

# Taxing Wealth and Capital Income When Returns are Heterogeneous

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Guvenen, Kambourov, Kuruscu, Ocampo

*Lisbon Macro Workshop*

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**Our earlier work:** **Quantitative analysis** of optimal capital income **versus** wealth tax  
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**This paper:** **Theoretical analysis** of optimal **combination** of taxes

- ▶ Analytical model with workers, heterogeneous entrepreneurs, and innovation
- ▶ **Find:** conditions for **(i)** efficiency gains **(ii)** welfare effects **(iii)** optimal taxes **(iv)** effects on innovation

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2. **Technical:** Capital taxes paid by the very wealthy.

- Models struggle to generate plausible wealth inequality.
- Return heterogeneity generates concentration at the very top, Pareto tail, and fast wealth growth

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- We need to provide better guidance to policy makers.

4. **Theoretical:** Interesting **new economic mechanisms** → Example next.

*Allais 1977, Piketty 2014, Guvenen, Kambourov, Kuruscu, Ocampo, Chen 2023*

## Return Heterogeneity: A Simple Example

- ▶ One-period model.
- ▶ Government taxes to finance  $G = \$50K$ .
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- ▶ Two brothers, Fredo and Mike, each with \$1M of wealth.
- ▶ **Key heterogeneity:** investment/entrepreneurial ability.
  - (Fredo) Low ability: earns  $r_f = 0\%$  rate of return.
  - (Mike) High ability: earns  $r_m = 20\%$  rate of return.

# Capital Income ( $\tau_k$ ) vs. Wealth Tax ( $\tau_a$ )

Capital income tax			Wealth tax
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	Fredo ( $r_f = 0\%$ )	Mike ( $r_m = 20\%$ )	
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- Replacing  $\tau_k$  with  $\tau_a \rightarrow$  **reallocates** assets to more productive agent (use it or lose it) + **increases dispersion** in after-tax returns & wealth.

# Baseline Model with **Exogenous** Entrepreneurial Productivity

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2. Heterogenous **entrepreneurs** (size 1)
  - Produce final goods using capital and labor ( $y_i = (z_i k_i)^\alpha n_i^{1-\alpha}$ ) + consume/save
  - Heterogeneity in
    - ▶ productivity ( $z_i \in \{z_\ell, z_h\}$ ) determined at birth:  $\mu$  ( $1 - \mu$ ) fraction w/ permanent  $z_h$  ( $z_\ell$ )
    - ▶ wealth ( $a$ )
  - Initial (inherited) wealth  $\bar{a}$  common across entrepreneurs ( $\bar{a}$  determined endogenously later)

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**Preferences** (of workers and entrepreneurs):  $E_0 \sum_{t=0}^{\infty} (\beta \delta)^t \log(c_t)$

where  $\beta < 1$  and  $\delta < 1$  is the conditional survival probability.

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**Aggregate output:**  $Y = \int y_i di = \int (z_i k_i)^\alpha n_i^{1-\alpha} di$

**Government:** Finances exogenous expenditure  $G$  with  $\tau_k$  and  $\tau_a$

## Financial markets:

- ▶ Collateral constraint ( $\lambda \geq 1$ ):  $k \leq \lambda a$ , where  $a$  is entrepreneur's wealth.
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  - low-type entrepreneurs bid down interest rate,  $r = \text{MPK}(z_\ell)$ .
  - **Unique steady state** with: return heterogeneity, misallocation of capital, wealth tax  $\neq$  capital income tax.
  - **Empirically relevant:**  $R_h > R_l$  and  $\frac{\text{Debt}}{\text{GDP}} \gg 1.5$  when  $\lambda = \bar{\lambda}$ .

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$$\text{▶ } (\lambda - 1)\mu A_h < (1 - \mu)A_\ell \iff \tau_a < \bar{\tau}_a = 1 - \frac{1}{\beta\delta} \left( 1 - \frac{1-\delta}{\delta} \frac{1-\lambda\mu}{(\lambda-1)\left(1-\frac{z_\ell}{z_h}\right)} \right)$$

Upper Bound on  $\tau_a$

## Equilibrium Values: Aggregation

**Lemma:** Aggregate output is

$$Y = (ZK)^\alpha L^{1-\alpha} \quad (Z^\alpha \text{ is measured TFP})$$

where

$$K \equiv \mu A_h + (1 - \mu) A_\ell$$

$K$  = Aggregate capital

$$Z \equiv s_h z_\lambda + (1 - s_h) z_\ell$$

$Z$  = Wealth-weighted productivity

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**Key variables:**

- ▶  $s_h = \frac{\mu A_h}{K}$ : wealth share of high-productivity entrepreneurs.
- ▶  $z_\lambda \equiv z_h + (\lambda - 1)(z_h - z_\ell)$ : effective productivity of high-productivity entrepreneurs.

