University of Minnesota
Math Refresher
Summer 2015

Problem Set 3

1. Prove or disprove: the convex hull of a compact set is compact.

2. Show that a function $f : \Re^n \to \Re$ is affine iff is convex and concave. (Exercise 5, Sundaram 7.8).

3. Consider a finite collection of sets $X_i \subset \Re^n_+$ for $i \in I = \{1, 2, ..., l\}$, for each set there is a preference relation characterized by $\succeq_i$, which implies that x is preferred to y iff $x \succeq_i y$. Suppose that $\succeq_i$ is reflexive, transitive and convex (see definition below) for all $i \in I$. Now suppose there exists $x \in X \subset \Re^{nl}_+$ such that $\sum_{i \in I} x_i = w$. Define $A_i(x_k) = \{y_i \in X_i : y_i \succ x_i\}$ for all $i \in I$ and $A = \sum_i A_i$. Show that there exists $p \in \Re^n$ such that $pz \geq pw$ for all $z \in A$.

   Convexity: A reflexive and transitive (see definitions in Exercise 6, PS0) preference relation $\succeq$ is convex on a set X if for all $x, y \in X$ and $\lambda \in (0,1)$, $y \succ x \Rightarrow \lambda x + (1-\lambda) y \succ x$.

4. Let $f : \Re^n \to \Re$ be a concave function stisfying $f(0) = 0$. Show that for all $k \geq 1$ we have $kf(x) \geq f(kx)$. What if $k \in [0, 1)$? (Exercise 3, Sundaram 7.8).

5. Let $\{f_i : i \in I\}$ a set of functions from $D \subset \Re^n$ to $\Re$ which are convex and bounded on D. Show that f is convex, for f defined as:

   $f(x) = sup_{i \in I} f_i$

   What if we replace sup by inf? (Exercise 8, Sundaram 7.8)

6. Are quasi-convex and quasi-concave functions always continuous in the interior of their domain?

7. Find the extrema of $F(x, y) = x^2 \cdot y - \log(x)$ subject to $0 = g(x, y) := 16x + 6y$.

8. Excercises 17, 18, 19, 20 and 23 Sundaram 7.8.

9. Exercises 2, 4 and 9, Sundaram 8.9.