University of Minnesota
Math Refresher
Summer 2015

Problem Set 0

1. Show by induction the following relations:
   - $1+2+3+...+n = \frac{1}{2}n\,(n+1)$.
   - $1+r+r^2+...+r^n = \frac{1-r^{n+1}}{1-r}$.
   - $1^3+2^3+...+n^3 = \left[\frac{1}{2}n\,(n+1)\right]^2$.
2. Show that there is no rational number p such that $p^2 = 2$.
3. Show that if $0 < a < b$ then $a < \sqrt{ab} < b$ and $0 < \frac{1}{b} < \frac{1}{a}$.
4. Let A, B, C and B_n be arbitrary sets, for $n \in I$. Show that:
   - $A \bigcup \left(\bigcap_{n\in I} B_n\right) = \bigcap_{n\in I}(A \bigcup B_n)$.
   - $A \bigcap \left(\bigcup_{n\in I} B_n\right) = \bigcup_{n\in I}(A \bigcap B_n)$.
   - $A - (B \bigcap C) = (A-B) \bigcup (A-C)$.
5. Show the following equivalences (Morgan's Laws):
   - $(A \bigcap B)' = A' \bigcup B'$.
   - $(A \bigcup B)' = A' \bigcap B'$.

   Note: E' represents the complement of set E.
6. Consider the following properties of the binary relation R on the set X:
   - R is *reflexive* if for every $x \in X$, xRx.
   - R is *transitive* if for x, y, z $\in X$, xRy and yRz implies xRz.
   - R is *complete* if for x,y $\in X$, xRy or yRx.

   Which of the properties above are satisfied by the following relations?
   - Let $X = \Re^n_+$ and R defined on X, where xRy iff $x \geq y$.
   - Let $X = \Re^2_+$ and R defined on X, where xRy iff $x_1 > y_1$ or if $x_1 = y_1$ and $x_2 > y_2$.

7. Consider the relation $\succeq$ defined on a set of alternatives X. This is known as a preference relation. Also, consider the following definitions:

   - *Strict preference* relation $\succ$: $x \succ y$ iff $x \succeq y$ but not $y \succeq x$.
   - *Indifference* relation $\succ$: $x \sim y$ iff $x \succeq y$ and $y \succeq x$.
   - A preference relation $\succeq$ is *rational* if it is *complete* and *transitive.*

   Suppose $\succeq$ is a rational preference. Show:

   - $\succ$ is transitive and irreflexive (irreflexive: $x \succ y$ never holds).
   - $\sim$ is reflexive, transitive and symmetric.

8. Show that if $\succeq$ is rational then the strict preference $\succ$ is *negatively transitive.*

   - $\succ$ is *negatively transitive* if $x \succ y$, for some x,y$\in X$, then for any other $z \in X$ either z or $x \succ z$.