Use it or lose it effect increases efficiency if  $s_h \uparrow (\longrightarrow Z \uparrow)$



# Steady State: Capital, Returns, and Taxes

**Steady State  $K$ :** Same as in Neoclassical Growth Model... but with endogenous  $Z$  (Moll, 2014)

$$(1 - \tau_a) + (1 - \tau_k) \overbrace{\alpha Z^\alpha (K/L)^{\alpha-1}}^{\text{MPK}} = \frac{1}{\beta\delta}$$

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**Steady State  $Z$ :** Returns and evolution of assets imply this quadratic equation:

$$(1 - \delta^2 \beta (1 - \tau_a)) Z^2 - [(1 - \delta)(\mu z_\lambda + (1 - \mu) z_\ell) + \delta (1 - \delta \beta (1 - \tau_a))(z_\lambda + z_\ell)] Z + \delta (1 - \delta \beta (1 - \tau_a)) z_\ell z_\lambda = 0.$$

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► **Wealth tax affects** returns, wealth shares, and productivity. **Capital income tax does not.**

# Main Result 1: Efficiency Gains from Wealth Taxation

## Proposition:

Proof

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- ▶ Wealth concentration:  $s_h \uparrow (Z \uparrow = s_h z_h + (1 - s_h) z_\ell)$
- ▶ Dispersion of after-tax returns rises:

$$\frac{dR_\ell}{d\tau_a} < 0 \quad \& \quad \frac{dR_h}{d\tau_a} > 0$$

## Government Budget and Aggregate Variables

$$G = \tau_k \alpha Y + \tau_a K.$$

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- **Key:** Higher  $\alpha \rightarrow$  Larger pass-through of productivity to  $K$ ,  $Y$ ,  $w$

$$\xi_K = \xi_Y = \xi_w = \alpha / (1 - \alpha) \quad \xi_x = \frac{d \log x}{d \log Z}$$

## Main Result 2: Welfare Gains by Type

### Proposition:

For all  $\tau_a < \bar{\tau}_a$ , a higher  $\tau_a$  changes welfare as follows:

- ▶ Workers: Higher welfare:  $\frac{dV_{workers}}{d\tau_a} > 0$
- ▶ High-z entrepreneurs: Higher welfare:  $\frac{dV_h(\bar{a})}{d\tau_a} > 0$  (since  $\xi_K + \frac{1}{1-\beta\delta}\xi_{R_h} > 0$ )
- ▶ Low-z entrepreneurs: Lower welfare ( $\frac{dV_\ell(\bar{a})}{d\tau_a} < 0$ ) iff  $\xi_K + \frac{1}{1-\beta\delta}\xi_{R_\ell} < 0$
- ▶ Entrepreneurs: Lower average welfare iff  $\xi_K + \frac{1}{1-\beta\delta}(\mu\xi_{R_h} + (1-\mu)\xi_{R_\ell}) < 0$

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- ▶ Low-z entrepreneurs: Lower welfare ( $\frac{dV_\ell(\bar{a})}{d\tau_a} < 0$ ) iff  $\xi_K + \frac{1}{1-\beta\delta}\xi_{R_\ell} < 0$
- ▶ Entrepreneurs: Lower average welfare iff  $\xi_K + \frac{1}{1-\beta\delta}(\mu\xi_{R_h} + (1-\mu)\xi_{R_\ell}) < 0$

**Note:** The last two conditions imply a threshold on  $\alpha$  for welfare gains that are high in practice, so average entrepreneur welfare is typically lowered when  $\tau_a$  increases.

$\alpha$  Thresholds

**Objective:** Choose taxes  $(\tau_a, \tau_k)$  to maximize newborn welfare

$$\mathcal{W} \equiv n_w V_w(w) + (1 - n_w) (\mu V_h(\bar{a}) + (1 - \mu) V_\ell(\bar{a}))$$

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$$\mathcal{W} = \frac{1}{1 - \beta\delta} \left\{ n_w \log w + (1 - n_w) \left( \log \bar{a} + \frac{\mu \log R_h + (1 - \mu) \log R_\ell}{1 - \beta\delta} \right) \right\} + \text{Constant}$$

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**Proposition:** There exists a **unique** optimal tax combination  $(\tau_a^*, \tau_k^*)$  that maximizes  $\mathcal{W}$ . An interior optimum ( $\tau_a^* < \bar{\tau}_a$ ) is the solution to:

$$0 = \left( \underbrace{n_w \xi_w^Z + (1 - n_w) \xi_K^Z}_{\text{Level Effect (+)}} + \underbrace{\frac{1 - n_w}{1 - \beta\delta} (\mu \xi_{R_h}^Z + (1 - \mu) \xi_{R_\ell}^Z)}_{\text{Return Productivity Effect (-)}} \right) \frac{d \log Z}{d \tau_a}$$

where  $\xi_x \equiv \frac{d \log x}{d \log Z}$  is the elasticity of variable  $x$  with respect to  $Z$ . **Furthermore,**

