Macroeconomics, Problem Set 1

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The solution of this problem consists of a PDF with all mathematical derivations and all graphs as well as julia or matlab script that produces the results.

1. Do exercise 10.1 of SLP.

   (a) Use the assumption that z is iid (drawn from the same distribution every period) to write down what the expectation in the sequential problem is about. This is simpler than the general Markov case because you do not have to keep track of previous realizations of z in the history. Put another way, the independence of z across time implies that the probability of $z^t = (z_0, z_1, z_2, \ldots, z_t)$ is just $1 \times \lambda(z_1) \times \lambda(z_2) \times \ldots \times \lambda(z_t)$.

   (b) Use Blackwell's conditions and the Contraction Mapping Theorem to verify that you have a contraction, then use the Theorem of Maximum to obtain characteristics of the policy.

   (c) Once again CMT and ToM. These operate the same as in the deterministic case after you understand that $E\left[v\left(y, z'\right)\right]$ is just another function (in particular it is a function of y alone because of the iid assumption on z, so the problem is really static)

   (d) Use the Euler equation to characterize the policy function. You can guess and then verify that the solution satisfies the desired form.

   (e) Envelope theorem (that applies to this problem the same as in deterministic problems)

   (f) Finally! Some real Markov processes! You can use the monotonicity of Q to show that $E\left[v\left(y, z'\right)|z\right]$ is increasing in z if v is increasing.

2. Chose at least one of exercises 2.3, 2.19 of Ljungqvist and Sargent

   (a) In Exercise 2.3 there is a typo and the expected value should be taken given $c_0 = \bar{c}_i$, where c_0 is the initial value of c and $\bar{c}_i$ is one of the values that c can take. The function v_i is the conditional expectation given and initial value $c_0 = \bar{c}_i$. The function V is the (unconditional) expected value when we do not know the value of c_0 but we do know that c_0 is distributed according to π_0. To solve part (a) express the problem in recursive form and solve for v directly. You can do it because there is not "max" as consumption itself is the stochastic variable. Part (b) just asks you to use the formulas you verified in part (a) and plug in numbers. Part (c) asks you to use the transition matrix to define the probability of a sequence (we have done this in the general case). Part (d) asks you to plug in the numbers of part (c) into Bayes rule. Part (e) is repeating the same computations with a different sequence of realizations (a different sample path).

   (b) In Exercise 2.19 you are asked to compute your best "guess" of θ given a sequence of noisy signals (y). The signals, y, are unbiased. Use formulas from bayesian updating to see how to update the beliefs about θ. Check out Laura Veldkamp's book Information Choice in Macroeconomics and Finance.

3. Durable Goods

Consider a single agent problem where each period, w total output is produced and can be divided into consumption of a perishable good, c_t and investment in a durable good, d_{xt}. The durable depreciates like a capital good, but is not directly productive. The stock of durables at any date, d_t, produces a flow of services that enters the utility function. Thus, the problem faced by the household with initial stock d_0 is:

$$\max_{c_t, d_t, d_{xt}} \sum_t \beta^t \{u_1(c_t) + u_2(d_t)\}$$

$$\text{s.t. } c_t + d_{xt} \leq w$$

$$d_{t+1} \leq (1-\delta) d_t + d_{xt}$$

$$c_t, d_t, d_{xt} \geq 0$$

$$d_0 \text{ given}$$

where both u_1 and u_2 are strictly increasing and continuous. Note: you can ignore non-negativity constraints on investment, d_{xt} in this problem.

(a) State a condition on either u_1 or u_2 (or both) such that you can write an equivalent problem in the following form:

$$\max_{\{d_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t F(d_t, d_{t+1})$$

$$\text{s.t.}$$

$$d_{t+1} \in \Gamma(d_t)$$

$$d_0 \text{given}$$

where $\Gamma(d) \in \mathbb{R}_+$. What is F? What is the correspondence Γ?

(b) Write the Bellman equation for this problem.

(c) State additional conditions on u_1 and u_2 such that the value function $v(d)$ is both strictly increasing and strictly concave. Prove these two properties.

(d) For the remaining questions, assume that both u_1 and u_2 satisfy the Inada conditions and are continuously differentiable. State the envelope and the FOC for the functional equation problem in (b)

(e) Show that there is a unique steady state value of the stock, $d^\star$, such that if $d_0 = d^\star$, then $d_t = d^\star$ for all t. Show that $d^\star > 0$.

(f) Show that the policy functions for the solution, $c^\star(d)$ and $d' = g^\star(d)$ are increasing.

(g) Show that the system is globally stable. You can assume that the policy functions are differentiable for this part. [Hint: Check Chapter 6 of SLP and use the Euler equation to show that g, the policy function, is increasing and concave. Use the fact that v is strictly concave when showing this.]

4. Recursive Competitive Equilibrium

Households:

- Consider an economy with two types of households indexed by $i \in \{a, b\}$ (arguers and bores).
- There is a continuum of each type of equal measure and they are both infinitely lived.
- Preferences for both households are given by the expected discounted value (both types have the same discount factor, β).
- The utility of bores is $u^b(c, h)$. The function is increasing in the first argument and decreasing in the second.
- Type a households like it more when other agents of their type are not working so that they can spend time together, and, you guessed it, argue. Their utility is $u^a(c, H^b, h)$, it is increasing in the first argument and decreasing in the other two arguments.

Production:

- There is a continuum of competitive firms.
- The labor from each type of household are not perfect substitutes in production.
- There is an aggregate production function $f(Z, K, H^a, H^b)$, where z is productivity, K is aggregate capital, and H^a and H^b are total hours worked by each type of household.
- The production function f has constant returns to scale in K, H^a, and H^b.
- Productivity shocks, Z, follow a Markov chain with transition $\Pi_{zz'}$
- Capital is rented from households at a rate R. There is no depreciation.
- Labor is hired at wage rates W^a and W^b.

Setup:

- Use the convention that big letters represent aggregate variables and small letters represent individual variables.
- The only shock in the economy is the productivity shock.

(a) Firm problem:

  i. Describe the demand for capital and the two types of labor.

  ii. What does the demand tell you about market clearing in these markets.

(b) Define Recursive Competitive Equilibrium. Make sure to include the goods market in your definition. Make sure that you not only define the required objects but also state the conditions that such objects must satisfy.

(c) Stochastic Processes.

  i. Is the stochastic process for output implied by the equilibrium a Markov process? Explain.

  ii. What variables form the state vector of the economy? Choose the minimal state vector.

  iii. A Markov process is characterized by its transition function $\mathcal{Q}$. What are the domain and the codomain of the Transition function for the state of this economy.

  iv. Construct the stochastic process for output implied by the equilibrium.

(d) Planner's problem:

  i. Pose the Planner's problem.

  ii. What are the optimality conditions that characterize the solution to the Planner's problem? List them all.

(e) Optimality of the equilibrium:

  i. Is the equilibrium optimal? Explain mathematically and intuitively why or why not.

  ii. How do the equilibrium and the planner's labor allocations differ?