

# **Book-Value Wealth Taxation, Capital Income Taxation, and Innovation**

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Fatih Guvenen, Gueorgui Kambourov, Burhan Kuruscu, Sergio Ocampo

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**This paper:** **Theoretical analysis** of optimal **combination** of taxes

- ▶ Characterize **(i)** innovation + productivity **(ii)** welfare **(iii)** optimal taxes
- ▶ Analytical model with workers, entrepreneurs, and innovation

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- **But** models struggle to generate plausible wealth inequality.

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- Return heterogeneity → concentration at very top + Pareto tail + fast wealth growth

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4. **Theoretical:** Interesting **new economic mechanisms** → Example next

*Allais (1977), Guvenen, Kambourov, Kuruscu, Ocampo, Chen (2023)*

## Return Heterogeneity: A Simple Example

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- ▶ **Objective:** illustrate key tradeoffs b/w capital income tax ( $\tau_k$ ) and wealth tax ( $\tau_a$ )

# Capital Income ( $\tau_k$ ) vs. Wealth Tax ( $\tau_a$ )

Capital Income Tax			
$a_{i,\text{after-tax}} = a_i + (1 - \tau_k)r_i a_i$			
	Fredo ( $r_f = 0\%$ )	Mike ( $r_m = 20\%$ )	
Wealth	\$1M	\$1M	
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- Replacing  $\tau_k$  with  $\tau_a \rightarrow$  **reallocates** assets to high-return agents (use it or lose it) + **increases dispersion** in after-tax returns & wealth.

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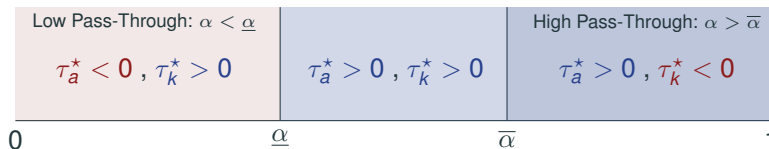
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Workers & Productive entrepreneurs **gain**      Unproductive entrepreneurs **lose**

3. **Optimal Taxes:** Depend on TFP pass-through to wages and  $K$  (given by capital intensity,  $\alpha$ )





1. **Benchmark model with endogenous entrepreneurial productivity distribution**
2. Innovation and efficiency gains from wealth taxation
3. Welfare and optimal taxation
4. Extension to managerial effort
5. Quantitative results (time allowing!)

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  - Produce final goods using capital and labor + consume/save
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**Common Preferences:** Discount  $\beta < 1$  and conditional survival probability  $\delta < 1$

$$E_0 \sum_{t=0}^{\infty} (\beta\delta)^t \log(c_t)$$

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**Entrepreneurial technology:**

$$y_i = (z_i k_i)^\alpha n_i^{1-\alpha}$$

- ▶ Key is constant-returns-to-scale



# Financial Markets & Entrepreneurs' Problem

## Financial markets:

- ▶ Collateral constraint:  $k \leq \lambda a$ , where  $a$  is entrepreneur's wealth and  $\lambda \geq 1$
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## Entrepreneurs' production decision:

[▶ details](#)

$$\Pi^*(z, a) = \max_{k \leq \lambda a, n} \{ (zk)^\alpha n^{1-\alpha} - rk - wn \} \quad \longrightarrow \quad \Pi^*(z, a) = \underbrace{\pi^*(z)}_{\text{Excess return above } r} \times a$$

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$$\text{If } \underbrace{(\lambda - 1) \mu A_h}_{K \text{ Demand from H-Type}} < \underbrace{(1 - \mu) A_\ell}_{K \text{ Supply from L-Type}} \iff \underbrace{\lambda < \bar{\lambda}}_{\text{Bound on Leverage}} \iff \tau_a < \bar{\tau}_a$$

# Entrepreneur's Dynamic Problem

$$V(a, z) = \max_{c, a'} \log(c) + \beta \delta V(a', z)$$
$$\text{s.t.} \quad c + a' = \underbrace{(1 - \tau_a) a + (1 - \tau_k)(r + \pi^*(z)) a}_{\text{After-tax Wealth}}.$$

► Define (after-tax) gross return as:

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**Note:** log utility  $\longrightarrow$  No behavioral response to taxes.  
 $\longrightarrow$  All effects come from use-it-or-lose-it (*conservative lower bound*)

# Entrepreneur's Innovation Effort Choice

Innovator's problem:

$$\max_e p(e) V_h(\bar{a}) + (1 - p(e)) V_\ell(\bar{a}) - \frac{1}{(1 - \beta\delta)^2} \Lambda(e)$$

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## Optimal innovation effort:

$$\underbrace{\Lambda'(e)}_{\text{Mrg. Cost of Effort}} = (1 - \beta\delta)^2 (V_h(\bar{a}) - V_\ell(\bar{a})) = \underbrace{\log R_h - \log R_\ell}_{\text{Mrg. Benefit: Return Gap}}$$

► Return dispersion incentivizes effort  $\rightarrow$  Return dispersion necessary for innovation!

# Aggregate Output and Taxes

## Aggregate output:

$$Y = \int y_i di = \int (z_i k_i)^\alpha n_i^{1-\alpha} di$$

- ▶ All output is produced by entrepreneurs
- ▶ **Equivalent:** Add corporate sector with  $Y_c = (z_c K_c)^\alpha N_c^{1-\alpha}$  and  $z_\ell \leq z_c < z_h$

**Government:** Finances exogenous expenditure  $G$  and transfers  $T$  with  $\tau_k$  and  $\tau_a$

$$G + T = \tau_k \alpha Y + \tau_a K$$

# Equilibrium Values: Aggregation

## Key variables:

- ▶  $s_h = \frac{\mu A_h}{\mu A_h + (1 - \mu) A_\ell}$ : wealth share of high-productivity entrepreneurs.
- ▶  $z_\lambda \equiv z_h + (\lambda - 1)(z_h - z_\ell)$ : effective productivity of high-productivity entrepreneurs.

