Macroeconomics, Problem Set 1

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The solution of this problem consists of a PDF with all mathematical derivations and all graphs as well as julia or matlab script that produces the results.

1. Do exercise 10.1 of SLP.

2. Do exercises 2.3, 2.19, 2.23 of Ljungqvist and Sargent

3. Durable Goods

   Consider a single agent problem where each period, w total output is produced and can be divided into consumption of a perishable good, c_t and investment in a durable good, d_{xt}. The durable depreciates like a capital good, but is not directly productive. The stock of durables at any date, d_t, produces a flow of services that enters the utility function. Thus, the problem faced by the household with initial stock d_0 is:

$$\max_{c_t,d_t,d_{xt}} \sum_t \beta^t \left\{u_1\left(c_t\right) + u_2\left(d_t\right)\right\}$$
$$\text{s.t.}$$
$$c_t + d_{xt} \leq w$$
$$d_{t+1} \leq \left(1-\delta\right) d_t + d_{xt}$$
$$c_t, d_t, d_{xt} \geq 0$$
$$d_0 \text{given}$$

   where both u_1 and u_2 are strictly increasing and continuous. Note: you can ignore non-negativity constraints on investment, d_{xt} in this problem.

   (a) State a condition on either u_1 or u_2 (or both) such that you can write an equivalent problem in the following form:

$$\max_{\{d_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t F\left(d_t, d_{t+1}\right)$$

$$\text{s.t.}$$

$$d_{t+1} \in \Gamma\left(d_t\right)$$

$$d_0 \text{given}$$

where $\Gamma(d) \in \mathbb{R}_+$. What is F? What is the correspondence Γ?

(b) Write the Bellman equation for this problem.

(c) State additional conditions on u_1 and u_2 such that the value function $v(d)$ is both strictly increasing and strictly concave. Prove these two properties.

(d) For the remaining questions, assume that both u_1 and u_2 satisfy the Inada conditions and are continuously differentiable. State the envelope and the FOC for the functional equation problem in (b)

(e) Show that there is a unique steady state value of the stock, $d^\star$, such that if $d_0 = d^\star$, then $d_t = d^\star$ for all t. Show that $d^\star > 0$.

(f) Show that the policy functions for the solution, $c^\star(d)$ and $d' = g^\star(d)$ are increasing.

(g) Show that the system is globally stable. You can assume that the policy functions are differentiable for this part.