The solution of this problem consists of a PDF with all mathematical derivations and all graphs as well as julia or matlab script that produces the results.

1   A closed-economy with span of control

Consider a static (one period) economy with infinitely many individuals. Individuals differ in their entrepreneurial (or managerial) talent denoted by z_i. Assume that talent is distributed Pareto with minimum value of $\underline{z} = 1$ and a Pareto coefficient of ξ, so that the CDF of the distribution is $\Pr(\tilde{z} \leq z) = 1 - z^{-\xi}$.

Individuals also have an endowment of assets that differs across agents and is distributed independently of their entrepreneurial talent. Assume that the aggregate assets are $\overline{K} = E[a_i] > 0$. There is a market for capital where individuals can lend their assets to firms at a market rate r.

Individuals face an occupational choice. They can use their entrepreneurial talent to start a firm, doing so has a fixed cost $\psi > 0$. If they start a firm they produce final goods with a technology $z_i g(F(k_i, n_i))$, where $g(x) = x^\alpha$ $\alpha \in (0,1)$ and $F(k,n) = k^\gamma n^{1-\gamma}$. If they do not start a firm they can work. Each individual has one unit of time that they offer inelastically. The market wage is w.

Individuals value consumption. The capital and labor markets are perfectly competitive.

1. Pose the profit maximization of an entrepreneur with talent z. Solve the problem. That is, find functions $\pi^\star(z)$, $n^\star(z)$, $k^\star(z)$ that characterize the optimal profits, and labor and capital demand of a entrepreneur. Take prices r and w as given.

2. Pose the occupational choice problem of individuals.

3. Define an equilibrium for this economy.

4. Solve the equilibrium in this economy. In particular, solve for the prices that clear the markets. You can make any assumptions you deem useful for this (for instance you can do away with capital or the fixed set up costs).

5. How does the distribution of firm productivity (talent) differ from the distribution of talent in the population?

6. How does talent translate into differences in income? For this compare the distribution of profits and the distribution of talent among entrepreneurs. In particular, if an entrepreneur has 10 times more talent than another, how much more income (profits) do they have?

7. How does the distribution of firm size by employment differ from the distribution by capital. Interpret the difference.

8. Analyze the solution. What do the comparative statics of the equilibrium prices and quantities tell you about the economy.

2 [60 points] Price Setting and Adjustment Costs

Consider an economy with a continuum of producers. All producers are identical except for the fact that they produce differentiated varieties of goods that are then aggregated into final consumption:

$$Y = \left(\int_0^1 y_i^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The demand for each producer's good is

$$\frac{y_i}{Y} = \left(\frac{P_i}{P} \right)^{-\varepsilon}.$$

1. Derive an expression for the aggregate price of the economy as a function of the prices of individual producers.

   The producers' problem is to maximize their profits choosing prices. Their output comes from using their linear-labor-technology: $y_i = zn_i$. Their problem is, in terms of output,

   $$\Pi_i = \max_{P_i} P_i y_i - \frac{W}{z} y_i$$

2. Derive the optimal pricing choice of firms.

3. What is the aggregate price and aggregate profits in the economy?

4. Real wages $w = W/P$:

   (a) What is the real wage in this economy? (how many units of final good can be bought with one unit of labor)

   (b) Are workers being paid the marginal product of their labor? Explain.

5. Is the aggregate (nominal) price level P defined in the model? What does this imply for Money Neutrality? Explain.

   Consider now that this economy operates for a second period. However, firms now face an adjustment cost that makes it costly to change their prices. They have to take this into

consideration when setting their prices. Their (discounted) profits are

$$\Pi = P_{i,1}y_{i,1} - \underbrace{\frac{w_1}{z_1}}_{\text{Real Mrg. Cost}} y_{i,1}P_1 - \underbrace{\frac{\lambda}{2}\left(\frac{P_{i,1}}{P_{i,0}}-1\right)^2}_{\text{Real Adjustment Costs}} P_1 + \beta\frac{P_1}{P_2}\left[P_{i,2}y_{i,2} - \underbrace{\frac{w_2}{z_2}}_{\text{Real Mrg. Cost}} y_{i,2}P_2 - \underbrace{\frac{\lambda}{2}\left(\frac{P_{i,2}}{P_{i,1}}-1\right)^2}_{\text{Real Adjustment Costs}} P_2\right]$$

6. Optimal prices:

   (a) Characterize the firm's optimal price P_1. Take as given $P_{i,0}$ and express the firms' first order condition in terms of relative prices (P_i/P) and inflation $(1 + \pi_t = P_t/P_{t-1})$. Verify that if $\lambda = 0$ then you get the same result as above when there were no adjustment costs.

   (b) Interpret the three terms of the first order condition. Hint: The first term is the same as in the normal monopolist problem. The other two terms are determined by the effects of price adjustment and depend on inflation.

7. Equilibrium. All the firms in this economy are identical. Further assume that $P_{i,0} = P_0$ for all firms. Then the equilibrium is symmetric, with all firms making the same choices and we have that $P_{i,t} = P_t$ for all firms.

   (a) Use this result to simplify the optimal pricing equation. You should obtain the following result:

   $$0 = -(\varepsilon - 1)\left(1 - \frac{\varepsilon}{\varepsilon - 1}\text{mc}_1\right)Y_1 - \lambda\left[\pi_1\left(1 + \pi_1\right) - \beta\pi_2\left(1 + \pi_2\right)\right]$$

   This is the New-Keynesian Phillips Curve. It establishes a relationship between inflation (and future inflation expectations) and the marginal cost of production (given here bye $\text{mc}_t = w_t/z_t$). Without adjustment costs for prices this equations simply determines the level of the real marginal costs (real wage). The New-Keynesian Phillips Curve implies a relationship between the marginal costs, output (demand), and inflation.

(b) What happens to inflation if the marginal cost goes up ($mc_1 \uparrow$)? How does the magnitude of the effect change with λ? Explain what is the mechanism for this in the model and its intuition.

(c) What happens to inflation if output demand goes up ($Y_1 \uparrow$)? Explain what is the mechanism for this in the model and its intuition.

In answering these questions you can set $\beta = 0$ so that you don't have to worry about the behavior of future inflation.