Taxing Wealth and Capital Income when Returns are Heterogeneous

Guvenen, Kambourov, Kuruscu, Ocampo

October, 2022

Taxing Capital

What is the optimal tax combination on capital income *(flow)* and wealth *(stock)* when returns are heterogeneous?

- Capital income tax: $a_{\text{after-tax}} = a + (1 - \tau_k) \cdot ra$
- Wealth tax: $a_{\text{after-tax}} = (1 - \tau_a) \cdot a + ra$

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1. Theoretical analysis of optimal combination of taxes *(Today)*
   - **Analytical** model entrepreneurs and workers
   - **Find:** conditions for **(i)** efficiency gains **(ii)** welfare gains *(ind.+overall)* **(iii)** optimal taxes

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1. Theoretical analysis of optimal combination of taxes *(Today)*
   - **Analytical** model entrepreneurs and workers
   - **Find:** conditions for **(i)** efficiency gains **(ii)** welfare gains *(ind.+overall)* **(iii)** optimal taxes
2. Quantitative analysis of optimal capital income **vs.** wealth tax *(new version!)*
   - **Rich OLG model** that matches both
     i. the distribution of cross-sectional and lifetime returns &
     ii. the extreme concentration and Pareto tail of the wealth distribution
   - **Find:** Large efficiency and welfare gains from wealth tax.

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2. **Technical:** Capital taxes paid by the very wealthy.

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4. **Theoretical:** Interesting **new economic mechanisms**. Example next.

Return Heterogeneity: A Simple Example

- One-period model.
- Government taxes to finance $G = \$50$.
- Two brothers, Fredo and Mike, each with \$1000 of wealth.

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- Two brothers, Fredo and Mike, each with $1000 of wealth.
- **Key heterogeneity**: investment/entrepreneurial ability.
  - (Fredo) Low ability: earns $r_f = 0\%$ rate of return.
  - (Mike) High ability: earns $r_m = 20\%$ rate of return.

Capital Income (τ_k) vs. Wealth Tax (τ_a)

	Capital income tax		Wealth tax
	$a_{i,\text{after-tax}} = a_i + (1 - \tau_k) r_i a_i$		
	Fredo $(r_f = 0\%)$	Mike $(r_m = 20\%)$	
Wealth	\$1000	\$1000	
Before-tax Income	**0**	**\$200**	
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Tax liability			
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- Market value internalizes inv. ability, taxing it weakens use it or lose it effect.

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What is next: Tractable dynamic model with entrepreneurs and workers

Results preview

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2. **Welfare Gain by Type:** With a marginal shift from capital income to wealth tax
   - Workers gain
   - High-productivity entrepreneurs "typically" gain
   - Low-productivity entrepreneurs "typically" lose
3. **Optimal Taxes:** Utilitarian welfare maximizing taxes depend on the elasticity of output with respect to capital (α)
   - If α is sufficiently high $\longrightarrow \tau_a^* > 0$ & $\tau_k^* < 0$
   - If α is sufficiently low $\longrightarrow \tau_a^* < 0$ & $\tau_k^* > 0$
   - If α is in between $\longrightarrow \tau_a^* > 0$ & $\tau_k^* > 0$.

Outline

1. **Model**
2. Efficiency gains from wealth taxation
3. Welfare gains from wealth taxation
4. Optimal taxation
5. Extensions

Theoretical Model

Two groups of infinitely-lived agents:

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- Workers' and entrepreneurs' preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t) \qquad \text{where } \beta < 1.$$

Theoretical Model

- Entrepreneurs' technology:

$$y = (zk)^{\alpha} n^{1-\alpha}$$

  - $z \in \{z_\ell, z_h\}$, where $z_h > z_\ell \geq 0$ with a transition matrix

$$\mathbb{P} = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix} \text{ with } 0 < p < 1.$$

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- Aggregate output:

$$Y = \int (zk)^{\alpha} n^{1-\alpha}$$

- Government finances exogenous expenditure G with τ_k and τ_a
  - τ_a on beginning-of-period wealth

Financial Markets & Entrepreneurs' Problem

Financial markets:

- Collateral constraint ($\lambda \geq 1$): $k \leq \lambda a$, where a is entrepreneur's wealth.
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$$\Pi^\star(z,a) = \max_{\mathbf{k} \leq \lambda \mathbf{a}, n} (zk)^\alpha n^{1-\alpha} - rk - wn$$

Solution: $$\Pi^\star(z,a) = \underbrace{\pi^\star(z)}_{\text{Excess return above } r} \times a$$

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Entrepreneurs' Dynamic Problem:

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- Letting $R_i \equiv (1-\tau_a) + (1-\tau_k)(r + \pi^\star(z_i))$ for $i \in \{l,h\}$,

  the savings decision (CRS + Log Utility):

$$a' = \beta R_i a \quad \longrightarrow \text{linearity allows aggregation}$$

Equilibrium Values: Aggregation

Lemma: Aggregate output is

$$Y = (ZK)^{\alpha} L^{1-\alpha} \qquad (Z^{\alpha} \text{ is measured TFP})$$

where

$$K \equiv A_h + A_\ell \qquad K = \text{Aggregate capital}$$

$$Z \equiv s_h z_\lambda + (1 - s_h) z_\ell \qquad Z = \text{Wealth-weighted productivity}$$

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Key variables:

- $s_h = \frac{A_h}{K}$: wealth share of high-productivity entrepreneurs.
- $z_\lambda \equiv z_h + (\lambda - 1)(z_h - z_\ell)$: effective productivity of high-type entrepreneurs.

