Robust Contracts in Common Agency*

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Abstract

We consider a game between several principals and a common agent, where principals design contracts that are robust to misspecification of the agent's technology. The principals know a subset of the actions available to the agent, but other unknown actions could exist. Principals demand robustness and evaluate contracts on the worst-case performance over all possible actions of the agent. We show that a pure strategy equilibrium always exists, by constructing a pseudo-potential for the game. Equilibrium contracts are linear in total output and imply that all players (the principals and the agent) receive a share of total output. The higher the share of total output accruing to the agent, the more efficient the outcome of the game. We also consider a game where principals collude and offer a joint contract. The efficiency of the competitive outcome relative to collusion depends crucially on the ability of principals to offer side-payments to one another through the agent. Lastly, we consider an application of the model to the taxation of multinational firms and study the effects of tax competition among countries. We show that a flat tax on domestic and foreign profits with a full deduction of foreign taxes provides the best worst-case guarantee for each country.

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Key Words: Common agency, robustness, worst case, taxation of multinationals, efficiency

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Strategic settings where several actors try to influence a common party have attracted abundant attention in many areas of economics. Examples include political economy, industrial organization, mechanism design, public finance, international trade, and auctions.[1] Starting with the work of Bernheim and Whinston (1986a,b) these settings have been modeled as a common agency game where several principals simultaneously and non-cooperatively contract with a single agent.

In this paper we present a general moral hazard common agency game, where the principals seek to design contracts that are *robust* to misspecification of the environment. More specifically, contracts that perform well if the principals have an incomplete knowledge of the agent's technology, or if principals cannot renegotiate contracts when technology changes.

The game has two stages. First, risk neutral principals simultaneously and non-cooperatively offer contracts to a risk neutral agent that is protected by limited liability.[2] Given the contracts, the agent chooses an action that induces a joint distribution over the output of each principal and a utility cost for the agent. Each principal observes her output realization along with that of the other principals, and can condition her contract on all these observations, as in Bernheim and Whinston (1986a). The action taken by the agent is not observed.

When designing the contracts the principals do not know the full set of actions available to the agent, similar to the setup in Carroll (2015). Principals demand robustness by evaluating contracts on their worst-case performance over all possible actions that the agent might have available. This modeling framework captures instances where the principals are contracting with a new agent, or where the agent's technology can change after the contracts have been set in place. In a lobbying game, this would imply that lobbyists (the principals) are unsure about the preferences and goals of new politicians. In the problem of taxing multinational firms, countries want to design tax policy taking into account that the production technology of multinational corporations might change, but they are unable to change their tax policy due to political constraints.

Our first result is showing that a pure strategy equilibrium always exist. We show that the game has a pseudo-potential, similar to that of the standard Cournot competition model.

[1] For example, in political economy, lobbying is modeled as a game between lobbyists (the principals) influencing a politician (the agent), see Grossman and Helpman (1994), Dixit et al. (1997), Le Breton and Salanie (2003), and Martimort and Semenov (2008). In public finance, a firm (the agent) is taxed and regulated by the local, state and federal government (the principals), or a multinational company (the agent) has to pay taxes in several countries (the principals), see Martimort (1996), and Bond and Gresik (1996). In combinatorial auctions, an auctioneer (the agent) wants to sell several items, and multiple bidders (the principals) bid on all or a subset of the items is studied as a common agency game (Milgrom, 2007). In the voluntary provision of public goods, the public good provider (the agent) elicits payment from consumers (the principals), see Laussel and Le Breton (1998).

[2] Limited liability implies that the aggregate payment to the agent has to be non-negative for any output realization, as in Martimort and Stole (2012).

The use of a pseudo-potential to prove existence of equilibrium is new in the common agency literature. This approach allows us to establish existence of equilibrium without imposing any assumptions over the action set of the agent, while previous papers can only obtain existence under restrictive assumptions (e.g. Bernheim and Whinston (1986a), Fraysse (1993) and Carmona and Fajardo (2009)).

In equilibrium, each principal offers a linear contract that is increasing in her output and decreasing in the other principals' output. Furthermore, contracts are such that the payoffs of all players are linearly tied, and payments depend only on total output. Specifically, all players (the principals and the agent) receive a share of total output. While previous papers on moral hazard common agency games, such as Dixit (1996) and Maier and Ottaviani (2009), restrict attention to linear contracts for tractability, we impose no such restrictions. Linear contracts arise as equilibrium outcomes.

Second, we show that the higher the share of total output accruing to the agent, the higher the surplus of the action chosen by the agent, as well as the sum of payoffs of all players. We then compare the outcome of the common agency game to that of of the game where principals collude and offer a joint contract. The comparison is tractable because under both scenarios the agent gets paid a share of total output. The share is weakly lower when the principals compete, thus leading to lower expected surplus and welfare than if the principals collude.

This result is similar to the one obtained in the moral hazard models of Bernheim and Whinston (1986a), Holmstrom and Milgrom (1988), and Martimort and Stole (2012), as well as the adverse selection models of Martimort and Stole (2012) and Bond and Gresik (1996), where the agent's effort is lower under competition than collusion, due to free-riding among principals.

In our framework principals induce an externality on each other, reducing the willingness of other principals to provide incentives to the agent. When a principal increases her own share of total output, she reduces the agent's share of total output. This in turn lowers the expected output under the agent's optimal action and reduces expected payment of the other principals. While each principal takes into account the effects on her own payoff and the payment of the agent, she does not take into account the effect on the payments of the other principals. Because of this, in equilibrium, the agent gets a lower share of total output relative to what he would get if the principals were to collude.

We also consider a stronger form of limited liability, where each principal's contract should specify non-negative payments to the agent for any output realization. We do this to better understand the role of limited liability in the provision of incentives to the agent. In this case, equilibrium contracts are such that each principal gets a share of total output for a fee. This fee is proportional to the share of total output that the principal appropriates for herself.

We compare the outcome of the game under this stronger version of limited liability to our previous results by studying the agent's share of total output. In this case the earlier result is overturned and the share of output accruing to the agent is higher than under collusion. The fee that principals pay for increasing their own share of total output makes them internalize the cost of such an increase on the other principals. As a result, the agent's equilibrium share of total output ends up being higher than the share the agent gets under collusion. If the agent receives a lower share than what he gets under collusion, we show that one of the principals is better off decreasing her own share to increase the agent's share to its collusion level. This lowers the fee the principal pays and increases expected total output enough to make up for the decrease in the principal's share of output.[3]

As noted above, we depart from the common agency literature by dropping usual assumptions on the information set of the principals. In particular, we deal with an extreme version of moral hazard of the type introduced in the principal-agent framework by Hurwicz (1977) and Hurwicz and Shapiro (1978), and recently explored by Chassang (2013), Frankel (2014), Garrett (2014), Antic (2014), Carroll (2015), and Carroll and Meng (2016). The information setup considered here most closely resembles the work of Carroll (2015).

This degree of informational asymmetry over the possible set of actions makes the design of incentive compatible contracts challenging. Instead, we look for robust contracts that maximize the minimum guaranteed payoff for the principals, as in Carroll (2015). Our work adds to this literature in a crucial way by allowing strategic interaction between several principals that simultaneously contract with a single agent. To our knowledge we are the first to study robust contracts in common agency. Dai and Toikka (2017) study an analogous problem of moral hazard in teams (one principal and multiple agents), where they find that the optimal contract for the principal is to give each agent a share of total output.

Robust contracts ensure performance over a wide range of possible settings that the principals may face (Chassang, 2013), for instance in the absence of a complete characterization of the agent's technology (action set), or a well-formed system of priors over possible technologies (Frankel, 2014). Furthermore, the sensitivity of some of the results in common agency to the details of the information structure (Martimort, 2006) justifies our focus on robust contracts. We find that, despite its increased complexity, our setting allows for a

[3] The results above can be easily understood with the following anecdote: Suppose that you are a child (the agent) and your parents (the principals) are incentivizing you to do well in school. If your parents are constrained to incentivize you only with rewards (the strong version of limited liability) then you would rather your parents be divorced than together, however if they can "punish" you (the weak version of limited liability) then you would rather your parents be together than divorced.

tractable yet general solution.

The theory we develop brings new insights to the role of the common agent in the alignment of incentives. This was originally studied in Bernheim and Whinston (1985, 1986a) in environments without limited liability, where incentives are aligned then by making the agent the residual claimant of all output. Each principal sells the firm to the agent.[4] In our setup, under the stronger version of limited liability, incentives align more intuitively since, instead of selling her firm, each principal buys a share of all the firms for a fee that she pays to the agent (the fee is indeterminate under the weaker version of limited liability). In equilibrium all players own a share of the 'conglomerate' of firms, caring only about aggregate output. The alignment of incentives in equilibrium takes the form of mergers and acquisitions facilitated by having a common agent.

Our comparative statics exercises, where we compare the outcome of the common agency game with the collusive outcome, has interesting welfare implications regarding mergers of upstream firms. When the upstream firms (the principals) can only reward the agent, we show that a merger (the collusive outcome) leads to lower surplus and flatter incentives for the downstream firm (the agent). However when the contracts of the upstream firms (the principals) have to satisfy joint limited liability (the weaker version of limited liability) then we show the opposite effect of the merger. A merger leads to higher surplus and steeper incentives for the downstream firm (the agent).

As a more natural application of our theoretical results we provide a common agency framework to study tax competition among countries in the presence of a multinational firm. Our robust contracting approach is especially relevant to the problem of taxing multinational companies, where the two primary concerns for policymakers are the eroding corporate tax base (due to aggressive profit shifting by multinational companies) and the complexity of tax law.

With the increase of globalization and advances in technology, the number of multinational corporations and their ability to shift profits to low-tax countries has increased tremendously. This issue has received considerable attention in the news and in political and economic debates in the United States and other developed countries. An estimated \$2 trillion dollars of U.S. multinational corporations' profits are "parked" overseas, mostly in tax havens like the Bahamas, Bermuda, and the Cayman Islands, which implies a loss in tax revenue of about \$50 billion dollars every year (Hungerford, 2014).[5] This issue has im-

[4]Without limited liability our solution converges to the Bernheim and Whinston (1986a) case where the agent becomes the residual claimant (Section 4.3). The only other moral hazard common agency paper that considers some form of limited liability on the agent is Martimort and Stole (2012). They extend the model of Innes (1990) to a common agency setting. Their model, without robustness concerns, is a special case of ours, where output is perfectly correlated between all principals.

[5]Using a different data source Zucman (2014) estimates that profit shifting activities have reduced the

plications beyond tax revenue. For example, Guvenen et al. (2017) show that profit shifting understates measured U.S. GDP in official statistics, helping to explain the slowdown in U.S. productivity and the decline in the labor share seen since the mid 2000s.

The debate among tax policy experts and lawmakers in the United States has centered on whether to adopt a territorial approach (taxing only profits generated in the U.S.) or a worldwide approach (taxing all profits, foreign and domestic, the same).[6] Our model provides a rationale for why a worldwide approach addresses the two primary concerns mentioned above, regarding corporate income tax. This approach coincides with the system proposed in the Bipartisan Tax Fairness and Simplification Act of 2011 by Sen. Wyden and Sen. Coats (Senate Bill 727, 2011). We show that taxing domestic and foreign profits at a flat rate, with a full deduction for taxes paid in the foreign country, provides the best guarantee for corporate tax revenues as well as a simplified tax code.

To summarize, in this paper we make the following contributions: First, we provide a model of common agency where principals seek robustness. Second, we characterize equilibrium contracts. Third, we establish how the surplus of the action chosen by the agent and the sum of payoffs of all players depend on the form of the equilibrium contracts, and compare the non-cooperative and cooperative solutions of the game. Fourth, we show how the outcome of the game depends on limited liability. Finally, we show how the theory developed here provides helpful insights and a rigorous framework for the policy debate on taxation of multinationals.

The remainder of this paper is organized as follows: Section 1 lays out the model. Section 2 compares our common agency results to the collusive outcome of the game. Section 3 develops the implication of our model to the taxation of multinational firms. Finally, Section 4 presents extensions.

1 Model

Consider a game played between two principals, indexed by $i \in \{1, 2\}$, and one agent A, all risk neutral.[7] The payoff space for the principals is $Y = Y_1 \times Y_2 \subset \mathbb{R}^2$. Y_i is compact with $\min\{Y_i\} = 0$ and $\max\{Y_i\} = \overline{y}_i$.[8] The agent has access to a compact technology set $\mathcal{A} \subset \Delta(Y) \times \mathbb{R}_+$. An action is a pair $(F, c) \in \mathcal{A}$, where F is a probability distribution

tax burden of corporations by about 20%.

[6]The current U.S. system is between the two approaches. Foreign profits are taxed (almost) the same as domestic profits, but not until they are paid to a U.S. parent company. This is known as deferral.

[7]All the results are extended to the general case with n principals in Section 4.2.

[8]This assumption can be relaxed by letting $Y \subseteq \mathbb{R}^2$ be an arbitrary compact set with $\min_{y \in Y} y_i = 0$ for $i \in \{1, 2\}$, allowing any degree of complementarity or substitutability, as long as $(0, 0) \in Y$.

over payoffs $y = (y_1, y_2)$ and $c \geq 0$ is the cost of the action. We endow the space of Borel distributions, $\Delta(Y)$ with the weak-$\star$ topology and $\Delta(Y) \times \mathbb{R}$ with the natural product topology.

The game has two stages. First both principals simultaneously and non-cooperatively offer contracts to the agent; this is done simultaneously and in a non-cooperative fashion. Second, the agent chooses an action in its technology set $\mathcal{A}$, and payments realize. Each principal observes her's and the other principals' output realization, and can condition her contract on all these observations as in Bernheim and Whinston (1986a). The action taken by the agent is not observed, moreover, the principals do not know the full set of actions available to the agent ($\mathcal{A}$). As in Carroll (2015) the principals both know a subset $\mathcal{A}_0$ of $\mathcal{A}$. We assume that both principals know the same $\mathcal{A}_0$ for notational convenience and to facilitate comparison across principals. Only three other assumptions are placed on the set $\mathcal{A}_0$. These assumptions are maintained throughout the paper:

Assumption 1. ***(Inaction)*** *The agent can always choose not to produce:* $(\delta_0, 0) \in \mathcal{A}_0$, *where* δ_0 *is the degenerate distribution on* $y = (0, 0)$.

Assumption 2. ***(Positive Cost)*** *For all* $\mathcal{A} \supseteq \mathcal{A}_0$, *If* $(F, c) \in \mathcal{A}$ *and* $c = 0$, *then* $F = \delta_0$.

Assumption 3. ***(Non-triviality)*** $\exists_{(F,c) \in \mathcal{A}_0} E_F [y_1 + y_2] - c > 0$.

Assumption 1 says that choosing the minimum output is costless for the agent, so that the agent can always choose not to produce. Assumption 2 is a technical assumption requiring the agent to pay a cost in order to produce. This cost can be arbitrarily small. Assumption 3 ensures that the principals and the agent will, potentially, find it beneficial to participate in the game.

Although no other assumptions are needed for our results, we can strengthen them when the following holds:

Assumption 4. ***(Full Support)*** *For all* $(F, c) \in \mathcal{A}_0$ *if* $(F, c) \neq (\delta_0, 0)$ *then* $supp(F) = Y$.

A contract is a continuous function $w_i : Y_1 \times Y_2 \to \mathbb{R}$, and a contract scheme is a vector of functions $w = (w_1, w_2)$. The agent has **limited liability** so that the aggregate payment to the agent has to be non-negative for any output realization, i.e. $w_1(y) + w_2(y) \geq 0$ for all $y \in Y$. A principal can charge the agent up to the amount that the other principal is paying. This is the same restriction imposed in Martimort and Stole (2012). We consider different limited liability assumptions in Sections 2.1, 3 and 4.3. Furthermore, in Section 4.1 we consider the *private* common agency case where principals are restricted to contract only on their own output, as opposed to the *public* common agency considered in this section.

