

# A Practitioners' Note on The Shapley-Owen-Shorrocks Decomposition\*

Richard Audoly   Rory M<sup>c</sup>Gee   Sergio Ocampo   Gonzalo Paz-Pardo

July 2025

## Abstract

Decomposing empirical or economic phenomena into the contributions of different inputs is a frequent goal of economic analysis. However, in many settings, the quantity of interest depends on many inputs which are aggregated non-linearly. In these settings, decompositions need not sum to one and often depend on the order in which inputs are “zero-ed out.” In this note we describe a simple, but convenient alternative. We show that using the [Shapley-Owen](#) value, extended to inequality decompositions in [Shorrocks](#) (1999, 2013), provides an additive decomposition that sums to one and is easily interpretable in terms of the contribution of different inputs (or groups of them) to some aggregate outcome. We provide several examples to help implement the approach. We believe this is exceptionally well-suited to decompositions in rich-structural models of economic phenomena which are typically non-linear.

---

\*Audoly: FRBNY; [audolyr@gmail.com](mailto:audolyr@gmail.com); Web: [www.newyorkfed.org/research/economists/Audoly](http://www.newyorkfed.org/research/economists/Audoly).

M<sup>c</sup>Gee: UWO & IFS; [rmcgee4@uwo.ca](mailto:rmcgee4@uwo.ca); Web: <https://sites.google.com/view/rorymcgee>.

Ocampo: UWO; [socampod@uwo.ca](mailto:socampod@uwo.ca); Web: <https://sites.google.com/site/sergiocampod>.

Paz-Pardo: ECB; [gonzalo.paz\\_pardo@ecb.europa.eu](mailto:gonzalo.paz_pardo@ecb.europa.eu); Web: <https://www.gonzalopazpardo.com>.

The views below are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York, the Federal Reserve System, the European Central Bank, or the Eurosystem.

We are grateful to David Wiczer for first suggesting this approach to us many years ago. We also thank Emmanuel Murray Leclair for research assistance. This previously appeared as an appendix to our paper "The Life-Cycle Dynamics of Wealth Mobility." Ocampo acknowledges financial support from the Research Council of Norway through the project TaxFair, Number 315765. M<sup>c</sup>Gee and Ocampo acknowledge financial support from the Social Science and Humanities Research Council of Canada through the Insight Development Grant Number 430-2022-00394.

In this short note, we aim to provide a simple overview of the Shapley-Owen-Shorrocks decomposition. The objective of the decomposition is to obtain values measuring the contribution of several inputs (or groups of them) to some aggregate outcome. When the outcome can be expressed as a linear function of inputs the decomposition is immediate: the value of each input can be obtained by setting all other inputs to their baseline value, say zero. But, when the outcome is the product of a non-linear aggregation, as is the case in structural models of economic decision making, the order in which inputs are “zero-ed out” matters.

The decomposition, developed in [Shorrocks \(1999, 2013\)](#) for the study of inequality, uses the concept of the Shapley value to treat each input (a variable, policy function, or price) as a player and the outcome of interest as the surplus being generated by the inputs collective “action.” In this way, the Shapley value of each input provides its *contribution*, adding up to the aggregate outcome. The same concept applies when a group of inputs moves together, as is the case when changing all prices or initial conditions in counterfactuals, following the generalization of [Shapley \(1953\)](#) in [Owen \(1977\)](#) to unions of players. We have found this decomposition useful in our own work, as well as suggesting to colleagues, collaborators, and students.

Despite being common in some areas of economics, or within some research networks, we believe it is an underused tool. We see this approach as providing an intuitive decomposition when there are non-trivial interactions between various factors, as is the case for virtually all economic phenomena. Our goal is to transparently define the decomposition and illustrate its uses through fully worked-out toy examples. We hope that this will contribute to wider use.