Use it or lose it effect increases efficiency if $s_h \uparrow (\longrightarrow Z \uparrow)$

Evolution of Aggregates

$$A_h' = \underbrace{p\beta R_h A_h}_{\text{stayers' savings}} + \underbrace{(1-p)\,\beta R_l A_l}_{\text{switchers' savings}}$$

A_h: High type wealth

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A_l: Low type wealth

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   - **Unique steady state** with:
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   - **Empirically relevant:** $R_h > R_l$ and $\frac{\text{Debt}}{\text{GDP}} \gg 1.5$ when $\lambda = \lambda^{\star}$.

Debt-GDP

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Steady State: 2 equations 2 unknowns

Using the law of motion for A_l and A_h we obtain two steady state equations:

Steady State K

$$(1-\tau_k)\overbrace{\alpha Z^{\alpha}\left({}^{K}/_{L}\right)^{\alpha-1}}^{\text{MPK}}-\tau_a=\frac{1}{\beta}-1.$$

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Steady State Z (depends on only τ_a!)

How τ_k disappears graph

$$h(Z)=(1-\rho\beta(1-\tau_a))Z^2-\frac{z_l+z_\lambda}{2}(1+\rho-2\rho\beta(1-\tau_a))Z+z_l z_\lambda\rho(1-\beta(1-\tau_a))=0.$$

- Simple graphical representation and analysis of the steady state!

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Main Result 1: Efficiency Gains from Wealth Taxation

Proposition:

Graph $\bar{\tau}_a$ graph

For all $\tau_a < \bar{\tau}_a$ ($\longleftrightarrow \lambda < \lambda^\star$), a marginal increase in τ_a **increases steady state Z iff** entrepreneurial productivity is autocorrelated, $\rho > 0$ ($p > 1/2$)

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G.E.

$$\frac{dR_\ell}{d\tau_a} = \underbrace{\left(\frac{z_\ell - Z}{Z}\right)}_{\text{use-it-lose-it}<0} \underbrace{- \left(\frac{1}{\beta} - (1 - \tau_a)\right) \frac{z_\ell}{Z^2} \frac{dZ}{d\tau_a}}_{\text{G.E. effect}<0} < 0$$

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G.E.

$$\frac{dR_\ell}{d\tau_a} = \underbrace{\left(\frac{z_\ell - Z}{Z}\right)}_{\text{use-it-lose-it}<0} \underbrace{-\left(\frac{1}{\beta} - (1 - \tau_a)\right)\frac{z_\ell}{Z^2}\frac{dZ}{d\tau_a}}_{\text{G.E. effect}<0} < \mathbf{0}$$

$$\frac{dR_h}{d\tau_a} = \underbrace{\left(\frac{z_\lambda - Z}{Z}\right)}_{\text{use-it-lose-it}>0} \underbrace{-\left(\frac{1}{\beta} - (1 - \tau_a)\right)\frac{z_\lambda}{Z^2}\frac{dZ}{d\tau_a}}_{\text{G.E. effect}<0} > \mathbf{0}$$

3. Ave. and log-ave. returns decrease with τ_a (use-it-or-lose-it)

Government Budget and Aggregate Variables

Government budget:

$$G = \tau_k \alpha Y + \tau_a K.$$

Assumption: G is a constant fraction $\theta\alpha$ of aggregate output: $G = \theta\alpha Y$.

- In what follows, τ_k adjusts in the background when $\tau_a \uparrow$

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- In what follows, τ_k adjusts in the background when $\tau_a \uparrow$

Lemma: For all $\tau_a < \overline{\tau}_a$, a marginal increase in $\boldsymbol{\tau_a}$

- **Increases** capital (K), output (Y), wage (w), h-type wealth (A_h), and **G iff $\rho > 0$**
  - **Key:** Higher $\alpha \longrightarrow$ Larger response of K, Y, w
  - $A_\ell = (1 - s_h) K \downarrow$ iff $\alpha z_\lambda < Z$ and $\rho > 0$

Outline

1. Model
2. Efficiency gains from wealth taxation
3. **Welfare gains from wealth taxation**
4. Optimal taxation
5. Extensions

Welfare gains (across steady states)

$\mathrm{CE}_{1,i}$ measure for agents of type i ($i \in \{\, workers\,,\ \ell ow\, prod.\,,\ high\, prod.\,\}$):

- (a, i) in **B**enchmark economy v.s.
  (a, i) in **C**ounterfactual economy with higher τ_a (lower τ_k)

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- (a, i) in **B**enchmark economy v.s.
  (a, i) in **C**ounterfactual economy with higher τ_a (lower τ_k)
- Welfare gains (**C**$\succ$**B**) if

$$\frac{\log\left(1 + \mathrm{CE}_{1,i}\right)}{1 - \beta} = V^{C}(a, i) - V^{B}(a, i) > 0$$

independent of a because $V(a, i) = m_i + \frac{1}{1-\beta}\log(a)\ i \in \{l, h\}$.

CE_1 Details

Welfare gains (across steady states)

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CE_1 Details

- Utilitarian welfare CE_1 depends on population shares n_i's:

$$\log\left(1 + \mathrm{CE}_1\right) = \sum_i n_i \log\left(1 + \mathrm{CE}_1(., i)\right)$$

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  CE_1 Details

- Utilitarian welfare CE_1 depends on population shares n_i's:

$$\log\left(1+\text{CE}_1\right) = \sum_i n_i \log\left(1+\text{CE}_1\left(.,i\right)\right)$$

- CE_1 does not account for changes in distribution of wealth.
  - Alternative measure CE_2 takes into account changes in wealth levels.