Given a contract scheme and a technology $\mathcal{A}$, the agent will choose an action to maximize his expected payoff. The set of optimal actions and the value they give to the agent are:

$$A^\star(w|\mathcal{A}) = \underset{(F,c)\in\mathcal{A}}{\operatorname{argmax}} \; E_F\left[w_1(y) + w_2(y)\right] - c \qquad V_A(w|\mathcal{A}) = \max_{(F,c)\in\mathcal{A}} \; E_F\left[w_1(y) + w_2(y)\right] - c. \tag{1}$$

We define the value of a principal, given a contract scheme w, as the minimum payoff guarantee offered by the contracts, as in Carroll (2015).[9] The payoff to the principal is:

$$V_i(w) = \inf_{\mathcal{A}\supseteq\mathcal{A}_0} V_i(w|\mathcal{A}) \tag{2}$$

where $V_i(w|\mathcal{A})$ is the value for a given technology $\mathcal{A}$, given by:

$$V_i(w|\mathcal{A}) = \min_{(F,c)\in A^\star(w|\mathcal{A})} E_F\left[y_i - w_i(y)\right]. \tag{3}$$

The principal doesn't know which action in $A^\star$ the agent will choose, so she assigns the value of the minimum payoff across those actions. In this we depart from what is usually assumed in the robustness literature, where the principal believes that the agent will take the best action for her among those in $A^\star(w|\mathcal{A})$ (see Frankel (2014) and Carroll (2015)). This change is motivated by the the principal's goal to maximize her guaranteed payoff. Any other tie-braking rule can potentially lead to cases where the expected payoff the principal would actually get is lower than $V_i(w|\mathcal{A})$.

The best response of principal i to a contract w_j is:

$$\mathrm{BR}_i(w_j) = \underset{w_i\geq 0}{\operatorname{argmax}} \quad V_i(w_1, w_2). \tag{4}$$

We call the contracts in the best response of the principal **robust**, since they maximize the guaranteed payoff of the principal across all possible technologies. We can now define an equilibrium:

Definition. A Nash equilibrium is a contract scheme $w^\star = (w_1^\star, w_2^\star)$ such that $w_i^\star \in \mathrm{BR}_i\left(w_j^\star\right)$, along with a best response of the agent $A^\star(w^\star|\mathcal{A})$ given the true technology set.

An implicit assumption in the definition of equilibrium is that principals correctly predict the behavior of the other principals and the only uncertainty is about the technology of the

[9]Behind the definition of the principal's guaranteed payoff there is a strong assumption on the type of technologies that the principals consider. All technologies $\mathcal{A}$ that contain $\mathcal{A}_0$ are allowed. This includes technologies for which the agent has almost zero cost of inducing distributions that are detrimental for the principal. It is possible to relax this assumption by allowing for a lower bound on the cost that the agent faces. Doing so does not change our main results. In particular a version of Theorem 1 can be proven and Proposition 1 goes unchanged. See online appendix.

agent. In a lobbying game, this would imply that lobbyists know each other but they are unsure about the preferences and goals of new politicians. In reality new politicians come every election cycle however lobbying firms are there longer term. In the problem of taxing multinational firms, countries know the tax policy of other countries, however they are unsure about future changes in the production technology of the multinational firm. Thus our assumption that worst case only considers the agent's technology (i.e. the multinational) and not the other principal's strategy (the other country's tax policy) is not very demanding. Dai and Toikka (2017) employ a similar informational structure, where the agents have complete knowledge about the true technology and the only misinformed party is the principal.

Note that we are restricting attention to pure strategies, and the equilibrium actions and payoffs of the principals are independent of the agent's true technology set, $\mathcal{A}$. They instead depend on $\mathcal{A}_0$.

1.1 Principal's best response

In this section we characterize the behavior of a principal who maximizes her guaranteed payoff, taking as given the contract of the other principal. We propose a set of contracts that imply linear revenue sharing between a principal and the agent and show that they are robust to misspecification of the agent's technology. That is, they maximize the principal's guaranteed payoff. The strategy to determine optimality is similar to that of Carroll (2015) and the detailed proofs can be found in Appendix A.1.

We proceed by defining the class of linear revenue sharing contracts:

Linear Revenue Sharing (LRS) Contracts: Given a contract w_j, a contract w_i is a LRS contract for principal i if it ties the principal's ex-post payoff linearly to the total revenue of the agent. That is, for some $\alpha \in (0, 1]$ and $k \in \mathbb{R}$:

$$y_i - w_i(y) = \frac{(1-\alpha)}{\alpha}(w_1(y) + w_2(y)) + k \tag{5}$$

this implies that w_i has the following form:

$$w_i(y) = \alpha y_i - (1-\alpha) w_j(y) - \alpha k \qquad \forall y \in Y \tag{6}$$

LRS contracts deal with the dual objective of the principal: providing incentives to the agent to increase her output and competing against the offers made by other principals. The contract rewards the agent when he produces for the principal, and partially undoes the payments the agent receives from the other principal.

The first part of the contract is reminiscent of the results in the literature on the max-min optimality of linear contracts in principal-agent settings (See Carroll (2015), Chassang (2013), Hurwicz (1977) and Hurwicz and Shapiro (1978)). The second part resembles the *principle of aggregate congruence* in Bernheim and Whinston (1986a), where the principals first offset the payments of the other principals and then design their preferred incentive scheme. However, under LRS contracts the payments of the other principal are only partially offset. The principal claims a fraction of his output from the agent and the same fraction of the payments of the other principal. This results in the sharing of the agent's total revenue.[10]

Moreover, the defining property of LRS contracts, i.e. the affine relationship between the ex-post payoffs of the agent and the principal, allows for linking the principal's guaranteed payoff to the agent's payoff under $\mathcal{A}_0$ in an affine way. Lemmas 6, 7 and 8 in Appendix A.1 show this explicitly. Given a contract scheme (w_1, w_2) where w_i is a LRS contract as in (6), the principal's guaranteed payoff is given by:

$$V_i\left(w\right) = \frac{1-\alpha}{\alpha} V_A\left(w|\mathcal{A}_0\right) - k \tag{7}$$

The affine link between the agent's payoff and the principal's payoff is a crucial element in providing incentives. Given the incomplete knowledge of the agent's set of actions the principals' optimal strategy is to tie their payoff to that of the agent, thus aligning the agent's objectives with their own. LRS contracts implement this strategy. This is the same mechanism at the heart of the optimal contracts in Carroll (2015) and Hurwicz and Shapiro (1978).

The main result of this section is summarized in Theorem 1. It states that offering a LRS contract is always a best response for the principal, that is: *a LRS contracts is always robust*. Furthermore, under Assumption 4 only LRS contracts are robust or the principal can only guarantee herself the payoff given by offering the agent zero aggregate incentives and inducing the agent to pick inaction.[11] The proof can be found in Appendix A.1.

Theorem 1. *For any contract w_j there exists an LRS contract $\overline{w}_i$ such that $\overline{w}_i \in BR_i\left(w_j\right)$, where $\min_{y \in Y}\left\{\bar{w}_i\left(y\right) + w_j\left(y\right)\right\} = 0$. That is, there is always a LRS contract that is **robust** for*

[10] Formally, the problem of a principal can then be thought of in two steps: first undoing the payments of other principals, and then offering the agent an aggregate contract satisfying limited liability. We call this aggregate contract $\tilde{w}_i$. Then the ex-post payoff of principal i is: $y_i + w_j\left(y\right) - \tilde{w}_i\left(y\right)$. Principal i's actual contract is of course: $w_i\left(y\right) = \tilde{w}_i\left(y\right) - w_j\left(y\right)$. When w_i is an LRS contract the implied aggregate contract is: $\tilde{w}_i\left(y\right) = \alpha\left(y_i + w_j\left(y\right)\right) - \alpha k$. Thus the ex-post payoffs of the principals are linearly tied to those of the agent, with principal i receiving $1-\alpha$ of the payoff $y_i + w_j\left(y\right)$ and the agent receiving α of it. k acts like a lump-sum transfer between the principal and the agent and is determined by limited liability.

[11] To offer the agent zero aggregate incentives principal i sets $w_i\left(y\right) = -w_j\left(y\right)$ and the agent picks $(F, c) = \left(\delta_{(0,0)}, 0\right)$.

principal i.

If $\mathcal{A}_0$ satisfies the full support property, then any robust contract for principal i is a LRS contract or $\max_{w_i} V_i(w_i, w_j) = w_j(0,0)$.

1.2 Equilibrium

Theorem 1 allows us to focus on equilibria where both principals offer LRS contracts. In this section we establish that an equilibrium in LRS contracts always exists and we characterize equilibrium payoffs in this case. In common agency with incomplete information, as in many sequential games, establishing existence under general conditions has proven difficult, mostly because of the failure of convexity of the principals' best responses (see Bernheim and Whinston (1986a), Fraysse (1993) and Carmona and Fajardo (2009)). However our robust approach allows us to prove existence of a pure strategy Nash equilibrium in a novel way, by showing that the common agency game has a pseudo-potential function as in Dubey et al. (2006). The proof for existence is presented at the end of the section.

Recall from (6) that an LRS contract depends on the contract offered by the other principal, by partially undoing the payments she makes to the agent. In equilibrium, when both principals play LRS contracts, that satisfy limited liability with equality, we obtain a sharper characterization of the form of the contract and of the principal's payoffs. Each principal gets a share of total output.

The following proposition characterizes contract schemes in LRS contracts precisely. If assumption 4 holds then all equilibria are in LRS contracts.

Proposition 1. *Let w be a LRS contract scheme satisfying limited liability. Then there exist $\{\theta_i, k_i\}_{i\in\{1,2\}}$ such that:*

$$w_i(y) = (1-\theta_i)\, y_i - \theta_i y_j - k_i \qquad \text{and} \qquad k_1 = -k_2 \qquad \theta_i \in [0, 1-\theta_j] \tag{8}$$

The guaranteed payoff of principal i is:

$$V_i(w) = \theta_i \max_{(F,c)\in\mathcal{A}_0} \left\{ E_F[y_1 + y_2] - \frac{c}{1-\theta_1-\theta_2} \right\} - k_i \tag{9}$$

Proof. Since both w_1 and w_2 are LRS contracts, there exist shares $\alpha_1, \alpha_2 \in [0,1]$ and constants k_1 and k_2 such that w_i is as in (6) for $i \in \{1,2\}$. Then the aggregate contract offered to the agent is:

$$w_1(y) + w_2(y) = \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2-\alpha_1\alpha_2}\,(y_1 + y_2 - k_1 - k_2)$$

In order to satisfy limited liability for all $y \in Y$ it must be that $k_1 = -k_2$.

We can further define $\theta_i = \frac{(1-\alpha_i)\alpha_j}{\alpha_1+\alpha_2-\alpha_1\alpha_2}$ to characterize the contracts:

$$w_i(y) = (1-\theta_i)\, y_i - \theta_i y_j - k_i$$

note that $\theta_i \in [0, 1-\theta_j]$ and that the aggregate contract faced by the agent is: $w_1(y) + w_2(y) = (1-\theta_1-\theta_2)(y_1+y_2)$. The principals' guaranteed payoffs are obtained from equation (7) (see Lemma 8 in Appendix A.1).

□

It is worthwhile noting that the transfers k_1 and k_2 do not affect the action chosen by the agent. In this sense the constants k_1 and k_2 act as transfers between the principals, channeled through the agent, as in Bernheim and Whinston (1985, 1986a). Moreover, since the effect of k_i on the payoff of principal i is independent of θ_i, the value of θ_1 and θ_2 can be computed separately from the value of the transfers (k_1 and k_2). In fact, the transfers are not pinned down in equilibrium.[12]

When both principals use LRS contracts the payoffs of all players depend only on aggregate output. In fact principal i receives a share θ_i of aggregate output. This is a more explicit form of the principle of aggregate congruence in Bernheim and Whinston (1986a):

> "[W]e underscore the need to make principals' objectives congruent in equilibrium: since all principals can effect the same changes in the aggregate incentive scheme, none must find any such change worthwhile. One can think of this congruence as being accomplished through implicit side payments among principals."

Under LRS contract one can equivalently cast the problem of each principal as choosing the share of output going to the agent. In an equilibrium all principals have to agree on the agent's share. Moreover, side payments are made explicit through the transfers k_1 and k_2, as mentioned above these transfers have to sum to zero, using the agent as a conduit for payments between principals.

LRS contracts maximize the guaranteed payoffs of the principals by balancing the dual objective of incentivizing the agent and competing with the other principal. The contract described in (8) gives the agent a fraction $(1-\theta_i)$ of principal i's output, and takes a share θ_i of principal j's output. Furthermore, the contract deals in an effective way with the distributional concerns that lie behind the competition between principals. Under LRS contracts it is irrelevant who the agent chooses to work for when determining realized payoffs.

[12]In Bernheim and Whinston (1986a) the value of transfers between principals is also indeterminate in equilibrium. A participation constraint is assumed for the agent and each principal can induce the agent not to participate. If that happens principals get some outside payoff. The value of transfers in an equilibrium with participation ensures that each principal receives at least her outside payoff.

Each player receives a share of total output and as a consequence the guaranteed payoffs of all principals are linearly tied with those of the agent (see (9)).

We now present the main result of this section. We use the results in Proposition 1 to show that an equilibrium in LRS contracts always exists. We do this by showing that our game allows for a pseudo-potential as in Dubey et al. (2006). The use of a potential function to show equilibrium existence is new to the common agency literature and can be useful in showing equilibrium existence in common agency games with incomplete information that do not take a robust contracting approach.

Theorem 2. *A pure strategy Nash Equilibrium in LRS contracts, with $\theta_1, \theta_2 > 0$, exists.*

Proof. First note from Proposition 1 that a pair of LRS contracts is characterized by two shares (θ_1, θ_2) and two transfers (k_1, k_2) satisfying $k_1 = -k_2$. The shares are chosen to maximize the principal's guaranteed payoff, as in equation (9). This is equivalent to maximizing:

$$\tilde{V}_i(\theta, \theta_j) = \max_{\theta \in [0, \theta_j]} \theta G(\theta + \theta_j) \tag{10}$$

where we define $G : \mathbb{R}_+ \to \mathbb{R}$ as follows:

$$G(x) = \frac{1}{1-x} \max_{(F,c) \in \mathcal{A}_0} \{(1-x) E_F[y_1 + y_2] - c\} \tag{11}$$

We adopt the convention that $G(1) = 0$. Then for $x \geq 1$, $G(x) = 0$. G is continuous.

We prove existence of an equilibrium in which $\theta_i > 0$ for $i \in \{1, 2\}$. As in Monderer and Shapley (1996) we consider an ordinal potential function for the game:

$$P(\theta_1, \theta_2) = \theta_1 \theta_2 G(\theta_1 + \theta_2) \tag{12}$$

The function P is an ordinal potential for the game if the shares θ_1, θ_2 are positive since the function P induces the same order over θ_i as the function V_i, that is for all $\theta_j > 0$ and $\theta, \theta' \in [0, 1]$:

$$\tilde{V}_i(\theta, \theta_j) - \tilde{V}_i\left(\theta', \theta_j\right) > 0 \quad \iff \quad P(\theta, \theta_j) - P\left(\theta', \theta_j\right) > 0 \tag{13}$$

However the strategy space that we are considering allows for θ_1 or θ_2 to be zero, so the function P is not an ordinal potential, however it is a pseudo-potential since its maxima in $[0,1]^2$ are interior. It is immediate from (13) that any maximum of P such that $\theta_1, \theta_2 > 0$ is a pure strategy equilibrium of the common agency game. Under assumption 3 such a maximum exist.

