delta}\log(w+T)$;

iii. Π is the solution to the entrepreneurs' production problem in (5), and k and n are the corresponding policy functions, given r and w;

iv. the labor markets clears: $L = \int n(a,z)\, d\Gamma$;

v. the capital (and bond) market clears: $K \equiv \int k(a,z)\, d\Gamma = \int a d\Gamma$;

vi. r and w satisfy the marginal conditions of the entrepreneurs' profit optimization problem (equations 21 and 20)

vii. the goods market clears:

$$G + wL + \int c(a,z)\, d\Gamma + \int (a^{'}(a,z)\, d\Gamma = K + \int (zk(a,z))^\alpha (n(a,z))^{1-\alpha} d\Gamma + (z_c K_c)^\alpha N_c^{1-\alpha};$$

viii. the government budget constraint is satisfied: $G + T = \tau_k \alpha Y + \tau_a K$;

ix. the distribution of wealth is constant over time and consistent with the saving choices of entrepreneurs, $a^{'}$, and the birth-death process.

X. The aggregate share of high-productivity entrepreneurs, $\mu^\star$ is consistent with the optimal effort choice $e(Z)$

$$\mu^\star = e(Z(\mu^\star)), \tag{72}$$

where$Z(\mu^\star)$ gives the stationary level of productivity given μ; that is, $Z(\mu^\star)$ is the solution to equation (35), for a given $\mu \in (0,1)$, and $e(Z)$ is the optimal effort choice defined in condition (i).

B Proofs: Benchmark Model

This appendix presents the proofs for the results listed in the paper, covering Sections 2 to 5. We reproduce the statement of all results for the reader's convenience.

B.1 Aggregation and Neutrality

Lemma 1. ***(Aggregate Variables in Equilibrium)*** *In equilibrium, aggregate output, the wage rate, the interest rate, and gross returns satisfy:*

$$\begin{aligned} Y &= (ZK)^{\alpha} L^{1-\alpha} \\ w &= (1-\alpha)\left({}^{ZK}\!/\!_{L}\right)^{\alpha} \\ r &= \alpha \left({}^{ZK}\!/\!_{L}\right)^{\alpha-1} z_c \\ R_\ell &= (1-\tau_a) + (1-\tau_k)\,\alpha \left({}^{ZK}\!/\!_{L}\right)^{\alpha-1} z_c \\ R_h &= (1-\tau_a) + (1-\tau_k)\,\alpha \left({}^{ZK}\!/\!_{L}\right)^{\alpha-1} z_\lambda . \end{aligned}$$

Proof. We start with labor market clearing, condition (24). Replacing in labor demand (6) we get

$$\left(\frac{1-\alpha}{w}\right)^{1/\alpha} \left(z_h \mu K_h + z_\ell (1-\mu) K_\ell\right) = L \quad \longrightarrow \quad \left(\frac{1-\alpha}{w}\right)^{1/\alpha} ZK = L$$

Manipulating this expression we get wages as in (20).

In the stationary heterogeneous-return equilibrium the interest rate is given by the returns of the corporate sector, so

$$r = \alpha \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} z_c = \alpha \left({}^{ZK}\!/\!_{L}\right)^{\alpha-1} z_c.$$

Then, the profit rate of the high-productivity entrepreneurs is (from 8)

$$\pi(z_h) = \left(\alpha \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha} z_h - r\right)\lambda = \alpha \left({}^{ZK}\!/\!_{L}\right)^{\alpha-1} (z_h - z_c)\,\lambda$$

Replacing into (10) gives the gross returns of entrepreneurs as in (22) and (23).

Finally, we consider aggregate output. We aggregate in terms of the aggregate capital of H-type entrepreneurs and the corporate sector because the ratio of labor to capital is constant across firms, equation (6). The output of a firm with productivity z and capital k is:

$$y(z,k) = \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha} zk = \left({}^{ZK}\!/\!_{L}\right)^{\alpha-1} zk,$$

where the second equality comes after replacing the equilibrium wage level. Aggregate output is the sum of the output produced by all firms:

$$Y = \left({}^{ZK}\!/\!_{L}\right)^{\alpha-1} \left(z_h \mu K_h + z_c (1-\mu) K_c\right) = (ZK)^{\alpha} L^{1-\alpha}.$$

This completes the derivation of the results.

□

Proposition 1. ***(Capital Income Tax is Neutral for Returns. Wealth Tax is Not)*** *In the stationary heterogeneous-return equilibrium, the after-tax returns of the H-type and L-type are independent of the capital income tax rate but do depend on the wealth tax rate:*

$$R_\ell = 1-\tau_a+\left(\frac{1}{\beta\delta}-(1-\tau_a)\right)\frac{z_c}{Z} \quad \text{and} \quad R_h = 1-\tau_a+\left(\frac{1}{\beta\delta}-(1-\tau_a)\right)\frac{z_\lambda}{Z}.$$

In particular, the wealth tax has a "use-it-or-lose-it" effect that changes the dispersion of returns and therefore the level of wealth inequality, whereas the capital income tax has no distributional effects.

Proof. The proof is immediate by replacing (31) into the expression for returns of low- and high-productivity entrepreneurs in terms of aggregate variables obtained in Lemma 1. Moreover, the wealth-weighted return depends only on the entrepreneurial saving rate,

$$s_h R_h+(1-s_h)R_\ell = (1-\tau_a)+\left(\frac{1}{\beta\delta}-(1-\tau_a)\right)\frac{s_h z_\lambda+(1-s_h)z_c}{Z}=\frac{1}{\beta\delta}.$$

□

B.2 Auxiliary results taking μ as given

The following two lemmas establish the existence of a stationary equilibrium and the effect of wealth taxes on productivity. The derivations follow from equation (35).

Lemma B.1. ***(Existence and Uniqueness of Stationary Heterogeneous-Return Equilibrium given μ)***

For $\mu \in (0,1)$, a stationary competitive equilibrium exists and is unique if and only if λ satisfies the condition in (39). This equilibrium is characterized by an endogenous productivity level Z that satisfies $z_c < Z < z_h$, and features return heterogeneity ($R_h > R_\ell$). In addition, the wealth share of the H-type satisfies $s_h < 1/\lambda$.

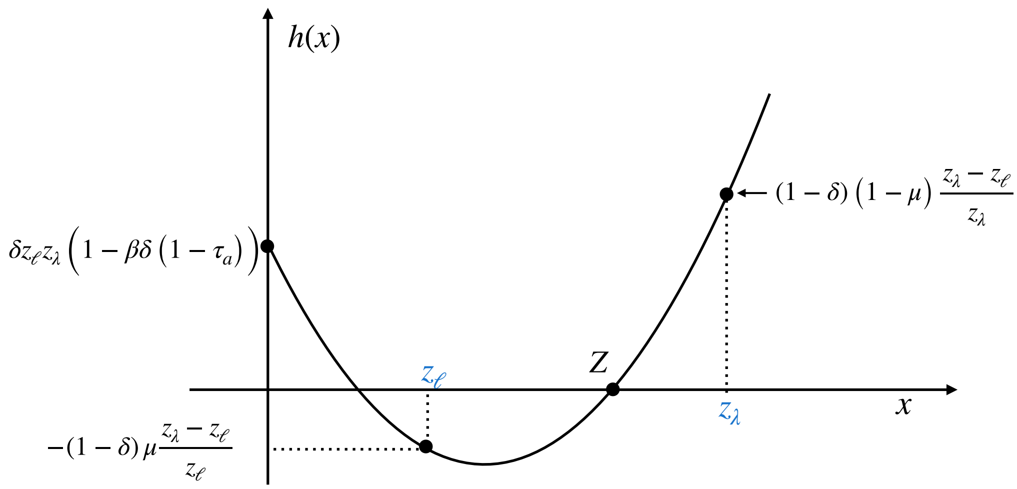
Proof. Equating the two conditions in (34) we obtain:

$$1=(1-\delta)\frac{1-\delta^2\beta\left((1-\mu)R_h+\mu R_\ell\right)}{\left(1-\delta^2\beta R_\ell\right)\left(1-\delta^2\beta R_h\right)}. \tag{73}$$

We then use the stationary value of returns from (33), which yields a quadratic equation that determines the steady state level of Z for a given value of μ as in (35).We characterize the equilibrium level of productivity, Z, by studying the behavior of the quadratic equation in (35), depicted in Figure B.1. Specifically, we show that there is a single admissible root in the interval

$$Z \in \left(\max\left\{z_c, \frac{\delta(1-\eta)}{1-\delta\eta}z_\lambda\right\}, z_\lambda\right).$$

Figure B.1: Stationary Competitive Equilibrium Productivity (Z)



Note: *The figure plots the polynomial* $h\left(x\right) = \left(1-\delta^2\beta\left(1-\tau_a\right)\right)x^2 - \left[\left(1-\delta\right)\left(\mu z_\lambda + \left(1-\mu\right)z_\ell\right) + \delta\left(1-\delta\beta\left(1-\tau_a\right)\right)\left(z_\lambda+z_\ell\right)\right]x + \delta\left(1-\delta\beta\left(1-\tau_a\right)\right)z_\ell z_\lambda = 0$ *that corresponds to equation (35). The stationary competitive equilibrium level of productivity corresponds to the larger root of h, marked with a circle on the horizontal axis.*

This interval is relevant for the proof of Lemma 6.

We start by defining the function

$$H\left(x\right) = \left(1-\delta\eta\right) - \frac{\left(1-\delta\right)\left(\mu z_\lambda + \left(1-\mu\right)z_c\right) + \delta\left(1-\eta\right)\left(z_\lambda + z_c\right)}{x} + \delta\left(1-\eta\right)\frac{z_c z_\lambda}{x^2}, \tag{74}$$

from the quadratic equation in (35), where $\eta \equiv \beta\delta\left(1-\tau_a\right)$. We verify directly that H has a root in the interval $\left(\max\left\{z_c, \frac{\delta(1-\eta)}{1-\delta\eta}z_\lambda\right\}, z_\lambda\right)$:

$$\begin{aligned} H\left(z_c\right) &= -\frac{\left(1-\delta\right)\mu}{z_c}\left(z_\lambda - z_c\right) < 0 \\ H\left(\frac{\delta\left(1-\eta\right)}{1-\delta\eta}z_\lambda\right) &= -\frac{\left(1-\delta\eta\right)\left(1-\delta\right)\mu}{\delta\left(1-\eta\right)}\frac{1}{z_\lambda}\left(z_\lambda - z_c\right) < 0 \\ H\left(z_\lambda\right) &= \frac{\left(1-\delta\right)\left(1-\mu\right)}{z_\lambda}\left(z_\lambda - z_c\right) > 0 \end{aligned}$$

The existence of the unique root is guaranteed by the intermediate value theorem and the fact that the function is quadratic.

Now we derive necessary and sufficient conditions for the equilibrium productivity level to satisfy $Z \in \left(z_c, z_h\right)$, so that the equilibrium features heterogenous returns $\left(R_h > R_\ell\right)$. Specifically we need $s_h < 1/\lambda$ to hold in equilibrium. This happens if and only if $Z < z_h$ (because $Z > z_c$ from its definition). So, we find a condition that guarantees that $H\left(z_h\right) > 0$ which implies that $Z < z_h$ because $H\left(Z\right) = 0$ and $H\left(z\right)$ is increasing in $z \geq Z$. The condition is

$$H\left(z_h\right) = \left(1-\delta\eta\right) - \frac{\left(1-\delta\right)\left(\mu z_\lambda + \left(1-\mu\right)z_c\right) + \delta\left(1-\eta\right)\left(z_\lambda + z_c\right)}{z_h} + \delta\left(1-\eta\right)\frac{z_c z_\lambda}{z_h^2} > 0,$$

after some manipulation this gives condition (39).

Finally, we verify that $z_h \geq \max\left\{z_c, \frac{\eta-\delta}{\eta} z_\lambda\right\}$. The first case is verified immediately, the second case applies if $\frac{\delta(1-\eta)}{1-\delta\eta} > \frac{z_c}{z_\lambda}$. A sufficient condition for $z_h \geq \frac{\delta(1-\eta)}{1-\delta\eta} z_\lambda$ is:

$$\begin{aligned}
z_h &\geq \frac{\delta(1-\eta)}{1-\delta\eta} z_\lambda \\
(1-\delta) &\geq \delta(1-\eta)(\lambda-1)\left(1-\frac{z_c}{z_h}\right) \\
(1-\delta) &\geq \delta(1-\eta)(\lambda-1)\left(1-\frac{\delta(1-\eta)}{1-\delta\eta}\right) \\
\frac{1-\delta\eta}{\delta(1-\eta)} &\geq \lambda-1
\end{aligned}$$

For this bound not to bind we need that it is above $\overline{\lambda}$:

$$\begin{aligned}
\frac{1-\delta\eta}{\delta(1-\eta)} &\geq \overline{\lambda}-1 \\
(1-\delta\eta)\left((1-\delta)\mu+\delta(1-\eta)\left(1-\frac{z_c}{z_h}\right)\right) &\geq (1-\delta)\delta(1-\eta)(1-\mu) \\
(1-\delta\eta)\delta(1-\eta)\left(1-\frac{z_c}{z_h}\right) &\geq (1-\delta)\left[\delta(1-\eta)-(\delta+1-2\delta\eta)\mu\right]
\end{aligned}$$

The condition is most stringent when $\mu = 0$ (counterfactually). This leads to a sufficient condition

$$\begin{aligned}
(1-\delta\eta)\left(1-\frac{z_c}{z_h}\right) &\geq (1-\delta) \\
\delta\frac{1-\eta}{1-\delta\eta} &\geq \frac{z_c}{z_h}
\end{aligned}$$

which is verified by assumption. So the upper bound $\overline{\lambda}$ is sufficient for $z_h \geq \max\left\{z_c, \frac{\delta(1-\eta)}{1-\delta\eta} z_\lambda\right\}$.

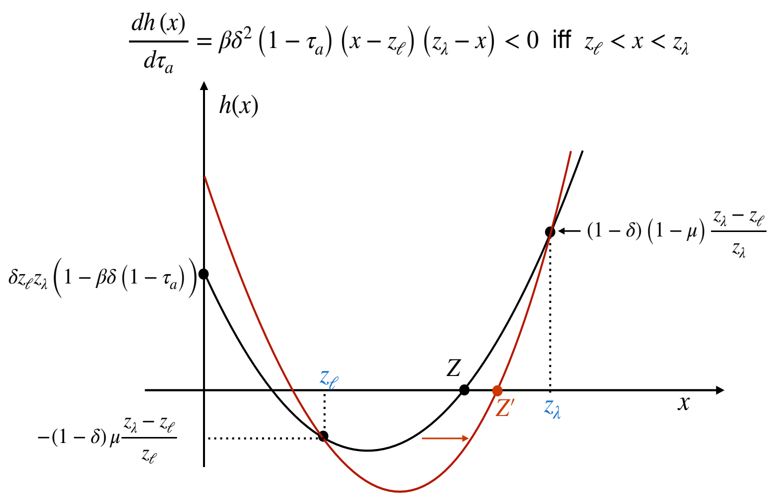
□

The next lemma establishes the effect of wealth taxes on productivity in the stationary equilibrium taking μ as given. We show directly the that an increase in wealth taxes raises aggregate productivity. The proof involves showing that the quadratic equation (35) discussed above that pins down Z shifts down and to the right when τ_a is raised, as shown in Figure B.2. The sketch of the proof is easy to see diagrammatically: the y-intercept of the polynomial h in Figure B.2 is given by $\delta z_c z_\lambda (1-\beta\delta(1-\tau_a))$, so it increases with τ_a, and the values of the parabola are fixed at z_c and z_λ (at values shown on the figure), forcing the x-intercepts (the roots of h) to shift to the right.