  CE_2 Details

Main Result 2: Welfare gains by type

Proposition:

For all $\tau_a < \overline{\tau}_a$, a marginally higher $\boldsymbol{\tau_a}$ changes welfare as follows **iff** $\boldsymbol{\rho > 0}$

- Workers: Higher $\text{CE}_{1,w} > 0$
- High-type entrepreneurs: Higher $\text{CE}_{1,h} > 0$ iff $R_h - R_\ell < \kappa_R(\beta, \rho)$
  - Taking wealth accumulation into account: $\text{CE}_{2,h} > 0$ always.
- Low-type entrepreneurs: Lower $\text{CE}_{1,l} < 0$
  - Taking wealth accumulation into account: $\text{CE}_{2,l} < 0$ if $\alpha z_\lambda < Z$.
- Lower average welfare of entrepreneurs: $\text{CE}_{1,\mathbf{E}} < 0$.

κ_R

Outline

1. Model
2. Efficiency gains from wealth taxation
3. Welfare gains from wealth taxation
4. **Optimal taxation**
5. Extensions

Optimal Taxation

Government chooses (τ_a, τ_k) to maximize the utilitarian social welfare CE_1 (or CE_2)

Key trade-off:

1. Higher wages (depends on α) v.s.
2. Lower (LOG) average return (higher return dispersion + negative GE effect)

   & changes in $\{A_l, A_h\}$ if CE_2 is the objective.

Main Result 3: Optimal Taxes

Graph | α thresholds

Proposition: There exists a unique optimal tax combination $(\tau_a^\star, \tau_k^\star)$ that maximizes CE_1. An interior optimum $(\tau_a^\star < \bar{\tau}_a)$ is the solution to:

$$\overbrace{n_w}^{\text{Share of Workers}} \underbrace{\xi_w}_{Z\text{-Elasticity of Wages}(=\alpha/(1-\alpha))} + \frac{1-n_w}{1-\beta} \underbrace{\left(\frac{\xi_{R_\ell} + \xi_{R_h}}{2}\right)}_{\text{Av. } Z\text{-Elasticity of Returns}<0} = 0$$

where $\xi_x \equiv \frac{d \log x}{d \log Z}$ is the elasticity of variable x with respect to Z.

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$$\tau_a^\star \in \left[1 - \frac{1}{\beta}, 0\right) \quad \text{and } \tau_k^\star > \theta \qquad \text{if } \alpha < \underline{\alpha}$$

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$$\tau_a^\star \in \left[0, \frac{\theta(1-\beta)}{\beta(1-\theta)}\right] \quad \text{and } \tau_k^\star \in [0, \theta] \qquad \text{if } \underline{\alpha} \le \alpha \le \bar{\alpha}$$

Main Result 3: Optimal Taxes

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$$\begin{array}{lll}
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\tau_a^\star > \frac{\theta(1-\beta)}{\beta(1-\theta)} & \text{and } \tau_k^\star < 0 & \text{if } \alpha > \bar{\alpha}
\end{array}$$

Main Result 3: Optimal Taxes

Graph | α thresholds

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$$\tau_a^\star > \frac{\theta(1-\beta)}{\beta(1-\theta)} \quad \text{and } \tau_k^\star < 0 \qquad \text{if } \alpha > \bar{\alpha}$$

Remark: Opt. $\tau_a^\star$ is independent of G but $\overline{\alpha}$ increases with G.

Outline

1. Model
2. Efficiency gains from wealth taxation
3. Welfare gains from wealth taxation
4. Optimal taxation
5. **Extensions**

Extensions

- Corporate sector that faces no borrowing constraint Details
  - If $z_\ell < z_C < z_h$, then low-productivity agents invest in the corporate sector.
- Rents: Return $\neq$ marginal productivity. Details
  - Introduce zero-sum return wedges so that $R_h <> R_\ell$.
  - Efficiency gains from $\tau_a \uparrow$ if $\rho > 0$ **and** $R_h > R_\ell$.
  - Efficiency gains from $\tau_a \uparrow$ if $\rho < 0$ **and** $R_h < R_\ell$.
- Entrepreneurial effort in production: Details
  - With GHH preferences, aggregate entrepreneurial effort increases with wealth tax.
- Perpetual youth and stationary distribution of agents: Details
  - $\text{CE}_{2,h} > \text{CE}_{1,h} > 0$ always.

Conclusions from theoretical analysis

Increasing τ_a (& reducing τ_k):

- Reallocates capital: less productive $\rightarrow$ more productive agents.
  - Higher TFP, output, and wages;
  - Higher dispersion in returns and wealth **iff $\rho > 0$**.
- Workers gain
- Entrepreneurs: High-productivity gain*, low-productivity lose*.

Optimal tax combination: depends on elasticity of output with respect to capital.

Thanks!