First note that P attains a maximum in $[0,1]^2$ by Weierstrass' theorem. Assumption 3 allows for an action that generates enough (expected) output to cover the cost of production, formally there exists $\theta_1, \theta_2 > 0$ such that $G(\theta_1 + \theta_2) > 0$, then $P(\theta_1, \theta_2) > 0$. Then for all $(\theta_1^\star, \theta_2^\star) \in \operatorname{argmax}_{(\theta_1, \theta_2) \in [0,1]^2} P(\theta_1, \theta_2)$, it holds that $\theta_1^\star, \theta_2^\star > 0$. All these pairs are Nash equilibria of the common agency game. If assumption 3 is violated then it is not possible to induce the agent to produce and the game has a trivial solution.[13]

$\square$

The potential function structure provides an interesting connection of the common agency game with the standard Cournot competition. Once we show that equilibrium contracts are LRS, the problem of each principal can be interpreted as maximizing profits $\left(\tilde{V}_i\right)$ by choosing a quantity of production (θ_i) and facing an inverse demand function given by G, defined in (11), and a constant marginal cost of zero.

Finally, it is possible to characterize an equilibrium in LRS contracts more tightly by analyzing the guaranteed payoff of the principals. An equilibrium is completely characterized by a pair of shares (θ_1, θ_2) and a pair of transfers (k_1, k_2). Interestingly the equilibrium has an anonymity property, pinning down the actions taken in equilibrium, but not the identity of the principal taking them. The conditions they satisfy are summarized in the following proposition:

Proposition 2. *A Nash equilibrium in LRS contracts is a pair of shares (θ^1, θ^2) and transfers (k_1, k_2), such that $k_1 = -k_2$ and there are actions $(F^1, c^1), (F^2, c^2) \in \mathcal{A}_0$ such that:*

$$\left(1-\theta^1-\theta^2\right)^2 = \frac{\left(1-\theta^j\right)c^i}{E_{F^i}\left[y_1+y_2\right]} \qquad \left(F^i, c^i\right) \in \underset{(F,c)\in\mathcal{A}_0}{argmax}\left\{\left(\sqrt{\left(1-\theta^j\right)E_F\left[y_1+y_2\right]}-\sqrt{c}\right)^2\right\}$$

Proof. From (9) in Proposition 1 we can find the shares and the transfers independently. The transfers don't have any constraint other than summing to zero, so $k_1 = -k_2$. The share θ_i of principal i is chosen to solve:

$$\max_{(F,c)\in\mathcal{A}_0} \max_{\theta\in[0,1-\theta_j]} \left\{\theta_i E_F\left[y_1+y_2\right] - \frac{\theta_i}{1-\theta_1-\theta_2}c\right\} \tag{14}$$

for a fixed $(F_i, c_i) \in \mathcal{A}_0$ the solution to this problem is characterized by:

$$\left(1-\theta_i-\theta_j\right)^2 = \frac{\left(1-\theta_j\right)c_i}{E_{F_i}\left[y_1+y_2\right]} \tag{15}$$

Since both principals satisfy this equation in equilibrium we have:

$$1-\theta_1-\theta_2 = \sqrt{\frac{\left(1-\theta_j\right)c_i}{E_{F_i}\left[y_1+y_2\right]}} = \sqrt{\frac{\left(1-\theta_i\right)c_j}{E_{F_j}\left[y_1+y_2\right]}}$$

[13] An equilibrium still exists if assumption 3 is violated, for instance it is a best response for both principals to set $\theta_i = 0$. That makes $\tilde{V}_i = 0$.

We obtain (F_i, c_i) by replacing (15) in (14):

$$(F_i, c_i) \in \underset{(F,c)\in\mathcal{A}_0}{\text{argmax}} \left\{ \left(\sqrt{(1-\theta_j)\, E_F\left[y_2 + y_2\right]} - \sqrt{c} \right)^2 \right\}$$

The problem of each principal does not depend on her identity, because of that the solution will be anonymous.

An equilibrium is then a pair of shares $\left(\theta^1, \theta^2\right)$ and actions $\left(\left(F^1, c^1\right), \left(F^2, c^2\right)\right)$ such that:

$$\left(1-\theta^1-\theta^2\right)^2 = \frac{\left(1-\theta^j\right) c^i}{E_{F^i}\left[y_1+y_2\right]} \qquad \left(F^i, c^i\right) \in \underset{(F,c)\in\mathcal{A}_0}{\text{argmax}} \left\{ \left(\sqrt{\left(1-\theta^j\right) E_F\left[y_1 + y_2\right]} - \sqrt{c} \right)^2 \right\}$$

We use superscripts to reinforce anonymity. So for every pair of shares identified there are two equilibria, one in which $\theta_1 = \theta^1$ and $\theta_2 = \theta^2$ and another one in which $\theta_1 = \theta^2$ and $\theta_2 = \theta^1$.

□

1.3 Collusion

When colluding, principals seek to maximize guaranteed joint payoff. They offer a single contract that satisfies limited liability of the form $w : Y_1 \times Y_2 \to \mathbb{R}_+$. The principals' problem is a generalization of the principal-agent problem studied in Carroll (2015) to a multi-task principal-agent model. In this case the agent controls two tasks or accounts (y_1, y_2).

Given technology $\mathcal{A}$, the agent's optimal actions and payoff are now given by:

$$A^\star(w|\mathcal{A}) = \underset{(F,c)\in\mathcal{A}}{\text{argmax}} \quad E_F\left[w(y)\right] - c \qquad\qquad V_A(w|\mathcal{A}) = \max_{(F,c)\in\mathcal{A}} E_F\left[w(y)\right] - c.$$

The guaranteed joint payoff for the principals is:

$$V_P(w) = \inf_{\mathcal{A}\supseteq\mathcal{A}_0} V_P(w|\mathcal{A}) \quad \text{where} \quad V_P(w|\mathcal{A}) = \min_{(F,c)\in A^\star(w|\mathcal{A})} E_F\left[y_1 + y_2 - w(y_1, y_2)\right].$$

The optimal contract under collusion is linear in total output $(y_1 + y_2)$. By making the contract dependent on total output the principals leave to the agent the decision of which task (y_1 or y_2) to favor when producing. The decision depends on the agent's true technology, which is unknown to the principals when contracting. Even if, under the known technology $(\mathcal{A}_0)$ the principals want to incentivize differently across tasks, say because the agent is more productive in generating y_1 than y_2, the same incentives do not generalize across all possible technologies, and thus do not provide the best guarantee for the principals.

The contract offered by the principals also ties linearly the value of the agent and the principal. We summarize these results in the following theorem. The proof can be found in the online appendix.

Theorem 3. *Let* $(F^\star, c^\star) \in \underset{(F,c)\in\mathcal{A}_0}{\text{argmax}}\left\{\left(\sqrt{E_F\left[y_1+y_2\right]}-\sqrt{c}\right)^2\right\}$*. When principals collude, the contract:*

$$w_c(y) = (1-\theta_c)(y_1+y_2) \qquad \textit{where} \quad 1-\theta_c = \sqrt{\frac{c^\star}{E_{F^\star}\left[y_1+y_2\right]}} \tag{16}$$

maximizes V_P*. Moreover, for any contract of the form* $w(y) = (1-\theta)(y_1+y_2)$ *that guarantees a positive payoff,* V_P *can be expressed as:*

$$V_P(w|\mathcal{A}_0) = \frac{\theta}{1-\theta}\max_{(F,c)\in\mathcal{A}_0}\left\{(1-\theta)E_F\left[y_1+y_2\right]-c\right\} \tag{17}$$

Finally, If $\mathcal{A}_0$ *satisfies assumption 4, then all optimal contracts are of this form.*

As mentioned above, when the principals collude, the model boils down to a multi-task principal-agent model. This type of problem has received extensive attention by the literature, most notably by Holmstrom and Milgrom (1987). A key question is how the incentives should depend on the different tasks controlled by the agent. In the model developed in Holmstrom and Milgrom (1987) an agent controls the drift of a multi-dimensional Brownian motion, the principal chooses how to reward the agent given the terminal value of the Brownian motion. Importantly they find that the optimal scheme is not generally linear in total output (principal's profits), instead it rewards the agent differently for different tasks. They specifically note (Holmstrom and Milgrom, 1987, p.306):

> "The optimal scheme for the multidimensional Brownian model is a linear function of the end-of-period levels of the different dimensions of the process. ...If... the compensation paid must be a function of profits alone (perhaps because reliable detailed accounts are unavailable), or if the manager has sufficient discretion in how to account for revenues and expenses then the optimal compensation scheme will be a linear function of profits. *This is a central result, because it explains the use of schemes which are linear in profits even when the agent controls a complex multi-dimensional process."* [Emphasis added]

However, in our model, robustness leads to linearity in profits no matter how complex the multi-dimensional process the agent controls is. The alignment of incentives between the principal and the agent requires linearity in profits.

2 Efficiency

In this section we examine the efficiency properties of the equilibrium, and compare them to those of the collusive outcome. In games of complete information the issue of efficiency was tackled by considering *truthful* equilibria (Bernheim and Whinston, 1986b), which are always efficient. However, in environments with asymmetric information competition among principals can lead to inefficiencies. In this section we show that *competition between principals always leads to a less efficient outcome than if principals were to collude.*

This efficiency result parallels finding in the literature, see for instance Bernheim and Whinston (1986a), Holmstrom and Milgrom (1988), and Martimort and Stole (2012), as well as the adverse selection models of Martimort and Stole (2015, 2012), Martimort and Moreira (2010) and Bond and Gresik (1996). As noted in Section 1.2 the objectives of all the principals are made congruent in the equilibrium (they all receive a share of total output), this gives rise to a "free-rider" problem.

The free riding problem appears here because each principal does not internalize the effect of an increase in her share of total output (which lowers the share of the agent) on the payoffs of the competing principal. This leads to the equilibrium share of output accruing to the agent to be lower relative to what he gets under collusion. As we show in Lemmas 1 and 2 this implies a less efficient equilibrium outcome.

We consider two notions of efficiency:

Total expected surplus (TES): Given a contract scheme w and a technology $\mathcal{A}$, total expected surplus measures the sum of the expected payoffs of all players. This is given by the difference between total expected output and the cost for the actions preferred action of the agent given the contract scheme w and technology $\mathcal{A}$:

$$\text{TES}\left(w|\mathcal{A}\right)=\left\{E_F\left[y_1+y_2\right]-c \mid \left(F,c\right)\in\underset{(F,c)\in\mathcal{A}}{\operatorname{argmax}}\left\{w_1\left(y\right)+w_2\left(y\right)-c\right\}\right\}$$

Total guaranteed surplus (TGS): Given a contract scheme w and a known technology set $\mathcal{A}_0$, total guaranteed surplus measures the surplus to be guaranteed to all players.

$$TGS\left(w\right)=V_1\left(w|\mathcal{A}_0\right)+V_2\left(w|\mathcal{A}_0\right)+V_A\left(w|\mathcal{A}_0\right)$$

and under collusion:

$$TGS\left(w\right)=V_P\left(w|\mathcal{A}_0\right)+V_A\left(w|\mathcal{A}_0\right)$$

The first notion of efficiency (TES) is standard in the literature, while the second one (TGS) is considered here because of the game's information structure. Given a technology set $\mathcal{A}$ it is possible to compute TES for any action on that set. Yet, the principals only know a minimal technology set $\mathcal{A}_0$. Given that knowledge and a contract scheme it is still possible to compute the guaranteed surplus of a principal, V_i.[14] This allows for a notion of ex-ante efficiency from the point of view of the principals.

Theorem 4 establishes the main result of this section, namely that collusion leads to a more efficient outcome than competition between principals. Its proof will be developed in lemmas 1, 2 and 3.

Theorem 4. *Total expected surplus and total guaranteed surplus are higher under collusion between the principals than under a Nash equilibrium in LRS.*

Note that under both competition and collusion the agent gets paid a share of total output. This will allow for a clean comparison of the efficiency properties of the two scenarios. Lemmas 1 and 2 show that total expected and total guaranteed surplus are increasing in the share of output that the agent gets. Then Lemma 3 shows that the agent will always get a higher share of output when principals collude than when they compete.

Recall that, given the equilibrium contracts (or contract in the case of collusion), the agent will choose an action (F, c) in order to maximize her payoff $V_A(w|\mathcal{A})$ given a technology set $\mathcal{A}$. Under both competition and collusion the agent's problem reduces to:

$$\tilde{V}_A(\theta|\mathcal{A}) = \max_{(F,c)\in\mathcal{A}} \{\theta E_F[y_1 + y_2] - c\} \tag{18}$$

for some share θ. Thus, the action taken by the agent is, in general, not efficient (in the sense that it does not maximize TES). Yet, we can establish how total (expected) surplus varies with the equilibrium contracts. Contracts for which the agent captures a larger share of realized output are more efficient. This is intuitive, since as θ goes to one, the agent's problem converges to that of maximizing total surplus. We formalize this argument in Lemmas 1 and 2 below.

Lemma 1. *Let w and w' be contract schemes such that the agent receives a share of total output given by θ and θ' respectively. Total expected surplus (TES) is weakly increasing in the share of total output going to the agent. That is, for any technology $\mathcal{A}$, let $s \in TES(w|\mathcal{A})$ and $s' \in TES(w'|\mathcal{A})$. If $\theta < \theta' \leq 1$ then $s \leq s'$.*

[14] We can think of V_i as the utility function of principal i, which is quasilinear in lump sum transfers. Because of the quasilinear environment we can consider the sum of utilities as a measure of welfare or efficiency.

Proof. Let w and $w^{'}$ be contract schemes such that the agent receives a share of total output given by θ and $\theta^{'}$ respectively, with $\theta < \theta^{'}$. Consider $(F_\theta, c_\theta) \in A^\star (w|\mathcal{A})$ and $\left(F_{\theta'}, c_{\theta'}\right) \in A^\star \left(w^{'}|\mathcal{A}\right)$, then:

$$\tilde{V}_A (\theta|\mathcal{A}) = \theta E_{F_\theta} [y_1 + y_2] - c_\theta < \theta^{'} E_{F_\theta} [y_1 + y_2] - c_\theta \leq \theta^{'} E_{F_{\theta^{'}}} [y_1 + y_2] - c_{\theta'} = \tilde{V}_A \left(\theta^{'}|\mathcal{A}\right) \quad (19)$$

where $y = y_1 + y_2$. The first inequality follows from $\theta < \theta^{'}$ and the second one from (F_θ, c_θ) being feasible at $\theta^{'}$. Furthermore, its easy to check that $E_{F_{\theta^{'}}} [y] \geq E_{F_\theta} [y]$, otherwise $(F_\theta, c_\theta) \notin A^\star (\theta y|\mathcal{A})$. Finally, using the second inequality in (19) we have:

$$\begin{aligned} c_{\theta'} - c_\theta &\leq \theta^{'} \left[E_{F_{\theta^{'}}} [y_1 + y_2] - E_{F_\theta} [y_1 + y_2]\right] \\ c_{\theta'} - c_\theta &\leq E_{F_{\theta^{'}}} [y_1 + y_2] - E_{F_\theta} [y_1 + y_2] \\ E_{F_\theta} [y_1 + y_2] - c_\theta &\leq E_{F_{\theta^{'}}} [y_1 + y_2] - c_{\theta'} \end{aligned}$$

Since the inequality is proven for arbitrary actions in the agent's best response This proves the monotonicity of expected total surplus surplus on θ.

□

Lemma 2. *Total guaranteed surplus (TGS) is weakly increasing in the share of total output going to the agent.*

Proof. Under LRS contracts total guaranteed surplus depends exclusively on the share of output going to the agent, regardless of whether or not the principals collude. Using (9) and (17) we have:[15]

$$\text{TGS}\,(w) = \frac{1}{\theta} \max_{(F,c) \in \mathcal{A}_0} \{\theta E_F [y_1 + y_2] - c\}$$

under both competition and collusion, where θ is the share of total output going to the agent.