Lemma B.2. ***(Efficiency Gains from Wealth Taxation given*** μ***)*** *For* $\mu \in (0,1)$, *and for all* $\tau_a < \overline{\tau}_a$, *a higher wealth tax increases the steady-state aggregate productivity level,* $\frac{dZ}{d\tau_a} > 0$.

Proof. We use the auxiliary function H defined in the proof of Lemma B.1, equation (74). Simple

Figure B.2: Efficiency Gains from Wealth Taxation



Note: The figure plots the quadratic polynomial on the left hand side of equation (35) for τ_a (black line) and $\tau_a' > \tau_a$ (red line). The equilibrium productivity levels are given by the larger root, marked with the two circles on the horizontal axis.

manipulation of the function gives:

$$H(x;\tau_a) = F(x) - \left(1-\frac{z_\ell}{x}\right)\left(1-\frac{z_\lambda}{x}\right)\delta^2\beta(1-\tau_a),$$

where $F(x)$ is a function of only x that does not depend on taxes. We now establish that H is decreasing in τ_a for $x \in (z_\ell, z_\lambda)$, which is the interval of the equilibrium value of Z:

$$\frac{d\tilde{H}(x,\tau_a)}{d\tau_a} = \underbrace{\left(1-\frac{z_\ell}{x}\right)}_{(+)}\underbrace{\left(1-\frac{z_\lambda}{x}\right)}_{(-)}\delta^2\beta < 0.$$

This implies that $\frac{dZ}{d\tau_a} > 0$ because, as we proved in proposition B.1, H is increasing in x for $x \in (z_\ell, z_\lambda)$. See Figure B.2 for a graphical version of this proof.

$\square$

B.3 Response of Returns to Productivity and Innovation

Lemma 2. ***(Returns, Productivity, and Innovation)*** *An increase in productivity, Z, increases the rate of return of the H-type and reduces that of the L-type, holding μ constant,*

$$\xi_Z^{R_h} \equiv \frac{d\log R_h}{d\log Z} > 0n \qquad \text{and} \qquad \xi_Z^{R_\ell} \equiv \frac{d\log R_\ell}{d\log Z} < 0.$$

Furthermore, the population-weighted average of returns and log returns decline with τ_a:

$$\frac{d(\mu\log R_h + (1-\mu)\log R_\ell)}{d\log Z} = \mu\xi_Z^{R_h} + (1-\mu)\xi_Z^{R_\ell} < 0.$$

An increase in innovation, μ, has the opposite effects, so that,

$$\xi_{\mu}^{R_h} \equiv \frac{d \log R_h}{d\mu} < 0, \qquad \xi_{\mu}^{R_\ell} \equiv \frac{d \log R_\ell}{d\mu} > 0, \qquad \textit{and} \qquad \mu \xi_{\mu}^{R_h} + (1-\mu)\, \xi_{\mu}^{R_\ell} > 0.$$

Proof. **Response to productivity:** From the stationary level of wealth of high-productivity entrepreneurs we know that:

$$R_h = \frac{1}{\beta \delta^2}\left(1 - \frac{(1-\delta)\,\mu}{s_h}\right) \quad \longrightarrow \quad \frac{dR_h}{dZ} = \frac{(1-\delta)\,\mu}{\beta\delta^2} \frac{1}{s_h^2} \frac{ds_h}{dZ} > 0$$

A similar calculation delivers:

$$R_\ell = \frac{1}{\beta \delta^2}\left(1 - \frac{(1-\delta)\,(1-\mu)}{(1-s_h)}\right) \quad \longrightarrow \quad \frac{dR_\ell}{dZ} = -\frac{(1-\delta)\,(1-\mu)}{\beta\delta^2} \frac{1}{(1-s_h)^2} \frac{ds_h}{dZ} < 0.$$

With this we get:

$$\frac{d\left(\mu R_h + (1-\mu)\, R_\ell\right)}{d\tau_a} = \frac{1-\delta}{\beta\delta^2}\left(\frac{\left(\mu\,(1-s_h) + (1-\mu)\, s_h\right)(\mu - s_h)}{s_h^2\,(1-s_h)^2}\right)\frac{ds_h}{d\tau_a} < 0.$$

The sign follows because $s_h > \mu$ as proven above.

The average return elasticity is negative. To see this consider the weighted product of returns:

$$\begin{aligned}
\frac{dR_h^{\mu} R_\ell^{1-\mu}}{d\tau_a} &= (1-\mu)\, R_h^{\mu} R_\ell^{-\mu} \frac{dR_\ell}{d\tau_a} + \mu R_h^{\mu-1} R_\ell^{1-\mu} \frac{dR_h}{d\tau_a} \\
&< R_h^{\mu} R_\ell^{1-\mu} \frac{(1-\delta)}{\beta\delta^2} R_\ell \left[\frac{\left(\mu\,(1-s_h) + (1-\mu)\, s_h\right)(\mu - s_h)}{s_h^2\,(1-s_h)^2}\right] \frac{ds_h}{d\tau_a}
\end{aligned}$$

The inequality follows because $s_h < \mu$. This result implies that the average elasticity is negative.

Response to innovation: We first get the derivative of returns with respect to μ:

$$\frac{\partial R_h}{\partial \mu} = -\frac{1}{\beta\delta^2}\frac{(1-\delta)\,(z_\lambda - z_\ell)}{Z - z_\ell} < 0 \qquad \text{and} \qquad \frac{\partial R_\ell}{\partial \mu} = \frac{1}{\beta\delta^2}\frac{(1-\delta)\,(z_\lambda - z_\ell)}{z_\lambda - Z} > 0.$$

The average return elasticity is positive:

$$\begin{aligned}
\mu \frac{1}{R_h}\frac{\partial R_h}{\partial \mu} + (1-\mu)\frac{1}{R_\ell}\frac{\partial R_\ell}{\partial \mu} &= \frac{(1-\delta)}{\beta\delta^2}\left[-\frac{1}{R_h}\mu\frac{z_\lambda - z_\ell}{Z - z_\ell} + \frac{1}{R_\ell}(1-\mu)\frac{(z_\lambda - z_\ell)}{z_\lambda - Z}\right] \\
&= (1-\delta)\left[\frac{(1-\mu)\frac{(z_\lambda - z_\ell)}{z_\lambda - Z}}{1 - (1-\delta)\frac{(1-\mu)(z_\lambda - z_\ell)}{z_\lambda - Z}} - \frac{\mu\frac{z_\lambda - z_\ell}{Z - z_\ell}}{1 - (1-\delta)\frac{\mu(z_\lambda - z_\ell)}{Z - z_\ell}}\right] \\
&= \frac{1}{\delta}\left(\frac{(1-\delta\eta)\, Z - \delta\,(1-\eta)\, z_\ell}{\eta Z + (1-\eta)\, z_\ell} - \frac{(1-\delta\eta)\, Z - \delta\,(1-\eta)\, z_\lambda}{\eta Z + (1-\eta)\, z_\lambda}\right) > 0.
\end{aligned}$$

where we make use of an expression for the equilibrium level of μ obtained from equation (35) that must be satisfied in equilibrium. We write μ as follows,

$$\mu=\frac{Z-z_{\ell}}{z_{\lambda}-z_{\ell}}\left(1-\frac{\delta\left(1-\eta\right)}{1-\delta}\frac{\left(z_{\lambda}-Z\right)}{Z}\right)\quad\text{and}\quad 1-\mu=\frac{z_{\lambda}-Z}{z_{\lambda}-z_{\ell}}\left(1+\frac{\delta\left(1-\eta\right)}{1-\delta}\frac{\left(Z-z_{\ell}\right)}{Z}\right).$$

With this expression for μ we establish the effect of Z and μ on returns.

□

B.4 Existence of Stationary Equilibrium

We establish the existence of a unique fixed point on innovation effort (equivalently on the share of H-types), where effort implies productivity that implies, in turn, the original level of effort. This is captured by a mapping $\varphi:\mathcal{M}\to\mathcal{M}$ that takes as an input a share of high-productivity entrepreneurs, $\mu\in\mathcal{M}$, and provides the implied level of effort; hence, $\varphi\left(\mu\right)\equiv e^{\star}\left(Z\left(\mu\right)\right)\in\mathcal{M}$. The existence of the fixed point for φ follows from standard fixed point arguments relying on Cellina's and Brouwer's fixed point theorems (Border, 1985, Thms. 15.1, 16.1).

Uniqueness of the equilibrium follows from the monotonicity of the equilibrium mapping φ and standard comparative statics results for fixed points. To see this, we first describe the mapping $Z\left(\mu\right)$ from μ to equilibrium Z and then how Z affects innovation effort in $e^{\star}\left(Z\right)$.

We state a series of intermediate lemmas and then join them to prove our main result.

We start by inspecting equation (35) and show that equilibrium productivity is increasing in the share of high-productivity entrepreneurs.

Lemma B.3. *The equilibrium level of productivity, $Z\left(\mu\right)$, is increasing in the share of high-productivity entrepreneurs, μ.*

Proof. Define the correspondence $\gamma\left(\mu\right)\equiv\min\left\{z_{h},\text{Roots}^{+}\left(H,\mu\right)\right\}$ as the largest admissible root of the quadratic function H, as defined in (35), that determines equilibrium productivity. We want to show that the function $\gamma\left(\mu\right)$ is increasing in μ. An increase in μ increases the magnitude of the linear term in . Because the linear term is always negative, the increase in magnitude increases the value of the highest root of H. This proves the result.

□

Then, from Proposition 1 we can see that steady-state returns are decreasing in Z, in such a way that return dispersion declines with productivity given a wealth tax rate τ_{a}. As a result, innovation effort declines in Z.

Lemma B.4. *Innovation effort, $e^{\star}\left(Z\right)$, is decreasing in the level of productivity Z.*

Proof. Define the function $f\left(\mu,Z\right)\equiv\max\left\{\min\left\{\left(\Lambda^{\prime}\right)^{-1}\left(\log R_{h}-\log R_{\ell}\right),1\right\},0\right\}$ as the solution to (14), where $\left(\Lambda^{\prime}\right)^{-1}$ is the inverse of the derivative of Λ. We want to show that the function $f\left(\mu,Z\right)$ is decreasing in Z (we already know it is independent of μ given Z and for a fixed τ_{a}). To get the result, we show that an increase in Z decreases the dispersion in (log) returns, $\frac{d}{dZ}(\log R_h - \log R_\ell) < 0$. We show this directly using the expression of equilibrium returns as a function of Z in equation (33),

$$\frac{d}{dZ}\log R_\ell = \frac{\frac{d}{dZ}\left(1+\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_c}{Z}\right)}{1+\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_c}{Z}} = -\frac{\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_c}{Z^2}}{1+\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_c}{Z}};$$

$$\frac{d}{dZ}\log R_h = \frac{\frac{d}{dZ}\left(1+\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_\lambda}{Z}\right)}{1+\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_\lambda}{Z}} = -\frac{\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_\lambda}{Z}}{1+\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_\lambda}{Z}}.$$

Joining

$$\frac{d(\log R_h - \log R_\ell)}{dZ} = \frac{\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_c}{Z^2}}{1+\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_c}{Z}} - \frac{\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_\lambda}{Z}}{1+\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_\lambda}{Z}} < \frac{\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{(z_c-z_\lambda)}{Z^2}}{1+\left(\frac{1}{\beta\delta(1-\tau_a)}-1\right)\frac{z_\lambda}{Z}} < 0.$$

The decrease in the dispersion of (log) returns implies lower effort from the solution to (14).

□

Remark. These results describe the mapping from an arbitrary level of Z to returns and innovation. This is the relevant mapping for constructing the fixed point that constitutes an equilibrium, when τ_a and all the model's parameters are held fixed. It is this mapping from productivity to return dispersion that is decreasing in productivity. This is different from the result established in Lemma 2 that takes into account the equilibrium conditions of the economy (that is, taking into account that Z and s_h adjust to satisfy equation 35 when τ_a changes).

Proposition 2. ***(Existence of a Unique Stationary Equilibrium with Innovation)*** *There exists an upper bound for the wealth tax such that, for $\tau_a < \overline{\tau}_a$, there is a unique stationary equilibrium that features heterogeneous returns ($R_h > R_\ell$). That is, there is a unique value of the share of H-type entrepreneurs, $\mu^\star$, such that the optimal level of effort exerted by innovators satisfies $\mu^\star = e(Z(\mu^\star))$, and $Z(\mu^\star) \in (z_c, z_h)$ satisfies equation (35). The upper bound for the wealth tax satisfies*

$$\overline{\tau}_a = 1 - \frac{1}{\beta\delta}\left(1 - \frac{1-\delta}{\delta}\frac{1-\lambda\overline{\mu}(\overline{\tau}_a)}{(\lambda-1)\left(1-\frac{z_c}{z_h}\right)}\right),$$

where we make the dependence of $\mu^\star$ on τ_a explicit.

Proof. We first tackle the existence and then the uniqueness of the equilibrium.

Existence We provide two proofs of this result. The first one is longer but proves to be instructive of the workings of the model. It relies on Cellina's fixed point theorem, as found in Border (1985). The second one is more direct and relies on Brouwer's fixed point theorem. The objective in both cases is to show that the mapping of the share of high-productivity entrepreneurs into itself, defined by (14) (and the other equilibrium conditions), has a fixed point in the space $\mathcal{M} \equiv [0,1]$.

We start by stating Cellina's fixed point theorem. The theorem breaks the construction of a mapping $\varphi : \mathcal{M} \to \mathcal{M}$ in two steps that capture how the share of high-productivity entrepreneurs

μ implies a level of productivity Z that in turn implies a share μ through the level of returns. The theorem is as follows:

Theorem. ***[Cellina 1969; Border 1985, Thm. 15.1]*** *Let $\mathcal{M} \subseteq \mathbb{R}^m$ be nonempty, compact, and convex. Let $\varphi : \mathcal{M} \rightrightarrows \mathcal{M}$ be a correspondence defined on K. Suppose there is a nonempty-, compact-, and convex-valued correspondence $\gamma : \mathcal{M} \rightrightarrows \mathcal{K}$ defined on $\mathcal{M}$ with values in $\mathcal{K} \subseteq \mathbb{R}^n$, a compact and convex set, and also a continuous function $f : \mathcal{M} \times \mathcal{K} \to \mathcal{M}$ such that, for every $\mu \in \mathcal{M}$, $\varphi(\mu) = \{f(\mu, Z) | Z \in \gamma(\mu)\}$. Then, φ has a fixed point.*

To apply Cellina's theorem we set $\mathcal{M} \equiv [0, 1]$ as the space of shares, with typical element μ, and $\mathcal{K} = [z_\ell, z_h]$ as the space of productivities with typical element Z. Both sets are nonempty, compact and convex, satisfying the theorem's requirements.