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**Lemma:** Aggregate output can be written as:

$$Y = (\textcolor{blue}{Z}\textcolor{red}{K})^\alpha L^{1-\alpha} \quad (Z^\alpha \text{ is measured TFP})$$

where

$$\textcolor{blue}{K} \equiv \mu A_h + (1 - \mu) A_\ell$$

$K$  = Aggregate capital

$$\textcolor{red}{Z} \equiv s_h z_\lambda + (1 - s_h) z_\ell$$

$Z$  = Wealth-weighted productivity

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$$\begin{aligned} \textcolor{blue}{K} &\equiv \mu A_h + (1 - \mu) A_\ell & K &= \text{Aggregate capital} \\ \textcolor{red}{Z} &\equiv s_h z_\lambda + (1 - s_h) z_\ell & Z &= \text{Wealth-weighted productivity} \end{aligned}$$

**Note:** **Use it or lose it effect** increases efficiency if  $s_h \uparrow (\longrightarrow Z \uparrow)$

## Steady State: Capital, Returns, and Taxes

**Steady State  $K$ :** Same as Neoclassical Growth Model... but endogenous  $Z$  (Moll, 2014)

$$(1 - \tau_a) + (1 - \tau_k) \overbrace{\alpha Z^\alpha (K/L)^{\alpha-1}}^{\text{MPK}} = \frac{1}{\beta\delta}$$

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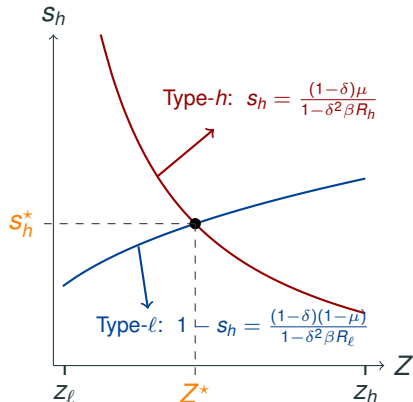
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**Steady State  $R$ :** Returns reflect MPK + effective entrepreneurial productivity  $z_i \in \{z_\ell, z_\lambda\}$

$$R_i = (1 - \tau_a) + (1 - \tau_k) \overbrace{\left( \alpha Z^\alpha (K/L)^{\alpha-1} \right)}^{\text{MPK}} \frac{z_i}{Z} \longrightarrow R_i = (1 - \tau_a) + \left( \frac{1}{\beta\delta} - (1 - \tau_a) \right) \frac{z_i}{Z}$$

# Steady State: Productivity and Returns



- $Z$  consistent with wealth accumulation

$$Z = s_h z_\lambda + (1 - s_h) z_c$$

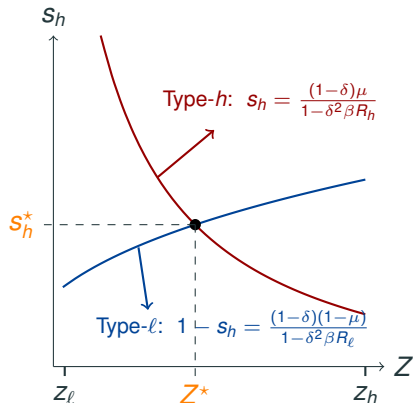
- Wealth distribution reflects returns

$$A'_i = \delta^2 \beta R_i A_i + (1 - \delta) \bar{a} \rightarrow \frac{A_i}{\bar{a}} = \frac{1 - \delta}{1 - \delta^2 \beta R_i}$$

- Equilibrium:  $Z \rightarrow \{R_h, R_\ell\} \rightarrow s_h \rightarrow Z$

■ Solution is quadratic!

# Steady State: Productivity and Returns



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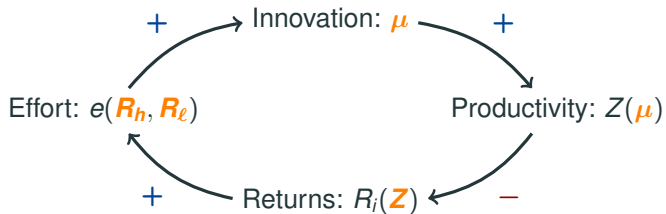
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■ Solution is quadratic!

- **Wealth tax affects** returns, productivity, and innovation. **Capital income tax does not.**
- Both taxes affect capital, output, wages...

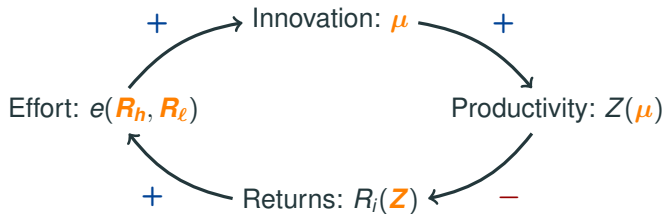
## Steady State: Innovation and Productivity Distribution

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**We show:** Existence and uniqueness of equilibrium with innovation.

