

# Book-Value Wealth Taxation, Capital Income Taxation, and Innovation

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Fatih Guvenen, Gueorgui Kambourov, Burhan Kuruscu, Sergio Ocampo

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**This paper:** **Theoretical analysis** of optimal **combination** of taxes

- ▶ Analytical model with workers, heterogeneous entrepreneurs, and innovation
- ▶ **Result:** characterize **(i)** productivity **(ii)** welfare **(iii)** optimal taxes **(iv)** innovation

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2. **Technical:** Capital taxes paid by the very wealthy.

- **But** models struggle to generate plausible wealth inequality.

Pareto Tail vs. Models

- Return heterogeneity → concentration at very top + Pareto tail + fast wealth growth

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4. **Theoretical:** Interesting **new economic mechanisms** → Example next

*Allais (1977), Guvenen, Kambourov, Kuruscu, Ocampo, Chen (2023)*

## Return Heterogeneity: A Simple Example

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- ▶ Government taxes to finance  $G = \$50K$ .
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- ▶ **Objective:** illustrate key tradeoffs b/w capital income tax ( $\tau_k$ ) and wealth tax ( $\tau_a$ )

# Capital Income ( $\tau_k$ ) vs. Wealth Tax ( $\tau_a$ )

| Capital Income Tax                            |                       |                       |  |
|---|-----------------------|-----------------------|--|
| $a_{i,after-tax} = a_i + (1 - \tau_k)r_i a_i$ |                       |                       |  |
|   | Fredo ( $r_f = 0\%$ ) | Mike ( $r_m = 20\%$ ) |  |
| Wealth  | \$1M                  | \$1M                  |  |
| Before-tax Income                             | \$0                   | \$200K                |  |
| Tax liability                                 |                       |                       |  |
| After-tax return                              |                       |                       |  |
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| After-tax wealth ratio                               | 1.15 (= $1150/1000$ )    |                                |  |



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- Replacing  $\tau_k$  with  $\tau_a \rightarrow$  **reallocates** assets to high-return agents (use it or lose it) + **increases dispersion** in after-tax returns & wealth.

1. **Benchmark model with exogenous entrepreneurial productivity process**
2. Efficiency gains from wealth taxation
3. Welfare and optimal taxation
4. Models with endogenous entrepreneurial productivity

# Perpetual Youth Model with Workers and Entrepreneurs

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  - Produce final goods using capital and labor + consume/save
  - Heterogeneity in productivity ( $z$ ) and wealth ( $a$ )
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**Preferences** (of workers and entrepreneurs):

$$E_0 \sum_{t=0}^{\infty} (\beta\delta)^t \log(c_t)$$

where  $\beta < 1$  and  $\delta < 1$  is the conditional survival probability

## Entrepreneurial technology:

$$y_i = (z_i k_i)^\alpha n_i^{1-\alpha}$$

- ▶ Productivity  $z_i \in \{z_\ell, z_h\}$ , where  $z_h > z_\ell \geq 0$
- ▶ Each entrepreneur draws  $z_i$  randomly at birth
  - $\mu$  fraction of entrepreneurs have  $z_i = z_h$ ,  $1 - \mu$  have  $z_i = z_\ell$
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# Technology, Production, and Taxes

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**Aggregate output:**  $Y = \int y_i di = \int (z_i k_i)^\alpha n_i^{1-\alpha} di$

**Government:** Finances exogenous expenditure  $G$  and transfers  $T$  with  $\tau_k$  and  $\tau_a$

# Financial Markets & Entrepreneurs' Problem

## Financial markets:

- ▶ Collateral constraint:  $k \leq \lambda a$ , where  $a$  is entrepreneur's wealth and  $\lambda \geq 1$
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## Entrepreneurs' production decision:

▶ details

$$\Pi^*(z, a) = \max_{k \leq \lambda a, n} \{ (zk)^\alpha n^{1-\alpha} - rk - wn \} \quad \longrightarrow \quad \Pi^*(z, a) = \underbrace{\pi^*(z)}_{\text{Excess return above } r} \times a$$

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Unique equilibrium with **return heterogeneity**, **capital misallocation** + Empirically relevant



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$$\text{If } \underbrace{(\lambda - 1) \mu A_h}_{K \text{ Demand from H-Type}} < \underbrace{(1 - \mu) A_\ell}_{K \text{ Supply from L-Type}} \iff \underbrace{\lambda < \bar{\lambda}}_{\text{Bound on Leverage}} \iff \tau_a < \bar{\tau}_a$$