Extra

Entrepreneur’s Problem

Financial Markets & Entrepreneurs’ Production Problem

Entrepreneurs’ Production Decision:

$$\Pi^{\star}(z, a) = \max_{\mathbf{k} \leq \lambda \mathbf{a}, n} (zk)^{\alpha} n^{1-\alpha} - rk - wn.$$

Financial Markets & Entrepreneurs' Production Problem

Entrepreneurs' Production Decision:

Solution: $\Pi^\star(z,a) = \underbrace{\pi^\star(z)}_{\text{Excess return above } r} \times a$

$$\pi^\star(z) = \begin{cases} (MPK(z) - r)\,\lambda & \text{if } MPK(z) > r \\ 0 & \text{otherwise.} \end{cases} \qquad k^\star(z) \begin{cases} = \lambda a & \text{if } MPK(z) > r \\ \in [0, \lambda a] & \text{if } MPK(z) = r \\ = 0 & \text{if } MPK(z) < r \end{cases}$$

- $(\lambda - 1)\,a$: amount of external funds used by type-z if $MPK(z) > r$.

back

Entrepreneur's Consumption-Saving Problem

$$V(a,z) = \max_{c,a'} \log(c) + \beta \sum_{z'} \mathbb{P}(z' \mid z) V(a', z')$$

$$\text{s.t. } c + a' = \underbrace{(1-\tau_a)\, a + (1-\tau_k)\,(r + \pi^\star(z))\, a}_{\text{After-tax wealth}}.$$

- Letting $R_i \equiv (1-\tau_a) + (1-\tau_k)(r + \pi^\star(z_i)) \quad \text{for } i \in \{l, h\}$,

  the savings decision (CRS + Log Utility):

$$a' = \beta R_i a \quad \longrightarrow \text{linearity allows aggregation}$$

back

Equilibrium

Unstable equilibrium

Back to Eq.

1. Can there be a steady state with $(\lambda - 1) A_h > A_\ell$? **NO.** In that case $R_h = R_\ell$,

$$\frac{A_h'}{A_\ell'} = \frac{pA_h + (1-p)A_\ell}{(1-p)A_h + pA_\ell} = \frac{A_h}{A_\ell},$$

which implies that $A_h = A_\ell$. But then $(\lambda - 1) A_h > A_\ell$ is violated because $\lambda < 2$.

Unstable equilibrium

Back to Eq.

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2. Can there be a steady state with $(\lambda - 1) A_h < A_\ell$? If the answer is yes, then we are already focusing on that SS and that SS implies that $\lambda < \lambda^\star$.

Unstable equilibrium

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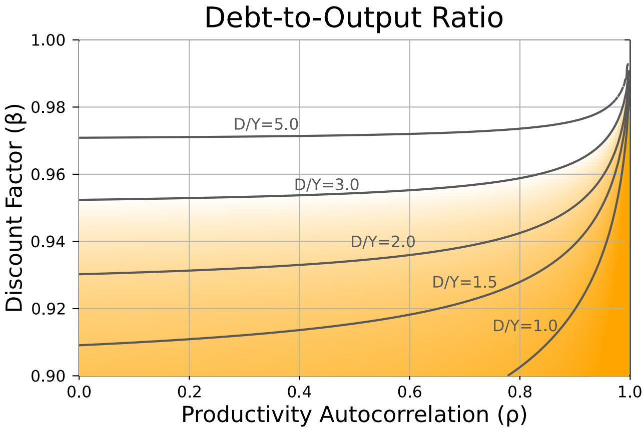
2. Can there be a steady state with $(\lambda - 1) A_h < A_\ell$? If the answer is yes, then we are already focusing on that SS and that SS implies that $\lambda < \lambda^\star$.

3. If $(\lambda - 1) A_h > A_\ell$ in the transition, then $A_h > A_\ell$ since $\lambda < 2$ and
$$\frac{A_h'}{A_\ell'} = \frac{pA_h + (1-p) A_\ell}{(1-p) A_h + pA_\ell} < \frac{A_h}{A_\ell}.$$
Then at some point, we will have $(\lambda - 1) A_h < A_\ell$ and we will be in the heterogenous-return case. If this converges to a a steady state, it is the one with $\lambda < \lambda^\star$.

Is λ^* too restrictive?

Back to Eq.

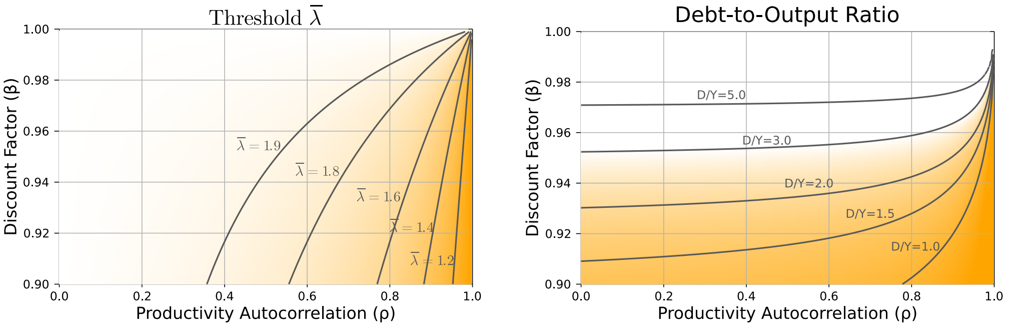


Debt-to-output ratio when $\lambda = \lambda^\star$ computed as ${}^{(\lambda^\star - 1)A_h}/_{Y}$.

Is λ^* too restrictive?

Back to Eq.

Figure 1: Conditions for Steady State with Heterogeneous Returns



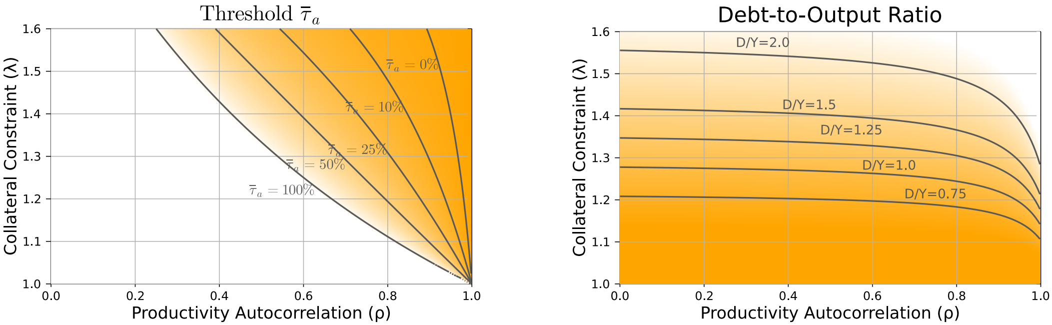
$z_\ell = 0$, $z_h = 2$, $\tau_k = 25\%$, and $\alpha = 0.4$.