Before proceeding we provide a short and in-exhaustive list of the uses of the decomposition in economics research. It has been used in for decomposing the  $R^2$  in regression analysis ([Israeli 2007](#); [Huettner and Sunder 2012](#)); and to decompose the contribution of groups of regressors to the variance of earnings as in [Allen and Arkolakis \(2014\)](#); in structural models, it has been used to quantify the roles of changing labor market and demographic conditions for the take up of disability insurance in a structural life cycle model ([Michaud and Wiczer 2018](#)); in the literature on earnings dynamics, it has been used to decompose higher order moments of earnings changes, distinguishing the roles of employer and occupational switchers ([Carrillo Tudela, Visschers, and Wiczer 2022](#)), and the role of changes in earnings risk for homeownership ([Paz Pardo 2024](#)); in [Hubmer, Halvorsen, Salgado, and Ozkan \(2024\)](#) the decomposition is used in accounting for the roles of returns, savings,

income, and inheritances in wealth accumulation; in [Audoly, M<sup>c</sup>Gee, Ocampo, and Paz Pardo \(2024\)](#) we use it to decompose the explanatory power of groups of regressors in a multinomial logit model; in firm dynamics it has been used to account for drivers of output volatility ([Kabir and Tan 2024](#)); [Kwon, Lee, and Pouliot \(2024\)](#) highlight the relationship between the Shapley value and least squares and perform the decomposition on groups of inputs in the context of a Roy model; in [Athreya, Gordon, Jones, and Neelakantan \(2025\)](#) it is used to decompose differences in wealth accumulation by race; [Millard \(2025\)](#) decomposes the gap in post-secondary education between individuals with and without an early-onset disability; [Freund \(2025\)](#) uses it to account for the change in the variance of log-wages over time in terms of structural parameters of an assignment model; and [Guler, Kuruscu, and Robinson \(2025\)](#) use it to separate the contribution of return heterogeneity, long-term debt, and the liquidity of housing wealth, to the marginal propensity to consume in a life-cycle housing model.

## 1. The Decomposition

Given an arbitrary function  $Y = f(X_1, X_2, \dots, X_n)$ , the Shapley-Owen-Shorrocks decomposition is a method to decompose the value of  $f(\cdot)$  into each of its arguments  $X_1, X_2, \dots, X_n$ . Intuitively, the contribution of each argument if it were to be “removed” from the function. However, because the function can be nonlinear, the order in which the arguments are removed matters in general for the decomposition. The function  $f$  can be the outcome of a regression, like the predicted values or sum of square residuals, or the output of a structural model, such as a counterfactual value for a variable given a list of model parameters or components, or a transformation of the sample, for example the Gini coefficient.

The Shapley-Owen-Shorrocks decomposition is the unique decomposition satisfying four important properties. **(i)** The decomposition is exact decomposition under addition, letting  $C_j$  denote the contribution of argument  $X_j$  to the value of the function  $f(\cdot)$ ,

$$\sum_{j=1}^n C_j = f(X_1, X_2, \dots, X_n), \quad (1)$$

so that  $C_j/f(\cdot)$  can be interpreted as the proportion of  $f(\cdot)$  that can be attributed to  $X_j$ .<sup>1</sup>

---

<sup>1</sup>The interpretation holds as long as  $f$  is non-negative. If  $f$  can take negative values, then the interpretation of  $C_j$  under the exact additive rule can be misleading as some arguments can have  $C_j < 0$ .

(ii) The decomposition is symmetric with respect to the order of the arguments. That is, the order in which the variable  $X_j$  is removed from  $f(\cdot)$  does not alter the value of  $C_j$ . (iii) The decomposition assigns zero contribution to factors that have a null-effect (an irrelevance normalization). If a factor  $X_j$  never changes the outcome of the function, in an abuse of notation  $\partial_j f(\cdot) = 0$  everywhere, then  $C_j = 0$ . (iv) The decomposition says that the attribution operator is linear in the index you are decomposing. This is a useful closure requirement that implies contributions rescale linearly with a rescaling of the outcome function and are linear for combinations of outcomes.