# Entrepreneur's Dynamic Problem

$$V(a, z) = \max_{c, a'} \log(c) + \beta \delta V(a', z)$$
$$\text{s.t.} \quad c + a' = \underbrace{(1 - \tau_a) a + (1 - \tau_k)(r + \pi^*(z)) a}_{\text{After-tax wealth}}.$$

► Define (after-tax) gross return as:

$$R_i \equiv (1 - \tau_a) + (1 - \tau_k)(r + \pi^*(z_i)) \quad \text{for } i \in \{\ell, h\}$$

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**Note:** log utility  $\rightarrow$  No behavioral response to taxes.  
 $\rightarrow$  All effects come from use-it-or-lose-it (*conservative lower bound*)

# Equilibrium Values: Aggregation

## Key variables:

- ▶  $s_h = \frac{\mu A_h}{\mu A_h + (1 - \mu) A_\ell}$ : wealth share of high-productivity entrepreneurs.
- ▶  $z_\lambda \equiv z_h + (\lambda - 1)(z_h - z_\ell)$ : effective productivity of high-productivity entrepreneurs.

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**Lemma:** Aggregate output can be written as:

$$Y = (\textcolor{blue}{Z}\textcolor{blue}{K})^\alpha L^{1-\alpha} \quad (Z^\alpha \text{ is measured TFP})$$

where

$$\textcolor{blue}{K} \equiv \mu A_h + (1 - \mu) A_\ell$$

$K$  = Aggregate capital

$$\textcolor{red}{Z} \equiv s_h z_\lambda + (1 - s_h) z_\ell$$

$Z$  = Wealth-weighted productivity

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where

$$\begin{aligned} K &\equiv \mu A_h + (1 - \mu) A_\ell & K &= \text{Aggregate capital} \\ Z &\equiv s_h z_\lambda + (1 - s_h) z_\ell & Z &= \text{Wealth-weighted productivity} \end{aligned}$$

**Note:** Use it or lose it effect increases efficiency if  $s_h \uparrow (\longrightarrow Z \uparrow)$

## Steady State: Capital, Returns, and Taxes

**Steady State  $K$ :** Same as Neoclassical Growth Model... but endogenous  $Z$  (Moll, 2014)

$$(1 - \tau_a) + (1 - \tau_k) \overbrace{\alpha Z^\alpha (K/L)^{\alpha-1}}^{\text{MPK}} = \frac{1}{\beta\delta}$$



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**Steady State  $R$ :** Returns reflect MPK + effective entrepreneurial productivity  $z_i \in \{z_\ell, z_\lambda\}$

$$R_i = (1 - \tau_a) + \overbrace{\left( \alpha Z^\alpha (K/L)^{\alpha-1} \right)}^{\text{MPK}} \frac{z_i}{Z} \longrightarrow R_i = (1 - \tau_a) + \left( \frac{1}{\beta\delta} - (1 - \tau_a) \right) \frac{z_i}{Z}$$

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**Steady State  $Z$ :** Returns + evolution of assets imply this quadratic equation:

$$(1 - \delta^2 \beta (1 - \tau_a)) Z^2 - [(1 - \delta) (\mu z_\lambda + (1 - \mu) z_\ell) + \delta (1 - \delta \beta (1 - \tau_a)) (z_\lambda + z_\ell)] Z + \delta (1 - \delta \beta (1 - \tau_a)) z_\ell z_\lambda = 0$$

- **Wealth tax affects** returns, wealth shares, productivity. **Capital income tax does not.**
- Both taxes affect capital, output, wages...

1. Benchmark model with exogenous entrepreneurial productivity process
2. **Efficiency gains from wealth taxation**
3. Welfare and optimal taxation
4. Models with **endogenous** entrepreneurial productivity

# Main Result 1: Efficiency Gains from Wealth Taxation

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Proof

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► Average return decreases:

$$\mu \frac{d \log R_h}{d\tau_a} + (1 - \mu) \frac{d \log R_\ell}{d\tau_a} < 0$$

# Government Budget and Aggregate Variables

$$G + T = \tau_k \alpha Y + \tau_a K.$$

- In what follows,  $\tau_k$  adjusts in the background when  $\tau_a \uparrow$  so that  $G + T = \theta \alpha Y$

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- **Increases** capital ( $K$ ), output ( $Y$ ), wage ( $w$ ), & high-type wealth ( $A_h$ )
- **Key:** Higher  $\alpha \longrightarrow$  Larger pass-through of productivity to  $K$ ,  $Y$ ,  $w$

$$\xi_Z^K = \xi_Z^Y = \xi_Z^w = \frac{\alpha}{1 - \alpha} \quad \xi_Z^x = \frac{d \log x}{d \log Z}$$