Let w and $w^{'}$ be LRS contract schemes such that the agent receives a share of total output given by θ and $\theta^{'}$ respectively, with $\theta < \theta^{'}$. Consider $(F_\theta, c_\theta) \in A^\star (w|\mathcal{A})$ and $\left(F_{\theta'}, c_{\theta'}\right) \in A^\star \left(w^{'}|\mathcal{A}\right)$, then:

$$\theta^{'} E_{F_\theta} [y_1 + y_2] - c_\theta \leq \theta^{'} E_{F_{\theta^{'}}} [y_1 + y_2] - c_{\theta'} \quad (20)$$

by optimality of the agent. Since $\theta < \theta^{'}$ we have:

$$\text{TGS}\,(w) = E_{F_\theta} [y_1 + y_2] - \frac{1}{\theta} c_\theta < E_{F_\theta} [y_1 + y_2] - \frac{1}{\theta'} c_\theta \leq E_{F_{\theta^{'}}} [y_1 + y_2] - \frac{1}{\theta'} c_{\theta'} = \text{TGS}\left(w^{'}\right)$$

The inequalities follow from $\theta < \theta^{'}$ and (20), respectively.

□

Finally we compare the share that the agent gets under collusion and in a Nash equilibrium. To do this we first note that the condition of the agent's share under collusion

[15] Note that this implies that under competition θ_i is not only the share of output going to the principal, but also the share of total guaranteed surplus.

given in (16) of Theorem 3 is equivalent to the problem of a principal under competition facing $\theta_j = 0$. This is immediate from equation (9) in Proposition 1. Then, in order to show that the share of the agent under competition is lower than the share under collusion, it is sufficient to show that the share that principal i wants to induce for the agent is decreasing in the share of principal j. Since in equilibrium $\theta_j \geq 0$ it follows that the share of the agent will be lower than under collusion.

Intuitively, as in Bernheim and Whinston (1986a), where principals only internalize "1/Jth" of the gain when making the principals' objectives congruent, here the principal only internalizes $1 - \theta_j$ of the increases in output. The principal's objective function is:

$$
\begin{aligned}
V_i(\theta_i, \theta_j) &= \theta_i \left(E_F[y_1 + y_2] - \frac{c}{1 - \theta_j - \theta_i} \right) + k_i \\
&= \tilde{\theta}_i \left(E_F[\tilde{y}_1 + \tilde{y}_2] - \frac{c}{1 - \tilde{\theta}_i} \right) + k_i
\end{aligned}
$$

where $\tilde{\theta}_i = {}^{\theta_i}/_{1-\theta_j}$ and $\tilde{y}_k = (1 - \theta_j) y_k$ for $k \in \{1, 2\}$. Then the problem of principal i of choosing $\theta_i \in [0, 1 - \theta_j]$ given θ_j, is equivalent to the problem of a single principal facing a multitasking agent over a "reduced" output space $\tilde{Y} = (1 - \theta_j) Y$, choosing $\tilde{\theta}_i \in [0, 1]$. The constant k_i plays no role in choosing θ_i. Since the principal does not internalize all of the output it also does not want to give as much incentives to the agent. This is the same force at the heart of the "free-rider" problem described in Bernheim and Whinston (1986a), Holmstrom and Milgrom (1988), Maier and Ottaviani (2009) and Martimort and Stole (2012).

The formal result is presented in the following Lemma:

Lemma 3. *Let $\theta_j^L < \theta_j^H$ and denote by θ_i^L and θ_i^H any elements of the best response of principal i to θ_j^L and θ_j^H respectively. It holds that:*

$$1 - \theta_i^L - \theta_j^L \geq 1 - \theta_i^H - \theta_j^H$$

Proof. Suppose for a contradiction that the best response of principal i implies a higher share for the agent when responding to θ_j^H than when responding to θ_j^L:

$$1 - \theta_i^L - \theta_j^L < 1 - \theta_i^H - \theta_j^H$$

Since θ_i^L is in the best response to θ_j^L it must give at least as much payoff to principal i as any other share, given a fixed level of transfers (k_1, k_2). In particular consider an alternative share for principal i given by: $\tilde{\theta}_i = \theta_i^H - \left(\theta_j^H - \theta_j^L\right)$. This alternative share implies that the share of the agent is the same as under the high share: $1-\theta_i^H-\theta_j^H$. It must be that:

$$\theta_i^L\left(E_{F^L}\left[y_1+y_2\right]-\frac{c^L}{1-\theta_i^L-\theta_j^L}\right)\geq\left(\theta_i^H-\left(\theta_j^H-\theta_j^L\right)\right)\left(E_{F^H}\left[y_1+y_2\right]-\frac{c^H}{1-\theta_i^H-\theta_j^H}\right)$$

Where $\left(F^l,c^l\right)\in\underset{(F,c)\in\mathcal{A}_0}{\operatorname{argmax}}\left\{\left(1-\theta_i^l-\theta_j^l\right)E_F\left[y_1+y_2\right]-c\right\}$ for $l\in\{L,H\}$. The pair (F,c) is determined by the agent's problem and thus depends only on the share of the agent.

Similarly, since θ_i^H is in the best response to θ_j^H we can consider an alternative share for principal i given by: $\tilde{\theta}_i^H=\theta_i^L-\left(\theta_j^H-\theta_j^L\right)$. As before, this alternative share implies that the share of the agent is $1-\theta_i^L-\theta_j^L$. It must be that:

$$\theta_i^H\left(E_{F^H}\left[y_1+y_2\right]-\frac{c^H}{1-\theta_i^H-\theta_j^H}\right)\geq\left(\theta_i^L-\left(\theta_j^H-\theta_j^L\right)\right)\left(E_{F^L}\left[y_1+y_2\right]-\frac{c^L}{1-\theta_i^L-\theta_j^L}\right)$$

By subtracting these inequalities we get:

$$\begin{gathered}\left(\theta_j^H-\theta_j^L\right)\left(E_{F^L}\left[y_1+y_2\right]-\frac{c^L}{1-\theta_i^L-\theta_j^L}\right)\geq\left(\theta_j^H-\theta_j^L\right)\left(E_{F^L}\left[y_1+y_2\right]-\frac{c^L}{1-\theta_i^H-\theta_j^H}\right)\\ \max_{(F,c)\in\mathcal{A}_0}\left\{E_F\left[y_1+y_2\right]-\frac{c}{1-\theta_i^L-\theta_j^L}\right\}\geq\max_{(F,c)\in\mathcal{A}_0}\left\{E_F\left[y_1+y_2\right]-\frac{c}{1-\theta_i^H-\theta_j^H}\right\}\\ \mathrm{TGS}\left(1-\theta_i^L-\theta_j^L\right)\geq\mathrm{TGS}\left(1-\theta_i^H-\theta_j^H\right)\end{gathered}$$

This contradicts Lemma 2 since $1-\theta_i^L-\theta_j^L<1-\theta_i^H-\theta_j^H$.

□

2.1 Limited liability and efficiency

In this section we provide a better understanding of the efficiency result derived in the previous section and in most of the common agency literature. Even though principals are competing among them trying to influence the agent, the agent gets a larger share of total output when the principals collude. This in turn leads to a more efficient action being taken by the agent. We show now that the ability of principals to implicitly make side-payments through the agent is crucial for this result. To see this we impose individual limited liability constraints on the contracts offered by the principals.

Up until now we have assumed that the agent has limited liability, implying that the aggregate payments to the agent must be non-negative for any realization of output. Critically, this allows the principals to extract payments from the agent. As mentioned above, the contract described in (8) allows principal i to take a share θ_i of principal j's output, thus receiving a share θ_i of total output. By doing this, one of the principals can end out receiving net payments from the agent, $w_i(y)<0$ for some $y\in Y$. Moreover, the contract in (8) stipulates transfers between the agent and the principals given by k_1 and k_2. In equilibrium these transfers are such that $k_1 = -k_2$, so that they have no effect over the agent's problem, they are in effect transfers between the principals.

Individual limited liability implies that the contract of each principal has to be non-negative, $w_i(y) \geq 0$ for all $y \in Y$ and $i \in \{1, 2\}$. This constrains the ability to transfer resources between principals, and to demand payments from the output produced for competitors. As we show below this will not change the optimality of LRS contracts for the principals, however it does affect the ability of a principal to free-ride on the incentives provided by her competitors. Limited liability forces each principal to internalize the externality imposed on the other principal. At the end of this section we show that under individual limited liability an equilibrium in LRS contracts is more efficient than the outcome under collusion.

Regardless of the limited liability constraints it is still optimal for the principals to tie their payoffs to the payoffs of the agent. The desire for robustness requires that link to ensure that the incentives of the agent and the principals are aligned. A version of Theorem 1 establishing the optimality of LRS contracts applies and is presented below, its proof is almost identical to the proof of Theorem 1 and is presented in the online appendix.

Theorem 5. *For any contract w_j there exists a LRS contract $\overline{w}_i$ such that $\overline{w}_i \in BR_i(w_j)$, where $\min_y \bar{w}_i(y) = 0$. That is, there is always a LRS contract that is* ***robust*** *for principal i. If $\mathcal{A}_0$ satisfies the full support property, then any robust contract for principal i is a LRS contract or principal i cannot guarantee a positive payoff.*

Theorem 5 suggests that since LRS contracts are always optimal for the principals it is worthwhile focusing in what happens when we restrict attention to this class of strategy, just as we did in Section 1.2. Unfortunately the added restrictions of individual limited liability do not allow for a general proof of existence as that in Theorem 2. We are nonetheless able to provide two different sufficient conditions for equilibrium existence and characterize an equilibrium in which both principals use LRS contracts. The exercise sheds light on the role of limited liability in the provision of incentives to the agent.

We proceed by characterizing equilibrium LRS contracts, obtaining a similar result to that of Proposition 1. The main difference is that individual limited liability pins down the transfer of the principal to the agent, this in turns allows for a nice interpretation of the contract: each principal gets a share of total output by rewarding the agent for not working for the other principal. Transfers are such that each principal pays the agent for the output that did not, but could have, produced for her competitors. The following proposition makes this precise:

Proposition 3. *Let w be a LRS contract scheme satisfying individual limited liability. Then there exist $(\theta_1, \theta_2) \in [0,1]$ such that:*

$$w_i(y) = (1-\theta_i) y_i + \theta_i \left(\overline{y}_j - y_j\right) \qquad \text{and} \qquad \theta_i \in [0, 1-\theta_j] \tag{21}$$

The guaranteed payoff of principal i is:

$$V_i(w) = \theta_i \max_{(F,c)\in\mathcal{A}_0} \left\{ E_F[y_1 + y_2] - \frac{c}{1-\theta_1-\theta_2} \right\} - \theta_i \overline{y}_j \tag{22}$$

Proof. Since both w_1 and w_2 are LRS contracts there are shares α_1 and α_2 and constants k_1 and k_2 such that w_i is as in (6) for $i \in \{1,2\}$. Then the aggregate contract offered to the agent is:

$$w_1(y) + w_2(y) = \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2-\alpha_1\alpha_2}(y_1 + y_2 - k_1 - k_2)$$

Defining $\theta_i = \frac{(1-\alpha_i)\alpha_j}{\alpha_1+\alpha_2-\alpha_1\alpha_2}$ this implies:

$$w_i(y) = (1-\theta_i) y_i - \theta_i (y_j - k_i - k_j) - k_i$$

where $\theta_i \in [0, 1-\theta_j]$. Note that the contract is increasing in y_i and decreasing in y_j, so $\min w_i(y)$ is attained at $y_i = 0$ and $y_j = \overline{y}_j$.[16] In order to satisfy limited liability for all $y \in Y$ it must be that $\min w(y) = w\left(0, \overline{y}_j\right) = 0$. This implies $k_i = -\frac{\theta_i}{1-\theta_1-\theta_2}\left((1-\theta_j)\overline{y}_j + \theta_j \overline{y}_i\right)$. Replacing we get:

$$w_i(y) = (1-\theta_i) y_i + \theta_i \left(\overline{y}_j - y_j\right)$$

The payoffs are obtained directly from equations (5) and (7). The aggregate contract faced by the agent is: $w_1(y) + w_2(y) = (1-\theta_1-\theta_2)(y_1+y_2) + \theta_1\overline{y}_2 + \theta_2\overline{y}_1$.

□

The first change induced by the strengthening of limited liability is that the upper bound of the output space plays now an explicit role in how incentives are provided to the agent. In this setting the dual objective of the principal (offering incentives to the agent and competing against the other principal) is served by giving the agent a fraction $(1-\theta_i)$ of her output, and also a share θ_i of the output, that he did not (but could have) produced for the other principal. Doing this ties the payoffs of all players, they all get a share of realized total output, but it also implies that transfers are now given entirely to the agent.

The equilibrium payment to the agent is then a combination of high and low powered

[16]The points $(0, \overline{y}_2)$ and $(\overline{y}_1, 0)$ are in Y by the assumption that Y is a cross product. This assumption is not a necessary one, and is just convenient for determining the values of (y_1, y_2) for which $\min_y w_i(y)$ is attained. If the assumption is lifted (as we do in the application in Section 3 and the examples of Section 1.2 of the online appendix), only the constants k_1 and k_2 are directly affected. For instance if output is perfectly and positively correlated $\min w_i(y)$ is attained when $y_1 = y_2 = 0$ and $k_1 = k_2 = 0$.

incentives, in the form of a share of total output and a constant fee. Notice that the fee that principal i pays ($\theta_i \bar{y}_j$) depends only on the maximum output of the other principal. Hence we can interpret $\bar{y}_j$ as the price per unit share of the conglomerate that principal i has to pay. A consequence of this is that increasing the share of output going to the principal comes at a cost, not only in terms of the share of output left to the agent (which potentially lowers production), but in terms of a fee payed with certainty.[17]

It is useful to understand the characteristics of the game that induce a principal to offer high powered versus low powered incentives. A lower θ gives the principal a lower share of total output, and, all else equal increases the share of the agent. It also reduces the fee that the principal pays. Hence incentives are 'high powered'. Conversely a higher θ gives the agent a smaller share of output, and it increases the fee the principal pays to the agent. Hence incentives are 'low powered'.[18] This allows for understanding the effect of competition and productivity on the use of 'high powered' incentives by simply analyzing how they affect the share of output θ.

Existence of an equilibrium as the one just described can be guaranteed under the following sufficient conditions:

Assumption 5. ***(Symmetry)*** *The output space is such that* $\max\{Y_1\} = \max\{Y_j\} = \overline{y}$.

Assumption 6. ***(Convexity of $\mathcal{A}_0$)*** *Consider the known technology set $\mathcal{A}_0$ and define a function $f : \mathbb{R} \to \mathbb{R}$ as:*

$$f(x) = \min\{c | (F, c) \in \mathcal{A}_0 \text{ and } E_F[y_1 + y_2] = x\}. \tag{23}$$

The set $\mathcal{F}_{\mathcal{A}_0} = \{F \in \Delta(Y) \,|\, (F, c) \in \mathcal{A}_0\}$ is convex, and the function f is continuous, and its square root is a convex function.

Theorem 6. *If assumption 5 or assumption 6 hold then a Nash equilibrium in LRS contracts that satisfy individual limited liability exists.*

The proof is in Appendix A.2.

Assumption 5 imposes a type of symmetry across principals. It allows to prove existence of equilibrium using the potential approach of Theorem 2. The symmetry imposed on the

[17] The dependency of the contract in the maximum output (size) of the competing principal also breaks the anonymity that characterized the equilibrium described in Section 1.2, see Proposition 2. It also implies stronger conditions on what a principal needs to guarantee herself a positive payoff. In particular Assumption 3 (non-triviality) is not enough, a necessary condition for principal i to guarantee herself a positive payoff is that there exists an action $(F, c) \in \mathcal{A}_0$ such that $E_F[y_1 + y_2] - c > \overline{y}_j$.

[18] The payment of fees to the agent implies that the ex post payoffs of the principals can be negative. Yet, our results do not rely on the ability of principals to make unbounded payments to the agent. In the online appendix we augment the model by adding limited liability on the principal's side.

principals is actually very mild, since only the maximum output that can be produced is required to be the same across principals. This leaves unconstrained the rest of the output space, and the known actions of the agent. In particular the agent can be known to favor production for one of the principals, or one of the principals can have just extreme realizations of output (only high and low values of y_i in Y_i), while the other one can have intermediate values of production.