The contribution of input  $X_j$  is then

$$C_j = \sum_{k=0}^{n-1} \frac{(n-k-1)!k!}{n!} \left( \sum_{s \subseteq S_k \setminus \{X_j\}; |s|=k} [f(s \cup X_j) - f(s)] \right), \quad (2)$$

where  $n$  is the total number of arguments in the original function  $f$ ,  $S_k \setminus \{X_j\}$  is the set of all “sub-models” that contain  $k$  arguments and exclude argument  $X_j$ .<sup>2</sup> For example,

$$\begin{aligned} S_{n-1} \setminus X_n &= f(X_1, X_2, \dots, X_{n-1}) \\ S_1 \setminus X_n &= \{f(X_1), f(X_2), \dots, f(X_{n-1})\}. \end{aligned}$$

The decomposition in (2) accounts for all possible permutations of the decomposition order. Thus,  $\frac{(n-k-1)!k!}{n!}$  can be interpreted as the probability that one of the particular sub-model with  $k$  variables is randomly selected when all model sizes are all equally likely. For example, if  $n = 3$ , there are sub-models of size  $\{0, 1, 2\}$ . In particular, there are  $2^2$  permutation of models that exclude each variable:  $\underbrace{\{(0, 0)\}}_{k=0}, \underbrace{\{(1, 0), (0, 1)\}}_{k=1}, \underbrace{\{(1, 1)\}}_{k=2}$ .

We provide a simple implementation of the decomposition in Matlab at the end of the document and outline the algorithm below.

---

<sup>2</sup>We abuse notation here. A “sub-model” is an evaluation of function  $f$  with only some of its arguments. This language is motivated by the function corresponding in practice to the outcome of a regression or structural model. Formally, when we write  $f(X_1)$ , we mean  $f(X_1, \emptyset_2, \dots, \emptyset_n)$ , where we assume the  $j$ -th argument of the function can always take on a null value denoted  $\emptyset_j$ . In our regression example below, this null value corresponds to a zero valued regressor or parameter. In the case of the structural model, this null value can correspond to setting some parameters to a predetermined value or excluding certain model components, like the adjustment of prices or a specific shock agents face.

---

**Shapley-Owen-Shorrocks Algorithm**

**Inputs:** Variable inputs and values across “sub-models.”

- (a)  $X$ : a binary  $2^n \times n$  matrix. Gives inputs in "sub-models."
- (b)  $F$ : a  $n \times 1$  vector of model values.

The rows of  $X$  and  $F$  corresponding to “sub-models”. A row of zeroes represents the reference value or null-model. A row of ones represent the full model with all inputs.

**Output:**  $C$ , a  $n \times 1$  vector of input contributions.

**Computing contribution for  $j^{th}$  input:**

- Initialize contribution to 0,  $C(j) = 0$ .
- Loop over sub-model sizes,  $k = 0, \dots, n - 1$ .
  - i Define weight for this class of sub-models  $\omega_k = \frac{(n-k-1)!k!}{n!}$ .
  - ii Find sub-models  $S_k \setminus \{X_j\}$ : Rows of  $X$  with  $k$  inputs and without  $j$ .
  - iii Loop over  $s \in S_k \setminus \{X_j\}$  updating the contribution of input  $j$ :

$$C(j) = C(j) + \omega_k \times (F(s \cup \{j\}) - F(s))$$

---

## 2. Linear Example

We begin with the benchmark case of a linearly-aggregated outcome. In this case the natural decomposition is just the individual value of each input as they are already additive. We take this case to illustrate how the weighting scheme in (2) works.

Consider a linear model with 3 variables:

$$Y = f(X_1, X_2, X_3) = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \quad (3)$$

Consider the the partial effect of  $X_3$  on  $Y$ . There are 4 possible models that exclude  $X_3$ , one with no variable, 2 with one variable and one with 2 variables

$$k = 0 : 0$$

$$k = 1 : \{\beta_0 + \beta_1 X_1, \beta_0 + \beta_2 X_2\}$$

$$k = 2 : \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

In all 4 models, the partial effect of including  $X_3$  is always  $f(s \cup X_3) - f(s) = \beta_3 X_3 \quad \forall s$ . Hence, linearity is such that the order in which variables are included to construct  $C_3$  does not matter:

$$\begin{aligned} C_3 &= \sum_{k=0}^2 \frac{(n-k-1)!k!}{n!} \sum_{s \subseteq S_k \setminus \{X_3\}: |s|=k} [f(s \cup X_j) - f(s)] \\ &= \sum_{k=0}^2 \frac{(3-k-1)!k!}{3!} \sum_{s \subseteq S_k \setminus \{X_3\}: |s|=k} \beta_3 X_3 = \beta_3 X_3 \end{aligned} \quad (4)$$