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## Main Result 3: Optimal Taxes

$\alpha$  thresholds

**Objective:** Choose taxes  $(\tau_a, \tau_k)$  to max newborn welfare ( $n_w = L/(1+L)$  pop. share of workers)

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### Key trade-off:

► Welfare by type

1. **Higher levels** of worker income ( $w + T$ ) and wealth ( $\bar{a} = K$ ) — Depends on  $\alpha$ !  
(higher welfare for workers and high- $z$  entrepreneurs)
2. **Lower wealth growth** over lifetime from lower average return — Depends on  $\tau_a$   
(lower welfare for low- $z$  entrepreneurs and entrepreneurs as a group)



## Main Result 3: Optimal Taxes

[▶ Diagram](#)[▶  \$\alpha\$  thresholds](#)[▶  \$\tau\_a^\*\$  level](#)

**Proposition:** There exists a **unique** optimal tax combination  $(\tau_a^*, \tau_k^*)$  that maximizes  $\mathcal{W}$ .

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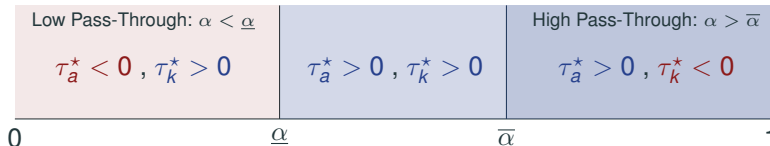
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## Model with Innovation Effort

- ▶ Interpret productivity  $z_i$  as the outcome of a **risky innovation** process
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### Innovator's problem:

$$\max_e \tilde{\mu}(e) V_h(\bar{a}) + (1 - \tilde{\mu}(e)) V_\ell(\bar{a}) - \frac{1}{(1 - \beta\delta)^2} \Lambda(e); \quad \Lambda(e) \text{ convex} + C^2; \tilde{\mu}(e) = e$$

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## Optimal innovation effort:

$$\underbrace{\Lambda'(e)}_{\text{Mrg. Cost of Effort}} = (1 - \beta\delta)^2 (V_h(\bar{a}) - V_\ell(\bar{a})) = \underbrace{\log R_h - \log R_\ell}_{\text{Mrg. Benefit: Return Gap}}$$

- Return dispersion incentivizes effort  $\rightarrow$  Return dispersion necessary for innovation!

# Stationary Equilibrium with Innovation

The stationary equilibrium share high-productivity entrepreneurs,  $\tilde{\mu}$ , solves

$$\tilde{\mu} = e(Z(\tilde{\mu})), \text{ where}$$

- i.  $Z(\tilde{\mu})$  gives the steady state productivity given  $\tilde{\mu}$ .
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**We show:**

- i. There exists a unique equilibrium with innovation.
- ii. An increase in wealth taxes  $\tau_a$  increase  $\tilde{\mu}$  and  $Z$  (+  $\tilde{\mu}$  and  $Z$  are independent of  $\tau_k$ )

$$\uparrow \tau_a \longrightarrow \uparrow Z \quad + \quad \uparrow \text{Return Dispersion} \longrightarrow \uparrow \text{Innovation (e)} \longrightarrow \uparrow \tilde{\mu} \longrightarrow \uparrow\uparrow Z$$



**Objective:** Choose  $(\tau_a^*, \tau_k^*)$  to maximize newborn welfare net of innovation costs

$$\mathcal{W} \equiv n_w V_w(w) + (1 - n_w) \left( \tilde{\mu} V_h(\bar{a}) + (1 - \tilde{\mu}) V_\ell(\bar{a}) - \frac{\Lambda(\tilde{\mu})}{(1 - \beta\delta)^2} \right)$$

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- **Innovation effect** increase lifetime wealth growth by increasing average return
- Optimal tax combination has higher wealth taxes:  $\tau_a^* \uparrow$



- Entrepreneurial effort in **production**: (maintain CRS)

$$y = (zk)^\alpha \mathbf{e}^\gamma n^{1-\alpha-\gamma} \longrightarrow \mathbf{e}: \text{effort}$$

- Entrepreneurial **preferences**: (avoid income effects)

$$u(c, \mathbf{e}) = \log(c - \psi \mathbf{e}) \quad \psi > 0$$

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Entrepreneurial problem becomes:

$$\hat{\pi}(z, k) = \max_{n, e} \left\{ y - wn - rk - \underbrace{\frac{\psi}{1 - \tau_k} e}_{\text{Effective Cost of Effort}} \right\}$$

- **Key:** Effective cost of effort *increases* with capital income tax  $\tau_k$  but not with  $\tau_a$ !