$$\tau_a^* < 0 \text{ and } \tau_k^* > 0$$

$$\text{if } \alpha < \underline{\alpha}$$

$$\tau_a^* > 0 \text{ and } \tau_k^* > 0$$

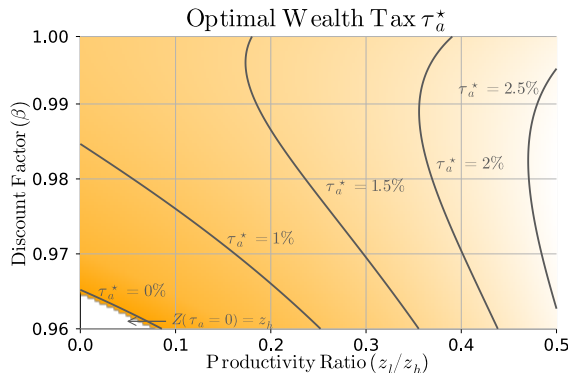
$$\text{if } \underline{\alpha} \leq \alpha \leq \bar{\alpha}$$

$$\tau_a^* > 0 \text{ and } \tau_k^* < 0$$

$$\text{if } \alpha > \bar{\alpha}$$

# How the Optimal Wealth Tax Varies with $\beta$ and productivity dispersion

Figure 1: Optimal Wealth Tax



**Note:** The figure reports the value of the optimal wealth tax for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_l/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_k = 25\%$ , and  $\alpha = 0.4$ .



# Baseline Model with **Innovation** and **Endogenous** Entrepreneurial Productivity

# Innovation Effort and Productivity

- ▶ We interpret productivity  $z_i$  as the outcome of a **risky innovation** process
- ▶ Innovation requires **costly effort**,  $e$ , and can end with a high- or low-productivity idea

## Innovator's problem:

$$\max_e \mu(e) V_h(\bar{a}) + (1 - \mu(e)) V_\ell(\bar{a}) - \frac{1}{(1 - \beta\delta)^2} \Lambda(e); \quad \Lambda(e) \text{ convex} + C^2; \mu(e) = e$$

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We can show:

- ▶ Unique equilibrium with innovation.
- ▶ Efficiency gains with wealth tax.
- ▶ Wealth tax **increases innovation**, hence fraction of high-type entrepreneurs.
- ▶ Optimal wealth tax is **higher**.

## Equilibrium with Innovation

**Steady State  $\mu^*$ :** For a given wealth tax level  $\tau_a \leq \bar{\tau}_a$ , the steady state share of high-productivity entrepreneurs,  $\mu^*$ , is determined by the solution to

$$\mu^* = e(Z(\mu^*)), \text{ where}$$

- i.  $Z(\mu)$  gives the steady state productivity given  $\mu$ .
- ii.  $e(Z)$  gives the optimal innovation effort given steady state productivity  $Z$ .

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**Corollary** (efficiency gains from wealth taxation):

The equilibrium  $Z^*$  is increasing in  $\tau_a$  (+ Both  $\mu^*$  and  $Z^*$  are independent of  $\tau_k$ ).

## Optimal taxes with innovation

**Objective:** Choose  $(\tau_a^*, \tau_k^*)$  to maximize newborn welfare net of innovation costs

$$\mathcal{W} \equiv n_w V_w(w) + (1 - n_w) \left( \mu V_h(\bar{a}) + (1 - \mu) V_\ell(\bar{a}) - \frac{\Lambda(\mu)}{(1 - \beta\delta)^2} \right)$$



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$$0 = \left( \underbrace{n_w \xi_w^Z + (1 - n_w) \xi_K^Z}_{\text{Level Effect (+)}} + \underbrace{\frac{1 - n_w}{1 - \beta\delta} (\mu \xi_{R_h}^Z + (1 - \mu) \xi_{R_\ell}^Z)}_{\text{Return Productivity Effect (-)}} \right) \frac{d \log Z}{d \tau_a} + \underbrace{\frac{1 - n_w}{1 - \beta\delta} (\mu \xi_{R_h}^\mu + (1 - \mu) \xi_{R_\ell}^\mu)}_{\text{New! Return Innovation Effect (+)}} \frac{d \mu}{d \tau_a}$$

where  $\xi_x^y \equiv \frac{d \log x}{d \log y}$  is the elasticity of variable  $x$  with respect to  $y$ .

# Extensions

## Extension: Infinite-Horizon Model with Mean-Reverting Productivity

- ▶ Entrepreneurial productivity follows Markov process with persistence  $\rho$  (first-order autocorrelation)
- ▶ All results hold as long as entrepreneurial productivity is persistent ( $\rho > 0$ ).

We further considered the following three extensions:

- ▶ **Corporate sector** that faces no borrowing constraint
  - If  $z_\ell < z_c < z_h$ , then low-productivity agents invest in the corporate sector.
- ▶ **Rents**: Return  $\neq$  marginal productivity.
  - Introduce **zero-sum return wedges** so that  $R_h < R_\ell$ .
  - Efficiency gains from  $\tau_a \uparrow$  if  $R_h > R_\ell$ .
- ▶ Per-period **entrepreneurial effort** in production (still exogenous  $z$ ):
  - With GHH preferences, **aggregate entrepreneurial effort increases** with wealth tax.