Debt-to-output ratio when $\lambda = \lambda^\star$ *computed as* $(\lambda^\star - 1) A_h / Y$

Bound on τ_a

Back to Eq. Back to Eff.

Figure 2: Conditions for Steady State with Heterogeneous Returns



$z_\ell = 0$, $z_h = 2$, $\tau_k = 25\%$, *and* $\alpha = 0.4$.

Debt-to-output ratio with $\tau_a = 0$ *(benchmark) computed as* $^{(\lambda^\star - 1)A_h}/_Y$

Steady State: 2 equations 2 unknowns

Back to ss Back to Eff.

SteadyState ***K*****:**
$$(1-\tau_a)+(1-\tau_k)\overbrace{\alpha Z^{\alpha}\left({}^{K}/_{L}\right)^{\alpha-1}}^{\text{Marginal Product } K}=\frac{1}{\beta}$$

Steady State ***R*****:**

$$R_i=(1-\tau_a)+(1-\tau_k)\overbrace{\alpha\left({}^{ZK}/_{L}\right)^{\alpha-1}}^{\text{Marginal Product } ZK} z_i \qquad \text{Equilibrium R}$$

$$R_i=(1-\tau_a)+(1-\tau_k)\,\alpha Z^{\alpha}\left({}^{K}/_{L}\right)^{\alpha-1}\frac{z_i}{Z} \qquad \text{Change to MPK}$$

$$R_i=(1-\tau_a)+\left(\frac{1}{\beta}-(1-\tau_a)\right)\frac{z_i}{Z} \qquad \text{Steady State}$$

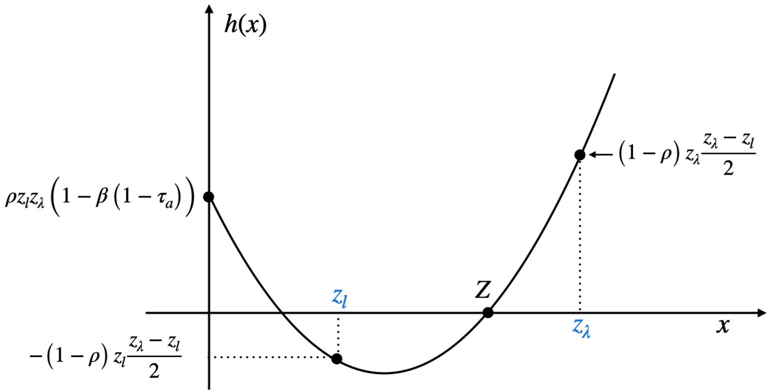
Key: Steady state K adjusts to maintain constant (after-tax) MPK:

$$(1-\tau_k)\,\text{MPK}=\frac{1}{\beta}-(1-\tau_a)$$

As in NGM τ_k affects level of K but not long run (after-tax) MPK $\left({}^{1}/_{\beta}-1+\tau_a\right)$.

Existence and Uniqueness of Steady State (when $\rho > 0$)

Back to ss

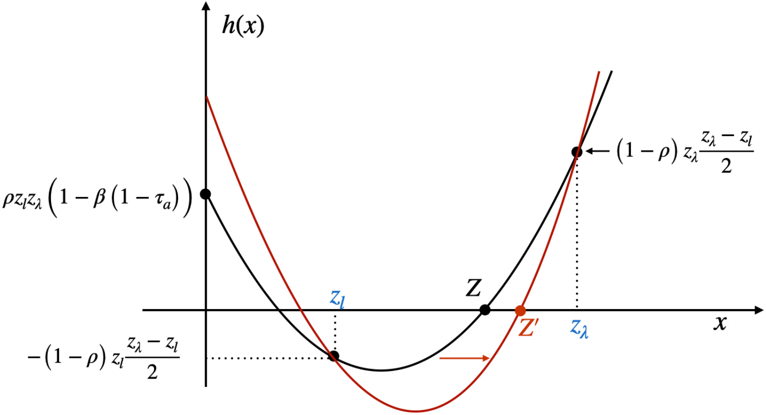


- $Z = s_h z_\lambda + (1 - s_h) z_\ell$ so $z_\ell \leq Z \leq z_\lambda$
- $R_h > R_\ell$ if and only if $Z < z_h \longrightarrow$ Characterization of bound $\lambda^\star$ so that $Z(\lambda^\star) = z_h$

What happens to Z if $\tau_a \uparrow$?