Notice that this would be the same if the original function took an arbitrary number of variables:  $Y = f(X_1, X_2, \dots, X_j) = \sum_{j=1}^n \beta_j X_j$ . The only difference is that the number of sub-models grows exponentially:  $2^{n-1}$ , but the partial effect of including  $X_j$  for some  $j \in \{0, 1, \dots, n\}$  is always  $\beta_j X_j$ .

Thus, in the linear case, the decomposition is mathematically identical to the usual regression decomposition. We average over the same object in each permutation because the effect does not depend on the order in this special case.

### 3. Nonlinear example I

We illustrate the value of this decomposition with a simple nonlinear model including  $n = 3$  variables:

$$Y = f(X_1, X_2, X_3) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 X_2. \quad (5)$$

The objective is to decompose the value of  $Y$  into the contribution (or partial effect) of each variable.

**Removing  $X_1$ .** There are four possible models that exclude  $X_1$ —one with no variable, two with one variable, and one with two variables:

$$\begin{aligned} k = 0 &: \beta_0 \\ k = 1 &: \{\beta_0 + \beta_2 X_2, \beta_0\} \\ k = 2 &: \beta_0 + \beta_2 X_2 + \beta_3 X_3 X_2 \end{aligned}$$

In all four models, the partial effect of including  $X_1$  is always  $f(s \cup X_1) - f(s) = \beta_1 X_1$ .

This reflects the fact that the order in which variables are included does not matter to construct  $C_1$ :

$$C_1 = \sum_{k=0}^2 \frac{(3-k-1)!k!}{3!} \left( \sum_{s \subseteq S_k \setminus \{X_3\}: |s|=k} [f(s \cup X_j) - f(s)] \right) = \beta_1 X_1 \quad (6)$$

This would be the same for any argument  $X_j$  entering linearly into  $f$  an arbitrary number of variables:  $Y = f(X_1, X_2, X_3, X_4, \dots, X_n) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 X_2 + \sum_{j=4}^n \beta_j X_j$ . The only difference is that the number of sub-models grows exponentially,  $2^{n-1}$ , but the partial effect of including  $X_j$  for some  $j \in \{4, \dots, n\}$  is always  $C_j = \beta_j X_j$ .

**Removing  $X_2$ .** In this case, the partial effect can be decomposed into all the possible ways  $X_2$  can be added into the model,  $f(s \cup X_2) - f(s)$ , these are

$$k=0 (\emptyset_1, \emptyset_3) : \beta_0 + \beta_2 X_2 - \beta_0 = \beta_2 X_2$$

$$k=1 (X_1, \emptyset_3) : \beta_0 + \beta_1 X_1 + \beta_2 X_2 - (\beta_0 + \beta_1 X_1) = \beta_2 X_2$$

$$k=1 (\emptyset_1, X_3) : \beta_0 + \beta_2 X_2 + \beta_3 X_2 X_3 - \beta_0 = \beta_2 X_2 + \beta_3 X_2 X_3$$

$$k=2 (X_1, X_3) : \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_2 X_3 - (\beta_0 + \beta_1 X_1) = \beta_2 X_2 + \beta_3 X_2 X_3$$

Here, the partial effects of adding  $X_2$  are not the same across sub-models because  $X_2$  enters nonlinearly into the original model. The symmetric property of the decomposition takes care of this.

$$\begin{aligned} C_2 &= \underbrace{\frac{1}{3} \beta_2 X_2}_{k=0} + \underbrace{\frac{1}{6} (\beta_2 X_2) + \frac{1}{6} (\beta_2 X_2 + \beta_3 X_2 X_3)}_{k=1} + \underbrace{\frac{1}{3} (\beta_2 X_2 + \beta_3 X_2 X_3)}_{k=2} \\ &= \beta_2 X_2 + \frac{1}{2} \beta_3 X_2 X_3 \end{aligned} \quad (7)$$

The result is quite intuitive.  $\beta_2 X_2$  appears in all sub-models; hence, its probability of appearing in the decomposition is 1.  $\beta_3 X_2 X_3$  appears in two of the four sub-models; hence, its probability of appearing is 1/2. Weighting each term by its probability of appearing in the decomposition ensures symmetry.