(Cellina's fixed point theorem + Monotonicity)

1. Benchmark model with endogenous entrepreneurial productivity distribution
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# Wealth Taxation and Returns

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**Lemma:** Partial response of returns to productivity and innovation

$$\xi_Z^{R_h} \equiv \frac{d \log R_h}{d \log Z} > 0, \quad \xi_Z^{R_\ell} \equiv \frac{d \log R_\ell}{d \log Z} > 0, \quad \& \quad \mu \xi_Z^{R_h} + (1 - \mu) \xi_Z^{R_\ell} < 0 \quad (\text{use-it-or-lose-it})$$

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# Main Result 1: Innovation & Efficiency Gains from Wealth Taxation

## Proposition:

Proof

For all  $\tau_a < \bar{\tau}_a$ , an increase in  $\tau_a$  increases  $\mu$  and  $Z$

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- ▶ Result from fixed-point comparative statics  $\rightarrow$  Partial responses are key
- ▶ Dispersion of after-tax returns **rises** (given  $\mu$ )

$$\frac{dR_h}{d\tau_a} > 0 \quad \& \quad \frac{dR_\ell}{d\tau_a} < 0$$

$\rightarrow$  Wealth concentration **rises**,  $s_h \uparrow$ , therefore  $Z \uparrow (= s_h Z_h + (1 - s_h) Z_\ell)$

Distribution

$\rightarrow$  Higher incentives for innovation effort  $(\Lambda'(e) = \log R_h - \log R_\ell)$

- ▶ Innovation, on its own, increases productivity:  $\frac{dZ}{d\mu} > 0$

# Government Budget and Aggregate Variables

$$G + T = \tau_k \alpha Y + \tau_a K.$$

- In what follows,  $\tau_k$  adjusts in the background when  $\tau_a \uparrow$  so that  $G + T = \theta \alpha Y$

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- **Key:** Higher  $\alpha \longrightarrow$  Larger pass-through of productivity to  $K$ ,  $Y$ ,  $w$

$$\xi_Z^K = \xi_Z^Y = \xi_Z^w = \frac{\alpha}{1 - \alpha} \quad \xi_Z^x = \frac{d \log x}{d \log Z}$$



1. Benchmark model with endogenous entrepreneurial productivity distribution
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## Main Result 3: Optimal Taxes

$\alpha$  thresholds

**Objective:** Choose taxes  $(\tau_a, \tau_k)$  to max newborn welfare ( $n_w = L/(1+L)$  pop. share of workers)

$$\mathcal{W} \equiv n_w V_w(w) + (1 - n_w) \left( \mu V_h(\bar{a}) + (1 - \mu) V_\ell(\bar{a}) - \frac{\Lambda(\mu)}{(1 - \beta\delta)^2} \right)$$

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### Key trade-off:

► Welfare by type

1. **Higher levels** of worker income  $(w + T)$  and wealth  $(\bar{a} = K)$  — Depends on  $\alpha$ !  
(higher welfare for workers and high- $z$  entrepreneurs)
2. **Lower wealth growth** over lifetime from lower average return — Depends on  $\tau_a$   
(lower welfare for low- $z$  entrepreneurs and entrepreneurs as a group)

## Main Result 3: Optimal Taxes

►  $\alpha$  thresholds

►  $\tau_a^*$  level

**Proposition:** There exists a **unique** optimal tax combination  $(\tau_a^*, \tau_k^*)$  that maximizes  $\mathcal{W}$ .

An interior optimum ( $\tau_a^* < \bar{\tau}_a$ ) is solution to:

$$0 = \left( \underbrace{n_w \xi_Z^{w+T} + (1 - n_w) \xi_Z^K}_{\text{Level Effect} = \frac{\alpha}{1-\alpha} (+)} + (1 - n_w) \underbrace{\xi_Z^g}_{\text{Growth Effect} (-)} \right) \frac{d \log Z}{d \tau_a} + (1 - n_w) \underbrace{\xi_\mu^g}_{\text{Innovation Effect} (+)} \frac{d \mu}{d \tau_a}$$

where  $\xi_Z^x \equiv \frac{d \log x}{d \log Z}$  is the **elasticity of  $x$**  with respect to  $Z$ .

# Main Result 3: Optimal Taxes

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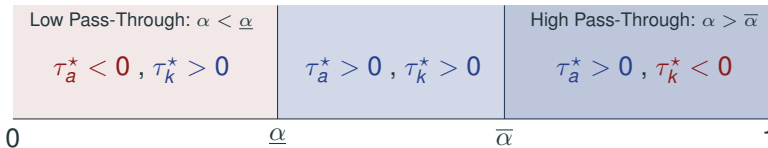
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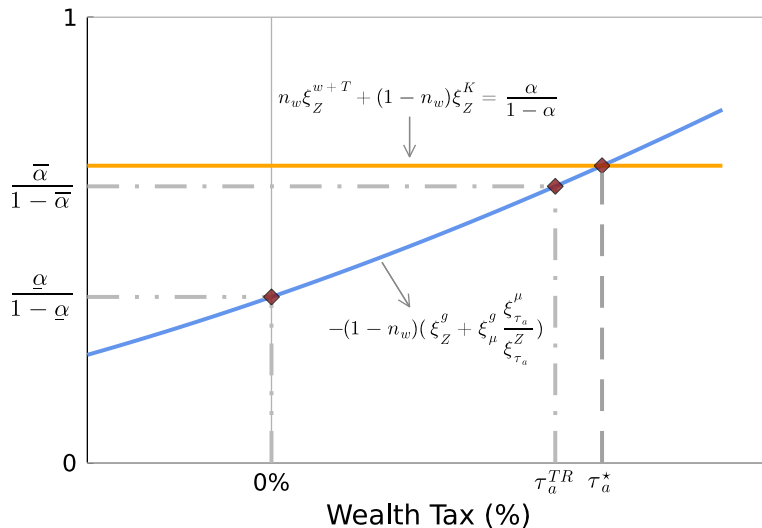
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# Optimal Tax and $\underline{\alpha}$ and $\bar{\alpha}$ Thresholds



1. Benchmark model with endogenous entrepreneurial productivity distribution
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4. **Extension to managerial effort**
5. Quantitative results (time allowing!)



# Managerial Effort

- ▶ Managerial effort in **production**: (maintain CRS)

$$y = (zk)^\alpha m^\gamma n^{1-\alpha-\gamma} \longrightarrow m : \text{managerial effort}$$

- ▶ Entrepreneurial **preferences**: (avoid income effects)

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Entrepreneurial problem becomes:

$$\hat{\pi}(z, k) = \max_{n, e} \left\{ y - wn - rk - \underbrace{\frac{\psi}{1 - \tau_k} m}_{\text{Effective Cost of Effort}} \right\}$$

- **Key**: Effective cost of effort *increases* with capital income tax  $\tau_k$  but not with  $\tau_a$ !