Details

Details

Details

## Increasing $\tau_a$ (& reducing $\tau_k$ ):

- ▶ **Reallocates capital:** less productive  $\rightarrow$  more productive agents.
  - Higher TFP, output, and wages;
  - Higher dispersion in returns and wealth
- ▶ Workers gain
- ▶ Entrepreneurs: High-productivity gain, low-productivity (typically) lose.
- ▶ Equilibrium innovation increases (when innovation is endogenous)

## Optimal taxes:

- ▶ Optimal tax combination depends on elasticity of output with respect to capital.
- ▶ Optimal wealth tax increases with capital share.
- ▶ Optimal wealth tax is higher with endogenous innovation.

# Extra

1. Benchmark model with exogenous entrepreneurial productivity process
2. Efficiency gains from wealth taxation
3. Welfare effects of wealth taxation
4. Optimal taxation
5. Model with **endogenous** entrepreneurial productivity
6. Extensions
7. **Quantitative Analysis**

# Entrepreneur's Problem

## Entrepreneurs' Production Decision:

$$\Pi^*(z, a) = \max_{k \leq \lambda a, n} (zk)^\alpha n^{1-\alpha} - rk - wn.$$



## Entrepreneurs' Production Decision:

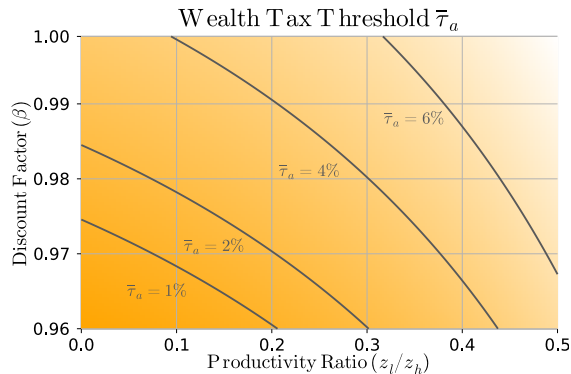
**Solution:**  $\Pi^*(z, a) = \underbrace{\pi^*(z)}_{\text{Excess return above } r} \times a$

$$\pi^*(z) = \begin{cases} (MPK(z) - r) \lambda & \text{if } MPK(z) > r \\ 0 & \text{otherwise.} \end{cases} \quad k^*(z) \begin{cases} = \lambda a & \text{if } MPK(z) > r \\ \in [0, \lambda a] & \text{if } MPK(z) = r \\ = 0 & \text{if } MPK(z) < r \end{cases}$$

- $(\lambda - 1) a$ : amount of external funds used by type- $z$  if  $MPK(z) > r$ .

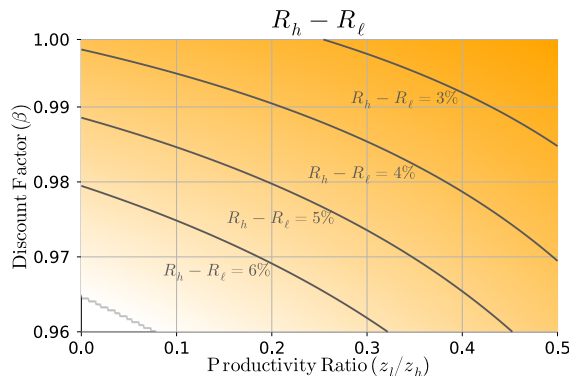
# FIGURES

Figure 2: Upper Bound Wealth Tax



**Note:** The figure reports the upper bound on wealth taxes for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_l/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_k = 25\%$ , and  $\alpha = 0.4$ .  $\lambda$  is such that the debt-to-output ratio in our baseline calibration is 1.5.

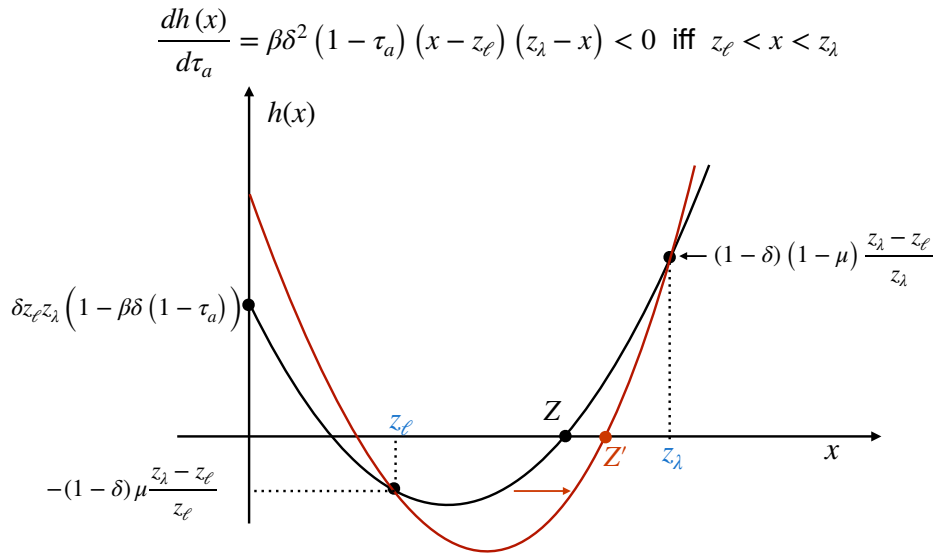
**Figure 3:** Dispersion of Returns in Steady State