We then define the correspondence $\gamma(\mu) \equiv \min\left\{z_h, \text{Roots}^+(H, \mu)\right\}$ as the largest admissible root of the quadratic function H, defined in (35). This correspondence determines the equilibrium productivity. From Lemma B.1 we know that γ is a function (a single-valued correspondence), and hence γ is nonempty-, compact-, and convex-valued.

Next, we define the function $f(\mu, Z) \equiv \max\left\{\min\left\{\left(\Lambda'\right)^{-1}(\log R_h - \log R_\ell), 1\right\}, 0\right\}$ as the solution to (14), where $\left(\Lambda'\right)^{-1}$ is the inverse of the derivative of Λ. This inverse exists and is continuous because Λ is convex and twice-continuously-differentiable. f takes as given μ and Z and provides a value of optimal effort, that gives a new value of μ. Notice that μ does not enter directly into f because returns are entirely determined given Z and τ_a, as seen in equation (33). So, the function is immediately (and vacuously) continuous in μ. The returns are themselves continuous in Z, see (33), so that f is continuous in Z.

Finally, we define the correspondence as $\varphi(\mu) \equiv \{f(\mu, Z) | Z = \gamma(\mu)\}$. All the conditions are satisfied and therefore a fixed point of φ exists. Any such fixed point is an equilibrium level for effort and the share of high-productivity entrepreneurs of the economy, $\mu^\star$. This level of μ in turn implies the equilibrium level of productivity and other aggregate variables.

We now provide an alternative, and more direct proof based on Brouwer's fixed point theorem.

Theorem. ***[Brouwer 1912; Border 1985, Thm. 6.1]*** *Let $\mathcal{M} \subseteq \mathbb{R}^m$ be nonempty, compact, and convex. Let $\varphi : \mathcal{M} \to \mathcal{M}$ be a continuous function defined on K. Then, φ has a fixed point.*

To apply Brouwer's theorem we need to show that the function

$$\varphi(\mu) \equiv f(\mu, \gamma(\mu)) = f\left(\mu, \min\left\{z_h, \text{Roots}^+(H, \mu)\right\}\right)$$

is continuous, with f and γ defined as above. This in fact the case because the roots of the quadratic equation H, the minimum, the maximum, and f are all continuous.

Uniqueness This follows from showing that innovation effort, $\varphi(\mu) = e^\star(Z(\mu))$, is decreasing in the share of H-type entrepreneurs, μ. The result is immediate from combining Lemmas B.3 and B.4 because Z is increasing in μ, and e is decreasing in Z. Therefore, $\varphi(\mu)$ is monotonically decreasing, implying that there can be at most one fixed point in $[0, 1]$.

Condition on wealth taxes The equilibrium solution requires that $\lambda < \overline{\lambda}$, defined in (39) and derived in Lemma B.1. This condition can be expressed as a condition on the wealth tax rate that gives $\tau_a < \overline{\tau}_a$. However, $\overline{\tau}_a$ depends on μ, which now responds endogenously to τ_a. This means that equation (38) defines the upper bound on τ_a, given by $\overline{\tau}_a$, implicitly, something we emphasize by writing $\mu^{\star}\left(\overline{\tau}_a\right)$ as the equilibrium level of μ when the wealth tax is $\overline{\tau}_a$.

□

B.5 Innovation, Productivity, Aggregates, and Welfare

Proposition 3. ***(Innovation Gains from Wealth Taxation) (Innovation and Productivity Gains from Wealth Taxation)*** *For all $\tau_a < \overline{\tau}_a$, an increase in the wealth tax (τ_a) increases the equilibrium share of high-productivity entrepreneurs, $\mu^{\star}$, and the equilibrium level of productivity $Z^{\star}$. Capital income taxes do not affect innovation or productivity.*

Proof. The proof uses Theorem 3 in Villas-Boas (1997):

Theorem. ***[Villas-Boas 1997, Thm. 3]*** *Consider the mapping $\varphi_1 : \mathcal{M} \to \mathcal{M}$, the mapping $\varphi_2 : \mathcal{M} \to \mathcal{M}$, and a transitive, and reflexive order $\geq$ on the set $\mathcal{M}$, such that both φ_1 and φ_2 have at least one fixed point in $\mathcal{M}$. If*

i. φ_1 is a weakly decreasing mapping, i.e., $\forall_{\mu',\mu\in\mathcal{M}}\, \mu' \geq \mu \longrightarrow \varphi_1\left(\mu'\right) \leq \varphi_1(\mu)$;

ii. φ_1 is higher than φ_2, that is $\varphi_1(\mu) > \varphi_2(\mu)$ for all $\mu \in \mathcal{M}$,

then, there is no fixed point $\mu_2^{\star}$ of φ_2 which is $>$ than a fixed point $\mu_1^{\star}$ of φ_1.

Remark. The theorem can be strengthened as it implies that the two mappings cannot have the same interior fixed point, so that we can conclude that for any (interior) fixed point $\mu_2^{\star}$ of φ_2 and any (interior) fixed point $\mu_1^{\star}$ of φ_1, it holds that $\mu_1^{\star} > \mu_2^{\star}$. To see this, consider a fixed point $\mu_2^{\star}$ of φ_2 a fixed point $\mu_1^{\star}$ of φ_1. We already know that $\mu_1^{\star} \geq \mu_2^{\star}$ from the Theorem. Now, suppose that $\mu_2^{\star} = \mu_1^{\star} = \mu^{\star}$ and that $\mu^{\star}$ is interior. Because φ_1 is higher than φ_2 and $\mu^{\star}$ is a common fixed point we have $\mu^{\star} = \varphi_1(\mu^{\star}) > \varphi_2(\mu^{\star}) = \mu^{\star}$, which is a contradiction.

We now turn to verify the conditions of the Theorem. Our space of interest is $\mathcal{M} \equiv [0,1]$, and so we take the order $\geq$ to be the natural order on $\mathbb{R}$, which is transitive and reflexive. We define the mappings as $\varphi_1(\mu) \equiv \varphi\left(\mu, \tau_a^1\right)$ and $\varphi_2(\mu) \equiv \varphi\left(\mu, \tau_a^2\right)$ with $\overline{\tau}_a^{\mu} > \tau_a^1 > \tau_a^2$ and φ as in Proposition 2. These mappings have each a unique fixed point in $K \equiv [0,1]$.

We know that φ is decreasing from Lemmas B.3 and B.4 and so φ_1 satisfies the first condition.

To verify the second condition of the theorem, we establish that an increase in the wealth tax increases effort for any given level of the share of high-productivity entrepreneurs. Crucially, this condition speaks to the behavior of φ for any fixed level of μ as τ_a changes. Thus, the setup of Section 4 applies. In particular, Lemmas B.2 and 2, which shows that $\frac{dZ}{d\tau_a} > 0$, $\frac{dR_h}{dZ} > 0$ and $\frac{dR_\ell}{dZ} < 0$, imply that the dispersion of returns increases with τ_a when holding μ fixed, that is, $d(\log R_h - \log R_\ell)/d\tau_a > 0$. This leads to a higher level of effort from equation (14).

All the conditions for the theorem are verified and so it must be that all the (interior) equilibrium shares of high-productivity entrepreneurs under the higher wealth tax, τ_a^1, are higher as the equilibrium shares under the low wealth tax, τ_a^2.

Remark. In establishing the second condition for the theorem we make use of Lemma 2 instead of Lemma B.4. The difference lies in the nature of the mapping being constructed. The mapping required for the construction of φ in this proof takes into account the equilibrium response of Z to μ and to τ_a, while the one constructed in Lemma B.4 captures how arbitrary levels of productivity affect returns, and, through them, the innovation effort, holding τ_a fixed.

$\square$

Lemma 3. *Under Assumption 2, the steady-state level of capital is*

$$K = \left(\alpha \frac{\beta\delta\,(1-\theta)}{1-\beta\delta}\right)^{\frac{1}{1-\alpha}} Z^{\frac{\alpha}{1-\alpha}} L.$$

The long-run elasticities of aggregate variables with respect to productivity, $\xi_Z^x \equiv \frac{d\log x}{d\log Z}$, are

$$\xi_Z^K = \xi_Z^Y = \xi_Z^w = \frac{\alpha}{1-\alpha}.$$

Proof. Using Assumption 2 we express the capital income tax rate in terms of the wealth tax rate and parameter θ as in (46). Using this expression and equation (31) we obtain the equilibrium level of capital as in (47), which is increasing in Z. From this, it is immediate that $Y = (ZK)^{\alpha} L^{1-\alpha}$ is also increasing in Z. Replacing these results in equilibrium wages (Lemma 1) we get

$$w = (1-\alpha)\left({}^{ZK}/{}_{L}\right)^{\alpha} = (1-\alpha)\frac{Y}{L} = (1-\alpha)\left(\alpha\frac{\beta\delta\,(1-\theta)}{1-\beta\delta}\right)^{\frac{\alpha}{1-\alpha}} Z^{\frac{\alpha}{1-\alpha}}.$$

The elasticities follow immediately.

$\square$

Lemma 4. *Assume that total government is fixed, $G + T = \overline{\theta}$. Then, the (semi-)elasticities of capital, output, and wages to a change in the wealth tax satisfy*

$$\xi_{\tau_a}^K, \xi_{\tau_a}^Y, \xi_{\tau_a}^w > \frac{\alpha}{1-\alpha}\frac{d\log Z}{d\tau_a}.$$

Proof. The proof is immediate and follows from the fact that the increase in Y under Lemma 3 also increases the revenue raised. Holding revenue constant allows for a larger decrease in capital income taxes in response to wealth taxes.

$\square$

Lemma 5. *For all $\tau_a < \overline{\tau}_a$, under Assumption 2, the values of workers and the ex-post and ex-ante values of entrepreneurs satisfy, respectively,*

$$\frac{dV_w}{d\tau_a} = \frac{1}{1-\beta\delta}\xi_{\tau_a}^{w+T} > 0;$$
$$\frac{dV_i\left(\overline{a}\right)}{d\tau_a} = \frac{1}{1-\beta\delta}\xi_{\tau_a}^{K} + \frac{1}{\left(1-\beta\delta\right)^2}\xi_{\tau_a}^{R_i};$$
$$\frac{d\mathbb{V}_0\left(\overline{a}\right)}{d\tau_a} = \frac{1}{1-\beta\delta}\xi_{\tau_a}^{K} + \frac{1}{\left(1-\beta\delta\right)^2}\left(\mu\xi_{\tau_a}^{R_h} + \left(1-\mu\right)\xi_{\tau_a}^{R_\ell}\right);$$

for $i \in \{h, \ell\}$ and $\xi_{\tau_a}^{x}$ is the (semi-)elasticity of x with respect to τ_a.

Proof. We begin with worker welfare. Recall that $V_w = \frac{1}{1-\beta\delta}\log\left(w+T\right)$ and so $dV_w/d\tau_a = \frac{1}{1-\beta\delta}\xi_Z^{w+T}d\log Z/d\tau_a$. Under Assumption (2), $w+T = \left(\left(1-\alpha\right)+\theta_T\alpha\right)Y/L$, and so the elasticity with respect to productivity is $\xi_Z^{w+T} \equiv d\log\left(w+T\right)/dZ = \frac{\alpha}{1-\alpha} > 0$. Finally, $d\log Z/d\tau_a > 0$ from proposition 3. This gives the result.

The value of a newborn entrepreneur with productivity z_i, for $i \in \{\ell, h\}$, is $V_i\left(\overline{a}\right) = \frac{1}{1-\beta\delta}\left(\log\left(\overline{a}\right) + \frac{1}{(1-\beta\delta)}\log R_i\right) + \nu$, where ν is a constant. See equation 12. Recall that $\overline{a} = K$. Hence, the change in the welfare of that entrepreneur when the wealth tax increases is

$$\frac{dV_i\left(\overline{a}\right)}{d\tau_a} = \frac{1}{1-\beta\delta}\left(\xi_{\tau_a}^{K} + \frac{1}{1-\beta\delta}\xi_{\tau_a}^{R_i}\right) = \frac{1}{1-\beta\delta}\left(\xi_Z^{K}\frac{d\log Z}{d\tau_a} + \frac{1}{1-\beta\delta}\left(\xi_Z^{R_i}\frac{d\log Z}{d\tau_a} + \xi_\mu^{R_i}\frac{d\mu}{d\tau_a}\right)\right).$$

For the ex-ante value of an entrepreneur,

$$\begin{aligned}
\frac{d\mathbb{V}_0\left(\overline{a}\right)}{d\tau_a} &= \frac{d}{d\tau_a}\left(\mu V_h\left(\overline{a}\right) + \left(1-\mu\right)V_\ell\left(\overline{a}\right) - \frac{1}{\left(1-\beta\delta\right)^2}\Lambda\left(\mu\right)\right) \\
&= \mu\frac{dV_h\left(\overline{a}\right)}{d\tau_a} + \left(1-\mu\right)\frac{dV_\ell\left(\overline{a}\right)}{d\tau_a} + \left(\underbrace{\left(V_h\left(\overline{a}\right) - V_\ell\left(\overline{a}\right)\right) - \frac{1}{\left(1-\beta\delta\right)^2}\Lambda'\left(\mu\right)}_{=0}\right)\frac{d\mu}{d\tau_a} \\
&= \frac{1}{1-\beta\delta}\xi_{\tau_a}^{K} + \frac{1}{\left(1-\beta\delta\right)^2}\left(\mu\xi_{\tau_a}^{R_h} + \left(1-\mu\right)\xi_{\tau_a}^{R_\ell}\right)
\end{aligned}$$

where the second term in the second step is equal to zero because effort is chosen optimally in equilibrium, equation (14).

□

B.6 Optimal combination of taxes

ProductivityThis result follows from Lemma B.2 and Proposition 3. Lemma B.2 establishes that Z is increasing in τ_a holding μ fixed, and Lemma B.3 establishes that Z is increasing in μ. Proposition 3 establishes that μ is increasing in τ_a for $\tau_a < \overline{\tau}_a^{\mu}$. Together they imply the result.