Back to eff. gain

$$\frac{dh(x)}{d\tau_a} = \rho\,(x - z_l)\,(z_\lambda - x) < 0 \text{ iff } \rho > 0 \text{ and } z_l < x < z_\lambda$$



Welfare Gains

Welfare gains (with changes in wealth)

Back to CE_1

$\text{CE}_{2,i}$ measure $(i \in \{w, l, h\})$**:**

- Evaluate welfare gain at average wealth levels for each economy.
- (A_i^B, i) in the **B**enchmark economy v.s. (A_i^C, i) in the **C**ounterfactual economy.
- Welfare gains (**C**$\succ$**B**) if

$$\frac{\log\left(1 + \text{CE}_{2,i}\right)}{1 - \beta} = V^C\left(A_i^C, i\right) - V^B\left(A_i^B, i\right) > 0 \qquad i \in \{w, l, h\}$$

  - Relation to CE_1:

$$\log\left(1 + \text{CE}_{2,i}\right) = \log\left(1 + \text{CE}_{1,i}\right) + \log\left(A_i^C / A_i^B\right)$$

Welfare gains: Functional Forms

Back to CE_1

- **Workers:** Value depends only on wages

$$\log\left(1 + \mathrm{CE}_{1,w}\right) = \log {}^{w_a}\!/_{w_k}$$

Welfare gains: Functional Forms

Back to CE_1

- **Workers:** Value depends only on wages

$$\log\left(1+\mathrm{CE}_{1,w}\right)=\log {w_a}/{w_k}$$

- **Entrepreneurs:** Value depends on assets and returns $V(a,i)=m_i\left(R_h,R_\ell\right)+\frac{\log(a)}{1-\beta}$

$$\log\left(1+\mathrm{CE}_{1,i}\right)=\frac{1}{(1-\beta)(1-\beta\rho)}\left[(1-\beta)\underbrace{\log\frac{R_{a,i}}{R_{k,i}}}_{\text{Own Return}}+\beta(1-p)\left(\underbrace{\log\frac{R_{a,l}}{R_{k,l}}+\log\frac{R_{a,h}}{R_{k,h}}}_{\text{Average (log) Returns}}\right)\right]$$

Welfare gains: Functional Forms

Back to CE_1

- **Workers:** Value depends only on wages

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- **Entrepreneurs:** Value depends on assets and returns $V(a,i)=m_i\left(R_h,R_\ell\right)+\frac{\log(a)}{1-\beta}$

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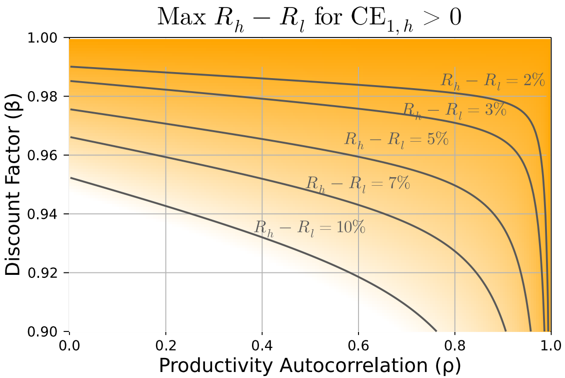
  - Total entrepreneurial value:

$$\log\left(1+\mathsf{CE}_1^{\mathsf{e}}\right)\equiv\sum_{i\in\{h,l\}}\frac{1}{2}\log\left(1+\mathsf{CE}_{1,i}\right)=\frac{1}{1-\beta}\left(\log\frac{R_{a,l}}{R_{k,l}}+\log\frac{R_{a,h}}{R_{k,h}}\right)$$

Return Dispersion for Welfare Gains of High-Type Entrepreneurs

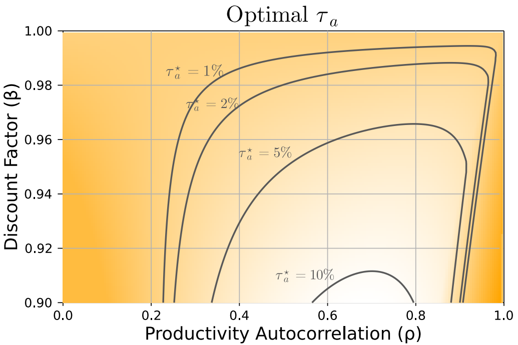
Back to CE_1

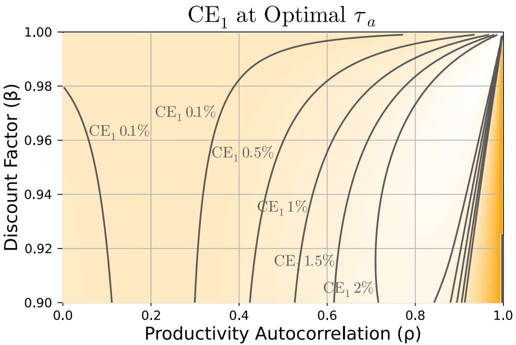


Optimal Taxes

Optimal Wealth Taxes and Welfare Gain

α-thresholds | Back to opt. tax

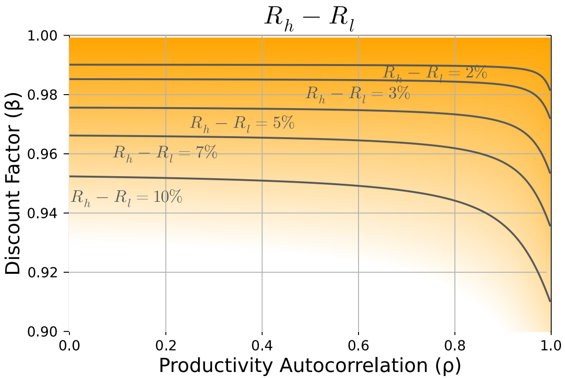




$z_\ell = 0$, $z_h = 2$, $\theta = 25\%$, and $\lambda = 1.3$.