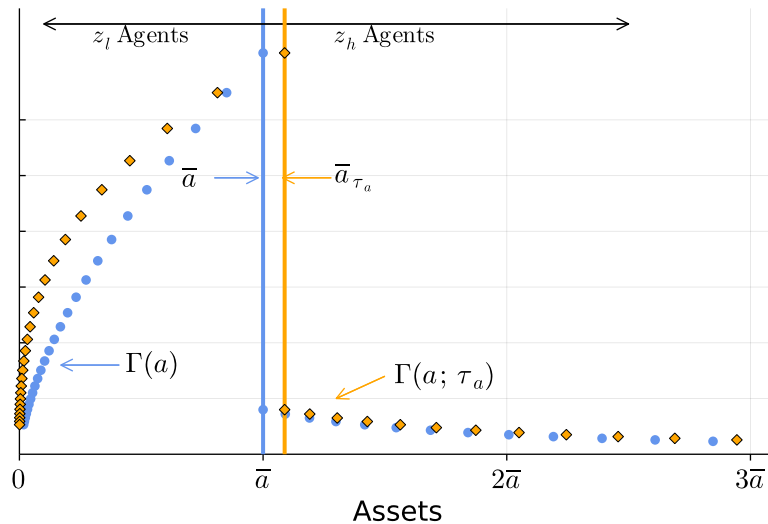


**Note:** The figure reports the value return dispersion in steady state for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_l/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_k = 25\%$ , and  $\alpha = 0.4$ .

# What happens to $Z$ if $\tau_a \uparrow$ ?

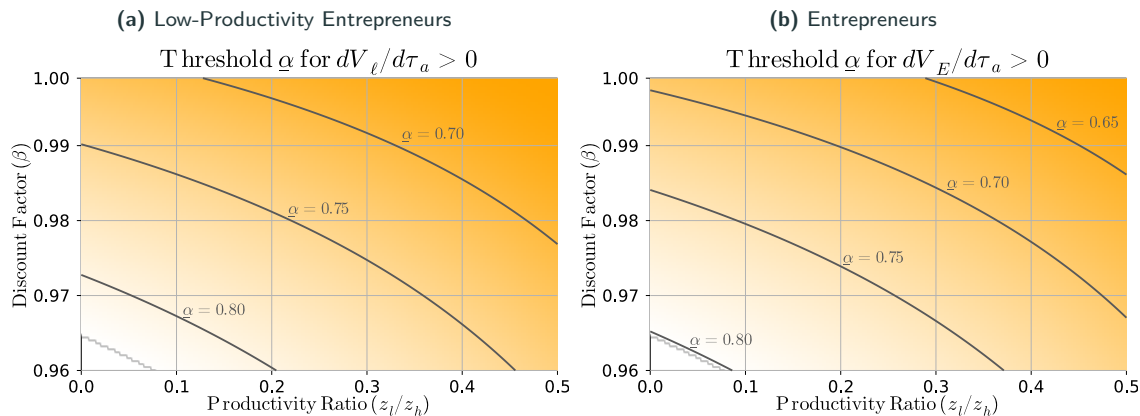
Back to eff. gain





# Welfare Gains

**Figure 4:**  $\alpha$  Thresholds for Entrepreneurial Welfare Gains

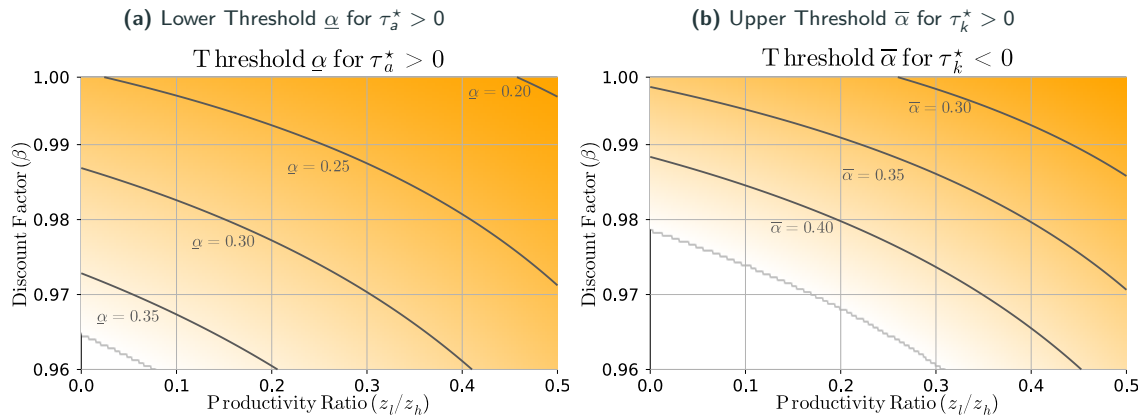


**Note:** The figures report the threshold value of  $\alpha$  above which entrepreneurial welfare increases after an increase in wealth taxes for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_\ell/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_k = 25\%$ , and  $\alpha = 0.4$ .