# Managerial Effort: Results

## 1. Efficiency gains from wealth taxation go through

- Neutrality holds  $\left( (1 - \tau_k) \text{MPK} = \frac{1}{\beta\delta} - (1 - \tau_a) \right) \rightarrow Z, R_h, R_\ell$  depend only on  $\tau_a$ !

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## 3. Optimal taxes: higher wealth tax and lower capital income tax

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5. **Quantitative results** (Is there any time left? ☐ Yes ☒ No)

# Conclusions

## Increasing $\tau_a$ (& reducing $\tau_k$ ):

- ▶ **Innovation Effect:** Provides incentives for innovation shaping productivity distribution
- ▶ **Use it or Lose it Effect:** Reallocates capital from less to more productive agents.
  - Higher innovation, productivity, output, and wages;
  - Higher dispersion in returns and wealth and lower average returns

## Optimal tax mix:

- ▶ Combination of taxes depends on pass-through of TFP to wages and wealth

# Extra



# Entrepreneur's Problem

## Entrepreneurs' Production Decision:

$$\Pi^*(z, a) = \max_{\substack{k \leq \lambda a, n}} (zk)^\alpha n^{1-\alpha} - rk - wn.$$

## Entrepreneurs' Production Decision:

**Solution:**  $\Pi^*(z, a) = \underbrace{\pi^*(z)}_{\text{Excess return above } r} \times a$

$$\pi^*(z) = \begin{cases} (MPK(z) - r) \lambda & \text{if } MPK(z) > r \\ 0 & \text{otherwise.} \end{cases} \quad k^*(z) = \begin{cases} \lambda a & \text{if } MPK(z) > r \\ \in [0, \lambda a] & \text{if } MPK(z) = r \\ 0 & \text{if } MPK(z) < r \end{cases}$$

►  $(\lambda - 1)a$ : amount of external funds used by type- $z$  if  $MPK(z) > r$ .

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- ▶ Low-productivity entrepreneurs bid down interest rate,  $r = \text{MPK}(z_\ell)$
- ▶ **Unique steady state** with:  
return heterogeneity, capital misallocation, wealth tax  $\neq$  capital inc tax
- ▶ **Empirically relevant:**  $R_h > R_l$  and  $\frac{\text{Debt}}{\text{GDP}} \gg 1.5$  when  $\lambda = \bar{\lambda}$

[▶ details](#)

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Condition implies an upper bound on wealth taxes:

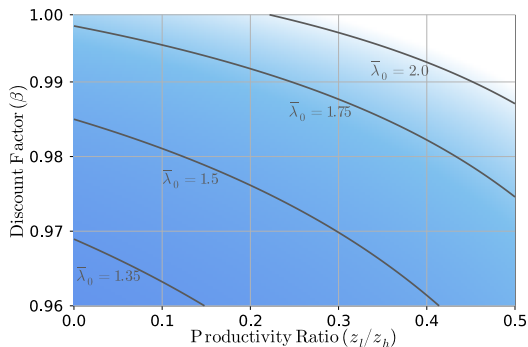
[▶ Upper Bound on  \$\tau\_a\$](#) 

$$(\lambda - 1)\mu A_h < (1 - \mu) A_\ell \iff \tau_a < \bar{\tau}_a = 1 - \frac{1}{\beta\delta} \left( 1 - \frac{1-\delta}{\delta} \frac{1-\lambda\mu}{(\lambda-1)\left(1-\frac{z_\ell}{z_h}\right)} \right)$$

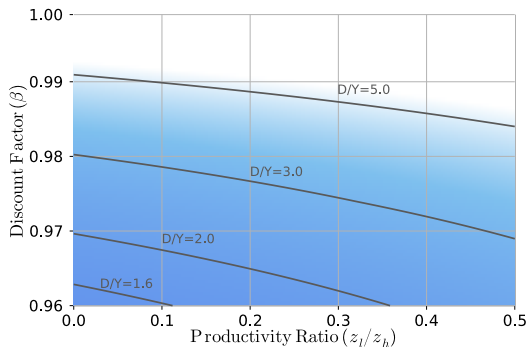
# FIGURES



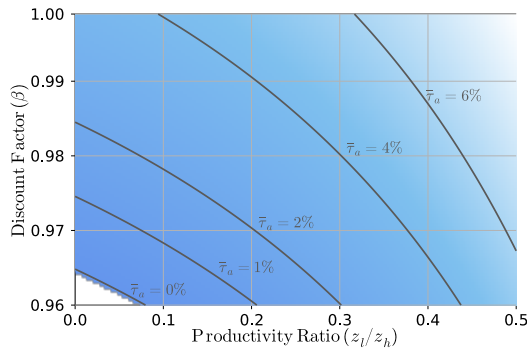
## Threshold $\bar{\lambda}_0$



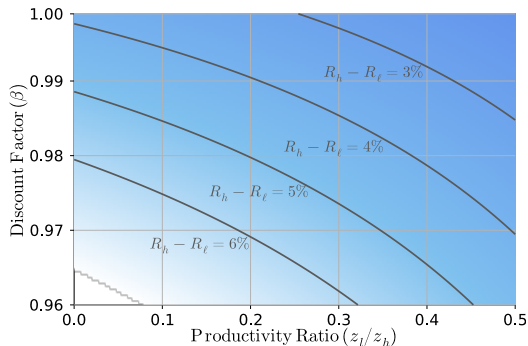
## Debt-to-Output Ratio ( $\lambda = \bar{\lambda}_0$ )