Proposition 4. ***(Optimal Taxes)*** *Under Assumption 2, there is a unique combination of tax instruments $(\tau_a^\star, \tau_k^\star)$ that maximizes utilitarian welfare $\mathcal{W}$. An interior solution $\tau_a^\star < \overline{\tau}_a$ solves:*

$$0 = \Bigg(\underbrace{n_w \xi_Z^{w+T} \; + \; (1-n_w)\, \xi_Z^K}_{\text{Level Effect } = \frac{\alpha}{1-\alpha} (+)} \; + \; (1-n_w) \underbrace{\xi_Z^g}_{\text{Growth Effect } (-)} \Bigg) \frac{d \log Z}{d\tau_a} \; + \; (1-n_w) \underbrace{\xi_\mu^g}_{\text{Innovation Effect } (+)} \frac{d\mu}{d\tau_a},$$

where $\xi_Z^x \equiv \frac{\partial \log x}{\partial \log Z}$ is the elasticity of x with respect to Z and $\xi_\mu^x \equiv \frac{\partial \log x}{\partial \mu}$ is the (semi-)elasticity with respect to μ. From Lemma 3, $\xi_Z^{w+T} = \xi_Z^K = \frac{\alpha}{1-\alpha}$, so that this condition can be restated as

$$\frac{\alpha}{1-\alpha} = -\left(1-n_w\right) \left[\xi_Z^g \; + \; \xi_\mu^g \times \frac{d\mu}{d\tau_a} \Bigg/ \frac{dlogZ}{d\tau_a} \right].$$

This implies two cutoff values for α, $\underline{\alpha}$ and $\overline{\alpha}$, such that $(\tau_a^\star, \tau_k^\star)$ satisfies:

$$\begin{aligned}
&\tau_a^\star \in \left[1 - \frac{1}{\beta\delta}, 0\right) \text{ and } \tau_k^\star > \theta && \text{if } \alpha < \underline{\alpha} \\
&\tau_a^\star \in \left[0, \frac{\theta\left(1-\beta\delta\right)}{\beta\delta\left(1-\theta\right)}\right] \text{ and } \tau_k^\star \in [0, \theta] && \text{if } \underline{\alpha} \le \alpha \le \bar{\alpha} \\
&\tau_a^\star \in \left(\frac{\theta\left(1-\beta\delta\right)}{\beta\delta\left(1-\theta\right)}, \tau_a^{\max}\right) \text{ and } \tau_k^\star < 0, && \text{if } \alpha > \bar{\alpha}
\end{aligned}$$

where $\tau_a^{\max} \ge 1$, $\underline{\alpha}$ and $\overline{\alpha}$ are the solutions to equation (61) with $\tau_a = 0$ and $\tau_a = \frac{\theta(1-\beta\delta)}{\beta\delta(1-\theta)}$, respectively. When $\theta = 0$, and there are no revenue needs, so $\underline{\alpha} = \overline{\alpha}$.

Proof. An interior solution to the government's problem in 58 satisfies the first order condition

$$0 = \frac{d\mathcal{W}}{d\tau_a} = n_w \frac{dV_w}{d\tau_a} + \left(1-n_w\right) \frac{d}{d\tau_a} \left(\mu V_h\left(\overline{a}\right) + \left(1-\mu\right) V_\ell\left(\overline{a}\right) - \frac{\Lambda\left(\mu\right)}{\left(1-\beta\delta\right)^2} \right).$$

Replacing for the values of workers and entrepreneurs as in Section 5,

$$\begin{aligned}
0 &= n_w \frac{d\log w + T}{d\tau_a} + \left(1-n_w\right) \frac{d\log\left(\overline{a}\right)}{d\tau_a} + \frac{1-n_w}{1-\beta\delta} \left(\frac{d\left(\mu \log R_h + \left(1-\mu\right)\log R_\ell\right)}{d\tau_a} - \frac{d\Lambda\left(\mu\right)}{d\tau_a} \right) \\
0 &= n_w \frac{d\log w + T}{d\tau_a} + \left(1-n_w\right) \frac{d\log\left(\overline{a}\right)}{d\tau_a} \\
&\quad + \frac{1-n_w}{1-\beta\delta} \left(\left(\mu \frac{d\log R_h}{d\tau_a} + \left(1-\mu\right) \frac{d\log R_\ell}{d\tau_a} \right) + \left[\underbrace{\left(\log R_h - \log R_\ell\right) - \Lambda^{'}\left(\mu\right)}_{=0} \right] \frac{d\mu}{d\tau_a} \right)
\end{aligned}$$

The last term is equal to zero because individuals already optimize over their innovation effort, so that, by the Pareto principle, taxes cannot improve on their choice. This leaves the local effects of the wealth tax taking μ as given (at its equilibrium level). We can further simplify these effects by noticing that, given μ, the effect of the wealth tax is only felt through the change in equilibrium productivity. This gives,

$$0 = \left(n_w \xi_Z^{w+T} \ + \ (1-n_w)\,\xi_Z^K\right) \frac{d\log Z}{d\tau_a} + (1-n_w)\left[\xi_Z^g \frac{d\log Z}{d\tau_a} + \xi_\mu^g \frac{d\mu}{d\tau_a}\right],$$

where $\xi_Z^{w+T} = \xi_Z^K = \alpha/1-\alpha > 0$, $\xi_Z^g = \frac{1}{1-\beta\delta}\left(\mu\xi_Z^{R_h} + (1-\mu)\,\xi_Z^{R_\ell}\right) < 0$, and $\xi_\mu^g = \frac{1}{1-\beta\delta}\left(\mu\xi_\mu^{R_h} + (1-\mu)\,\xi_\mu^{R_\ell}\right) > 0$. The signs of the wealth growth effect, $\xi_Z^g < 0$, and innovation effect, $\xi_\mu^g > 0$, come from Lemma 2.

$\square$

C Distribution of Wealth

The following lemma characterizes the gross saving rates of entrepreneurs.

Lemma 6. ***(Saving and Dissaving in the Stationary Equilibrium)*** *In the stationary heterogeneous-return equilibrium, the rates of return of the L-type and the H-type satisfy the following inequalities: $\beta\delta R_\ell < 1 < \beta\delta R_h < 1/\delta$. As a result, the wealth of the L-type (H-type) shrinks (grows) with age. Therefore, the H-type is wealthier than the L-type: $s_h > \mu$.*

Proof. We start by showing that $R_\ell < 1/\beta\delta < R_h$. We verify this directly using the expression for the returns of H- and L-types, the fact that $z_\ell < Z < z_\lambda$, and the equilibrium condition for the return on capital:

$$R_\ell = (1-\tau_a) + (1-\tau_k)\,\alpha Z^\alpha \left(\frac{K}{L}\right)^{\alpha-1} \frac{z_\ell}{Z} < (1-\tau_a) + (1-\tau_k)\,\alpha Z^\alpha \left(\frac{K}{L}\right)^{\alpha-1} = \frac{1}{\beta\delta},$$

and

$$\frac{1}{\beta\delta} = (1-\tau_a) + (1-\tau_k)\,\alpha Z^\alpha \left(\frac{K}{L}\right)^{\alpha-1} < (1-\tau_a) + (1-\tau_k)\,\alpha Z^\alpha \left(\frac{K}{L}\right)^{\alpha-1} \frac{z_\lambda}{Z} = R_h,$$

Letting $\eta \equiv \delta\beta\,(1-\tau_a)$, we can also show that $\beta\delta R_h < 1/\delta$ if $\frac{\delta(1-\eta)}{1-\delta\eta} z_\lambda < Z$. Thus,

$$\beta\delta R_\ell < 1 < \beta\delta R_h < 1/\delta \longleftrightarrow Z \in \left(\max\left\{z_\ell, \frac{\delta\,(1-\eta)}{1-\delta\eta} z_\lambda\right\}, z_\lambda\right).$$

The interval for Z is non-empty. This is immediate because:

$$z_\ell < z_\lambda \qquad \text{and} \qquad \frac{\delta\,(1-\eta)}{1-\delta\eta} < 1.$$

Moreover, the lower bound depends on the ratio of productivities: $\max\left\{z_\ell, \frac{\delta(1-\eta)}{1-\delta\eta} z_\lambda\right\} = z_\ell$ if and only if $\frac{\delta(1-\eta)}{1-\delta\eta} \leq \frac{z_\ell}{z_\lambda}$. In the proof of Lemma B.1 we establish that Z lies in the desired interval.

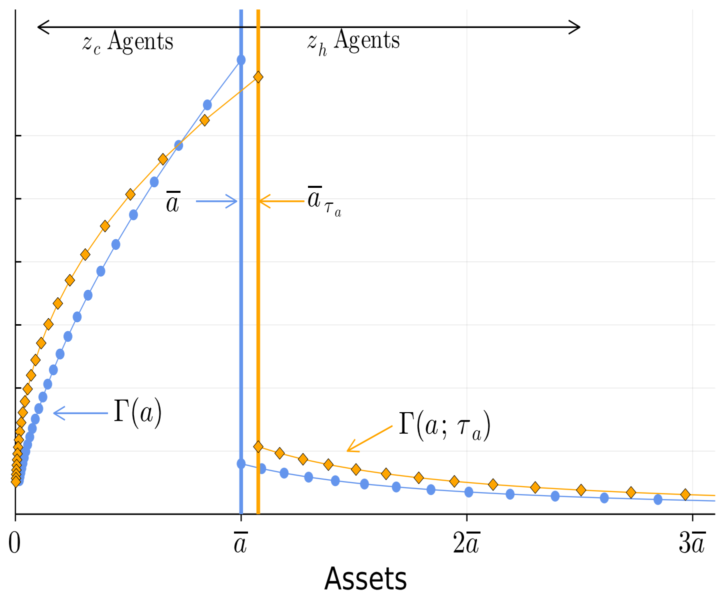
Finally, we prove that $s_h > \mu$. We know that $s_h = \frac{Z-z_\ell}{z_\lambda - z_\ell}$, so $s_h > \mu$ is equivalent to $Z > \mu z_\lambda + (1-\mu)\,z_\ell$. We can verify if this is the case by evaluating at $\mu z_\lambda + (1-\mu)\,z_\ell$ the residual of the quadratic equation H defined in (35):

$$H\left(\mu z_\lambda + (1-\mu)\,z_\ell\right) = -\delta\,(1-\eta)\,(1-\mu)\,\mu \left(\frac{z_\lambda - z_\ell}{\mu z_\lambda + (1-\mu)\,z_\ell}\right)^2 < 0$$

The residual is always negative. So it must be that $Z > \mu z_\lambda + (1-\mu)\,z_\ell$ and thus $s_h > \mu$.

$\square$

Figure C.3: Stationary Wealth Distribution and Wealth Taxes



Note: The blue line marked with circles is the stationary wealth distribution for an economy with a zero wealth tax, and the orange line with diamonds is the corresponding distribution with a positive wealth tax ($\tau_a > 0$). The vertical lines mark the levels of $\overline{a}$ in the respective economy. The wealth distribution of low-productivity entrepreneurs is to the left of $\overline{a}$ since they dissave and the distribution of high-productivity entrepreneurs is to the right. The wealth tax economy has a higher level of $\overline{a}$ and different mass-points for the distribution as a result. The share of high-productivity entrepreneurs, μ is held constant.

Using this Lemma, we characterize the stationary wealth distribution as in Section 3.3. The wealth distribution has a geometric distribution with parameter δ:

$$\Gamma_i\left((\beta\delta R_i)^t\,\overline{a}\right) = \Pr(z = z_i)\Pr(\text{age} = t) = \Pr(z = z_i)\,\delta^t\,(1-\delta)\,. \tag{75}$$

This structure allows us to define a measure of wealth concentration at the top, by dividing the total wealth of H-type entrepreneurs older than age t :

$$A_{h,t} \equiv (1-\delta)\sum_{s=t}^{\infty}\left(\beta\delta^2 R_h\right)^s \mu\overline{a} = \left(\beta\delta^2 R_h\right)^t \mu A_h, \tag{76}$$

by aggregate wealth. Hence, the wealth share of the top x percent is:

$$s(x) \equiv \frac{\left(\beta\delta^2 R_h\right)^{t(x)}\mu A_h}{K} = \left(\beta\delta^2 R_h\right)^{t(x)} s_h, \tag{77}$$

where $t(x)$ corresponds to the age above which agents are in the top x percent of the wealth distribution.[35] It is easy to see from (77) that the wealth distribution has a Pareto right tail, which is one of the most salient features of the wealth distribution in modern economies (Vermeulen, 2018). To see this, let $S(x) \equiv s(x)/s(10x)$ be the share of the wealth held by the top $10x$ percent

[35] Formally, the wealth share of the top x percent corresponds to the wealth share of H-type entrepreneurs of age $t = \log x/\log\delta$.

that is held by the top x percent. From (77), this is:

$$S(x) = \left(\beta\delta^2 R_h\right)^{-\frac{\log 10}{\log \delta}}, \tag{78}$$

which is independent of x and increasing in the returns of high-productivity entrepreneurs. Hence, the distribution is Pareto, with the *inverse* of the tail index given by $\eta = -\log(\beta\delta^2 R_h)/\log(\delta)$. Using Lemma 6, η satisfies $0 < \eta < 1$ and increases (inequality is higher) with R_h as expected.[36]

What happens to the wealth distribution when τ_a increases? The whole wealth distribution shifts after an increase in the wealth tax, an outcome that reflects the increase in aggregate wealth and the change in returns. The increase in aggregate capital shifts all mass points to the right, as they are proportional to $\overline{a} = K$. They are further affected by the compounding effect of returns—see Proposition 2. The resulting shift is shown in Figure C.3. When we take into account the changes in innovation effort, there is additional change in the distribution following an increase in the wealth tax. This is because the share of high-productivity entrepreneurs increases, shifting the mass of the distribution towards them.

[36] Although, technically speaking, the Pareto distribution is continuous, Γ can be thought of a discrete counterpart, with the same fractal property as the Pareto.

D Managerial Effort

Consider a model like that in Section 2 where entrepreneurs can exert effort to better manage their firms every period and in that way increase their level of productivity. We capture the effect of managerial effort, m, as modifying the production function of entrepreneurs to:

$$y = (zk)^{\alpha} g(m)^{\gamma} n^{1-\alpha-\gamma}.$$

where $\gamma \in [0,1)$. Exerting effort has a utility cost of $h(m)$, where $h'(m) > 0$ and $h''(m) \geq 0$. The utility function is now

$$u(c,m) = \log(c - h(m)).$$

To better focus on how managerial effort choices are affected by taxation we change Assumption 1 to make L-types operate firms instead of the corporate sector.

Assumption C.1. *Corporate and entrepreneurial productivity satisfy* $z_c < z_\ell < z_h$.