# Optimal Taxes

**Figure 5:**  $\alpha$  Thresholds for Optimal Wealth Taxes



**Note:** The figures report the threshold value of  $\alpha$  for the optimal wealth taxes to be positive (left) and capital income taxes to be positive (right) for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_l/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_k = 25\%$ , and  $\alpha = 0.4$ .

# Extensions

- ▶ Corporate sector produces final goods using CRS technology:

$$Y_c = (z_c K_c)^\alpha L_c^{1-\alpha}$$

- No financial constraints!

- ▶ Corporate sector imposes lower bound on  $r$ :

$$r \geq \alpha z_c \left( \frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}}.$$

**Interesting case:**  $z_\ell < z_c < z_h$

- ▶ Corporate sector and high-productivity entrepreneurs produce
- ▶ Low-productivity entrepreneurs lend all of their funds.
- ▶ No real changes in the aggregates of the economy!  $z_c$  takes the place of  $z_\ell$

$$Y = (ZK)^\alpha L^{1-\alpha}$$

but now  $Z = s_h z_\lambda + s_l z_c$ , where  $z_\lambda = z_h + (\lambda - 1)(z_h - z_c)$ .

- Introduce wedge for returns above/below productivity:

$$R_i = (1 - \tau_a) + (1 - \tau_k) \underbrace{(1 + \omega_i)}_{\text{Return Wedge}} \alpha (ZK/L)^{\alpha-1} z_i$$

- Zero-sum condition on wedges:  $\omega_l z_\ell A_\ell + \omega_h z_h A_h = 0$
- Characterization of eq. in terms of “effective productivity”  $\tilde{z}_i = (1 + \omega_i) z_i$

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### Proposition:

For all  $\tau_a < \bar{\tau}_a$ , a marginal increase in wealth taxes ( $\tau_a$ ) increases  $Z$ ,  $\frac{dZ}{d\tau_a} > 0$ , **iff**

1.  $\rho > 0$  and  $R_h > R_\ell \longrightarrow$  Same mechanism as before
2.  $\rho < 0$  and  $R_h < R \longrightarrow$  Reallocates wealth to the true high types next period

► Entrepreneurial production:

$$y = (zk)^\alpha e^\gamma n^{1-\alpha-\gamma} \longrightarrow e : \text{effort}$$

- Production functions is CRS  $\longrightarrow$  Aggregation

► Entrepreneurial preferences:

$$u(c, e) = \log(c - \psi e) \quad \psi > 0$$

- GHH preferences with no income effects  $\longrightarrow$  Aggregation
- $\psi$  plays an important role: Cost of effort in consumption units

Problem is isomorphic to having preferences

$$u(\hat{c}) = \log \hat{c} \quad \text{where } \hat{c} = c - \psi e$$

and modifying entrepreneurial problem to:

$$\hat{\pi}(z, k) = \max_{n, e} y - wn - rk - \underbrace{\frac{\psi}{1 - \tau_k}}_{\text{Effective cost of effort}} e$$

- ▶ Solution is just as before (linear policy functions  $a'$ ,  $n$ , and  $e$ )
- ▶ **Key:** Effective cost of effort depends on capital income tax  $\tau_k$ !
  - Effort affects entrepreneurial income
  - Income subject to capital income taxes but not to **book value** wealth taxes



- Aggregate effort:

$$E = \left( \frac{(1 - \tau_k) \gamma}{\psi} \right)^{\frac{1}{1-\gamma}} (ZK)^{\frac{\alpha}{1-\gamma}} L^{\frac{1-\alpha-\gamma}{1-\gamma}}$$

- Comparative statics:  $K \uparrow$ ,  $Z \uparrow$ , and  $\tau_k \downarrow$
- New wedge from capital income taxes on aggregate output and wages!
- Effort affects marginal product of capital  $\rightarrow$  Affects  $K_{ss}$

### A neutrality result:

- **No changes to steady state productivity!**
- Steady state capital adjusts in background to satisfy:

$$(1 - \tau_k) \text{MPK} - \tau_a = \frac{1}{\beta} - 1$$

### Results:

1. Efficiency gains from wealth taxation remain
2. Effect on aggregates is stronger if capital income taxes go down
  - **Effort increases with wealth taxes** (if  $\rho > 0$ )!
3. Characterization of optimal taxes is similar but  
higher wealth taxes and lower capital incomes taxes are optimal

# Quantitative Framework with **New** Results

- ▶ **OLG** demographic structure.
- ▶ **Uncertain lifetimes:** individuals face mortality risk every period.
- ▶ **Bequest motive**, inheritance goes to (newborn) offspring.