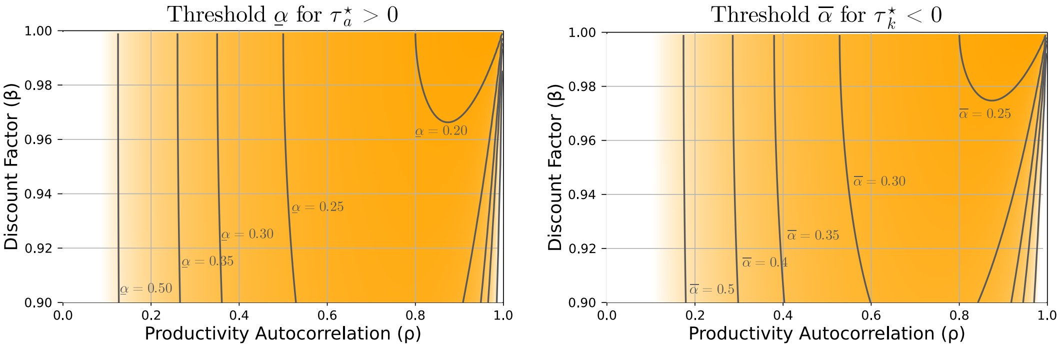
Return dispersion $R_h - R_\ell$:

Back to α-thresholds



α-thresholds for Optimal Wealth Taxes

Back to opt. tax

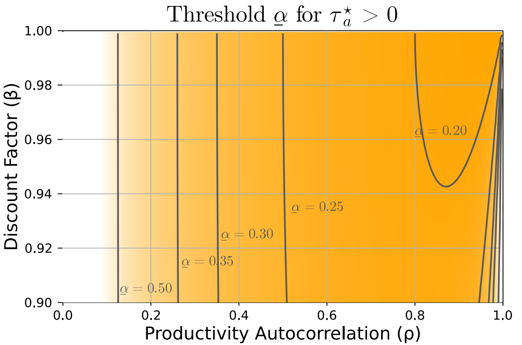


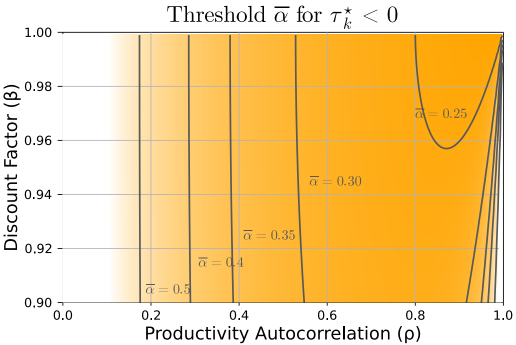
$z_\ell = 0$, $z_h = 2$, $\lambda = 1.3$, and $\theta = 25\%$.

Alt. Parameters | $R_h - R_l$ | Opt. Tax and Welfare Gains

α-thresholds for Optimal Wealth Taxes (alternative parameters)

Back to opt. tax

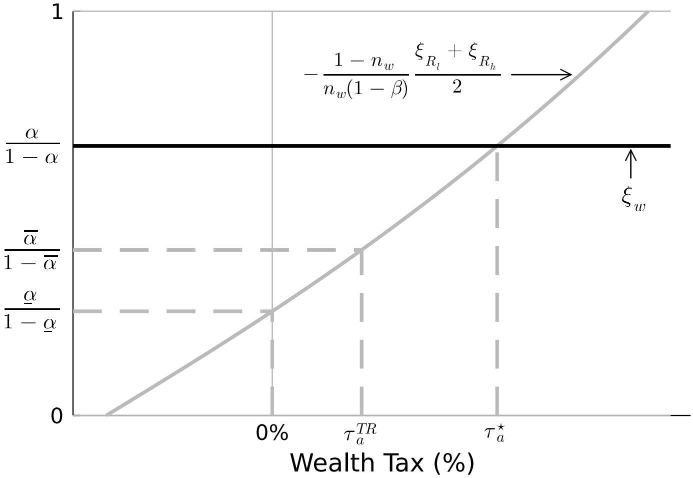




$z_\ell = 0.5$, $z_h = 1.5$, $\lambda = 1.2$, *and* $\theta = 25\%$.

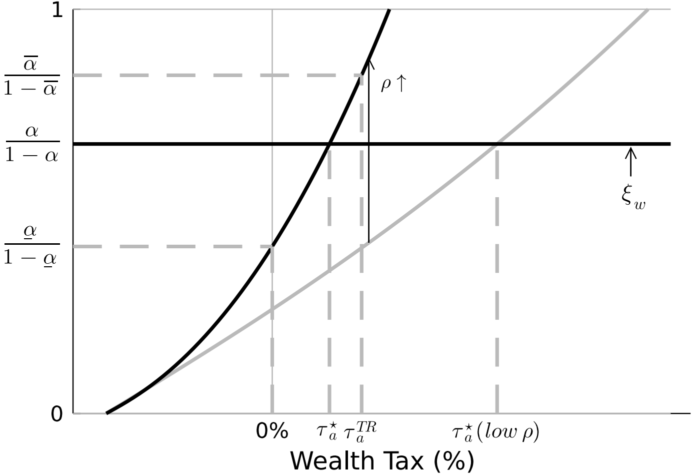
Optimal Wealth Taxes and α Thresholds

Back to opt. tax



Optimal Wealth Taxes and α Thresholds

Back to opt. tax



Extensions

Extension: Corporate sector

Back to extensions

- Corporate sector produces final goods using CRS technology:

$$Y_c = (z_c K_c)^{\alpha} L_c^{1-\alpha}$$

  - No financial constraints!
- Corporate sector imposes lower bound on r:

$$r \geq \alpha z_c \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} .$$

Interesting case: $z_\ell < z_c < z_h$

- Corporate sector and high-productivity entrepreneurs produce
- Low-productivity entrepreneurs lend all of their funds.
- No real changes in the aggregates of the economy! z_c takes the place of z_ℓ

$$Y = (ZK)^{\alpha} L^{1-\alpha}$$

but now $Z = s_h z_\lambda + s_l \mathbf{z_c}$, where $z_\lambda = z_h + (\lambda - 1)(z_h - \mathbf{z_c})$.