D.1 Entrepreneurial Problem with Effort in Production

Entrepreneurial Production. The first order conditions of the entrepreneur's static managerial effort and labor demand choices are

$$u_m h'(m) = (1-\tau_k) u_c \cdot \gamma (zk)^{\alpha} g(m)^{\gamma-1} n^{1-\alpha-\gamma} g'(m); \qquad w = (1-\alpha-\gamma)(zk)^{\alpha} g(m)^{\gamma} n^{-\alpha-\gamma};.$$

The second condition implies that

$$n = \left[\frac{(1-\alpha-\gamma)(zk)^{\alpha} g(m)^{\gamma}}{w}\right]^{\frac{1}{\alpha+\gamma}}.$$

Replacing back in the first order condition we obtain

$$\frac{u_m}{u_c}\frac{h'(m)}{g'(m)} = (1-\tau_k)\gamma (zk)^{\frac{\alpha}{\alpha+\gamma}} g(m)^{\frac{-\alpha}{\alpha+\gamma}} \left(\frac{1-\alpha-\gamma}{w}\right)^{\frac{1-\alpha-\gamma}{\alpha+\gamma}}.$$

For tractability we impose that $\frac{h'(m)}{g'(m)} = \psi$ is constant, say with $h(m) = \psi m$ and $g(m) = m$. This gives

$$m = \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\alpha+\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{w}\right)^{\frac{1-\alpha-\gamma}{\alpha}} zk; \qquad n = \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{w}\right)^{\frac{1-\gamma}{\alpha}} zk.$$

Profits are then:

$$\pi(z,k) = (zk)^{\alpha} g(m)^{\gamma} n^{1-\alpha-\gamma} - wn - rk = \left[\underbrace{(\alpha+\gamma)\left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{w}\right)^{\frac{1-\alpha-\gamma}{\alpha}} z - r}_{\pi^\star(z)}\right] k.$$

Crucially, profits, labor, and managerial effort are proportional to the level of capital the entrepreneur uses. The entrepreneur will only demand capital and operate their firm if the (after-tax) profits net of the effort cost are positive, that is:

$$k \geq 0 \longleftrightarrow (1-\tau_k)\,\pi^\star(z) - \underbrace{\frac{u_m h'(m)}{u_c}}_{\text{Shadow Price}=\psi}\varepsilon(z) \geq 0,$$

where the shadow price of the effort cost is equal to ψ given our assumptions and

$$\frac{m(z,k)}{k} = \left(\frac{(1-\tau_k)\,\gamma}{\psi}\right)^{\frac{\alpha+\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{w}\right)^{\frac{1-\alpha-\gamma}{\alpha}} z.$$

Entrepreneur demand capital if their after-tax profits cover the cost of effort:

$$k(z,a) = \begin{cases} \lambda a & \text{if } \alpha\left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{w}\right)^{\frac{1-\alpha-\gamma}{\alpha}} z > r \\ [0,\lambda a] & \text{if } \alpha\left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{w}\right)^{\frac{1-\alpha-\gamma}{\alpha}} z = r \\ 0 & \text{if } \alpha\left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{w}\right)^{\frac{1-\alpha-\gamma}{\alpha}} z < r \end{cases}.$$

We replace $k(z,a)$ back and get the optimal profits, effort and labor demand.

Before proceeding to the optimal savings problem, we need to determine the level of capital demand for H- and L-types. The relevant case has H-types demanding $k(z_h,a) = \lambda a$ for a total demand of $K_h = \lambda\mu A_h$. The remaining assets are used by the L-types, $K_L = (1-\mu)\,A_L - (\lambda-1)\,\mu A_h$, who will be indifferent between any production level. Let $\lambda_{\ell,\iota} \equiv \frac{k_\iota}{a_\iota}$ be the ratio of capital to assets of low-productivity entrepreneur ι, for $\iota \in [\mu,1]$. We show below that the savings choice of the entrepreneur is independent of the value of $\lambda_{\ell,\iota}$.

Entrepreneurial Savings.

$$V_\iota(a,z) = \max_{\{c,a'\}} \ln(c - \psi e_\iota(z,a)) + \beta\delta E\left[V_\iota\left(a',z'\right)|z\right] \qquad \text{s.t. } c + a' = R_\iota(z)\,a$$

where $R(z) \equiv (1-\tau_a) + (1-\tau_k)\,(r + \pi^\star(z)\,\lambda_\iota(z))$, $e_\iota(z,a) = \varepsilon(z)\,\lambda_\iota(z)\,a$, and

$$\lambda_\iota(z) = \begin{cases} \lambda & \text{if } z = z_h \\ \lambda_{\iota,\ell} & \text{if } z = z_\ell. \end{cases}$$

We guess that the value function of an entrepreneur with productivity z_i, $i \in \{\ell,h\}$, has the form $V_{i,\iota}(a) = m_{i,\iota} + n\log(a)$, where $\{m_{\ell,\iota}, m_{h,\iota}\}_{\iota\in\{0,1\}}, n \in \mathbb{R}$ are coefficients. Under this guess, the optimal savings choice of the entrepreneur is characterized by

$$\frac{1}{(R_{i,\iota} - \psi\varepsilon_i\lambda_{i,\iota})\,a - a_i'} = \frac{\beta\delta n}{a_i'} \longrightarrow a_i' = \frac{\beta\delta n}{1+\beta\delta n}\,(R_{i,\iota} - \psi\varepsilon_i\lambda_{i,\iota})\,a.$$

Replacing the savings rule into the value function gives:

$$m_{i,\iota} + n\log(a) = \log\left((R_{i,\iota} - \psi\varepsilon_i\lambda_{i,\iota})\,a - a_i'\right) + \beta\delta m_{i,\iota} + \beta\delta n\log\left(a_i'\right)$$

Matching coefficients delivers the optimal saving decision of the entrepreneur:

$$a' \quad = \quad \beta\delta \left(R_\iota (z) - \psi\varepsilon (z) \lambda_\iota (z)\right) a.$$

Finally, we solve for the remaining coefficients noting that returns are independent of the identity of the entrepreneur and depend only on productivity when H-types are constrained and L-types are indifferent between any level of production,

$$R_\iota (z) - \psi\varepsilon (z) \lambda_\iota (z) = R (z) - \psi\varepsilon (z) \lambda \equiv \hat{R} (z) .$$

This allows us to solve for m_ℓ and m_h as:

$$m_i = \frac{1}{(1-\beta\delta)^2} \left(\log \left((\beta\delta)^{\beta\delta} (1-\beta\delta)^{1-\beta\delta}\right)\right) + \frac{1}{(1-\beta\delta)^2} \log \hat{R} (z) .$$

D.2 Equilibrium and Aggregation

In equilibrium, the interest rate is such that L-types are indifferent between lending their assets or using them in their own firm. Lending the assets gives them a (before-tax) return of r, using them gives them $\pi^\star (z_\ell)$ but it also entails a utility cost because of effort, which we know from the previous results is proportional to assets. Indifference requires that

$$0 = (1-\tau_k) \pi^\star (z_\ell) - \underbrace{\frac{u_e h'(e)}{u_c}}_{\text{Shadow Price}=\psi} \varepsilon (z_\ell) .$$

Replacing for the optimal solution of the entrepreneur's problem:

$$r = \alpha \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{w}\right)^{\frac{1-\alpha-\gamma}{\alpha}} z_\ell$$

We can then exploit the linearity of the savings function to aggregate.

Lemma 7. *If $s_h < 1/\lambda$, output, wages, interest rate, and gross returns on savings are:*

$$\begin{aligned}
Y &= \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}} (ZK)^{\frac{\alpha}{1-\gamma}} L^{\frac{1-\alpha-\gamma}{1-\gamma}} \\
M &= \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{1}{1-\gamma}} (ZK)^{\frac{\alpha}{1-\gamma}} L^{\frac{1-\alpha-\gamma}{1-\gamma}} \\
w &= (1-\alpha-\gamma) \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{ZK}{L}\right)^{\frac{\alpha}{1-\gamma}} \\
r &= \alpha \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}} z_\ell \\
R_\ell &= (1-\tau_a) + (1-\tau_k) \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}} (\alpha + \gamma\lambda_\iota) z_\ell \\
R_h &= (1-\tau_a) + (1-\tau_k) \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}} (\alpha z_\lambda + \gamma\lambda z_h)
\end{aligned}$$

and the returns net of managerial effort costs are

$$\hat{R}(z) \equiv R(z) - \psi \frac{m(z,k)}{k}\lambda = \begin{cases} (1-\tau_a) + (1-\tau_k)\,\alpha\left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}} z_\ell & \textit{if } z = z_\ell \\ (1-\tau_a) + (1-\tau_k)\,\alpha\left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}} z_\lambda & \textit{if } z = z_h \end{cases}$$

Proof. The labor market clearing condition, $n^\star(z_h, K_h) + n^\star(z_\ell, K_\ell) = L$, and the optimal labor demand derived above give the wage. Using the market clearing wage we obtain the expression for the interest rate by substituting in the condition above this lemma. Total managerial effort is,

$$\left(\frac{M}{ZK}\right)^\alpha = \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\alpha+\gamma}\left(\frac{1-\alpha-\gamma}{w}\right)^{1-\alpha-\gamma} \quad \longrightarrow \quad M = \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{1}{1-\gamma}}(ZK)^{\frac{\alpha}{1-\gamma}} L^{\frac{1-\alpha-\gamma}{1-\gamma}}.$$

We can use this expression to get the usual Cobb-Douglas expressions for w and r:

$$w = (1-\alpha-\gamma)\frac{(ZK)^\alpha M^\gamma L^{1-\gamma-\alpha}}{L}; \qquad r = \alpha\frac{(ZK)^\alpha M^\gamma L^{1-\gamma-\alpha}}{ZK} z_\ell.$$

These two expressions also let us rewrite the profit rate (of capital) of entrepreneurs:

$$\pi^\star(z) = \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}}(\alpha(z-z_\ell)+\gamma z) > 0.$$

Profits are positive for both types of entrepreneurs, reflecting the managerial effort costs.

We use the equilibrium profit rates of entrepreneurs to rewrite the gross returns,

$$\begin{aligned} R(z) &= (1-\tau_a) + (1-\tau_k)(r + \pi^\star(z)\lambda); \\ &= (1-\tau_a) + (1-\tau_k)\left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}}(\alpha(z_\ell + \lambda(z-z_\ell)) + \gamma\lambda z); \\ &= \begin{cases} (1-\tau_a) + (1-\tau_k)\left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}}(\alpha+\gamma\lambda) z_\ell & \text{if } z = z_\ell \\ (1-\tau_a) + (1-\tau_k)\left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}}(\alpha z_\lambda + \gamma\lambda z_h) & \text{if } z = z_h \end{cases}. \end{aligned}$$

The return net of effort cost is

$$\hat{R}(z) = R(z) - \psi\varepsilon(z)\lambda = \begin{cases} (1-\tau_a) + (1-\tau_k)\,\alpha\left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}} z_\ell & \text{if } z = z_\ell \\ (1-\tau_a) + (1-\tau_k)\,\alpha\left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}} z_\lambda & \text{if } z = z_h \end{cases}.$$

We aggregate output in terms of total capital using the constant ratio of labor to capital across entrepreneurs. The output of an individual entrepreneur with productivity z and capital k is

$$y(z,k) = \left(\frac{(1-\tau_k)\gamma}{\psi}\right)^{\frac{\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{w}\right)^{\frac{1-\alpha-\gamma}{\alpha}} zk.$$

Aggregate output is the sum of the total output produced by all entrepreneurs,

$$Y=\left(\frac{(1-\tau_k)\,\gamma}{\psi}\right)^{\frac{\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{w}\right)^{\frac{1-\alpha-\gamma}{\alpha}}(z_hK_h+z_\ell K_\ell)=\left(\frac{(1-\tau_k)\,\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}}(ZK)^{\frac{\alpha}{1-\gamma}}L^{\frac{1-\alpha-\gamma}{1-\gamma}}$$

For completeness we also consider the aggregate effort of high- and low-productivity entrepreneurs:

$$M_i\equiv\int m\left(z,k_{\iota,i}\right)d\iota=\left[\frac{(1-\tau_k)\,\gamma}{\psi}ZK^{-(1-\alpha-\gamma)}L^{1-\alpha-\gamma}\right]^{\frac{1}{1-\gamma}}z_iK_i$$

This completes the derivation of the results.

□

Evolution of aggregates and stationary equilibrium: Using the savings decision rules, we obtain the law of motion for the wealth held by low- and high-productivity entrepreneurs:

$$A_i^{'}=\delta^2\beta\hat{R}_iA_i+(1-\delta)\,\overline{a},$$

where $\overline{a}\equiv K=(1-\mu)\,A_\ell+\mu A_h$ is the endowment of a newborn entrepreneur, equal to the total (average) wealth in the economy. Combining these we obtain the low of motion of capital:

$$\frac{K^{'}}{K}=\delta^2\beta\left(s_h\hat{R}_h+(1-s_h)\,\hat{R}_\ell\right)+(1-\delta)\,.$$

From this we see that the wealth weighted returns net of effort costs are constant in steady state,

$$s_h\hat{R}_h+(1-s_h)\,\hat{R}_\ell=\frac{1}{\beta\delta}$$

$$(1-\tau_a)+(1-\tau_k)\,\alpha\left(\frac{(1-\tau_k)\,\gamma}{\psi}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{L}{ZK}\right)^{\frac{1-\alpha-\gamma}{1-\gamma}}Z=\frac{1}{\beta\delta}$$

This is similar to the result in (31) but it includes the distortionary effect of capital income taxes on effort. As in Proposition 1, this result implies neutrality of the capital income tax. Returns (now net of effort cost) are:

$$\hat{R}\left(z\right)=\begin{cases}(1-\tau_a)+\left(\frac{1}{\beta\delta}-(1-\tau_a)\right)\frac{z_\ell}{Z} & \text{if } z=z_\ell\\ (1-\tau_a)+\left(\frac{1}{\beta\delta}-(1-\tau_a)\right)\frac{z_\lambda}{Z} & \text{if } z=z_h\end{cases}.$$

The equations for the evolution of assets $\left(A_i^{'}\right)$ and the steady state of returns (above) imply that equation (35) applies unchanged and determines the stationary level of productivity as in Section 2.2. Consequently, Proposition B.1, Lemma 2, and Propositions 3 and B.2 apply to this economy without modifications.

Effect of taxes on aggregates: The difference between the benchmark model (Section 2) and the model with managerial effort is in the response of aggregate variables other than Z to changes in taxes. All directions are maintained, but there is now an additional source of changes on aggregates: a direct effect of taxes on the managerial effort of entrepreneurs. Shifting the tax mix toward the wealth tax reduces capital income taxes, and because of that it also reduces the distortions on the managerial effort choice of entrepreneurs.

The Government's budget is still given by 24 and Assumption 2 still implies the link between capital income and wealth taxes in equation (46).Then, equilibrium capital is

$$K = \left(\alpha\beta\delta\frac{1-\theta}{1-\beta\delta}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}} \left(\frac{(1-\tau_k)\,\gamma}{\psi}\right)^{\frac{\gamma}{1-\alpha-\gamma}} Z^{\frac{\alpha}{1-\alpha-\gamma}} L.$$

Crucially, capital depends directly on capital income taxes through their effect on managerial effort.

Lemma 8. *If $\tau < \overline{\tau}_a$ and under Assumption 2, an increase in the wealth tax (τ_a) increases aggregate entrepreneurial effort, capital, output, and wages, $\frac{dE}{d\tau_a}, \frac{dK}{d\tau_a}, \frac{dY}{d\tau_a}, \frac{dw}{d\tau_a} > 0$. .*

Proof. Total capital increases with the wealth tax:

$$\frac{d\log K}{d\log \tau_a} = \frac{\gamma}{1-\alpha-\gamma}\frac{\beta\delta\tau_a}{1-\beta\delta\,(1-\tau_a)} + \frac{\alpha}{1-\alpha-\gamma}\frac{d\log Z}{d\log \tau_a} > 0.$$

It follows immediately that output, wages, and total effort increase since they depend positively on ZK and negatively on capital income taxes τ_k.

□

D.3 Optimal Taxes

Managerial effort changes the choice of optimal taxes in two ways. First, aggregate wage and savings now depend on taxes directly through effort (while before they only changed through changes in productivity). Second, entrepreneurial welfare depends now on after-tax returns net of effort cost. However, only aggregates affect optimal taxes. This is because, in equilibrium, the after-tax returns net of effort cost behave exactly like after-tax returns did in Section 2.