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## Individuals:

- ▶ Have preferences over consumption, **leisure** and bequests

- ▶ Make three decisions:

consumption-savings || **labor supply** || portfolio choice

- ▶ Two exogenous characteristics:

$y_{ih}$  (**labor market productivity**) ||  $z_{ih}$  (entrepreneurial productivity)

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## Entrepreneurs: monopolistic competition → **decreasing returns to scale**

► Idiosyncratic wage risk :

- Modeled in a rich way, but does not turn out to be critical.

[Details](#)

- ▶ Idiosyncratic wage risk :
  - Modeled in a rich way, but does not turn out to be critical. [Details](#)
- ▶ Entrepreneurial productivity,  $z_{ih}$ , varies
  1. permanently across individuals
    - ▶ imperfectly correlated across generations
  2. stochastically over the life cycle



## Government budget balances:

- ▶ **Outlays:** Expenditure ( $G$ ) + Social Security pensions
- ▶ **Revenues:** tax on consumption ( $\tau_c$ ), labor income ( $\tau_\ell$ ), bequests ( $\tau_b$ ) plus:
  1. tax on capital income ( $\tau_k$ ), or
  2. tax on wealth ( $\tau_a$ ).

Choose parameters of

- ▶ Bequest motive →

- level and concentration of bequests

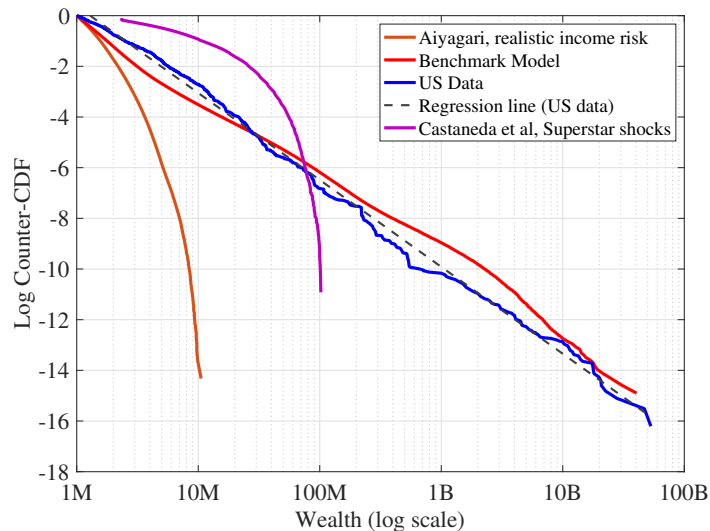
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- ▶ Bequest motive →
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- ▶ Entrepreneurial productivity →
  - top wealth concentration (overall and in the hands of entrepreneurs)
  - shares of entrepreneurs and **self-made billionaires**

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  - shares of entrepreneurs and self-made billionaires
- ▶ Entrepreneurs' collateral constraint →
  - Business debt plus external funds/GDP

[Details](#)



Note: Both axes are in natural logs.

**Table 1:** Distribution of Rates of Return (Untargeted) in the Model and the Data

	Annual Returns			Persistent Component of Returns					
	Std dev	P90-P10	Kurtosis	Std dev	P90-P10	Kurtosis	P90	P99	P99.9
Data (Norway)	8.6	14.2	47.8	6.0	7.7	78.4	4.3	11.6*	23.4*
Data (Norway, bus. own.)	–	–	–	4.8	10.9	14.2	10.1	–	–
Data (US, private firms)	17.7	33.8	8.3	–	–	–	–	–	–
Benchmark Model	8.4	17.1	7.6	4.1	9.2	6.1	5.8	13.9	19.7
L-INEQ Calibration	6.7	13.1	9.2	3.8	9.2	4.3	5.6	11.2	15.8

*Notes: Returns on wealth in percentage points. All cross-sectional returns are value weighted. \*The statistics for Norway are for individual returns on wealth (net worth) taken from Fagereng, Guiso, Malacrino, and Pistaferri (2020). The US statistics are from Smith, Zidar, and Zwick (2021) and are for S-corps' returns on investment; they also report statistics for partnerships, which are very similar (std dev of 17.8% and P90-P10 of 27.9). For each individual, the persistent component of returns is calculated following Fagereng et al as the unweighted average of annual, before-tax, returns between ages 25 and 75, after taking out the average return by age.*

	$\tau_k$	$\tau_\ell$	$\tau_a$	$\Delta\text{Welfare}$
Benchmark	25%	22.4%	–	–
RN Tax reform	–	22.4%	1.19%	7.2
<b>Opt. <math>\tau_a</math></b>				
<b>Opt. <math>\tau_k</math></b>				

	$K$	$Q$	TFP	$L$	$Y$	$w$	$w$ (net)
% change							
Tax reform	16.4	22.6	2.1	1.2	9.2	8.0	8.0
Optimal $\tau_a$		.		.	.	.	.
Optimal $\tau_k$		.		.	.	.	.