Assumption 6 imposes more structure over the known set of actions. This structure adds enough convexity to ensure that the principals' best responses are single valued. As mentioned before failure of convexity in the principals' best responses is prevalent in common agency games.

It is worthwhile mentioning that these conditions are only sufficient, we can also show existence of equilibrium in special cases. For instance when the cost function is linear, or when the agent is indifferent between actions. This latter case has been used, for example, in Bernheim and Whinston (1986a) to establish that an equilibrium of the common agency game exists and implements the efficient outcome.[19] Although the assumption is restrictive, it is well suited to describe situations such as auctions or lobbying, were the agent is expected not to have preferences over the actions.

We can now turn to the final result of this section. We show that under individual limited liability the efficiency result established in Theorem 4 is overturned. We do this by showing that the agent receives a higher share of total output when principals compete than when they collude. Or else, there exist a profitable deviation for one of the principals. The deviation consists in reducing her own share of output so that the agent receives the same share as under collusion. The critical step is that doing this increases expected output and reduces the fees that the principal has to pay, increasing the principal's own payoff.

Theorem 7. *Total expected surplus and total guaranteed surplus are higher under a Nash equilibrium in LRS contracts satisfying individual limited liability than under collusion.*

Proof. First note that Lemmas 1 and 2 still apply since fees do not play a role in the agent's decisions and are cancelled out across players. Then it is only left to show that the share of output accruing to the agent in a Nash equilibrium in LRS contracts is higher than in collusion.

Let w^N be a contract scheme in LRS contracts as the one in equation (21) characterized by shares $\left(\theta_1^N, \theta_2^N\right)$. The share of output going to the agent is: $\theta_A^N = 1 - \theta_1^N - \theta_2^N$. Under collusion the principals get a share θ^C of output and the agent a share $\theta_A^C = 1 - \theta^C$, see Theorem 3. The problem of the agent is then equivalent to that in (18). To simplify notation we define the following

[19] We reproduce Bernheim and Whinston (1986a) results under this condition in the examples of Section 1.2 of the online appendix.

objects:

$$\tilde{V}_A^N = \tilde{V}_A\left(\theta_A^N|\mathcal{A}_0\right) \qquad \tilde{V}_A^C = \tilde{V}_A\left(\theta_A^C|\mathcal{A}_0\right)$$

It will be also useful to define the value of principal i under LRS contracts as in (22), given his share (θ_i) and that of the agent (θ_A):

$$V_i\left(\theta_i, \theta_A\right) = \frac{\theta_i}{\theta_A}\tilde{V}_A\left(\theta_A\right) - \theta_i \bar{y}_j$$

Suppose that w^N is such that $\theta_A^N < \theta_A^C$, where θ_A^C is the highest share that the agent gets if principals were to collude. We will show that this leads to a contradiction. There are five cases to consider.

Case 1. Both principals can reduce their share of output so as to give the agent the same share of output as under collusion. This is:

$$\theta_1^N \geq \theta_A^C - \theta_A^N \qquad \text{and} \qquad \theta_2^N \geq \theta_A^C - \theta_A^N$$

Thus any principal can unilaterally deviate and induce the collusive outcome. Now, let $V_i\left(\theta_i^C, \theta_A^C\right) - V_i\left(\theta_i^N, \theta_A^N\right) \leq 0$ be the gain that principal i gets by unilaterally deviating to the collusion outcome, i.e. by reducing his share to $\theta_i^C = \theta_i^N - \left(\theta_A^C - \theta_A^N\right)$. Since w^N is a Nash equilibrium the gain must be non-positive. Letting $V_i\left(\theta_i^C, \theta_A^C\right) = V_i^C$ and $V_i\left(\theta_i^N, \theta_A^N\right) = V_i^N$ we can write

$$V_i^C - V_i^N = \left(\theta_A^C - \theta_A^N\right)\bar{y}_j + \frac{\theta_i^C}{\theta_A^C}\tilde{V}_A^C - \frac{\theta_i^N}{\theta_A^N}\tilde{V}_A^N$$

Summing across the two principals we get:

$$\begin{aligned}
0 &\geq \left(V_1^C - V_1^N\right) + \left(V_2^C - V_2^N\right) \\
&= \left(\theta_A^C - \theta_A^N\right)\left(\bar{y}_1 + \bar{y}_2\right) + \frac{\theta_1^C + \theta_2^C}{\theta_A^C}\tilde{V}_A^C - \frac{\theta_1^N + \theta_2^N}{\theta_A^N}\tilde{V}_A^N \\
&= \left(\theta_A^C - \theta_A^N\right)\left(\bar{y}_1 + \bar{y}_2\right) + \left(\frac{1-\theta_A^C}{\theta_A^C} - \frac{\theta_A^C - \theta_A^N}{\theta_A^C}\right)\tilde{V}_A^C - \frac{1-\theta_A^N}{\theta_A^N}\tilde{V}_A^N \\
&= \left(\theta_A^C - \theta_A^N\right)\left(\bar{y}_1 + \bar{y}_2 - \frac{\tilde{V}_A^C}{\theta_A^C}\right) + \frac{1-\theta_A^C}{\theta_A^C}\tilde{V}_A^C - \frac{1-\theta_A^N}{\theta_A^N}\tilde{V}_A^N
\end{aligned}$$

Where the third equality follows since

$$\begin{aligned}
\frac{\theta_1^C + \theta_2^C}{\theta_A^C} &= \frac{\theta_1^N - \left(\theta_A^C - \theta_A^N\right)}{\theta_A^C} + \frac{\theta_2^N - \left(\theta_A^C - \theta_A^N\right)}{\theta_A^C} \\
&= \frac{1 - \theta_A^N - 2\left(\theta_A^C - \theta_A^N\right)}{\theta_A^C} \\
&= \frac{1-\theta_A^C}{\theta_A^C} - \frac{\theta_A^C - \theta_A^N}{\theta_A^C}
\end{aligned}$$

Now note that

$$\frac{1-\theta_A^C}{\theta_A^C}\tilde{V}_A^C - \frac{1-\theta_A^N}{\theta_A^N}\tilde{V}_A^N \geq 0$$

by the fact that when the principals act collusively they maximize $\frac{1-\theta}{\theta}\tilde{V}_A(\theta|\mathcal{A}_0)$, see (17) in Theorem 3. Also note that

$$\bar{y}_1+\bar{y}_2-\frac{\tilde{V}_A^C}{\theta_A^C}=\min_{(F,c)\in\mathcal{A}_0}\left\{(\bar{y}_1+\bar{y}_2-E_F[y_1+y_2])+\frac{c}{\theta_A^C}\right\}>0$$

since the highest action possible that the agent can choose is to put full support on $(\bar{y}_1,\bar{y}_2)$, and by assumption 2 $c>0$ in $\mathcal{A}_0$. This leads to a contradiction since it violates the assumption that w^N is a Nash equilibrium contract scheme. At least one principal has a profitable deviation.

Case 2. Only one principal can unilaterally deviate to collusion, say principal i, and $\theta_j>0$.

$$\theta_i^N\geq\theta_A^C-\theta_A^N\qquad\text{and}\qquad 0<\theta_j^N<\theta_A^C-\theta_A^N$$

As in Case 1, it is still the case that

$$\left(V_i^C-V_i^N\right)+\left(V_j^C-V_j^N\right)>0$$

However to get a contradiction we must show that $\left(V_j^C-V_j^N\right)\leq 0$:

$$\begin{aligned}V_j^C-V_j^N&=\left(\theta_A^C-\theta_A^N\right)\bar{y}_i+\frac{\theta_j^C}{\theta_A^C}\tilde{V}_A^C-\frac{\theta_j^N}{\theta_A^N}\tilde{V}_A^N\\&=\left(\theta_A^C-\theta_A^N\right)\left(\bar{y}_i-\frac{\tilde{V}_A^C}{\theta_A^C}\right)+\theta_j^N\left(\frac{\tilde{V}_A^C}{\theta_A^C}-\frac{\tilde{V}_A^N}{\theta_A^N}\right)\\&=\left(\theta_A^C-\theta_A^N\right)\left(\bar{y}_i-\frac{\tilde{V}_A^N}{\theta_A^N}\right)+\left(\theta_A^C-\theta_A^N-\theta_j^N\right)\left(\frac{\tilde{V}_A^N}{\theta_A^N}-\frac{\tilde{V}_A^C}{\theta_A^C}\right)\leq 0\end{aligned}$$

The inequality follows since both terms are non-positive. $\left(\theta_A^C-\theta_A^N-\theta_j^N\right)>0$ by assumption and $\left(\bar{y}_i-\frac{\tilde{V}_A^N}{\theta_A^N}\right)\leq 0$ since it must be the case that $V_j^N\geq 0$ if w^N is a Nash equilibrium. Finally, $\left(\frac{\tilde{V}_A^N}{\theta_A^N}-\frac{\tilde{V}_A^C}{\theta_A^C}\right)\leq 0$, this follows from the proof of lemma 2 since $\theta_A^N\leq\theta_A^C$. Hence it must be that $V_i^C-V_i^N\geq 0$, proving that principal i has a profitable deviation. Then w^N is not an equilibrium.

Case 3. None of the principals can unilaterally deviate to induce the collusive outcome, and $\theta_1,\theta_2>0$. Then $\theta_1^N<\theta_A^C-\theta_A^N$ and $\theta_2^N<\theta_A^C-\theta_A^N$. As shown in Case 2 this implies that: $V_1^C-V_1^N\leq 0$ and $V_2^C-V_2^N\leq 0$, but this leads to a contradiction since, as in Case 1, it still holds that: $\left(V_i^C-V_i^N\right)+\left(V_j^C-V_j^N\right)>0$.

Case 4. Finally we consider the case where one principal, say j, has no share of the surplus, so that: $\theta_j^N=0$. This case can be shown directly. Recall the problem of principal i given $\theta_j=0$:

$$\theta_A^N=\underset{\theta_A\in[0,1]}{\operatorname{argmax}}\left\{\frac{1-\theta_A}{\theta_A}\tilde{V}_A(\theta_A)-(1-\theta_A)\bar{y}_j\right\}$$

Note that for all $\theta_A < \theta_A^C$:

$$\frac{1-\theta_A}{\theta_A}\tilde{V}_A(\theta_A) - (1-\theta_A)\overline{y}_j \leq \frac{1-\theta_A^C}{\theta_A^C}\tilde{V}_A\left(\theta_A^C\right) - \left(1-\theta_A^C\right)\overline{y}_j$$

Since $\frac{1-\theta_A}{\theta_A}\tilde{V}_A(\theta_A) \leq \frac{1-\theta_A^C}{\theta_A^C}\tilde{V}_A\left(\theta_A^C\right)$ from the collusion problem (17), and $-(1-\theta_A)\overline{y}_j \leq -\left(1-\theta_A^C\right)\overline{y}_j$ by assumption. This implies that $\theta_A^N \geq \theta_A^C$.

Case 5. Finally we consider the case where both principals have no share of the surplus in equilibrium, so that $\theta_1^N = \theta_2^N = 0$. This cannot be since it would imply that $\theta_A^N = 1$, contradicting $\theta_A^C > \theta_A^N$.

□

The crucial element that overturns the efficiency result is that individual limited liability forces the principals to internalize the externality that they impose on the other principals. The fees $\left(\theta_i\overline{y}_j\right)$ force each principal to lower the share of total output that she claims in equilibrium implying a higher share for the agent and a more efficient outcome under competition than under collusion.

3 Robust Taxation of multinationals

We now show how the setup developed in Sections 1 and 2 can be applied to study the problem of taxing multinational companies. There is a big debate among tax policy experts and lawmakers on how to reform the corporate income tax with a particular focus on foreign profits. The debate in the United States has centered on whether to adopt a territorial approach–taxing only the profits generated in the U.S.–or a worldwide approach–taxing all profits, foreign and domestic, the same.

The need for tax systems to be robust to profit shifting strategies is evident. Tax reforms are slow, complicated and expensive processes, hard to adapt to changes in the strategies used by firms. Our common agency approach characterizes the main features of a robust tax system for multinationals. We show that a worldwide tax with a deduction paid for taxes in the foreign countries is indeed robust. This is the tax system proposed in the Bipartisan Tax Fairness and Simplification Act of 2011 by Senators Wyden and Coats (Senate Bill 727, 2011).

The theoretical literature on taxation of multinational corporations has primarily focused on the issue of how taxes affect multinational corporations who allocate their business operations and capital abroad (Feldstein and Hartman, 1979). The main concern of that literature is to achieve neutrality in investment allocation, and they abstract from informational asymmetries. However, today's biggest multinational corporations (like Google and Apple) rely heavily on intangible capital (patents, brands, etc.), which can easily shift ownership to a subsidiary in a tax haven without affecting the firm's productivity. These transfer pricing practices have allowed firms like Microsoft and Google to pay an overall tax rate of less than 3% on non-U.S. profits (Zucman, 2014).

The literature has addressed profit shifting by multinationals in the context of an adverse selection problem–see Bond and Gresik (1996) and Olsen and Osmundsen (2001). They use the revelation principle to deal with profit shifting strategies. As noted in Martimort (2006) their solution is highly sensitive to the information structure of the problem, while our focus is on solutions that are robust to potential misspecification of the environment.

We develop a similar problem to the common agency problem in Section 1. There are two countries $i \in \{1, 2\}$ and a multinational firm denoted by A. Let π_i be the firm's profit in country i. The set of possible profits that can be declared in country i is $\Pi_i \subset \mathbb{R}$ with $\min \Pi_i = 0$ and $\max \Pi_i = \bar{\pi}_i$. Also $\Pi = \Pi_1 \times \Pi_2$. The firm's actions are then distributions (F) over the profits in Π, and a cost (c) associated with each distribution.[20] The firm's action set $(\mathcal{A})$ is then composed by pairs $(F, c) \in \Delta(\Pi) \times \mathbb{R}_+$.

Each country's government chooses a tax function to maximize their guaranteed corporate tax revenue when they only know a subset $\mathcal{A}_0 \subset \mathcal{A}$, all assumptions on $\mathcal{A}$ and $\mathcal{A}_0$ are as in Section 1. The tax function for country i is a continuous function $t_i : \Pi \to \mathbb{R}$.

We consider two different restrictions over the range of the taxes which are equivalent to the two versions of limited liability imposed in the common agency game. We refer to them as weak and strong enforceability:

Weak Enforceability: Countries have weak enforceability if they can only tax up to the amount of profits declared in their respective territories. This implies: $t_i(\pi_1, \pi_2) \leq \pi_i$.

For small countries that have a subsidiary of a big multinational firm this is a reasonable restriction[21].

Strong Enforceability: Countries have strong enforceability if they can collect taxes on all profits generated by the firm. This implies: $t_1(\pi_1, \pi_2) + t_2(\pi_1, \pi_2) \leq \pi_1 + \pi_2$.

For large countries like the United States where the multinational corporation has most of its activity this restriction is more reasonable.

[20]The cost can be interpreted as an economic cost (after accounting costs are deducted) of engaging in transfer pricing between the firm's subsidiaries in each country. Alternatively, the cost can be interpreted as unobservable effort from the firm's manager as in Laffont and Tirole (1986).

[21]Weak enforceability is equivalent to the strong version of limited liability assumption imposed in the main section . It does not amount to a territorial approach to taxation. A territorial approach would amount to restricting the domain of the taxes, so that $t_i(\pi_1, \pi_2) = t_i(\pi_i)$.

The firm's problem is to maximize after tax profits:

$$A^{\star}\left(t|\mathcal{A}\right) = \underset{(F,c)\in\mathcal{A}}{\operatorname{argmax}} E_F\left[\left(\pi_1 - t_1\left(\pi_1,\pi_2\right)\right) + \left(\pi_2 - t_2\left(\pi_1,\pi_2\right)\right)\right] - c \tag{24}$$

where $t = (t_1, t_2)$ is a tax scheme.