**Removing  $X_3$ .** We proceed in the same way for  $X_3$  as we did for  $X_2$ . There are four sub-models. In two of them, the effect of adding  $X_3$  is null, because  $X_2$  is not in the

model. In the two remaining sub-models, the effect is  $\beta_3 X_2 X_3$ . Hence,

$$C_3 = \frac{1}{2} \beta_3 X_2 X_3. \quad (8)$$

Finally, we verify the decomposition:

$$\begin{aligned} C_1 + C_2 + C_3 &= \beta_1 X_1 + \left( \beta_2 X_2 + \frac{1}{2} \beta_3 X_2 X_3 \right) + \left( \frac{1}{2} \beta_3 X_2 X_3 \right) \\ &= \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_2 X_3 \\ &= f(X_1, X_2, X_3) - \beta_0 \\ &= f(X_1, X_2, X_3) - f(\emptyset_1, \emptyset_2, \emptyset_3). \end{aligned}$$

**Reference value for the decomposition.** The decomposition is additive with respect to the “null” model where none of the variables is included. This is made apparent in the previous result, where the decomposition does not include the value of  $\beta_0$ .

#### 4. Nonlinear example II: $\mathbf{R}^2$

Finally, we consider a decomposition of the coefficient of determination in the linear model. Our own use of the decomposition applies this for a nonlinear model, combining the insights from this and the preceding example (see [Audoly et al. 2024](#)). We note that this application of the decomposition was first proposed by [Israeli \(2007\)](#) and [Huettnner and Sunder \(2012\)](#). Recent applications include [Nikolova and Cnossen \(2020\)](#); [Engstrom, Hersh, and Newhouse \(2021\)](#); [Biasi and Ma \(2022\)](#); and [Biasi, Lafortune, and Schönholzer \(2025\)](#); among others.

Consider a linear regression model with  $n$  regressors and  $i = 1, \dots, M$  observations,

$$y_i = \mathbf{x}_i' \beta + u_i = \beta_0 + \sum_{j=1}^n \beta_j x_{ij} + u_i, \quad (9)$$

and define the average value of  $y$  as  $\bar{y} \equiv \sum_{i=1}^M y_i / M$  and the predicted value

$$\hat{y}_i = \mathbf{x}_i' \hat{\beta} = \hat{\beta}_0 + \sum_{j=1}^n \hat{\beta}_j x_{ij}, \quad (10)$$

where we assume that all regressors have zero mean so that  $\hat{\beta}_0 = \bar{y}$ .



The function of interest is  $f(X_1, \dots, X_K) = R^2$ , defined as the explained sum of squares  $SSE$  over the total sum of squares  $SST$

$$R^2(X_1, X_2, \dots, X_n) = \frac{SSE}{SST} = \frac{\sum_{i=1}^M (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^M (y_i - \bar{y})^2}. \quad (11)$$

This makes it clear that the function being decomposed is nonlinear even though the model that generates it is itself linear.

**Reference value for the decomposition.** The reference value for the  $R^2$  in the Shapley-Owen-Shorrocks decomposition is given by the model without regressors, satisfying

$$R^2(\emptyset) = \frac{\sum_i^M (\hat{\beta}_0 - \bar{y})^2}{\sum_i^M (y_i - \bar{y})^2} = 0, \quad (12)$$

so that, in this case, the decomposition recovers the level of the  $R^2$  of the full model (with all variables), unlike the previous example.