Extension: Rents

Back to extensions

- Introduce wedge for returns above/below productivity:

$$R_i = (1-\tau_a) + (1-\tau_k) \underbrace{(1+\omega_i)}_{\text{Return Wedge}} \alpha \left({}^{ZK}/{}_{L}\right)^{\alpha-1} z_i$$

- Zero-sum condition on wedges: $\omega_l z_\ell A_\ell + \omega_h z_\lambda A_h = 0$
- Characterization of eq. in terms of "effective productivity" $\tilde{z}_i = (1+\omega_i)\, z_i$

Extension: Rents

Back to extensions

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Proposition:

For all $\tau_a < \overline{\tau}_a$, a marginal increase in wealth taxes (τ_a) increases Z, $\frac{dZ}{d\tau_a} > 0$, **iff**

1. $\rho > 0$ and $R_h > R_\ell \longrightarrow$ Same mechanism as before
2. $\rho < 0$ and $R_h < R \longrightarrow$ Reallocates wealth to the true high types next period

Extension: Entrepreneurial Effort

Back to extensions

- Entrepreneurial production:

$$y = (zk)^{\alpha} e^{\gamma} n^{1-\alpha-\gamma} \quad \longrightarrow \quad e: \text{ effort}$$

  - Production functions is CRS $\longrightarrow$ Aggregation

- Entrepreneurial preferences:

$$u(c, e) = \log(c - \psi e) \qquad \psi > 0$$

  - GHH preferences with no income effects $\longrightarrow$ Aggregation
  - ψ plays an important role: Cost of effort in consumption units

Extension: Entrepreneurial Effort

Back to extensions

Problem is isomorphic to having preferences

$$u\left(\hat{c}\right) = \log \hat{c} \qquad \text{where } \hat{c} = c - \psi e$$

and modifying entrepreneurial problem to:

$$\hat{\pi}\left(z, k\right) = \max_{n,e} y - wn - rk - \underbrace{\frac{\psi}{1-\tau_k}}_{\text{Effecive cost of effort}} e$$

- Solution is just as before (linear policy functions a', n, **and e**)
- **Key:** Effective cost of effort depends on capital income tax τ_k!
    - Effort affects entrepreneurial income
    - Income subject to capital income taxes but not to **book value** wealth taxes

Extension: Entrepreneurial Effort

Back to extensions

- Aggregate effort:

$$E = \left(\frac{(1 - \tau_k)\,\gamma}{\psi} \right)^{\frac{1}{1-\gamma}} (ZK)^{\frac{\alpha}{1-\gamma}} L^{\frac{1-\alpha-\gamma}{1-\gamma}}$$

  - Comparative statics: $K \uparrow$, $Z \uparrow$, and $\tau_k \downarrow$
- New wedge from capital income taxes on aggregate output and wages!
- Effort affects marginal product of capital $\longrightarrow$ Affects K_{ss}

A neutrality result:

- **No changes to steady state productivity!**
- Steady state capital adjusts in background to satisfy:

$$(1 - \tau_k)\,\mathrm{MPK} - \tau_a = \frac{1}{\beta} - 1$$

Extension: Entrepreneurial Effort

Back to extensions

Results:

1. Efficiency gains from wealth taxation remain
2. Effect on aggregates is stronger if capital income taxes go down
   - **Effort increases with wealth taxes** (if $\rho > 0$)**!**
3. Characterization of optimal taxes is similar but
   higher wealth taxes and lower capital incomes taxes are optimal

Extension: Perpetual Youth

Back to extensions

- Baseline model has no stationary distribution

Perpetual youth: Entrepreneurs die with probability $1 - \delta$

- Replaced by new entrepreneur with assets $\overline{a}$ and productivity z_i $(i \in \{h, l\})$
- $\overline{a}$ endogenous: Average bequest (= average wealth).

Extension: Perpetual Youth

Back to extensions

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Solution:

- Entrepreneur's savings choice: $a' = \beta\delta R(z)\,a$.
- Aggregate law of motion: $A_i' = \beta\delta^2 R_i A_i + (1-\delta)\,\overline{a}$
  - Depends only on R_i!
- Similar characterization of SS and aggregates

Extension: Perpetual Youth

Back to extensions

Effects of wealth taxation:

- Efficiency gains from wealth taxation "always" (bc productivity is persistent)
- Increase return dispersion: $R_\ell \downarrow + R_h \uparrow$

Extension: Perpetual Youth

Back to extensions

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Welfare and optimal taxes:

$$\sum_a \left(V_k(a,i) + \frac{\log\left(1 + \mathrm{CE}_{2,i}\right)}{1-\beta\delta} \right) \Gamma_k(a,i) \quad = \quad \sum_a V_a(a,i)\,\Gamma_a(a,i)$$

- Consumption equivalent measure takes into account asset levels!

$$\log\left(1 + \mathrm{CE}_{2,i}\right) = \frac{1-\beta\delta^2}{(1-\delta)(1-\beta\delta)} \log \frac{R_{a,i}}{R_{k,i}} + \log \frac{K_a}{K_k}.$$

Extension: Perpetual Youth

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- High-productivity entrepreneurs always benefit from wealth taxes
- Optimal taxes are higher $\longrightarrow$ Include gains of capital accumulation

Extension: Perpetual Youth

Back to extensions