## Upper Bound on Wealth Tax $\bar{\tau}_a$



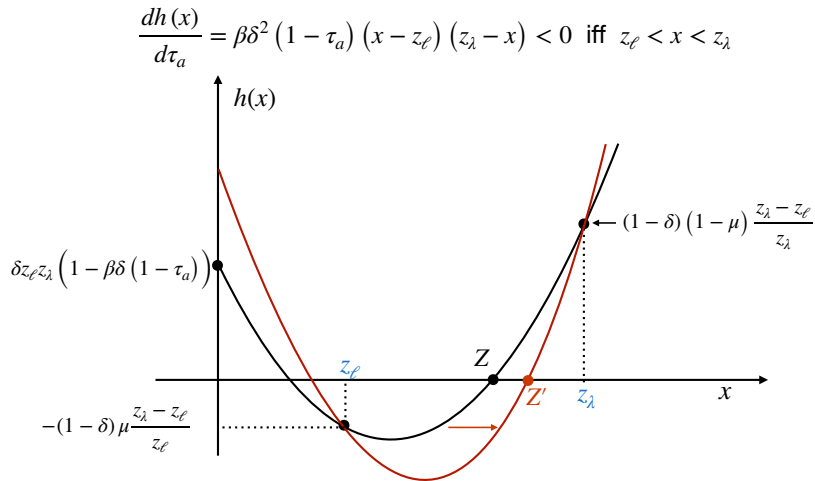
## Dispersion of Returns in Equilibrium, $R_h - R_\ell$



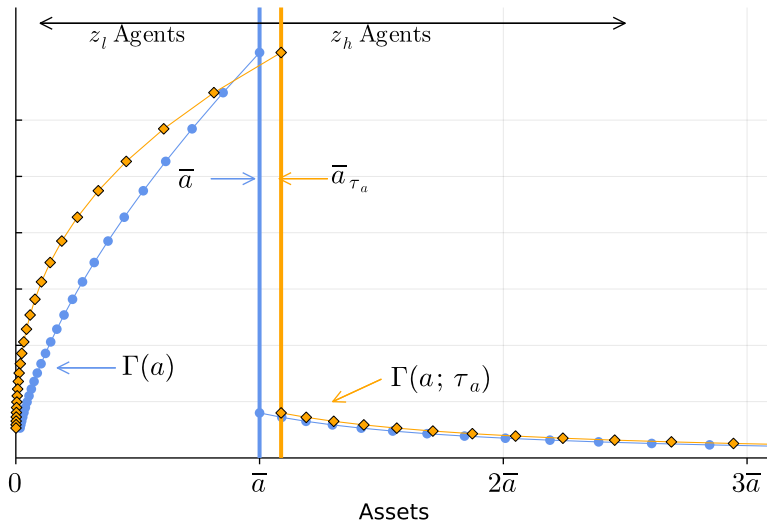
**Note:** The figure reports the value return dispersion in steady state for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_\ell/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_K = 25\%$ , and  $\alpha = 0.4$ .

# What happens to $Z$ if $\tau_a \uparrow$ ?

Back to eff. gain



# Stationary wealth distribution and wealth taxes

[back](#)

# Welfare Gains

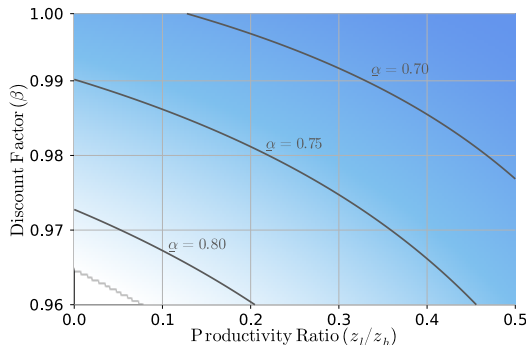
### Proposition:

[▶  \$\alpha\$  Thresholds](#)

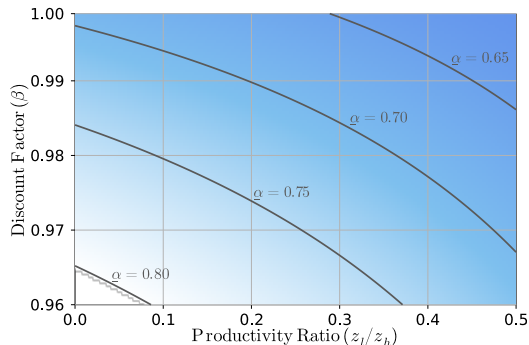
For all  $\tau_a < \bar{\tau}_a$ , a higher  $\tau_a$  changes welfare as follows:

- ▶ Workers: Higher welfare:  $\frac{dV_{workers}}{d\tau_a} > 0$
- ▶ High-z entrepreneurs: Higher welfare  $\left(\frac{dV_h(\bar{a})}{d\tau_a} > 0\right)$  because  $\xi_Z^K + \frac{1}{1-\beta\delta}\xi_Z^{R_h} > 0$
- ▶ Low-z entrepreneurs: Lower welfare  $\left(\frac{dV_\ell(\bar{a})}{d\tau_a} < 0\right)$  iff  $\xi_Z^K + \frac{1}{1-\beta\delta}\xi_Z^{R_\ell} < 0$ ;  $\alpha < \underline{\alpha}_\ell$
- ▶ Entrepreneurs: Lower average welfare iff  $\xi_Z^K + \frac{1}{1-\beta\delta} \left( \mu\xi_Z^{R_h} + (1-\mu)\xi_Z^{R_\ell} \right) < 0$ ;  $\alpha < \underline{\alpha}_E$