**Decomposition when  $n = 3$ .** We explicitly calculate the decomposition for  $n = 3$  regressors. As before, we abuse notation by only listing the arguments being included in each sub-model. The contribution of each variable is:

$$R_1^2 = \frac{1}{3} \left[ R^2(X_1) - R^2(\emptyset) \right] + \frac{1}{6} \left( \left[ R^2(X_1, X_2) - R^2(X_2) \right] + \left[ R^2(X_1, X_3) - R^2(X_3) \right] \right) + \frac{1}{3} \left[ R^2(X_1, X_2, X_3) - R^2(X_2, X_3) \right]; \quad (13)$$

$$R_2^2 = \frac{1}{3} \left[ R^2(X_2) - R^2(\emptyset) \right] + \frac{1}{6} \left( \left[ R^2(X_1, X_2) - R^2(X_1) \right] + \left[ R^2(X_2, X_3) - R^2(X_3) \right] \right) + \frac{1}{3} \left[ R^2(X_1, X_2, X_3) - R^2(X_1, X_3) \right]; \quad (14)$$

$$R_3^2 = \frac{1}{3} \left[ R^2(X_3) - R^2(\emptyset) \right] + \frac{1}{6} \left( \left[ R^2(X_3, X_2) - R^2(X_2) \right] + \left[ R^2(X_1, X_3) - R^2(X_1) \right] \right) + \frac{1}{3} \left[ R^2(X_1, X_2, X_3) - R^2(X_2, X_1) \right]. \quad (15)$$

Summing across all the contributions we obtain back  $R^2(X_1, X_2, X_3)$ ,

$$R_1^2 + R_2^2 + R_3^2 = R^2 = f(X_1, X_2, X_3). \quad (16)$$

***Decomposing the  $R^2$  versus the partial  $R^2$ .*** The value of the contribution differs from the standard definition of partial  $R^2$ . This is because the partial  $R^2$  is an all-else-being-equal comparison of excluding regressor  $X_j$ . However, it is worth noting that the partial  $R^2$  does not satisfy the exact decomposition requirement or (when applied iteratively) the symmetry requirement.

## 5. Summary

The Shapley-Owen-Shorrocks decomposition provides an effective alternative when studying non-linear outcomes that come from the interaction of various factors, as is the case in most economic applications. The decomposition is additive, symmetric with respect to factors, and it allows naturally for groups of factors that move together.

While the decomposition is already used by some authors, and in many contexts, there are still examples where it may be useful. We conclude by highlight some of these examples. [De Nardi, French, Jones, and M<sup>c</sup>Gee \(2025\)](#) decompose the contribution of various factors to retirement savings in a structural life cycle model, but do not provide an additive decomposition. [Nakajima and Telyukova \(2020\)](#) provide a decomposition under one order of elimination in their main text, but (commendably!) provide robustness to various alternative orders of elimination. Although the cost of implementing the Shapley-Owen-Shorrocks decomposition grows substantially with the number of factors, judiciously grouping them can help minimize costs. Additionally, alternative approaches also impose large computational burdens on researchers.

Finally, the decomposition can also be useful in the context of welfare decompositions from counterfactual exercises, like those in [Flodén \(2001\)](#), [Conesa, Kitao, and Krueger \(2009\)](#), and [Moschini and Tran Xuan \(2025\)](#) that separate the roles of changes in the aggregate level and distributions of consumption and leisure for welfare. The value of these decompositions depends on the order in which consumption and leisure are incorporated, limiting their interpretability. The symmetry of the Shapley-Owen-Shorrocks decomposition provides a viable alternative in this context at little additional cost.