Proposition 5. *Under Assumption 2, there exists a unique tax combination $\left(\tau^\star_{a,m}, \tau^\star_{k,m}\right)$ that maximizes $\mathcal{W}$. An interior solution $\tau^\star_{a,m} < \overline{\tau}_a$ solves*

$$0 = \left(\frac{\alpha}{1-\alpha-\gamma} + (1-n_w)\,\xi^g_Z \right) \frac{d\log Z}{d\tau_a} + (1-n_w)\;\xi^g_\mu \frac{d\mu}{d\tau_a} + \frac{\gamma}{1-\alpha-\gamma}\beta\delta$$

where $\xi^x_Z \equiv \frac{\partial \log x}{\partial \log Z}$ is the elasticity of x with respect to Z and $\xi^x_\mu \equiv \frac{\partial \log x}{\partial \mu}$ is the (semi-)elasticity with respect to μ. Moreover, $\tau^\star_{a,m} > \tau^\star_a$, with $\tau^\star_a$ defined in (61).

Proof. Newborn welfare, $\mathcal{W}$, is the same as in (57) but with $\hat{R}_i$ taking the place of R_i. The first order condition of the government's problem is then

$$0 = n_w\frac{d\log(w+T)}{d\tau_a} + (1-n_w)\,\frac{d\log K}{d\tau_a} + \frac{1-n_w}{1-\beta\delta}\left(\mu\frac{d\log \hat{R}_h}{d\tau_a} + (1-\mu)\,\frac{d\log \hat{R}_\ell}{d\tau_a}\right),$$

where we have, under Assumption 2,

$$\frac{d\log K}{d\tau_a} = \frac{d\log w + T}{d\tau_a} = \frac{\gamma}{1-\alpha-\gamma}\beta\delta + \frac{\alpha}{1-\alpha-\gamma}\frac{d\log Z}{d\tau_a};$$

$$\mu\frac{d\log \hat{R}_h}{d\tau_a} + (1-\mu)\frac{d\log \hat{R}_\ell}{d\tau_a} = \xi_Z^g \frac{d\log Z}{d\tau_a} + \xi_\mu^g \frac{d\mu}{d\tau_a}.$$

Joining gives

$$0 = \left[\frac{\gamma}{1-\alpha-\gamma}\frac{\beta\delta}{\frac{\log Z}{d\tau_a}} + \frac{\alpha}{1-\alpha-\gamma} - \frac{\alpha}{1-\alpha}\right] + \left[\frac{\alpha}{1-\alpha} + \frac{1-n_w}{1-\beta\delta}\left(\xi_Z^g + \xi_\mu^g \frac{\frac{d\mu}{d\tau_a}}{\frac{d\log Z}{d\tau_a}}\right)\right]$$

The second term is the same as in Proposition 4, were the elasticity of capital and workers' income and capital were equal to $\alpha/1-\alpha$. The average elasticity of returns net of effort cost is equal to the elasticity of returns in Proposition 4. The first term is positive because $\frac{\log Z}{d\tau_a} > 0$ and $\frac{\alpha}{1-\alpha-\gamma} \geq \frac{\alpha}{1-\alpha}$. This implies that the optimal wealth tax is weakly higher than in Proposition 4, and it is equal if and only if $\gamma = 0$.

□

E Alternative Modeling: Fluctuating Entrepreneurial Productivity

Finally, we consider the role of *fluctuations of entrepreneurial productivity* in shaping our results on productivity and welfare gains from wealth taxation. The models studied so far assume that entrepreneurs have the same productivity throughout their lives. In them, an increase in the wealth tax benefits H-type entrepreneurs whose returns and wealth increase permanently, reducing misallocation. However, fluctuations in individual entrepreneurial productivity increase misallocation as wealthy (formerly productive) entrepreneurs lose their productivity. Nevertheless, we show that entrepreneurial productivity needs only to be persistent (i.e., positively autocorrelated) in order to preserve our main results.

To study the role of productivity persistence, we put forth a model where entrepreneurial productivity follows a Markov process and entrepreneurs are infinitely lived. This model remains tractable while allowing for fluctuations in individual productivity, and provides a clear cut answer to the conditions under which wealth taxes increase productivity and welfare. As in Section 2, there are two types of agents, homogeneous workers of size L and heterogenous entrepreneurs of size 1, but they are now infinitely-lived. This amounts to setting $\delta = 1$.[37] Preferences are as in Section 2, as is the behavior of workers. The entrepreneurs' production problem is given by (5). Thus, we can aggregate as in Lemma 1.

The main change comes from having entrepreneurial productivity, $z \in \{z_\ell, z_h\}$, follow a Markov process with transition matrix

$$\mathbb{P} = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}, \tag{79}$$

where $p \in (0,1)$ is the probability that an entrepreneur retains their productivity across periods. The autocorrelation coefficient of productivity is $\rho \equiv 2p - 1$, so that productivity is persistent if $p > 1/2$ ($\rho > 0$). The symmetry in transition probabilities ensures that half of the entrepreneurs have high-productivity at any point in time, $\mu = 1/2$.

The dynamic problem of the entrepreneurs is now

$$V(a,z) = \max_{a'} \log\left(R(z)\,a - a'\right) + \beta \sum_{z'} \mathbb{P}\left(z' \mid z\right) V\left(a', z'\right), \tag{80}$$

where $R(z)$ is as in equation (10). The solution to this problem gives the same savings rule as before,

$$a' \quad = \quad \beta R(z)\,a. \tag{81}$$

This structure leads to equilibrium conditions paralleling those in Section 3. In particular, the neutrality result in Proposition 1 is preserved and productivity in the stationary equilibrium is endogenous and determined by a quadratic equation that is now

$$0 = \left(1 - \rho\beta\left(1-\tau_a\right)\right) Z^2 - \left(1 + \rho\left(1 - 2\beta\left(1-\tau_a\right)\right)\right) \frac{z_h + z_\ell}{2} Z + \rho\left(1 - \beta\left(1-\tau_a\right)\right) z_h z_\ell. \tag{82}$$

[37] Alternatively, one can think of dynasties where offspring inherit the totality of the previous generation's wealth. This is similar to the formulation in Benhabib, Bisin and Zhu (2011). This model does not admit a stationary wealth distribution but remains tractable by focusing on the behavior of aggregates and wealth shares across entrepreneurial types.

Aggregate productivity in the stationary equilibrium depends now on ρ, the persistence of the entrepreneurial productivity process.

The main result out of this model is that the effects of wealth taxes on Z also depend on the persistence of productivity. We show that Z is increasing in the wealth tax if and only if entrepreneurial productivity is persistent, $\rho > 0$. As in Section 4, an increase in the wealth tax increases the returns of high-productivity entrepreneurs and reduces those of low-productivity entrepreneurs (see Lemma 10). This translates into a higher wealth share of high-productivity entrepreneurs (s_h) if and only if *current* high-productivity entrepreneurs are expected to remain so in the future.[38]

Proposition 6. ***(Efficiency Gains from Wealth Taxation)*** *For all $\tau_a < \overline{\tau}_{a,\rho}$, an increase in the wealth tax (τ_a) increases aggregate productivity, $\frac{dZ}{d\tau_a} > 0$, if and only if entrepreneurial productivity is persistent, $\rho > 0$.*

The remainder of our results also have parallels in this model. Wealth taxes reduce average entrepreneurial returns and so entrepreneurs as a group see their welfare decrease when wealth taxes increase. Workers benefit through the increase in their income following the increase in productivity, as they did before. The choice of optimal taxes takes a familiar form, balancing the positive level effect on aggregate variables, with the decrease in returns.

E.1 Entrepreneurial Problem

We solve the entrepreneurs' dynamic programming problem in (80) guessing that the value function has the form $V_i(a) = m_i + n\log(a)$, where $m_\ell, m_h, n \in \mathbb{R}$ are coefficients. Under this guess, the optimal savings choice is the solution to the following first order condition:

$$\frac{1}{R_i a - a_i^{'}} = \frac{\beta n}{a_i^{'}} \qquad \longrightarrow \qquad a_i^{'} = \frac{\beta n}{1+\beta n} R_i a.$$

Replacing the savings rule into the value function gives:

$$\begin{aligned} V_i(a) &= \log\left(R_i a - a_i^{'}\right) + \beta\left(pV_i\left(a_i^{'}\right) + (1-p)V_j\left(a_i^{'}\right)\right) \\ m_i + n\log(a) &= \log\left(R_i a - a_i^{'}\right) + \beta\left(pm_i + (1-p)m_j\right) + \beta n\log\left(a_i^{'}\right) \\ m_i + n\log(a) &= \beta n\log(\beta n) + (1+\beta n)\log\left(\frac{R_i}{1+\beta n}\right) + \beta\left(pm_i + (1-p)m_j\right) + (1+\beta n)\log(a) \end{aligned}$$

Matching coefficients we obtain

$$n = 1+\beta n \qquad \text{and} \qquad m_i = \beta n\log(\beta n) + (1+\beta n)\log\left(\frac{R_i}{1+\beta n}\right) + \beta\left(pm_i + (1-p)m_j\right),$$

where $j \neq i$. The solution to the first equation implies $n = \frac{1}{1-\beta}$. This in turn delivers the optimal saving decision of the entrepreneur in (81).Finally, we solve for the remaining coefficients from the

[38]Our results on the interplay of the persistence of entrepreneurial productivity and wealth taxes in determining aggregate productivity extend those of Moll (2014). We show that wealth taxes reduce misallocation through their heterogeneous effect on returns (for a given degree of persistence) resulting in asset accumulation by high-productivity entrepreneurs in a similar way that higher persistence does.

system of linear equations in m_ℓ and m_h to obtain

$$m_i = \frac{\log(1-\beta)}{1-\beta} + \frac{\beta}{(1-\beta)^2}\log(\beta) + \frac{(1-\beta p)\log R_i + \beta(1-p)\log R_j}{(1-\beta)^2(1-\beta(2p-1))}.$$

E.2 Stationary Recursive Competitive Equilibrium

We are interested in the equilibrium where the interest rate is determined by the return of the corporate sector, or equivalently by the L-types. Recall that the transition matrix for entrepreneurial productivity ensures that $\mu = 1/2$. Using the saving rules in equation (81), we derive the law of motion for the aggregate wealth of each group

$$\mu A_h^{'} = p\beta R_h \mu A_h + (1-p)\beta R_\ell (1-\mu) A_\ell \qquad \text{and} \qquad (1-\mu) A_\ell^{'} = (1-p)\beta R_h \mu A_h + p\beta R_\ell (1-\mu) A_\ell,$$

and for the aggregate capital ($K \equiv (1-\mu) A_\ell + \mu A_h$), where $s_h = \mu A_h / K$

$$\frac{K^{'}}{K} = \beta(s_h R_h + (1-s_h) R_\ell) = \beta\left((1-\tau_a) + (1-\tau_k)\alpha Z^\alpha K^{\alpha-1} L^{1-\alpha}\right).$$

where, as in Section 2.2, we use Lemma 1

In the stationary equilibrium a version of Proposition (1) applies:

$$R_\ell = (1-\tau_a) + \left(\frac{1}{\beta} - (1-\tau_a)\right)\frac{z_\ell}{Z} \qquad \text{and} \qquad R_h = (1-\tau_a) + \left(\frac{1}{\beta} - (1-\tau_a)\right)\frac{z_\lambda}{Z}. \tag{83}$$

Then, we use the law of motion of assets to obtain

$$\frac{(1-p)\beta R_\ell}{1-p\beta R_h} = \frac{s_h}{1-s_h} = \frac{1-p\beta R_\ell}{(1-p)\beta R_h}. \tag{84}$$

Replacing R_ℓ and R_h using (83) we get the quadratic equation in (82), similar to (35).

Studying equation (82), we show that there is a unique stationary equilibrium and obtain necessary and sufficient conditions for it to feature heterogeneous returns. We first discuss the logic behind the proof. For $\rho \leq 0$, there is a unique solution. For $\rho > 0$, there are two roots. However, only the larger root satisfies $z_\ell < Z < z_\lambda$. Then, there is always a unique equilibrium. For the equilibrium to feature return heterogeneity with $R_h > R_\ell$ it must be that $Z < z_h$. We obtain an upper bound on the collateral constraint parameter, $\overline{\lambda}_\rho$, that guarantees this.

Proposition 7. *There exists a unique stationary competitive equilibrium that features heterogenous returns ($R_h > R_\ell$), characterized by a productivity level $Z \in (z_\ell, z_h)$, if and only if the collateral constraint is not "too loose," that is, λ satisfies*

$$\lambda < \overline{\lambda}_\rho \equiv 1 + \frac{1-\rho}{1+\rho\left(1-2\left(\beta(1-\tau_a) + (1-\beta(1-\tau_a))\frac{z_\ell}{z_h}\right)\right)}.$$

Moreover, this condition, stated in terms of an upper bound on λ can be restated as an upper bound on the wealth tax, given λ:

$$\lambda < \overline{\lambda}_\rho \longleftrightarrow \tau_a \leq \overline{\tau}_{a,\rho} \equiv 1 - \frac{1}{\beta\left(1-\frac{z_\ell}{z_h}\right)}\left[\frac{(\lambda-1)(\rho+1)-(1-\rho)}{2(\lambda-1)\rho} - \frac{z_\ell}{z_h}\right].$$

Proof. For the heterogeneous return equilibrium to arise it must be that $(\lambda-1)\,\mu A_h < (1-\mu)\,A_\ell$. First, we show that such an equilibrium is unique and then that it exists under conditions on λ. The equilibrium Z corresponds to the largest root of equation (82). Define the function $h(z)$ as

$$\begin{aligned} h(z) = (1-\beta(1-\tau_a)(2p-1))\,z^2 - (z_\ell+z_\lambda)(p-\beta(1-\tau_a)(2p-1))\,z \\ +(2p-1)\,z_\ell z_\lambda(1-\beta(1-\tau_a)) \quad = \quad 0. \end{aligned}$$

It is easy to show that $h(z_\ell) = (1-p)\,z_\ell(z_\ell - z_\lambda) < 0$ and $h(z_\lambda) = (1-p)\,z_\lambda(z_\lambda - z_\ell) > 0$. Hence, there is a single root satisfying $z_\ell < Z < z_\lambda$ because $h(z)$ is a quadratic function.