The payoff of government i is given by:

$$R_i\left(t_1,t_2\right) = \inf_{\mathcal{A}\supseteq\mathcal{A}_0}\left\{\min_{(F,c)\in A^{\star}(t|\mathcal{A})} E_F\left[\rho_i ATP + t_i\left(\pi_1,\pi_2\right)\right]\right\} \tag{25}$$

where ATP $= \left(\pi_1 - t_1\left(\pi_1,\pi_2\right)\right) + \left(\pi_2 - t_2\left(\pi_1,\pi_2\right)\right)$ is the after tax profits of the firm.

Each government's utility is a weighted sum of net tax revenue and the profits of the multinational. $\rho_i \in [0,1]$ is the weight each country puts on the profits of the multinational company. If $0 < \rho_i < 1$ country i cares about raising some distortionary taxes, so that the shadow value of a tax dollar exceeds that of a unit of factor income. See Bond and Gresik (1996) for a justification of that objective function for the governments. Also note that when $\rho_1 = \rho_2 = 0$ the problem is isomorphic to the common agency problem considered in the previous sections. Similarly to Theorem 1 we can show that given the tax system of country j, country i's best response contains the following tax:[22]

Worldwide Tax: A tax function t_i is a worldwide (flat) tax rate if the firm's global profits are taxed at a constant rate $\frac{\alpha_i}{1-\alpha_i\rho_i}$, allowing for the full deduction of taxes payed to country j, and a potential tax incentive (in the form of a lump sum subsidy). That is, for some $\alpha_i \in (0,1]$ and $k_i \in \mathbb{R}$:[23]

$$t_1\left(\pi_1,\pi_2\right) = \left(1 - \frac{\alpha_i}{1-\alpha_i\rho_i}\right)\left(\pi_1 + \pi_2 - t_2\left(\pi_1,\pi_2\right)\right) + k_i \tag{26}$$

The tax proposed by Senators Wyden and Coats has this form. It proposes a flat tax rate for all profits independently of country of origin. While the model does not provide a literal description of reality, it provides a robustness property for the Wyden-Coats tax reform, that a territorial tax system does not have. This property has been informally articulated among tax policy experts (Hungerford, 2014), and thus the model provides a rigorous treatment of the policy debate. Note that a worldwide tax is not just an equilibrium outcome of the game. A worldwide tax is a best response for country i to any arbitrary tax system of country j.

Interestingly, the worldwide tax has the same form as the taxes found by Feldstein and Hartman (1979). Unlike us they have a complete information setup and restrict attention to

[22]Formal proofs are provided in the online appendix.

[23]Note that the enforceability regime, i.e. limited liability only changes the constant k_i, it does not change the structure of the tax system.

linear tax functions, and their "full taxation after deduction" result rests on concerns on the optimal allocation of capital between countries.

Another important issue is that of the effects of tax competition and the welfare implications of a tax treaty between the countries. As shown in Section 2, competition between countries - the common agency setup- would lead to a lower (higher) overall tax rate on the multinational, relative to cooperation between countries through a tax treaty - the collusion setup - when countries have weak (strong) enforceability and it would lead to a higher (lower) overall tax rate on the multinationals.

4 Extensions

4.1 Private common agency

In this section we consider the case where principals are restricted to contract only on their own output.[24] The study of private common agency is appealing when considering certain applications of common agency. For instance, when home buyers and sellers hire a realtor, they do not explicitly reward him for not working for other home buyers and sellers. Celebrities or professional athletes usually simply give their agents a share of their earnings, regardless of the earnings of others that are also represented by the same agent. The equilibrium contract that arises in this setting provides a rationale for this behavior.

The essential feature of the LRS contracts in Section 1 was that they allowed each principal to tie their payoff to the payoff of the agent in an affine way. The principals did so by partially offsetting the contract given to the agent by their competitors. In the private common agency framework such contracts are not allowed.

A contract now is a continuous function $w_i^r : Y_i \to \mathbb{R}_+$. The best response for a principal is to give the agent a share of her output and offset the payment of the other principal by charging the agent the maximum value of the other principal's contract. However, limited liability does not allow the principal to charge the agent, so the best response is a **linear** contract; she gives the agent a share of her output.

We formalize this in the following theorem. All the proofs of this section can be found in the online appendix.

Theorem 8. *For any contract w_j^r, the best response of principal i contains a linear contract.*

[24] This can be due to their inability to observe the other principal's output, or because of regulation that prohibits contracting on output other than your own.

i.e. there exists $\theta_i \in [0,1]$ *such that:*

$$w_i^r(y_i) = (1-\theta_i)\, y_i \qquad \text{and} \qquad w_i^r \in BR_i\left(w_j^r\right)$$

If $\mathcal{A}_0$ *has the full support property then, for any* w_j^r*, if* $w_i^r \in BR_i\left(w_j^r\right)$ *then* $w_i^r(y_i) = (1-\theta_i)\, y_i$ *for some* $\theta_i \in [0,1]$*. All best responses are linear contracts, or principal* i *cannot guarantee a positive payoff.*

This does not imply that the problem is reduced to a simple principal agent problem like the one studied in Carroll (2015). Because of the interaction between principals, the share of output that each principal offers in equilibrium is affected by the need to compete for the agent's services.

We can now further characterize the equilibrium. When both principals play linear contracts as in (8) the best response of principal i is:

$$\mathrm{BR}_i(\theta_j) = \underset{\theta_i \in [0,1]}{\operatorname{argmax}} \left[\max_{(F,c)\in\mathcal{A}_0} \left\{ \frac{\theta_i}{1-\theta_i} E_F\left[(1-\theta_i)\, y_i - (1-\theta_j)\left(\overline{y}_j - y_j\right) - c\right] \right\} \right] \tag{27}$$

In an interior equilibrium (i.e. $\theta_i \in (0,1)$) we have that:

$$1-\theta_i = \frac{c + (1-\theta_j)\left(\overline{y}_j - E_F[y_j]\right)}{(1-\theta_i)\, E_F[y_i]} \tag{28}$$

where $(F,c) \in A^\star((\theta_i,\theta_j)\,|\mathcal{A}_0)$ [25]

The numerator in (28) is the opportunity cost of the agent (as perceived by the principal) of taking action (F,c). This cost is formed by the accounting cost of the action (c), plus the expected forgone earnings from the other principal. The share of output that a principal gives to the agent is then equal to the ratio between this cost and the expected payment that the agent receives from the principal.

The principal increases the share of output given to the agent as the forgone earnings from the other principal increase. This resembles the second term in the equilibrium contract (21) found in Proposition 3. When contracts were not restricted, each principal was able to compensate the agent for the forgone earnings from the other principal. Under the restricted contracting domain this explicit form of competition is not possible. Instead, principals implicitly compete with each other by offering higher shares of their own output to the agent.

[25] We slightly abuse the notation by writing (θ_i, θ_j) instead of (w_i, w_j).

4.2 Multiple Principals

The model considered in Section 1 can be extended to multiple principals. Our main results are preserved in this case. Below we summarize them and we leave all details in the online appendix. We denote the number of principals by N, and we define the vector of competing contracts as $w_{-i}(y) = (w_1(y), \ldots, w_{i-1}(y), w_{i+1}(y), \ldots, w_N(y))$. The definition of a LRS contract is the natural extension to that in Section 1:

Linear Revenue Sharing (LRS) Contracts: A contract w_i is a LRS contract for principal i if, given a vector of competing contracts w_{-i}, it ties the principal's ex-post payoff linearly to the total revenue of the agent. That is, for some $\alpha \in (0, 1]$ and $k \in \mathbb{R}$:

$$y_i - w_i(y) = \frac{(1-\alpha)}{\alpha}\left(\sum_{n=1}^{N} w_n(y)\right) - k \tag{29}$$

Just as before we can show that that there is always a LRS contract in each principal's best response. And if all principals play LRS contracts then we can characterize them as in Propositions 1 and 3 depending on the limited liability restrictions:

$$w_i(y) = (1-\theta_i) y_i - \theta_i \sum_{j \neq i} y_j - k_i \tag{30}$$

$$w_i(y) = (1-\theta_i) y_i + \theta_i \sum_{j \neq i} \left(\overline{y}_j - y_j\right) \tag{31}$$

where $\sum_{i=1}^{N} k_i = 0$ when limited liability is placed over the aggregate payment to the agent. When limited liability is imposed over each contract, so that $w_i(y) \geq 0$ for all $y \in Y$ and all $i \in \{1, \ldots, N\}$, the principal's best response functions are more sensitive to competition than before. In fact, we show in the online appendix that, in order to have $\theta_i > 0$ in equilibrium, the principal must expect the agent to choose an action (F, c) such that:

$$E_F[y_i] > \sum_{j \neq i} E_F\left[\overline{y}_j - y_j\right]$$

This condition is stronger than non-triviality and increasingly difficult to satisfy as the number of principals increases. Intuitively the LRS contract is compensating the agent for her forgone earnings from other principals, this compensation requires principal i's payoff to be large enough in order to guarantee a positive payoff.

4.3 No limited liability on the agent

If we dispense with the limited liability on the agent and instead impose a participation constraint on the agent, guaranteeing the agent a given expected payoff (normalized here to 0), then we get the Bernheim and Whinston (1986b) solution where each principal sells "her firm" to the agent.

Suppose that $\mathcal{A}_0$ is common knowledge among the principals. Also let s_0 equal the total surplus under $\mathcal{A}_0$, that is:

$$s_0 = \max_{(F,c)\in\mathcal{A}_0} \{E_F [y_1 + y_2] - c\}$$

Now consider the best response of principal i to the strategy of principal j that sells the firm to the agent for a price $s_j \leq s_0$ (i.e. $w_j (y) = y_j - s_j$). Principal i cannot be guaranteed a payoff higher than $s_0 - s_j$. Otherwise the participation constraint of the agent would be violated. This payoff is achieved if principal i offers $w_i (y) = y_i - (s_0 - s_j)$. Thus selling the firm is a best response of principal i. There is an indeterminacy in how the total surplus is divided between the principals as is the case in Bernheim and Whinston (1986b).

Furthermore it is obvious the same equilibria are also valid under private common agency, since optimal contracts do not depend on the other principal's output.

5 Conclusion

Taking a robust contracting approach provides a crisp characterization of equilibrium strategies and payoffs in the complicated problem of common agency. The central issue in the literature of how competition among principals affects the efficient provision of incentives can be easily pinned down to one component, namely the share of total output that the agent receives in equilibrium. We show that when principals can make side payments (through the agent) to each other a free-riding problem appears. Free riding leads to lower incentives given to the agent, compared to the collusive outcome. When such side payments are not possible because of limited liability, then principals are forced to internalize their externality, which leads to the competitive outcome being more efficient than the collusive outcome.

We also show how our model provides a rigorous framework for analyzing taxation of multinationals. In the policy debate on whether the United States should adopt a territorial or worldwide approach to taxation, our model establishes some desirable properties that the worldwide system has, but the territorial one lacks. We show that a worldwide flat tax is a robust solution to the problem of the erosion of the corporate tax base.

A Proofs

A.1 Best Response - 2 Principals

First consider the implications of the common limited liability assumption. Under this assumption contracts have to guarantee that $w_1(y) + w_2(y) \geq 0$. From the point of view of an individual principal this allows to charge the agent up to the amount that the opposing party is paying. From the point of view of the equilibrium this allows for transfers between principals (through the agent), as in Bernheim and Whinston (1986a,b). The problem of a principal can then be thought of in two steps: first undoing the payments of other principals, and then offering the agent an aggregate contract satisfying limited liability. We call this aggregate contract $\tilde{w}_i$. Then the ex-post payoff of principal i is: $y_i + w_j(y) - \tilde{w}_i(y)$. Principal i's actual contract is of course: $w_i(y) = \tilde{w}_i(y) - w_j(y)$.

One option that is always available to a principal when facing a competing contract is to undo all payments and offer the agent the "zero contract", i.e. $\tilde{w}_i(y) = 0$. Under assumption (2) the agent's unique optimal action, under any technology set, given the zero contract is to choose inaction. This allows us to define a lower bound on the payoff of the principal. We call a contract scheme **eligible for principal i,** if it guarantees the principal a payoff higher than the zero (aggregate) contract[26]. Unless specified an eligible contract scheme will always be for principal i:

$$V_i(w) > w_j(0,0) \tag{32}$$

The following proposition relates the expected payments to the agent under any technology with its value under $\mathcal{A}_0$.

Proposition 4. *Let $(F, c) \in A^{\star}(w|\mathcal{A})$. For $\mathcal{A} \supseteq \mathcal{A}_0$, it holds that:*

$$E_F[w_1(y) + w_2(y)] \geq V_A(w|\mathcal{A}_0)$$

Moreover, if $(F, c) \in A^{\star}(w|\mathcal{A})$ then $F \in \mathcal{F}$ where:

$$\mathcal{F} = \{F \in \Delta(Y) \,|E_F[w_1(y) + w_2(y)] \geq V_A(w|\mathcal{A}_0)\}$$

[26] In the case of a single principal dealing with a single agent this is similar to allowing the output of the principal $(\tilde{y})$ to go from $\underline{\tilde{y}} = \min_{y \in Y}\{y_i + w_j(y)\}$ to $\bar{\tilde{y}} = \max_{y \in Y}\{y_i + w_j(y)\}$. In that case a contract is eligible if it gives a guaranteed payoff above the minimum possible output $(\underline{\tilde{y}})$.

Proof. To see the first inequality let $(F,c) \in A^{\star}(w|\mathcal{A})$ for $\mathcal{A} \supseteq \mathcal{A}_0$:

$$E_F[w_1(y)+w_2(y)] \geq E_F[w_1(y)+w_2(y)] - c \geq V_A(w|\mathcal{A}) \geq V_A(w|\mathcal{A}_0)$$

Then $F \in \mathcal{F}$.

$\square$

Lemma 4 characterizes the principal's payoff for a given contract scheme using the set $\mathcal{F}$ (defined in the proposition above). Note that $\mathcal{F}$ only depends on the contract scheme and the known set of actions $\mathcal{A}_0$. In this way we replace the complexity of the definition of $V_i(w)$ in (2) with an object that depends only on known elements.

Lemma 4. *Let w be an eligible contract scheme for principal i. Then $V_i(w) = \min_{F\in\mathcal{F}} E_F[y_i - w_i(y)]$. Moreover if $F \in \underset{F\in\mathcal{F}}{argmin} E_F[y_i - w_i(y)]$ then $E_F[w_1(y)+w_2(y)] = V_A(w|\mathcal{A}_0)$.*

Proof. We first establish the first claim: Let w be an eligible contract scheme then: $V_i(w) = \min_{F\in\mathcal{F}} E_F[y_i - w_i(y)]$.

It must be that: $V_i(w) \geq \min_{F\in\mathcal{F}} E_F[y_i - w_i(y)]$. Using the definition of $V_i(w)$:

$$V_i(w) = \inf_{\mathcal{A}\supseteq\mathcal{A}_0} \min_{(F,c)\in A^{\star}(w|\mathcal{A})} E_F[y_i - w_i(y)] \geq \min_{F\in\mathcal{F}} E_F[y_i - w_i(y)]$$

Where the inequality follows because if $(F,c) \in A^{\star}(w|\mathcal{A})$ then $F \in \mathcal{F}$.

To prove equality suppose that $V_i(w) > \min_{F\in\mathcal{F}} E_F[y_i - w_i(y)]$, and let $F' \in \underset{F\in\mathcal{F}}{\text{argmin}} E_F[y_i - w_i(y)]$. Note that $E_{F'}[w_1(y)+w_2(y)] \geq V_A(w|\mathcal{A}_0)$ from Proposition 4. There are two options:

1. F' does not place full support on the values of y that maximize $w_1 + w_2$.

   Let $\hat{y} \in \text{argmax}\{w_1(y)+w_2(y)\}$, and $\hat{F} = \delta_{\hat{y}}$ be a distribution with full mass on $\hat{y}$. Let $\epsilon \in [0,1]$ and $F_\epsilon = (1-\epsilon)F' + \epsilon\hat{F}$.