## References

- Allen, Treb and Costas Arkolakis. 2014. "Trade and the topography of the spatial economy." *The Quarterly Journal of Economics* 129 (3):1085–1140. URL <https://doi.org/10.1093/qje/qju016>.
- Athreya, Kartik, Grey Gordon, John Bailey Jones, and Urvi Neelakantan. 2025. "Decomposing wealth differences: earnings, demographics, and rates of return." *Unpublished Manuscript, Federal Reserve Bank of Richmond* URL <https://drive.google.com/file/d/1LM4KYG2B8WSJCsg-Fnr4B9fV-r237XB9/view>.
- Audoly, Richard, Rory McGehee, Sergio Ocampo, and Gonzalo Paz Pardo. 2024. "The life-cycle dynamics of wealth mobility." Working Paper 24/12, Institute for Fiscal Studies. URL <https://ifs.org.uk/publications/life-cycle-dynamics-wealth-mobility>.
- Biasi, Barbara, Julien Lafortune, and David Schönholzer. 2025. "What works and for whom? Effectiveness and efficiency of school capital investments across the US." *The Quarterly Journal of Economics* 140 (3):2329–2379. URL <https://doi.org/10.1093/qje/qjaf013>.
- Biasi, Barbara and Song Ma. 2022. "The education-innovation gap." Working Paper 29853, National Bureau of Economic Research. URL <http://www.nber.org/papers/w29853>.
- Carrillo Tudela, Carlos, Ludo Visschers, and David Wiczer. 2022. "Cyclical earnings, career and employment transitions." Tech. Rep. 15603, IZA. URL <https://www.iza.org/publications/dp/15603/cyclical-earnings-career-and-employment-transitions>.
- Conesa, Juan Carlos, Sagiri Kitao, and Dirk Krueger. 2009. "Taxing capital? Not a bad idea after all!" *American Economic Review* 99 (1):25–48. URL <https://doi.org/10.1257/aer.99.1.25>.
- De Nardi, Mariacristina, Eric French, John Bailey Jones, and Rory McGehee. 2025. "Why do couples and singles save during retirement? Household heterogeneity and its aggregate implications." *Journal of Political Economy* 133 (3):750–792. URL <https://doi.org/10.1086/733421>.
- Engstrom, Ryan, Jonathan Hersh, and David Newhouse. 2021. "Poverty from Space: Using High Resolution Satellite Imagery for Estimating Economic Well-being." *The World Bank Economic Review* 36 (2):382–412. URL <https://doi.org/10.1093/wber/lhab015>.
- Flodén, Martin. 2001. "The effectiveness of government debt and transfers as insurance." *Journal of Monetary Economics* 48 (1):81–108. URL [https://doi.org/10.1016/S0304-3932\(01\)00064-2](https://doi.org/10.1016/S0304-3932(01)00064-2).
- Freund, Lukas. 2025. "Superstar teams." Tech. rep., Boston College. URL [https://lukasbfreund.github.io/files/freund\\_superstarTeams.pdf](https://lukasbfreund.github.io/files/freund_superstarTeams.pdf).
- Guler, Bulent, Burhan Kuruscu, and Baxter Robinson. 2025. "The composition and concentration of wealth: Fiscal and monetary policy implications." Tech. rep., University of Western Ontario. URL [https://bguler.pages.iu.edu/files/GKR\\_June2025\\_Submit.pdf](https://bguler.pages.iu.edu/files/GKR_June2025_Submit.pdf).
- Hubmer, Joachim, Elin Halvorsen, Sergio Salgado, and Serdar Ozkan. 2024. "Why are the wealthiest so wealthy? A longitudinal empirical investigation." Working Paper 2024-013A, Federal Reserve Bank of St. Louis. URL <https://doi.org/10.20955/wp.2024.013>.
- Huettner, Frank and Marco Sunder. 2012. "Axiomatic arguments for decomposing goodness of fit according to Shapley and Owen values." *Electronic Journal of Statistics* 6:1239–1250. URL <https://doi.org/10.1214/12-EJS710>.
- Israeli, Osnat. 2007. "A Shapley-based decomposition of the R-Square of a linear regression." *Journal of Economic Inequality* 5:199–212. URL <https://doi.org/10.1007/s10888-006-9036-6>.