Next, we prove that $(\lambda-1)\,\mu A_h < (1-\mu)\,A_\ell$ (excess supply of funds) iff $\lambda < \overline{\lambda}_\rho$. First, we show that $(\lambda-1)\,\mu A_h < (1-\mu)\,A_\ell$ iff $Z < z_h$. To see this substitute the definition of $Z = \frac{(z_h+(\lambda-1)(z_h-z_\ell))\mu A_h + z_\ell(1-\mu)A_\ell}{\mu A_h+(1-\mu)A_\ell}$ into $Z < z_h$, some algebra gives $(\lambda-1)\,\mu A_h < (1-\mu)\,A_\ell$. Second, we derive the condition on λ so that $h(z_h) > 0$ and thus $Z < z_h$:

$$h(z_h)/z_h^2 = 1-(2p-1)\,\beta(1-\tau_a) - \frac{(z_\ell+z_\lambda)}{z_h}(p-(2p-1)\,\beta(1-\tau_a)) + (2p-1)\,\frac{z_\ell z_\lambda}{z_h^2}(1-\beta(1-\tau_a)).$$

Inserting $z_\lambda = z_h + (\lambda-1)(z_h - z_\ell)$ and combining the terms that include $\lambda-1$ gives

$$h(z_h)/z_h^2 = \frac{(1-p)(z_h-z_\ell)}{z_h} - \frac{(\lambda-1)(z_h-z_\ell)}{z_h}\left(p-(2p-1)\left(\beta(1-\tau_a)+(1-\beta(1-\tau_a))\,\frac{z_\ell}{z_h}\right)\right).$$

Since $p-(2p-1)\left(\beta(1-\tau_a)+(1-\beta(1-\tau_a))\,\frac{z_\ell}{z_h}\right) > 0$ for all p, then, $h(z_h) > 0$ iff $\lambda - 1 < \frac{1-p}{p-(2p-1)\left(\beta(1-\tau_a)+(1-\beta(1-\tau_a))\frac{z_\ell}{z_h}\right)}$. Finally, recall that this equilibrium can only exist if $\lambda \le 2$ (this gives $K_\ell \ge 0$). Inspecting the previous result it is immediate that $\overline{\lambda} \le 2$ iff $p \ge 1/2$.

$\square$

Proposition 8. ***(Efficiency Gains from Wealth Taxation)*** *For all $\tau_a < \overline{\tau}_{a,\rho}$, an increase in the wealth tax (τ_a) increases aggregate productivity, $\frac{dZ}{d\tau_a} > 0$, if and only if entrepreneurial productivity is persistent, $\rho > 0$.*

Proof. The equilibrium level of Z is given by the solution of $h(Z) = 0$ where $h(z)$ is defined in equation (82). Differentiating $h(z)$ with respect to τ_a gives

$$\begin{aligned} \frac{d}{d\tau_a}h(z) &= (2p-1)\,\beta z^2 - (2p-1)\,\beta(z_\ell+z_\lambda)\,z + (2p-1)\,\beta z_\ell z_\lambda \\ &= (2p-1)\,\beta z_\ell z_\lambda (z-z_\ell)(z-z_\lambda). \end{aligned}$$

The equilibrium Z satisfies $z_\ell < Z < z_\lambda$, so $(z-z_\ell)(z-z_\lambda) < 0$. Thus, $\frac{d}{d\tau_a}h(z) < 0$ if and only if $p > 1/2$. Moreover, $\frac{d}{d\tau_a}h(z) < 0$ for all τ_a if $z_\ell < Z < z_\lambda$. Thus, $\frac{dZ}{d\tau_a} > 0$ as long as $\tau_a \le \overline{\tau}_{a,\rho}$.

$\square$

We now provide additional results that aid in the explanation of Proposition 6.

Lemma 9. ***(Savings Rates and Wealth Shares)*** *For all $\tau_a < \overline{\tau}_{a,\rho}$, the stationary saving rate of high-productivity entrepreneurs is positive and the saving rate of low-productivity entrepreneurs is negative: $\beta R_h > 1 > \beta R_\ell$. Furthermore, $s_h > 1/2$ if and only if $\rho > 0$.*

Proof. We first show that an entrepreneur's gross saving rate satisfies $\beta R_i > 1$ if and only if $z_i > Z$. This follows immediately by substituting R_i's from equation (83):

$$\beta R_i > 1 \longleftrightarrow \beta(1-\tau_a) + (1-\beta(1-\tau_a))\, z_i/Z > 1 \longleftrightarrow z_i > Z.$$

The result then follows because $z_\ell < Z < z_\lambda$.

Now, consider $s_h \geq 1/2$. We know that $s_h = \frac{Z-z_\ell}{z_\lambda - z_\ell}$, so $s_h > 1/2$ is equivalent to $Z > \frac{z_\lambda + z_\ell}{2}$. We can verify if this is the case by evaluating the residual of (82) at $\frac{z_\lambda+z_\ell}{2}$:

$$\begin{aligned}
h\left(\frac{z_\lambda+z_\ell}{2}\right) &= -(2p-1)(1-\beta(1-\tau_a))\left(\frac{z_\lambda+z_\ell}{2}\right)^2 + (2p-1)(1-\beta(1-\tau_a))\, z_\ell z_\lambda \\
&= -(2p-1)(1-\beta(1-\tau_a))\left[\left(\frac{z_\lambda+z_\ell}{2}\right)^2 - z_\ell z_\lambda\right] \\
&= -(2p-1)(1-\beta(1-\tau_a))\left(\frac{z_\lambda-z_\ell}{2}\right)^2 < 0
\end{aligned}$$

The residual is negative if and only if $p \geq 1/2$, $\rho > 0$. So, $Z > \frac{z_\lambda+z_\ell}{2}$ and thus $s_h > 1/2$ for $p \geq 1/2$. $\square$

Lemma 10. ***(Wealth Shares and Returns)*** *For all $\tau_a < \overline{\tau}_{a,\rho}$, the following equations and inequalities hold in equilibrium:*

$$\begin{aligned}
s_h &= \frac{1-\beta R_\ell}{\beta(R_h - R_\ell)} = \frac{Z-z_\ell}{z_\lambda - z_\ell} & \qquad \frac{ds_h}{dZ} &= \frac{1}{z_\lambda - z_\ell} > 0 \\
R_h &= \frac{1}{\beta(2p-1)}\left(1 - \frac{1-p}{s_h}\right) & \qquad \frac{dR_h}{dZ} &> 0 \\
R_\ell &= \frac{1}{\beta(2p-1)}\left(1 - \frac{1-p}{1-s_h}\right) & \qquad \frac{dR_h}{dZ} &< 0.
\end{aligned}$$

Moreover, the average returns are always decreasing with productivity, $\frac{d(R_\ell+R_h)}{dZ} < 0$, and the geometric average of returns decreases, $\frac{d(R_h R_\ell)}{dZ} < 0$, if and only if $\rho > 0$.

Proof. The wealth share is $s_h = \frac{Z-z_\ell}{z_\lambda - z_\ell}$ from the definition of Z in (16), so it is increasing in Z, $\frac{ds_h}{dZ} = \frac{1}{z_\lambda - z_\ell} > 0$. For returns consider the evolution equation for A_h in steady state

$$(1-p\beta R_h)\,\mu A_h \;=\; (1-p)\,\beta R_\ell\,(1-\mu)\, A_\ell.$$

Manipulating this expression (and the equivalent expression for L-types) gives

$$R_h = \frac{1}{p\beta} - \left(\frac{1-p}{p}\right)\left(\frac{1-s_h}{s_h}\right) R_\ell; \qquad \text{and} \qquad R_\ell = \frac{1}{p\beta} - \left(\frac{1-p}{p}\right)\left(\frac{s_h}{1-s_h}\right) R_h.$$

Replacing we can solve for R_h and R_ℓ as a function of s_h,

$$R_h = \frac{1}{\beta(2p-1)}\left(1 - \frac{1-p}{s_h}\right); \qquad \text{and} \qquad R_\ell = \frac{1}{\beta(2p-1)}\left(1 - \frac{1-p}{1-s_h}\right). \tag{85}$$

Their derivates with respect to Z are,

$$\frac{dR_h}{dZ} = \frac{1-p}{\beta(2p-1)} \frac{1}{s_h^2} \frac{ds_h}{dZ} > 0; \qquad \frac{dR_\ell}{dZ} = -\frac{(1-p)}{\beta(2p-1)} \frac{1}{(1-s_h)^2} \frac{ds_h}{dZ} < 0. \tag{86}$$

Using the results in (85) and (86) we obtain expressions for the derivative of the sum and product of returns with respect to the wealth tax:

$$\frac{d(R_h+R_\ell)}{dZ} = \frac{-(2s_h-1)(1-p)}{\beta(2p-1)\left((1-s_h)^2 s_h^2\right)} \frac{ds_h}{dZ}; \qquad \frac{d(R_hR_\ell)}{dZ} = \frac{-(2s_h-1)\,p\,(1-p)}{[(1-s_h)\,s_h\beta(2p-1)]^2} \frac{ds_h}{dZ}$$

$\frac{d(R_h+R_\ell)}{dZ}$ is always negative because $s_h \geq 1/2$ if and only if $p \geq 1/2$, as we proved in the previous Lemma. $\frac{d(R_hR_\ell)}{dZ}$ is negative if and only if $s_h \geq 1/2$, again, this happens if and only if $p \geq 1/2$.

□

E.3 Optimal Taxes

We first discuss the welfare measure we use as the government's objective. Because there is no stationary wealth distribution, it is not possible to compute aggregate welfare directly. However, it is possible to define policy so as to maximize the welfare change with respect to a benchmark economy. Let B denote the initial benchmark economy with a given level of capital income and wealth taxes and C denote a counterfactual economy with a higher wealth tax and a lower capital income tax, satisfying Assumption 2. Define $\left\{c_t^{\mathrm{j}}(a,i)\right\}$ as the consumption path and $V^{\mathrm{j}}(a,i)$ as the value function of an individual of type $i \in \{w,h,\ell\}$ under economy $\mathrm{j} \in \{\mathrm{B},\mathrm{C}\}$. We ask each individual how much they value being dropped from B to C in terms of a consumption-equivalent welfare measure $\mathrm{CE}_1(a,i)$, which is defined by

$$\mathbb{E}_0 \sum_t \beta^{t-1} \log\left((1+\mathrm{CE}_1(a,i))\, c_t^{\mathrm{B}}(a,i)\right) \quad = \quad \mathbb{E}_0 \sum_t \beta^{t-1} \log\left(c_t^{\mathrm{C}}(a,i)\right).$$

Solving for $\mathrm{CE}_1(a,i)$ all terms containing wealth cancel, so we drop wealth from the arguments,

$$\log(1+\mathrm{CE}_{1,i}) \quad = \quad \begin{cases} \log\left(\frac{w^{\mathrm{C}}+T^{\mathrm{C}}}{w^{\mathrm{B}}+T^{\mathrm{B}}}\right) & \text{if } i = w \\ \frac{(1-\beta)\log\left(\frac{R_i^{\mathrm{C}}}{R_i^{\mathrm{B}}}\right)+\beta(1-p)\left(\log\left(\frac{R_\ell^{\mathrm{C}}}{R_\ell^{\mathrm{B}}}\right)+\log\left(\frac{R_h^{\mathrm{C}}}{R_h^{\mathrm{B}}}\right)\right)}{(1-\beta)(1-\beta(2p-1))} & \text{if } i \in \{\ell,h\}\,. \end{cases} \tag{87}$$

The aggregate welfare gain is the population-weighted average of welfare gains,

$$\log(1+\mathrm{CE}_1) = \sum_{i\in\{w,h,\ell\}} n_i \log(1+\mathrm{CE}_{1,i})\,, \tag{88}$$

where $n_w \equiv L/(L+1)$ is the population share of workers and $n_h = n_\ell \equiv 1/(L+1)$ the share of entrepreneurs. We also define the average entrepreneurial welfare gain ($\mathrm{CE}_1^{\mathrm{e}}$) as

$$\log(1+\mathrm{CE}_1^{\mathrm{e}}) = \mu\log(1+\mathrm{CE}_{1,h}) + (1-\mu)\log(1+\mathrm{CE}_{1,\ell}) = \frac{1}{1-\beta}\left(\mu\log\left(\frac{R_h^{\mathrm{C}}}{R_h^{\mathrm{B}}}\right) + (1-\mu)\log\left(\frac{R_\ell^{\mathrm{C}}}{R_\ell^{\mathrm{B}}}\right)\right). \tag{89}$$

Workers gain from an increase in the wealth tax because their income increase. For entrepreneurs, the welfare effects of the increase in the wealth tax come from changes in after-tax returns. There are two effects. First, a higher wealth tax reduces the current returns of low-productivity entrepreneurs and increase those of high-productivity entrepreneurs. Second, (log-)average of returns decrease with the wealth tax, decreasing entrepreneurs' expectations over future returns and reducing their welfare. The net result of these effects is a lower welfare for the low-productivity entrepreneurs and for entrepreneurs as a group.

The welfare gain for the H-type depends on the magnitude of the decrease in average returns, that in turn depends on the initial return dispersion. There is an upper bound on the dispersion of returns (κ_R) that ensures that the loss from lower expected returns is low relative to the increase in R_h. The upper bound is a function of only β and ρ and does not change with the wealth tax. However the ambiguity over welfare gains is explained because the $\mathrm{CE}_{1,h}$ welfare measure ignores the gains in wealth growth for H-types brought about by the increase in the wealth tax. Taking this into account makes the welfare change unambiguously positive for them.

Lemma 11. ***(Welfare Gain by Agent Type)*** *For all $\tau_a < \overline{\tau}_{a,\rho}$, if Assumption 2 holds and $\rho > 0$, a marginal increase in the wealth tax (τ_a) increases the welfare of workers ($CE_{1,w} > 0$) and decreases the welfare of low-productivity entrepreneurs ($CE_{1,\ell} < 0$) and the average welfare of entrepreneurs ($CE_1^e < 0$). Furthermore, there exists an upper bound on the dispersion of returns (κ_R) such that an increase in the wealth tax increases the welfare of high-productivity entrepreneurs ($CE_{1,h} > 0$) if and only if $R_h - R_\ell < \kappa_R$.*

Proof. For workers' welfare,

$$\frac{d\log\left(1+\mathrm{CE}_{1,w}\right)}{d\tau_a} = \frac{\alpha}{1-\alpha}\frac{d\log Z}{d\tau_a} > 0 \longleftrightarrow \rho > 0$$

The welfare gain is positive if and only if productivity is persistent because of Proposition 6. The welfare of low-productivity entrepreneurs decreases,

$$\frac{d\log\left(1+\mathrm{CE}_{1,\ell}\right)}{d\tau_a} \propto (1-\beta)\frac{d\log R_\ell}{d\tau_a} + \beta(1-p)\frac{d\log R_\ell R_h}{d\tau_a} < 0,$$

following from Lemma (10) $\left(\frac{dR_\ell}{d\tau_a}, \frac{dR_\ell R_h}{d\tau_a} < 0\right)$. The average welfare of entrepreneurs also decreases,

$$\frac{d\log\left(1+\mathrm{CE}_1^e\right)}{d\tau_a} = \frac{\beta(1-p)}{1-\beta}\frac{1}{R_\ell R_h}\frac{dR_\ell R_h}{d\tau_a} < 0.$$

Finally, for the high-productivity entrepreneurs:

$$\frac{d\log\left(1+\mathrm{CE}_{1,h}\right)}{d\tau_a} \propto \frac{1-\beta}{R_h}\frac{dR_h}{d\tau_a} + \frac{\beta(1-p)}{R_\ell R_h}\frac{dR_\ell R_h}{d\tau_a} = \left[(1-\beta) - \frac{\beta\left(2s_h-1\right)p(1-p)}{\left(p-s_h\right)\left(1-s_h\right)}\right]\frac{(1-p)}{\beta(2p-1)s_h^2 R_h}\frac{ds_h}{d\tau_a}.$$

We maintain the assumption that $\rho > 0$, and from Lemma (10) we know that $\frac{ds_h}{d\tau_a} > 0$. So, the sign of derivative of interest depends on the sign of the term in square brackets,

$$\frac{d\log\left(1+\mathrm{CE}_{1,h}\right)}{d\tau_a} \geq 0 \longleftrightarrow 1-\beta \geq \frac{\beta\left(2s_h-1\right)p(1-p)}{\left(p-s_h\right)\left(1-s_h\right)}.$$

We verify that $s_h < p$ in equilibrium, which together with Lemma (9) implies that the right hand side of the inequality is always positive. To verify that $s_h < p$, note that this condition is equivalent to $Z < pz_\lambda + (1-p)\, z_\ell$, then evaluate function h defined in (82) at $pz_\lambda + (1-p)\, z_\ell$. The value of h is always positive, so it must be that $Z < pz_\lambda + (1-p)\, z_\ell$ and thus $s_h < p$.