Low-Productivity Entrepreneurs:  $dV_\ell/d\tau_a > 0$



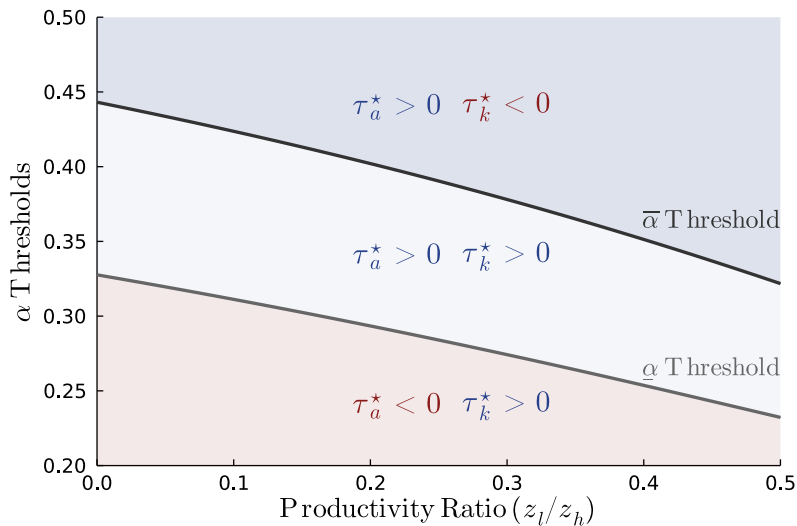
Average Entrepreneur:  $dV_E/d\tau_a > 0$



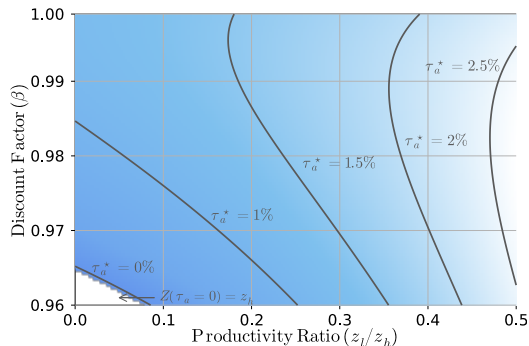
**Note:** The figures report the threshold value of  $\alpha$  above which entrepreneurial welfare increases after an increase in wealth taxes for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_\ell/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_k = 25\%$ , and  $\alpha = 0.4$ .



# Optimal Taxes



Optimal Wealth Tax  $\tau_a^*$



**Note:** The figure reports the value of the optimal wealth tax for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_l/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_k = 25\%$ , and  $\alpha = 0.4$ .

# Extensions

- ▶ Technology:  $Y_c = (z_c K_c)^\alpha L_c^{1-\alpha}$ 
  - No financial constraints!
- ▶ Corporate sector imposes lower bound on  $r$ :

$$r \geq \alpha z_c \left( \frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}}.$$

**Interesting case:**  $z_\ell < z_c < z_h$

- ▶ Corporate sector and high-productivity entrepreneurs produce
- ▶ Low-productivity entrepreneurs lend all of their funds.
- ▶ No real changes in the aggregates of the economy!  $z_c$  takes the place of  $z_\ell$

$$Y = (ZK)^\alpha L^{1-\alpha}$$

but now  $Z = s_h z_\lambda + s_l z_c$ , where  $z_\lambda = z_h + (\lambda - 1)(z_h - z_c)$ .

- Introduce wedge for returns above/below productivity:

$$R_i = (1 - \tau_a) + (1 - \tau_k) \underbrace{(1 + \omega_i)}_{\text{Return Wedge}} \alpha (ZK/L)^{\alpha-1} z_i$$

- Zero-sum condition on wedges:  $\omega_l z_\ell A_\ell + \omega_h z_h A_h = 0$
- Characterization of eq. in terms of “effective productivity”  $\tilde{z}_i = (1 + \omega_i) z_i$

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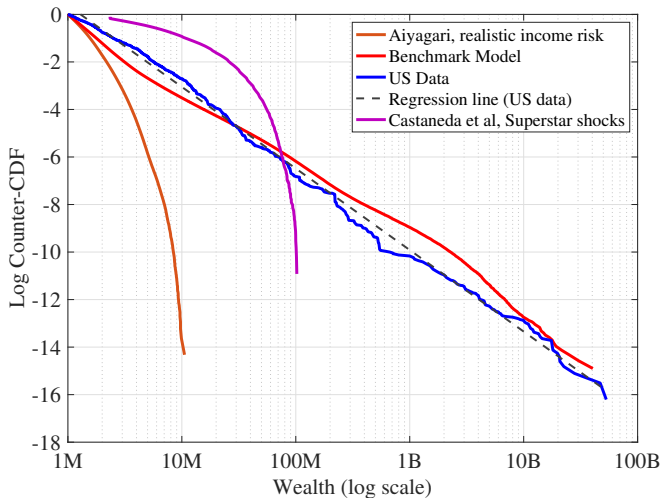
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## Proposition:

For all  $\tau_a < \bar{\tau}_a$ , a marginal increase in wealth taxes ( $\tau_a$ ) increases  $Z$ ,  $\frac{dZ}{d\tau_a} > 0$ , **iff**

1.  $\rho > 0$  and  $R_h > R_\ell \longrightarrow$  Same mechanism as before
2.  $\rho < 0$  and  $R_h < R \longrightarrow$  Reallocates wealth to the true high types next period

# Pareto Tail of Wealth Distribution: Model vs. Data

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**Note:** Both axes are in natural logs.

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# Policy Implications

## Individuals: OLG demographic structure (*retirement, mortality risk*)

- Preferences over consumption, leisure and bequests (*inheritances go to newborn offspring*)

- Make three decisions:

consumption-savings || labor supply || portfolio choice

- Two exogenous characteristics:

[► Details](#)

$y_{ih}$  (labor market productivity) ||  $z_{ih}$  (entrepreneurial productivity)

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$y_{ih}$  (labor market productivity) ||  $z_{ih}$  (entrepreneurial productivity)

**Markets:** monopolistic competition → decreasing returns to scale

**Government:** Expenditures:  $G$  + SS pensions || Taxes: Consumption ( $\tau_c$ ), Labor income ( $\tau_\ell$ ), Bequests ( $\tau_b$ ) +  $\tau_k$  or  $\tau_a$

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1. Capital income taxes much more distorting than what we believed.