- Kabir, Poorya and Eugene Tan. 2024. "Maintenance volatility, firm productivity, and the user cost of capital." Tech. rep., University of Toronto, Rotman School of Management. URL [https://drive.google.com/file/d/1jL6\\_Ew21UVKCO1Qc\\_5psqgZzRzBprMQJ/view](https://drive.google.com/file/d/1jL6_Ew21UVKCO1Qc_5psqgZzRzBprMQJ/view).
- Kwon, Yongchan, Sokbae Lee, and Guillaume A. Pouliot. 2024. "Group shapley value and counterfactual simulations in a structural model." URL <https://arxiv.org/abs/2410.06875>.
- Michaud, Amanda and David Wiczer. 2018. "The disability option: Labor market dynamics with macroeconomic and health risks." Tech. Rep. 2018-7, Centre for Human Capital and Productivity (CHCP), University of Western Ontario. URL <https://www.econstor.eu/handle/10419/197793>.
- Millard, Robert G. 2025. "Early-onset disability, education investments, and social insurance." Tech. rep., State University of New York, Stony Brook. URL <http://dx.doi.org/10.2139/ssrn.5233570>.
- Moschini, Emily G. and Monica Tran Xuan. 2025. "Family policies and child skill accumulation." *Review of Economic Dynamics* 56:101270. URL <https://doi.org/10.1016/j.red.2025.101270>.
- Nakajima, Makoto and Irina A Telyukova. 2020. "Home equity in retirement." *International Economic Review* 61 (2):573–616. URL <https://doi.org/10.1111/iere.12435>.
- Nikolova, Milena and Femke Cnossen. 2020. "What makes work meaningful and why economists should care about it." *Labour Economics* 65:101847. URL <https://www.sciencedirect.com/science/article/pii/S0927537120300518>.
- Owen, Guillermo. 1977. "Values of Games with a Priori Unions." In *Mathematical Economics and Game Theory*, edited by Rudolf Henn and Otto Moeschlin. Berlin, Heidelberg: Springer Berlin Heidelberg, 76–88. URL [https://doi.org/10.1007/978-3-642-45494-3\\_7](https://doi.org/10.1007/978-3-642-45494-3_7).
- Paz Pardo, Gonzalo. 2024. "Homeownership and portfolio choice over the generations." *American Economic Journal: Macroeconomics* 16 (1):207–237. URL <https://doi.org/10.1257/mac.20200473>.
- Shapley, L. S. 1953. "A value for N-person games." In *Contributions to the Theory of Games, Volume II*, edited by H. W. Kuhn and .A. W. Tucker. Princeton, New Jersey: Princeton University Press, 307–318. URL <https://doi.org/10.1515/9781400881970-018>.
- Shorrocks, Anthony F. 1999. "Decomposition procedures for distributional analysis: A unified framework based on the Shapley value." Tech. rep., University of Essex.
- . 2013. "Decomposition procedures for distributional analysis: A unified framework based on the Shapley value." *Journal of Economic Inequality* 11 (1):99. URL <https://doi.org/10.1007/s10888-011-9214-z>.

## Shapley-Owen-Shorrocks Implementation in Matlab

---

```
function C = shapley_owen_shorrocks(X_ind, f_vals)
    % X_ind: binary matrix of size ((2^n) x n)
    % f_vals: 2^n vector of function values
    n      = size(X_ind, 2) ; % Number of variables
    C      = zeros(n, 1)    ; % Store contributions
    fact_vec = factorial(0:n); % Precompute factorial terms

    for j = 1:n % Loop over all inputs
        contrib = 0; % Initialize input j contribution

        for k = 0:n-1 % Loop over sub-models without input j
            % Weight for size-k sub-models--adj. by 1 for indexing
            w = fact_vec(n-k-1+1)*fact_vec(k+1)/fact_vec(n+1);
            % Find rows of X_ind with k inputs excluding j
            mask_k = sum(X_ind, 2) == k ; % k inputs
            mask_j = X_ind(:, j) == 0 ; % Excluding j
            idx_base = find(mask_k & mask_j) ; % Both

            for idx = idx_base' % Loop over sub-models (k, no j)
                base = X_ind(idx, :); % Reference sub-model
                base_val = f_vals(idx) ; % Reference value

                % Add variable j to form S U {j}
                with_j = base; with_j(j) = 1;
                % Find index of "with_j" sub-model and its value
                idx_with_j = find(ismember(X_ind, with_j, 'rows'));
                if isempty(idx_with_j);
                    error('sub-model not found');
                end
                with_j_val = f_vals(idx_with_j);

                % Update contribution
                contrib = contrib + w*(with_j_val - base_val);
            end
        end
        C(j) = contrib; % Update output
    end
end
```

---