Then, the H-type's welfare gain is positive if and only if

$$g(s_h) \equiv (1-\beta)(p-s_h)(1-s_h) - \beta(2s_h-1)\, p\,(1-p) \geq 0. \tag{90}$$

We know that for the interval $s_h \in [1/2, p]$ g is continuous and monotonically decreasing,

$$g(1/2) = (1-\beta)\left(p - \frac{1}{2}\right)\frac{1}{2} > 0; \qquad \text{and} \qquad g(p) \equiv -\beta(2p-1)\, p\,(1-p) < 0..$$

So, there exists an upper bound $\bar{s}_h$ such that

$$\frac{d \log(1 + \mathrm{CE}_{1,h})}{d\tau_a} \geq 0 \;\longleftrightarrow\; s_h \in \left[\frac{1}{2}, \bar{s}_h\right],$$

characterized by the solution to

$$(p - \bar{s}_h)(1-\bar{s}_h) - \beta(2\bar{s}_h - 1)\, p\,(1-p) = 0.$$

Alternatively, we can make us of the link between s_h and the dispersion of returns:

$$R_h - R_\ell = \frac{(1-p)(2s_h-1)}{\beta(2p-1)(1-s_h)\, s_h}.$$

So the high-productivity entrepreneurs benefit from an increase in the wealth tax if and only if the dispersion of returns is low enough:

$$\frac{d \log(1 + \mathrm{CE}_{1,h})}{d\tau_a} \geq 0 \;\longleftrightarrow\; s_h \in \left[\frac{1}{2}, \bar{s}_h\right] \;\longleftrightarrow\; R_h - R_\ell \in [0, \kappa_R],$$

where $\kappa_R \equiv \frac{(1-p)(2\bar{s}_h-1)}{\beta(2p-1)(1-\bar{s}_h)\bar{s}_h}$. Note that $\bar{s}_h$, and therefore κ_R, depend only on p and β.

□

We now characterize the optimal tax combination $\left(\tau^\star_{a,\rho}, \tau^\star_{k,\rho}\right)$ that maximizes the utilitarian welfare measure CE_1. Proposition 11 makes clear the key tradeoff when considering the welfare effects of wealth taxation: A higher wealth tax increases the welfare of workers by increasing wages through productivity gains, but they reduce the welfare of entrepreneurs by increasing the dispersion of returns and decreasing their expected value. This tradeoff is captured by the elasticities of wages and returns to changes in productivity. The welfare gain of workers is proportional to the wage elasticity with respect to productivity, $\xi_Z^{w+T} = \frac{\alpha}{1-\alpha}$, while the welfare loss of entrepreneurs is proportional to the average elasticity of returns with respect to productivity, $\mu\xi_Z^{R_h} + (1-\mu)\,\xi_Z^{R_\ell}$.

Proposition 9. ***(Optimal*** $\boldsymbol{CE_1}$ ***Taxes)*** *Under Assumption 2, there exist a unique tax combination* $\left(\tau^\star_{a,\rho}, \tau^\star_{k,\rho}\right)$ *that maximizes the utilitarian welfare measure* CE_1*. An interior solution,*

$\tau^{\star}_{a,\rho} < \overline{\tau}_{a,\rho}$, *is the solution to:*

$$0 = n_w \xi_Z^{w+T} + \frac{1-n_w}{1-\beta}\left(\mu \xi_Z^{R_h} + (1-\mu)\,\xi_Z^{R_\ell}\right), \tag{91}$$

where $\xi_x^Z \equiv \frac{d\log x}{d\log Z}$ *is the elasticity of variable* x *with respect to* Z.

Similar to Proposition (4), this condition implies threshold values for α (that determines ξ_Z^{w+T}) corresponding to the values for which $\tau^{\star}_{a,\rho} = 0$ and $\tau^{\star}_{a,\rho} = \tau_a^{TR}$ are optimal.

As an alternative to CE_1, we consider the welfare gain of a stand-in representative low- and high-productivity entrepreneur. We compare the values of the entrepreneurs between being in the Benchmark or Counterfactual economy while holding the average wealth of a low- or high-productivity entrepreneur. We denote this welfare measure as $\mathrm{CE}_{2,i}$:

$$\log\left(1+\mathrm{CE}_{2,i}\right) = (1-\beta)\left(V^{\mathrm{C}}\left(A_i^{\mathrm{C}}, i\right) - V^{\mathrm{B}}\left(A_i^{\mathrm{B}}, i\right)\right) = \log\left(1+\mathrm{CE}_{1,i}\right) + \log\left({}^{A_i^{\mathrm{C}}}/{}_{A_i^{\mathrm{B}}}\right). \tag{92}$$

We can also ask each entrepreneur how much they value being in the counterfactual economy with its average wealth $\left(K^{\mathrm{C}}\right)$ relative to being in the benchmark economy with its average wealth $\left(K^{\mathrm{C}}\right)$. The welfare gain for a type-i entrepreneur is

$$\log\left(1+\widetilde{\mathrm{CE}}_{2,i}\right) = (1-\beta)\left(V^{\mathrm{C}}\left(K^{\mathrm{C}}, i\right) - V^{\mathrm{B}}\left(K^{\mathrm{C}}, i\right)\right) = \log\left(1+\mathrm{CE}_{1,i}\right) + \log\left({}^{K^{\mathrm{C}}}/{}_{K^{\mathrm{B}}}\right), \tag{93}$$

and the aggregate (or expected) welfare is

$$\log\left(1+\widetilde{\mathrm{CE}}_{2}\right) = \sum_i n_i \log\left(1+\widetilde{\mathrm{CE}}_{2,i}\right) = n_w \log\left(1+\mathrm{CE}_1\right) + (1-n_w)\log\left({}^{K^{\mathrm{C}}}/{}_{K^{\mathrm{B}}}\right). \tag{94}$$

This gives a similar welfare measure to the one used in Section 5. The optimal taxes are similarly given as:

Proposition 10. ***(Optimal*** $\widetilde{\boldsymbol{CE}}_2$ ***Taxes)*** *Under Assumption 2, there exist a unique tax combination* $\left(\tau^{\star}_{a,2}, \tau^{\star}_{k,2}\right)$ *that maximizes the utilitarian welfare measure* $\widetilde{CE}_2$*, an interior solution* $\tau^{\star}_{a,2} < \overline{\tau}_{a,\rho}$ *is the solution to:*

$$0 = n_w \xi^Z_{w+T} + (1-n_w)\,\xi^Z_K + \frac{1-n_w}{1-\beta}\left(\mu \xi^Z_{R_h} + (1-\mu)\,\xi^Z_{R_\ell}\right) \tag{95}$$

where $\xi_x^Z \equiv \frac{d\log x}{d\log Z}$ *is the elasticity of variable* x *with respect to* Z.

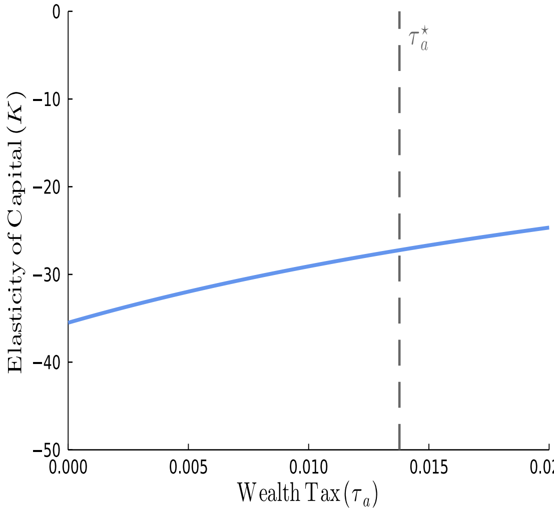
Taking into account the role of capital accumulation results in a higher optimal tax level, and lower thresholds $\underline{\alpha}$ and $\overline{\alpha}$:

Corollary 2. ***(Comparison of*** $\boldsymbol{CE}_1$ ***and*** $\boldsymbol{CE}_2$ ***Taxes)*** *The optimal wealth tax is higher when taking the wealth accumulation into account* $\left(\tau^{\star}_{a,2} > \tau^{\star}_{a,\rho}\right)$*. Moreover, the* α*-thresholds are lower* $\underline{\alpha}_2 < \underline{\alpha}_\rho$ *and* $\overline{\alpha}_2 < \overline{\alpha}_\rho$.

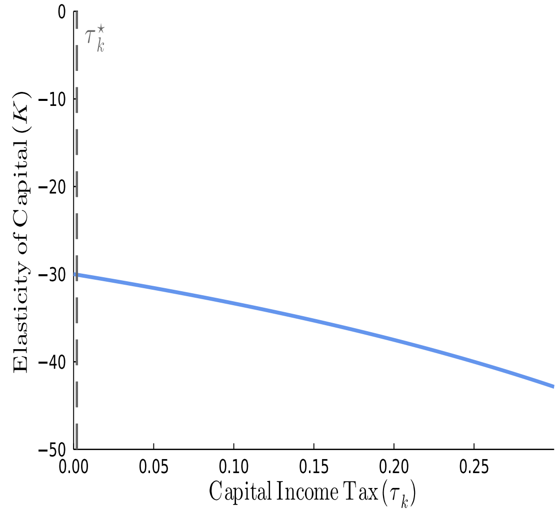
F Additional Figures

Figure F.4: Elasticity of Capital to Taxes

(a) Elasticity w.r.t. Wealth Tax, $\xi_K^{\tau_a}$



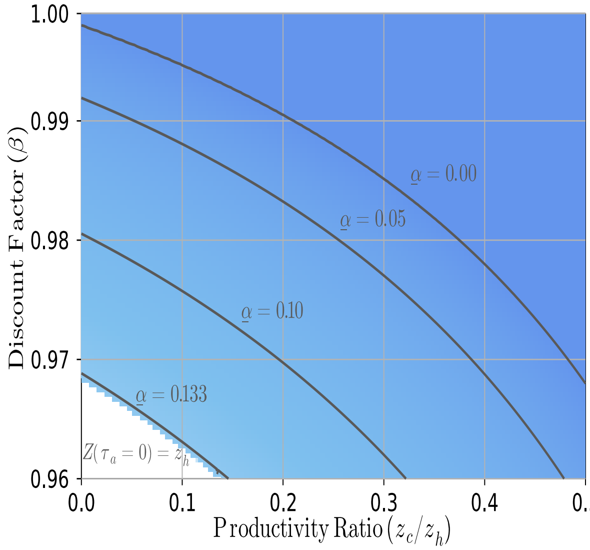
(b) Elasticity w.r.t. Capital Income Tax, $\xi_K^{\tau_k}$



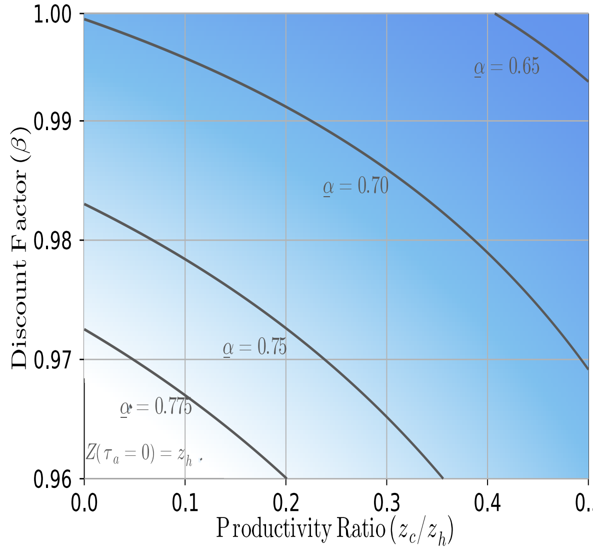
Note: The figures report the elasticity of K to τ_a (left) and τ_k (right) computed as in (36) and (37). To make the magnitudes comparable, the elasticity of K with respect to τ_k is re-scaled by $^{(1-\theta)\beta\delta}/_{(1-\beta\delta)}$ to reflect the change in τ_k that matches a one percentage point change in τ_a under Assumption 2. . Other parameters are: $\delta = {}^{49}/_{50}$, $\beta\delta = 0.96$, $z_h = 1$, $\theta = 0.25$, and $\alpha = 0.4$. λ is such that the debt-to-output ratio is 1.5 when $\tau_a = 0$.

Figure F.5: α Thresholds for Entrepreneurial Welfare Gains

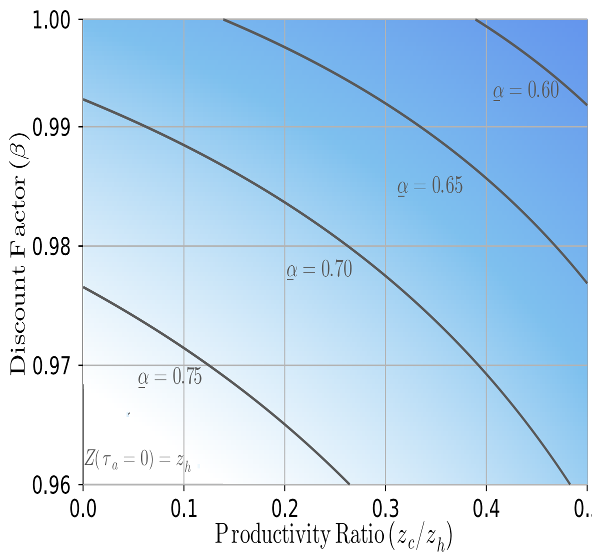
(a) High-Productivity Entrepreneurs: ${}^{dV_h}/_{d\tau_a} > 0$



(b) Low-Productivity Entrepreneurs: ${}^{dV_\ell}/_{d\tau_a} > 0$



(c) Average Entrepreneur: ${}^{dV_E}/_{d\tau_a} > 0$



Note: The figures report the threshold value of α above which entrepreneurial welfare increases after an increase in the wealth tax for combinations of the discount factor (β) and productivity dispersion (${}^{z_\ell}/_{z_h}$). Other parameters are: $\delta = {}^{49}/_{50}$, $z_h = 1$, $\theta = 0.25$, and $\alpha = 0.4$. λ is such that the debt-to-output ratio is 1.5 when $\tau_a = 0$.