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  - Optimal wealth tax delivers both efficiency and distributional gains.
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3. Due to higher wages, most people benefit from switch to wealth tax.
  - Optimal wealth tax delivers both efficiency and distributional gains.
  - No equity-efficiency trade-off.
4. Gains from optimal wealth tax come from reallocation, not capital accumulation.
  - Hence, gains remain even after taking the transition into account.

► **Model the current US tax system with four taxes on:**

1. Capital income
2. Labor income
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- ▶ **Model the current US tax system with four taxes on:**
  1. Capital income
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  3. Consumption
  4. Bequests.
  
- ▶ **Wealth Tax Reform:** Replace  $\tau_k$  with  $\tau_a$  so as to keep government revenue constant.
  - First: Compare across steady states.
  - Then: Compare with transition after reform.

## Taxes and welfare:

	$\tau_k$	$\tau_\ell$	$\tau_a$	$\Delta$ Welfare
Benchmark	25%	22.4%	–	–
Tax reform	–	22.4%	1.19%	7.2

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## Aggregate variables: (% change)

	$K$	$Q = ZK$	TFP	$L$	$Y$	$w$
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**Key:** Tax reform **replaces**  $\tau_k$  with  $\tau_a$ . This is  $\neq$  from adding wealth taxes.

- Adding wealth taxes reduces welfare by  $-6\%$  to  $-9\%$

	Benchmark US Economy	Tax Reform	
		Comparison Across Steady-States	Full Transition Equilibrium
<i>Tax Rates</i>			
$\tau_k$	25.0	—	—
$\tau_a$	—	1.19	
$\tau_\ell$	22.4	22.4	
<i><math>\Delta</math>Welfare</i>	—	7.2	

**Note:** Percentage changes are computed with respect to the US benchmark

	Benchmark US Economy	Tax Reform	Opt $\tau_a$
		Comparison Across Steady-States	Full Transition Equilibrium
<i>Tax Rates</i>			
$\tau_k$	25.0	—	—
$\tau_a$	—	1.19	3.03
$\tau_\ell$	22.4	22.4	15.4
<i><math>\Delta</math> Welfare</i>	—	7.2	8.7

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	Benchmark US Economy	Tax Reform	Opt $\tau_a$	Opt $\tau_k$	
		Comparison Across Steady-States			Full Transition Equilibrium
<i>Tax Rates</i>					
$\tau_k$	25.0	—	—	−13.6%	
$\tau_a$	—	1.19	3.03	—	
$\tau_\ell$	22.4	22.4	15.4	31.2	
$\Delta$ Welfare	—	7.2	8.7	5.1	

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	Benchmark US Economy	Tax Reform	Opt $\tau_a$	Opt $\tau_k$	Opt $\tau_a$ Transition
		Comparison Across Steady-States			Full Transition Equilibrium
Tax Rates					
$\tau_k$	25.0	—	—	−13.6%	—
$\tau_a$	—	1.19	3.03	—	3.80
$\tau_\ell$	22.4	22.4	15.4	31.2	14.4
$\Delta$ Welfare	—	7.2	8.7	5.1	6.0

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	Benchmark US Economy	Tax Reform	Opt $\tau_a$	Opt $\tau_k$	Opt $\tau_a$ Transition	Opt $\tau_k$ Transition
		Comparison Across Steady-States			Full Transition Equilibrium	
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$\tau_\ell$	22.4	22.4	15.4	31.2	14.4	31.2
$\Delta$ Welfare	—	7.2	8.7	5.1	6.0	−8.4

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# Policy Implications - Extra

## Idiosyncratic wage risk:

- ▶ Modeled in a rich way, but does not turn out to be critical.

[▶ Details](#)

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## Entrepreneurial productivity, $z_{ih}$ , varies

1. permanently across individuals:  $z_i^p$  (*imperfectly correlated across generations*)
2. stochastically over the life cycle

$$z_{ih} = f(z_i^p, \mathbb{I}_{ih}) = \begin{cases} (z_i^p)^\lambda & \text{if } \mathbb{I}_{ih} = H \\ z_i^p & \text{if } \mathbb{I}_{ih} = L \\ z_{min} & \text{if } \mathbb{I}_{ih} = 0 \end{cases} \quad \text{where } \lambda > 1$$

$\lambda$ : degree of **superstar productivity** (*consistent w/ Halvorsen, Hubmer, Ozkan, Salgado, 2024*).

- ▶ Labor market efficiency of household  $i$  at age  $h$  is

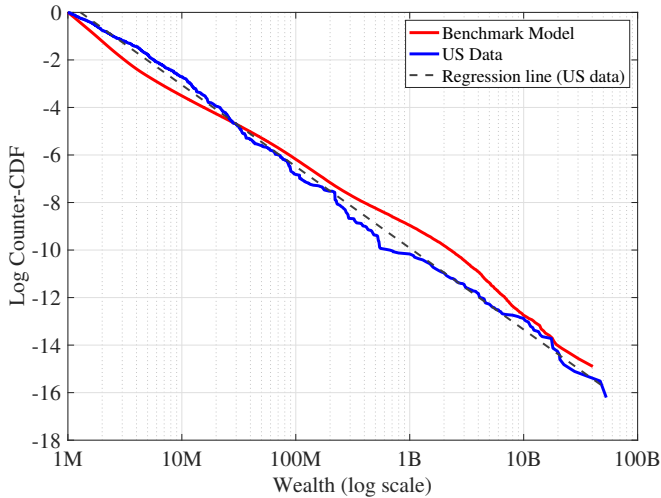
$$\log y_{ih} = \underbrace{\kappa_h}_{\text{life cycle}} + \underbrace{\theta_i}_{\text{permanent}} + \underbrace{\eta_{ih}}_{\text{AR}(1)}$$