Average (consumption equivalent) **welfare gain** by age-productivity groups:

Age	Productivity group (Percentile)					
	0-40	40-80	80-90	90-99	99-99.9	99.9+
20	<b>6.7</b>	<b>6.3</b>	<b>6.8</b>	<b>8.5</b>	<b>11.5</b>	<b>13.4</b>
21-34						
35-49						
50-64						
65+						

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35-49	4.9	3.8	3.3	3.3	3.1	2.8
50-64	2.2	1.5	1.1	0.9	0.4	<b>-0.2</b>
65+	<b>-0.2</b>	<b>-0.3</b>	<b>-0.4</b>	<b>-0.4</b>	<b>-0.7</b>	<b>-1.0</b>

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BB tax reform turns welfare losses of retirees to gains, ranging from 2.3% to 6.5%.

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RN Tax reform	–	22.4%	1.19%	7.2
<b>Opt. <math>\tau_a</math></b>				
<b>Opt. <math>\tau_k</math></b>				

	$\tau_k$	$\tau_\ell$	$\tau_a$	$\Delta\text{Welfare}$
Benchmark	25%	22.4%	–	–
RN Tax reform	–	22.4%	1.19%	7.2
<b>Opt. <math>\tau_a</math></b>	–	<b>15.4%</b>	<b>3.03%</b>	<b>8.7</b>
<b>Opt. <math>\tau_k</math></b>				

	$\tau_k$	$\tau_\ell$	$\tau_a$	$\Delta\text{Welfare}$
Benchmark	25%	22.4%	–	–
RN Tax reform	–	22.4%	1.19%	7.2
<b>Opt. <math>\tau_a</math></b>	–	<b>15.4%</b>	<b>3.03%</b>	<b>8.7</b>
<b>Opt. <math>\tau_k</math></b>	<b>–13.6%</b>	<b>31.2%</b>	–	5.1

	$K$	$Q$	TFP	$L$	$Y$	$w$	$w$ (net)
% change							
Tax reform	16.4	22.6	2.1	1.2	9.2	8.0	8.0
Optimal $\tau_a$	<b>2.6</b>	<b>10.5</b>	3.1	3.3	6.1	<b>2.8</b>	<b>12.0</b>
Optimal $\tau_k$							

	$K$	$Q$	TFP	$L$	$Y$	$w$	$w$ (net)
% change							
Tax reform	16.4	22.6	2.1	1.2	9.2	8.0	8.0
Optimal $\tau_a$	<b>2.6</b>	<b>10.5</b>	3.1	3.3	6.1	<b>2.8</b>	<b>12.0</b>
Optimal $\tau_k$	<b>38.6</b>	<b>46.1</b>	2.2	-1.0	15.7	<b>16.8</b>	<b>3.6</b>



Welfare gain comes from changes in consumption ( $c$ ) and leisure( $\ell$ ).

- How much comes from changes in the **level** vs **distribution** of  $c$  and  $\ell$ ?

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	Tax Reform	Opt. $\tau_k$	Opt. $\tau_a$
$CE_2$ (NB)	7.2	5.1	8.7
Level $(\bar{c}, \bar{\ell})$	8.9		
Dist. $(c, \ell)$	-1.5		

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	Tax Reform	Opt. $\tau_k$	Opt. $\tau_a$
$CE_2$ (NB)	7.2	5.1	8.7
Level $(\bar{c}, \bar{\ell})$	8.9	14.7	
Dist. $(c, \ell)$	-1.5	-8.3	

Welfare gain comes from changes in consumption ( $c$ ) and leisure( $\ell$ ).

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	Tax Reform	Opt. $\tau_k$	Opt. $\tau_a$
$CE_2$ (NB)	7.2	5.1	8.7
Level ( $\bar{c}, \bar{\ell}$ )	8.9	14.7	5.9
Dist. ( $c, \ell$ )	-1.5	-8.3	2.6

# Optimal taxes with transition

- ▶ Fix opt. tax level ( $\tau_k$  or  $\tau_a$ ) and solve transition to new steady state
- ▶ Use labor income tax ( $\tau_\ell$ ) to finance debt from deficits during transition

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- Use labor income tax ( $\tau_\ell$ ) to finance debt from deficits during transition

	$\tau_k$ Transition	$\tau_a$ Transition
$\tau_k$	-13.6*	0.00
$\tau_a$	0.00	3.03*
$\tau_\ell$	39.90	17.01
$\overline{CE}_2$ (newborn)	<b>-8.4</b> (5.1)	<b>6.0</b> (8.7)
$\overline{CE}_2$ (all)	<b>-6.1</b> (4.5)	<b>3.5</b> (4.3)