   Note then that for all ϵ there exists a $\xi_\epsilon > 0$ such that: $E_{F_\epsilon}[w_1(y)+w_2(y)] - \xi_\epsilon > V_A(w|\mathcal{A}_0)$.

   Define and $\mathcal{A}_\epsilon = \mathcal{A}_0 \cup \{(F_\epsilon, \xi_\epsilon)\}$. It follows that the unique optimal action of the agent in $\mathcal{A}_\xi$ is $(F_\epsilon, \xi_\epsilon)$. Then:

   $$V_i(w) \leq V_i(w|\mathcal{A}_\epsilon) = E_{F_\epsilon}[y_i - w_i(y)] = (1-\epsilon)E_{F'}[y_i - w_i] + \epsilon E_{\hat{F}}[y_i - w_i]$$

   This condition holds for all $\epsilon > 0$. Letting $\epsilon \to 0$ we arrive at a contradiction:

   $$V_i(w) \leq E_{F'}[y_i - w_i] = \underset{F\in\mathcal{F}}{\text{argmin}} E_F[y_i - w_i]$$

2. F' places full support on the values of y that maximize $w_1 + w_2$. There are still two possible cases:

(a) $E_{F'}[w_1 + w_2] > V_A(w|\mathcal{A}_0)$. Then there exists $\xi > 0$ and a technology $\mathcal{A}^{'} = \mathcal{A}_0 \cup \left\{\left(F^{'}, \xi\right)\right\}$ such that $\left(F^{'}, \xi\right)$ is the unique optimal action for the agent in $\mathcal{A}^{'}$. Then we arrive at a contradiction:

$$V_i(w) \leq V_i\left(w|\mathcal{A}^{'}\right) = E_{F'}[y_i - w_i] = \underset{F \in \mathcal{F}}{\operatorname{argmin}} E_F[y_i - w_i]$$

(b) $E_F[w_1 + w_2] = V_A(w|\mathcal{A}_0)$. This implies $V_A(w|\mathcal{A}_0) = \max_{y \in Y}\{w_1 + w_2\}$ which can only be satisfied if $F^{'}$ is available in $\mathcal{A}_0$ at zero cost. By assumption 2 this implies that $F = \delta_{(0,0)}$ and that $w_1(0,0) + w_2(0,0) = \max_{y \in Y}\{w_1(y) + w_2(y)\}$. In this case the unique optimal action for the agent under any technology is $(\delta_0, 0)$, so the value of the principal is $V_i(w) = -w_i(0,0) \leq w_j(0,0)$, where the inequality follows from limited liability. This contradicts eligibility.

Now we establish the second claim: Let w be an eligible contract scheme for principal i. If $F \in \underset{F \in \mathcal{F}}{\operatorname{argmin}} E_F[y_i - w_i]$ then $E_F[w_1 + w_2] = V_A(w|\mathcal{A}_0)$.

To prove this, let $F^{'} \in \underset{F \in \mathcal{F}}{\operatorname{argmin}} E_F[y_i - w_i(y)]$ and suppose for a contradiction that $E_F[w_1(y) + w_2(y)] > V_A(w|\mathcal{A}_0)$.

Let $\epsilon \in [0,1]$ and define $F_\epsilon = (1-\epsilon)F^{'} + \epsilon\delta_0$.

For low enough ϵ it holds that: $E_{F_\epsilon}[w_1(y) + w_2(y)] > V_A(w|\mathcal{A}_0)$. Then there exists $\xi_\epsilon > 0$ such that $\{(F_\epsilon, \xi_\epsilon)\} = A^\star(w|\mathcal{A}_\epsilon)$ where $\mathcal{A}_\epsilon = \mathcal{A}_0 \cup \{(F_\epsilon, \xi_\epsilon)\}$. The payoff to the principal is then:

$$\begin{aligned}
V_i(w|\mathcal{A}_\epsilon) &= (1-\epsilon)E_F[y_i - w_i(y)] + \epsilon(-w_i(0,0)) \\
&= (1-\epsilon)V_i(w) + \epsilon(w_j(0,0) - (w_1(0,0) + w_2(0,0))) \\
&= V_i(w) - \epsilon(V_i(w) - w_j(0,0) + (w_1(0,0) + w_2(0,0))) \\
&\leq V_i(w) - \epsilon(V_i(w) - w_j(0,0)) \\
&< V_i(w)
\end{aligned}$$

This gives a contradiction.

□

Given the known action set $\mathcal{A}_0$, the next lemma links the principal's guaranteed payoff to the agent's payoff in an affine way. This link allows the principal to increase her own guaranteed payoff by controlling the payoff given to the agent. Lemma 5 also offers a relation between any contract w_i, the outcome y_i and the contract w_j offered by the other principal.

The affine link between the agent's payoff and the principal's payoff is a crucial element in providing incentives for the agent. Given the partial knowledge over the agent's set of actions the principals' optimal strategy is to tie their payoff to that of the agent, thus aligning the agent's objectives with their own. This is the same mechanism at the heart of the optimal contracts in Hurwicz and Shapiro (1978) and Carroll (2015), and will be crucial in establishing the optimality of affine (LRS) contracts in the setting we develop.

Lemma 5. *Let w be an eligible contract scheme. There exits k, λ with $\lambda > 0$ such that for all $y \in Y$:*

$$\begin{aligned} w_i(y) &\leq \frac{1}{1+\lambda} y_i - \frac{\lambda}{1+\lambda} w_j(y) - \frac{1}{1+\lambda} k && (33) \\ V_i(w) &= k + \lambda V_A(w|\mathcal{A}_0) && (34) \end{aligned}$$

Proof. For the proof define the following two sets:

1. Let $S \subseteq \mathbb{R}^2$ be the convex hull of all points $(w_1(y) + w_2(y), y_i - w_i(y))$ for $y \in Y$.
2. Let $T \subseteq \mathbb{R}^2$ be the set of all pairs (u, v) such that $u > V_A(w|\mathcal{A}_0)$ and $v < V_i(w)$.

We first establish that $S \cap T = \emptyset$. Let $(u, v) \in T$ then let $F \in \underset{F \in \mathcal{F}}{\operatorname{argmin}} E_F[y_i - w_i(y)]$, by definition of T and Lemma (4):

$$\begin{aligned} u &> V_A(w|\mathcal{A}_0) = E_F[w_1(y) + w_2(y)] \\ v &< V_i(w) = E_F[y_i - w_i(y)] \end{aligned}$$

now, suppose for a contradiction that $(u, v) \in S$, then there exists $F' \in \Delta(Y)$ such that:

$$u = E_{F'}[w_1(y) + w_2(y)] \qquad \text{and} \qquad v = E_{F'}[y_i - w_i(y)]$$

Note that F' guarantees a payoff to the agent larger than $V_A(w|\mathcal{A}_0)$ so $F' \in \mathcal{F}$ but:

$$E_F[y_i - w_i(y)] > E_{F'}[y_i - w_i(y)]$$

which contradicts minimality of F. Then $S \cap T = \emptyset$

Second, since $S \cap T = \emptyset$ we can apply the separating hyperplane theorem which implies that there exist constants (k, λ, μ) such that $(\lambda, \mu) \neq (0, 0)$ and:

$$\begin{aligned} k + \lambda u - \mu v \leq 0 \qquad (u, v) \in S && (35) \\ k + \lambda u - \mu v \geq 0 \qquad (u, v) \in T && (36) \end{aligned}$$

Let $F^\star \in \underset{F \in \mathcal{F}}{\operatorname{argmin}} E_F[y_i - w_i(y)]$. Note that the pair $(E_{F^\star}[w_1(y) + w_2(y)], E_{F^\star}[y_i - w_i(y)])$ lies in the closures of both S and T. Then:

$$k + \lambda E_{F^\star}[w_1(y) + w_2(y)] - \mu E_{F^\star}[y_i - w_i(y)] = 0 \tag{37}$$

It is left to show that $\lambda, \mu > 0$.

Note that $(u, v) \in T$ admits u arbitrarily high and v arbitrarily low. So for (36) to hold it must be that $\lambda \geq 0$ and $\mu \geq 0$. There are then two cases to rule out:

1. Suppose $\mu = 0$, then it must be that $\lambda > 0$ (since $(\lambda, \mu) \neq (0, 0)$). From (35) and (36)

$$u \leq -\frac{k}{\lambda} \quad (u, v) \in S \qquad \text{and} \qquad u \geq -\frac{k}{\lambda} \quad (u, v) \in T$$

So $\max_{y\in Y}\left[w_1(y)+w_2(y)\right]=\max_{u\in S} u \le -\frac{k}{\lambda} \le \inf_{u\in T} u = V_A(w|\mathcal{A}_0)$. Which implies:

$$\max_{y\in Y}\left[w_1(y)+w_2(y)\right]=V_A(w|\mathcal{A}_0)$$

This can only be satisfied if the agent takes an action with zero cost. By assumption 2 this implies that $F=\delta_{(0,0)}$ and that $w_1(0,0)+w_2(0,0)=\max_{y\in Y}\{w_1(y)+w_2(y)\}$. In this case the unique optimal action for the agent under any technology is $(\delta_0,0)$, so the value of the principal is $V_i(w)=-w_i(0,0)\le w_j(0,0)$, where the inequality follows from limited liability. This contradicts eligibility. Then $\mu>0$.

2. Suppose $\lambda=0$, then it must be that $\mu>0$ (since $(\lambda,\mu)\neq(0,0)$). From (35) and (36)

$$v\ge\frac{k}{\mu}\quad (u,v)\in S \qquad \text{and} \qquad v\le\frac{k}{\mu}\quad (u,v)\in T$$

So $\min_{y\in Y}\left[y_i-w_i(y)\right]=\min_{v\in S} v\ge\frac{k}{\mu}\ge\sup_{v\in T} v=V_i(w)$, then:

$$V_i(w)\le\min_{y\in Y}\left[y_i-w_i(y)\right]\le\min_{y\in Y}\left[y_i+w_j(y)\right]\le w_j(0,0)$$

which violates eligibility (the second inequality follows from limited liability). So $\lambda>0$.

Note that since λ and μ are greater than zero μ can be normalized to 1, giving from (35):

$$k+\lambda\left(w_i(y)+w_j(y)\right)-\left(y_i-w_i(y)\right)\le 0$$

And from (37):

$$V_i(w)=k+\lambda V_A(w|\mathcal{A}_0)$$

□

The following two lemmas (6 and 7) use the relation between the principals' contracts derived in Lemma 5 to construct an alternative contract that dominates the original one, in the sense that it weakly increases principal i's guaranteed payoff. Since the relation obtained in (33) is affine in output and the other principal's contract, the alternative contract constructed below will inherit that form. These contracts form the LRS contracts defined in (6).

Lemma 6. *Let $w=(w_i,w_j)$ be an eligible contract scheme that satisfies limited liability and $w_1(0,0)+w_2(0,0)<\max_{y\in Y}\{w_1(y)+w_2(y)\}$. Then there exists $\lambda>0$ and k such that the contract*

$$w_i^{'}(y)=\frac{1}{1+\lambda}y_i-\frac{\lambda}{1+\lambda}w_j(y)-\frac{1}{1+\lambda}k \tag{38}$$

satisfies $V_i\left(w_i^{'},w_j\right)\ge V_i(w)$, moreover it is also eligible and satisfies limited liability.

Proof. From Lemma 5 w_i satisfies equations (33) and (34). Clearly $w_i^{'}$ satisfies (33) as an equality, rearrange it as:

$$\left(y_i - w_i^{'}(y)\right) = k + \lambda\left(w_i^{'}(y) + w_j(y)\right)$$

then let $(F, c) \in A^{\star}\left(w_i^{'}, w_j | \mathcal{A}\right)$ for any $\mathcal{A} \supseteq \mathcal{A}_0$ and taking expectations one gets:

$$E_F\left[y_i - w_i^{'}(y)\right] \geq k + \lambda V_A\left(\left(w_i^{'}, w_j\right) | \mathcal{A}_0\right) \tag{39}$$

This applies to any (F, c) under any technology, so this guarantees a payoff for principal i.

Note that $w_i^{'}(y) \geq w_i(y)$ for all $y \in Y$ so the agent is always at least as well off under $w_i^{'}$. Moreover, $w_i^{'}$ satisfies limited liability. Then from equations (34) and (39):

$$E_F\left[y_i - w_i^{'}(y)\right] \geq k + \lambda V_A(w | \mathcal{A}_0) = V_i(w)$$

Since this holds for all $(F, c) \in A^{\star}\left(w_i^{'}, w_j | \mathcal{A}\right)$, by Lemma 4:

$$V_i\left(\left(w_i^{'}, w_j\right) | \mathcal{A}\right) = \min_{F \in A^{\star}\left(w_i^{'}, w_j | \mathcal{A}\right)} E_F\left[y_i - w_i^{'}(y)\right] \geq V_i(w)$$

Then $V_i(w)$ is a lower bound for $V_i\left(\left(w_i^{'}, w_j\right) | \mathcal{A}\right)$ under arbitrary $\mathcal{A} \supseteq \mathcal{A}_0$. Thus $V_i\left(w_i^{'}, w_j\right) \geq V_i(w)$ by definition. Finally since w is an eligible contract scheme, so is $\left(w_i^{'}, w_j\right)$.

□

Lemma 7. *Let (w_i, w_j) be a contract scheme satisfying limited liability strictly:*

$$\min_{y \in Y}\{w_i(y) + w_j(y)\} = \beta > 0$$

The alternative contract $w_i^{'}(y) = w_i(y) - \beta$ outperforms w_i for principal i:

$$V_i\left(w_i^{'}, w_j\right) > V_i(w_i, w_j)$$

Proof. Note that by limited liability $\min_{y \in Y}\{w_i(y) + w_j(y)\} = \beta > 0$. Let $w_i^{'}(y) = w_i(y) - \beta$, this contract satisfies limited liability with equality: $w_i^{'}(y) + w_j(y) = 0$. Note that $A^{\star}\left(\left(w_i^{'}, w_j\right) | \mathcal{A}\right) = A^{\star}((w_i, w_j) | \mathcal{A})$ for all $\mathcal{A} \supseteq \mathcal{A}_0$. This implies $V_i\left(w_i^{'}, w_j\right) = V_i(w_i, w_j) + \beta \geq V_i(w_i, w_j)$.

□

From the previous two lemmas we see that an eligible contract that satisfies limited liability is weakly dominated by an LRS contract of the form:

$$w_i(y) = \alpha y_i - (1 - \alpha) w_j(y) - \alpha k_i \qquad \forall y \in Y$$

satisfying limited liability with equality. For an LRS contract to satisfy limited liability with equality it must be that:

$$k = \min_{y \in Y} \{y_i + w_j(y)\}$$

The last two lemmas (8 and 9) establish the form of the principal's payoffs under LRS contracts and the existence of an optimal contract in that class.

Lemma 8. *Let w an eligible contract scheme, such that w_i is an LRS contract given w_j, i.e. $w_i = \alpha y_i - (1-\alpha) w_j(y) - \alpha k$ for some $\alpha \in (0,1]$ and some $k \in \mathbb{R}$ that makes limited liability bind with equality for some output level. Then:*

$$V_i(w) = \frac{1-\alpha}{\alpha} V_A(w|\mathcal{A}_0) + k = \max_{(F,c)\in\mathcal{A}_0} \left((1-\alpha) E_F[y_i + w_j(y)] - \frac{1-\alpha}{\alpha} c \right) + \alpha k \qquad (40)$$

This also holds for $\alpha = 0$ if we interpret the term $\frac{1-\alpha}{\alpha} c$ as 0 when $c = 0$ and ∞ for $c > 0$.