- ▶ Permanent component  $\theta_i$  is imperfectly inherited from parents:

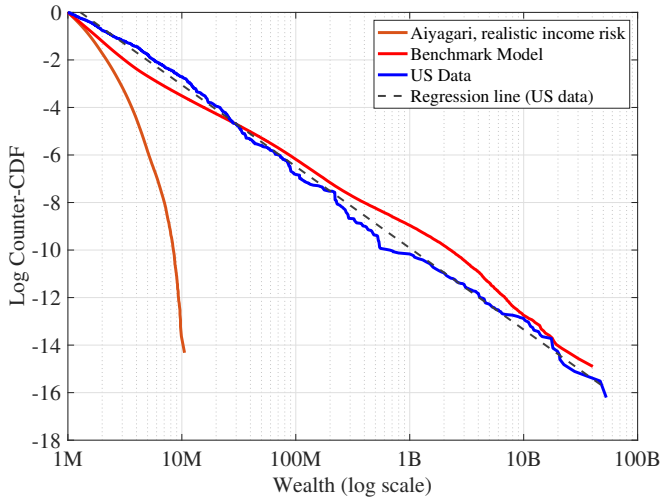
$$\theta_i^{child} = \rho_\theta \theta_i^{parent} + \varepsilon_\theta$$

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# Pareto Tail of Wealth Distribution: Model vs. Data

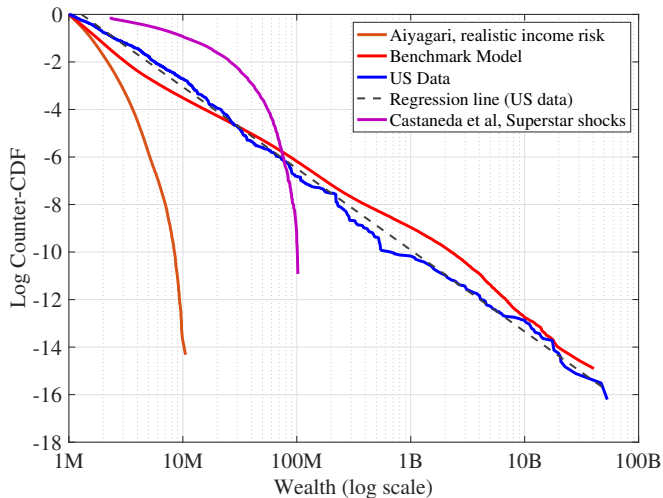


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# Pareto Tail of Wealth Distribution: Model vs. Data

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**Table 1:** Distribution of Rates of Return (Untargeted) in the Model and the Data

	Annual Returns			Persistent Component of Returns					
	Std dev	P90-P10	Kurtosis	Std dev	P90-P10	Kurtosis	P90	P99	P99.9
Data (Norway)	8.6	14.2	47.8	6.0	7.7	78.4	4.3	11.6*	23.4*
Data (Norway, bus. own.)	—	—	—	4.8	10.9	14.2	10.1	—	—
Data (US, private firms)	17.7	33.8	8.3	—	—	—	—	—	—
Benchmark Model	8.4	17.1	7.6	4.1	9.2	6.1	5.8	13.9	19.7
L-INEQ Calibration	6.7	13.1	9.2	3.8	9.2	4.3	5.6	11.2	15.8

*Notes: Returns on wealth in percentage points. All cross-sectional returns are value weighted. \*The statistics for Norway are for individual returns on wealth (net worth) taken from Fagereng, Guiso, Malacrino, and Pistaferri (2020). The US statistics are from Smith, Zidar, and Zwick (2021) and are for S-corps' returns on investment; they also report statistics for partnerships, which are very similar (std dev of 17.8% and P90-P10 of 27.9). For each individual, the persistent component of returns is calculated following Fagereng et al as the unweighted average of annual, before-tax, returns between ages 25 and 75, after taking out the average return by age.*

Average (consumption equivalent) **welfare gain** by age-productivity groups:

Age	<i>Productivity group (Percentile)</i>					
	0-40	40-80	80-90	90-99	99-99.9	99.9+
20	<b>6.7</b>	<b>6.3</b>	<b>6.8</b>	<b>8.5</b>	<b>11.5</b>	<b>13.4</b>
21-34						
35-49						
50-64						
65+						

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35-49	4.9	3.8	3.3	3.3	3.1	2.8
50-64	2.2	1.5	1.1	0.9	0.4	<b>-0.2</b>
65+	<b>-0.2</b>	<b>-0.3</b>	<b>-0.4</b>	<b>-0.4</b>	<b>-0.7</b>	<b>-1.0</b>

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Adjusting pensions turns welfare losses of retirees to gains, ranging from 2.3% to 6.5%.

Welfare gain comes from changes in consumption ( $c$ ) and leisure( $\ell$ ).

- ▶ How much comes from changes in the **level** vs **distribution** of  $c$  and  $\ell$ ?

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	Tax Reform	Opt. $\tau_a$	Opt. $\tau_k$
$CE_2$ (NB)	7.2	8.7	5.1
Level ( $\bar{c}, \bar{\ell}$ )	8.9		
Dist. ( $c, \ell$ )	-1.5		

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	Tax Reform	Opt. $\tau_a$	Opt. $\tau_k$
$CE_2$ (NB)	7.2	8.7	5.1
Level ( $\bar{c}, \bar{\ell}$ )	8.9	5.9	
Dist. ( $c, \ell$ )	-1.5	2.6	

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	Tax Reform	Opt. $\tau_a$	Opt. $\tau_k$
$CE_2$ (NB)	7.2	8.7	5.1
Level ( $\bar{c}, \bar{\ell}$ )	8.9	5.9	14.7
Dist. ( $c, \ell$ )	-1.5	2.6	-8.3