Proof. Let $F \in \underset{F\in\mathcal{F}}{\operatorname{argmin}} E_F[y_i - w_i(y)]$ by Lemma 4 one has:

$$V_i(w) = E_F[y_i - w_i(y)] = \frac{1-\alpha}{\alpha} E_F[w_1(y) + w_2(y)] + k = \frac{1-\alpha}{\alpha} V_A(w|\mathcal{A}_0) + k$$

The second equality follows by replacing $V_A(w|\mathcal{A}_0)$ and the fact that that $w_i = \alpha y_i - (1-\alpha) w_j(y) - \alpha k$.

□

Its worthwhile to highlight that when $\alpha = 0$ the LRS contract offsets the other principal's payments to the agent and implies the zero aggregate contract, i.e. $w_i(y) = -w_j(y)$. In this case Lemma (8) gives $V_i(w) = w_j(0,0)$, corresponding to $(F,c) = (\delta_0, 0)$. Note that this is the only optimal action for the agent under any technology under assumption (2) and a zero aggregate contract.

Lemma 9. *In the class of LRS contracts that satisfy limited liability with equality there exists an optimal one for principal i.*

Proof. From Lemma 8 we can express $V_i(w)$ directly as a function of α as in (40). Recall that $k = \min_{y\in Y}\{y_i + w_j(y)\}$ is independent of α. Moreover, The function $(1-\alpha) E_F[y_i + w_j(y)] - \frac{1-\alpha}{\alpha} c$ is continuous in α, thus its maximum over $\mathcal{A}_0$ is continuous as well. Since the RHS in equation (40) is continuous in α it achieves a maximum in $[0,1]$. This α gives the optimal guarantee over all contracts of this class.

□

Theorem 1. *For any contract w_j there exists LRS contract $\overline{w}_i$ such that $\overline{w}_i \in BR_i(w_j)$, where $\min_{y \in Y} \{\bar{w}_i(y) + w_j(y)\} = 0$. That is, there is always a LRS contract that is **robust** for principal i.*

Proof. Consider a contract w_j by the competing principal. By Lemma 9 among the class of LRS contracts satisfying limited liability with equality there is an optimal one, call it $w_i^{\star}$. There are two cases to consider:

1. The contract $w_i^{\star}$ is eligible.

   Suppose for a contradiction that there is an arbitrary contract w_i that satisfies limited liability and that does strictly better than $w_i^{\star}$: $V_i(w_i, w_j) > V_i(w_i^{\star}, w_j)$. This contract is itself eligible. Then by Lemmas 5, 6 and 7 there exists an LRS contract $w_i^{'}$ that satisfies limited liability with equality such that $V_i\left(w_i^{'}, w_j\right) \geq V_i(w_i, w_j)$. This contradicts $w_i^{\star}$ being optimal among LRS contracts that satisfy limited liability with equality.

2. The contract $w_i^{\star}$ is not eligible, i.e. $V_i(w_i^{\star}, w_j) \leq w_j(0,0)$.

   Note that since $w_i(y) = -w_j(y)$ is a LRS contract that satisfies limited liability we know that:

   $$V_i(w_i^{\star}, w_j) \leq w_j(0,0) = V_i((-w_j, w_j)) \leq V_i(w_i^{\star}, w_j)$$

   Then $w^{\star}$ attains the bound: $V_i(w_i^{\star}, w_j) = w_j(0,0)$. We now claim that $w_i^{\star} \in \mathrm{BR}_i(w_j)$, if it were not then there exists a contract w_i that satisfies limited liability and $V_i(w_i, w_j) > V_i(w_i^{\star}, w_j)$, then this contract is eligible. Just as in the first case this leads to a contradiction of $w_i^{\star}$ being optimal among LRS contracts that satisfy limited liability with equality.

□

Corollary 1. *If $\mathcal{A}_0$ has the full support property (Assumption 4) then any robust contract for principal i is a LRS contract, or there are no eligible contracts.*

Proof. Consider a contract w_j by the competing principal. Suppose that there exists an eligible contract, then any contract in the best response is eligible. Suppose w_i is an optimal contract for principal i. Define $w_i^{'}$ as in Lemma 6. Note that $w_i^{'}$ satisfies:

$$E_F\left[y_i - w_i^{'}(y)\right] \geq k + \lambda V_A\left(\left(w_i^{'}, w_j\right)|\mathcal{A}_0\right)$$

Since w_i satisfies Equation (34) from Lemma 5 we can replace for k to obtain:

$$E_F\left[y_i - w_i^{'}(y)\right] \geq V_i(w) + \lambda\left(V_A\left(\left(w_i^{'}, w_j\right)|\mathcal{A}_0\right) - V_A((w_i, w_j)|\mathcal{A}_0)\right) \tag{41}$$

Because of full support, since $w_i^{'}(y) \geq w_i(y)$ pointwise and any action under $\mathcal{A}_0$ gives a (weakly) higher payoff to the agent under $w_i^{'}$ than under w_i, it follows that $V_A\left(\left(w_i^{'}, w_j\right)|\mathcal{A}_0\right) \geq V_A((w_i, w_j)|\mathcal{A}_0)$, with strict inequality unless $w_i^{'}$ is identical to w_i.

Since the equation (41) holds for all F we get: $V_i\left(w_i^{'}, w_j\right) \geq V_i(w)$, with strict inequality when w_i is not identical to $w_i^{'}$. Then $w_i = w_i^{'}$, or else optimality would be contradicted. Moreover, w_i has to be a LRS contract, or else by Lemma 7 there is a LRS contract that strictly improves on w_i.

□

A.2 Existence of Nash Equilibrium in LRS contracts under strong limited liability

Existence of an equilibrium can be guaranteed under the following sufficient conditions:

Theorem 2. *If assumption 5 or assumption 6 hold then a Nash equilibrium in LRS contracts that satisfy individual limited liability exists.*

Proof. Consider first Assumption 5. Then as in the proof of existence in the main text we show that the game has a pseudo-potential.

Let $G : \mathbb{R}_+ \to \mathbb{R}$ be defined as:

$$G(\theta_P) = \frac{1}{1-\theta_P} \max_{(F,c)\in\mathcal{A}_0} \left\{ (1-\theta_P) E_F[y_1 + y_2] - c \right\}$$

Note that if $\theta_P \geq 1$ then $G(\theta_P) = 0$, G is continuous.

Each principal plays an LRS contract characterized by a share $\theta \in [0,1]$. Contracts must satisfy individual limited liability so that:

$$w_i(y_1, y_2) = (1-\theta_i) y_i + \theta_i (\overline{y} - y_j)$$

The payoff of principal i is a function $V_i : [0,1]^2 \to \mathbb{R}$:

$$V_i(\theta_i, \theta_j) = \theta_i (G(\theta_1 + \theta_2) - \overline{y})$$

We want to establish the existence of a pure strategy Nash equilibrium for the game. We will do it by showing the existence of an equilibrium where $\theta_i > 0$ for $i \in \{1, 2\}$. In what follows we only consider best responses to strictly positive actions.

As in Monderer and Shapley (1996) we first define an ordinal potential function. We want a function $P : [0,1]^2 \to \mathbb{R}$ such that for $\theta_j > 0$:

$$V_i(x, \theta_j) - V_i(z, \theta_j) > 0 \quad \iff \quad P(x, \theta_j) - P(z, \theta_j) > 0$$

Define P as in Monderer and Shapley (1996):

$$P(\theta_1, \theta_2) = \theta_1 \theta_2 (G(\theta_1 + \theta_2) - \overline{y}) \tag{42}$$

We list the properties of P in the following proposition:

Proposition 5. *The function P defined in (42) satisfies the following properties:*

1. For $\theta_j > 0$ the function P induces the same order over θ_i as the function V_i.

2. P attains a maximum in $[0,1]^2$.

Its immediate from the first property that any maximum of P such that $\theta_1, \theta_2 > 0$ is a pure strategy equilibrium of the common agency game. Under the following condition such a maximum exist:

Assumption 7. *(Strong Non-Triviality) There exists an action $(F,c) \in \mathcal{A}_0$ such that $E_F[y_1 + y_2] - c > \overline{y}$.*

This condition allows for an action that generates enough (expected) surplus to cover the cost that the principals pay in fees. If such an action does not exist then the principals cannot guarantee themselves a positive payoff. A trivial equilibrium exists then, where $\theta_1 = \theta_2 = 0$.

The following lemma finishes the proof under Assumption 5:

Lemma 10. *Under assumptions 5 7 a pure strategy Nash Equilibrium in LRS contracts, where both principals get positive guaranteed payoffs, exists .*

Proof. Under assumption 7 there exists $\theta_1, \theta_2 > 0$ such that $G(\theta_1 + \theta_2) - \overline{y} > 0$, then $P(\theta_1, \theta_2) > 0$. Then the maximum of P over $(\theta_1, \theta_2) \in [0,1]^2$ is not attained in the boundary. Then for all $(\theta_1^\star, \theta_2^\star) \in \underset{(\theta_1,\theta_2)\in[0,1]^2}{\operatorname{argmax}} P(\theta_1, \theta_2)$, it holds that $\theta_1^\star, \theta_2^\star > 0$. All these pairs are Nash equilibria of the common agency game by proposition 5.

□

If Assumption 6 holds then we can also show existence of equilibrium.

First note that if both principals use LRS contracts, only expected total output is relevant in determining payoffs due to the form of LRS contracts. Hence, it is without loss to have the agent choose expected total output, x, and an associated cost, c. Moreover, if two actions have the same expected total output the agent will always pick the one with lower cost. These actions form the lower envelope of the action set in the (x,c) space and imply the cost function of the agent given by Equation 23:

The next lemma finishes the proof establishing existence of an equilibrium under Assumption 6.

Lemma 11. *Let $\mathcal{A}_0$ be the agent's technology set and f the cost function, defined as in (23). If $\mathcal{F}_{\mathcal{A}_0} = \{F \in \Delta(Y) \,|\, (F,c) \in \mathcal{A}_0\}$ is convex, f is continuous, and the square root of f is a convex function, then there exists a pure strategy equilibrium in LRS contracts.*

Proof. Assume that principal j plays a contract of the form $w_j(y) = (1-\theta_j)\,y_j + \theta_j(\overline{y}_i - y_i)$, and that principal i best responds with an LRS contract that satisfies limited liability with equality: $w_i = \alpha y_i - (1-\alpha)\,w_j(y) + \alpha k$. Then $k = \frac{1-\alpha}{\alpha}\left((1-\theta_j)\,\overline{y}_j + \theta_j \overline{y}_i\right)$, replacing gives: $w_i(y) = (1-\theta_i)\,y_i + \theta_i\left(\overline{y}_j - y_j\right)$, where $\theta_i = (1-\alpha)(1-\theta_j)$.

The value of principal i can be written as:

$$V_i(w) = \max_{(F,c)\in\mathcal{A}_0} \max_{\theta_A \in [0, 1-\theta_j]} \left\{ (1-\theta_j-\theta_A)\left(E_F[y_j + y_i] - \overline{y}_j\right) - \frac{1-\theta_j-\theta_A}{\theta_A} c \right\}$$

Where $\theta_A = 1 - \theta_j - \theta_i$. Note that given an action (F, c) there is a unique solution for θ_A given by:

$$\theta_A = \begin{cases} \sqrt{\frac{(1-\theta_j)c}{E_F[y_j+y_i]-\overline{y}_j}} & \text{if} \quad (1-\theta_j)\left(E_F\left[y_j+y_i\right]-\overline{y}_j\right) \geq c \\ 1-\theta_j & \text{otw} \end{cases}$$

Replacing back into the objective function gives:

$$V_i(w) = \max_{x \in \mathcal{F}_{\mathcal{A}_0}} \left\{ \left(\max\left\{ \sqrt{(1-\theta_j)\left(x-\overline{y}_j\right)} - \sqrt{f(x)} \quad , \quad 0 \right\} \right)^2 \right\}$$

As noted we can consider the max over the choices of expected total output available in $\mathcal{A}_0$, the relevant cost is given by the cost function f. Since $\sqrt{f}$ is continuous and convex the function $\sqrt{(1-\theta_j)\left(x-\overline{y}_j\right)} - \sqrt{f(x)}$ is continuous and strictly concave and admits a unique global maximum on $\mathcal{F}_{\mathcal{A}_0}$, call it $\tilde{x}(\theta_j)$. Without loss we define the argmax of V_i as: $x^\star(\theta_j) = \max\{\tilde{x}(\theta_j), \underline{x}\}$, where $\underline{x}$ is the lowest value of x for which $\sqrt{(1-\theta_j)\left(x-\overline{y}_j\right)} - \sqrt{f(x)} = 0$ necessary for a positive payoff. The Theorem of the Maximum implies that $x^\star(\theta_j)$ is a continuous function. The best response of principal i is then:

$$\theta_i = \mathrm{BR}_i(\theta_j) = (1-\theta_j) - \sqrt{\frac{(1-\theta_j) f(x^\star(\theta_j))}{x^\star(\theta_j) - \overline{y}_j}}$$

This is also a continuous function. Now consider the function $g : [0,1]^2 \to [0,1]^2$ defined by:

$$g(\theta_1, \theta_2) = (\mathrm{BR}_1(\theta_2), \mathrm{BR}_2(\theta_1))$$

This function is continuous and maps a compact convex subset of an Euclidean space into itself. By Brouwer's fixed point theorem it has a fixed point. That is $(\theta_1^\star, \theta_2^\star) \in [0,1]$ such that $\theta_i^\star = \mathrm{BR}_i\left(\theta_j^\star\right)$. This is an equilibrium of the game in LRS contracts.

□

□

Assumption 6 amounts to having the agent's cost function be sufficiently convex. Typical cost functions like the power function $(f(x) = kx^p; p \geq 2)$ and the exponential function $\left(f(x) = k_1 e^{k_2 x}\right)$ satisfy this assumptions. Yet, the conditions we state are not necessary. Consider the following linear cost function:

For any $x \in [0, \bar{y}_1 + \bar{y}_2]$ there exists $(F, c) \in \mathcal{A}_0$ such that $E_F[y_1 + y_2] = x$ and

$$\gamma x = \min\{c | (F, c) \in \mathcal{A}_0 \text{ and } E_F[y_1 + y_2] = x\}$$

where $\gamma < 1$ is the marginal cost.

The cost function above does not satisfy Assumption 6, but, as we show in the online appendix, the best response of each principal is still single valued, and the existence of an equilibrium is guaranteed.

Proof. ksjhgakjghak

$\square$

We can also show existence of an equilibrium when the agent is indifferent between actions. This assumption has been used in the literature, for example, Bernheim and Whinston (1986a) establish that an equilibrium of the common agency game exists and implements the efficient outcome when the agent is indifferent between actions (their condition (ii)). We reproduce Bernheim and Whinston (1986a) results under this condition in the online appendix. Although the assumption is restrictive, it is well suited to describe situations such as auctions or lobbying, were the agent is expected not to have preferences over the actions.

A.3 Private Common Agency

In the case of private output we employ the same procedure as in the case of the public common agency with the appropriate changes. To avoid repetition we provide only the statements of the crucial lemma in this new environment.

Lemma 12. *Let w be an eligible contract. There exits k, λ with $\lambda > 0$ such that for all $y \in Y$:*

$$\begin{aligned} w_i\left(y_i\right) &\leq \frac{1}{1+\lambda} y_i - \frac{\lambda}{1+\lambda} \overline{w}_j - \frac{1}{1+\lambda} k && (43) \\ V_i\left(w\right) &= k + \lambda V_A\left(w | \mathcal{A}_0\right) && (44) \end{aligned}$$

where $\overline{w}_j = \max_{y_j \in Y_j} w_j\left(y_j\right)$.

As noted earlier in the private common agency framework the principal gives the agent a share of his output and punishes the agent based on the maximum value of the other principal's contract. (i.e. the only difference is the use of $\bar{w}_j$ instead of $w_j\left(y\right)$).

Lemmas similar to (6), (7), (8), and (9) provide the optimality of linear contracts. Details are available in the online appendix.

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