# Model with Entrepreneurial Effort: Results

## 1. Efficiency gains from wealth taxation go through

- Neutrality holds  $\left( (1 - \tau_k) \text{MPK} = \frac{1}{\beta\delta} - (1 - \tau_a) \right) \rightarrow Z, R_h, R_\ell$  depend only on  $\tau_a$ !

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3. Optimal taxes: **higher wealth tax** and **lower capital income tax**

# Conclusions

## Increasing $\tau_a$ (& reducing $\tau_k$ ):

- ▶ **Use it or Lose it Effect:** Reallocates capital from less to more productive agents.
  - Higher TFP, output, and wages;
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## Extensions:

Stochastic Productivity

Corporate Sector

Rents

# Extra

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6. Extensions

# Entrepreneur's Problem

# Financial Markets & Entrepreneurs' Production Problem

## Entrepreneurs' Production Decision:

$$\Pi^*(z, a) = \max_{\substack{k \leq \lambda a, n}} (zk)^\alpha n^{1-\alpha} - rk - wn.$$



## Entrepreneurs' Production Decision:

**Solution:**  $\Pi^*(z, a) = \underbrace{\pi^*(z)}_{\text{Excess return above } r} \times a$

$$\pi^*(z) = \begin{cases} (MPK(z) - r) \lambda & \text{if } MPK(z) > r \\ 0 & \text{otherwise.} \end{cases} \quad k^*(z) \begin{cases} = \lambda a & \text{if } MPK(z) > r \\ \in [0, \lambda a] & \text{if } MPK(z) = r \\ = 0 & \text{if } MPK(z) < r \end{cases}$$

►  $(\lambda - 1)a$ : amount of external funds used by type- $z$  if  $MPK(z) > r$ .

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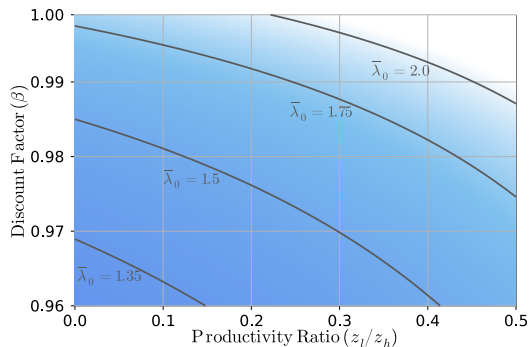
Condition implies an upper bound on wealth taxes:

[▶ Upper Bound on  \$\tau\_a\$](#) 

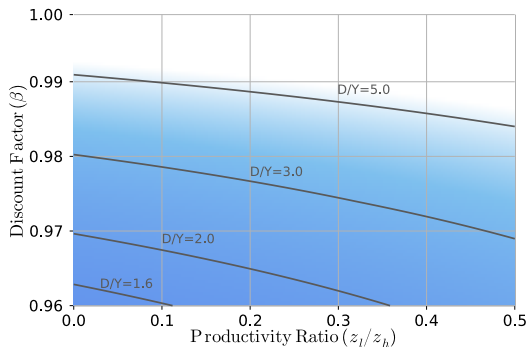
$$(\lambda - 1)\mu A_h < (1 - \mu) A_\ell \iff \tau_a < \bar{\tau}_a = 1 - \frac{1}{\beta\delta} \left( 1 - \frac{1-\delta}{\delta} \frac{1-\lambda\mu}{(\lambda-1)\left(1-\frac{z_\ell}{z_h}\right)} \right)$$

# FIGURES

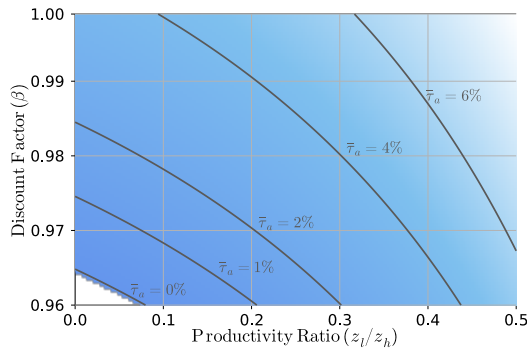
## Threshold $\bar{\lambda}_0$



## Debt-to-Output Ratio ( $\lambda = \bar{\lambda}_0$ )

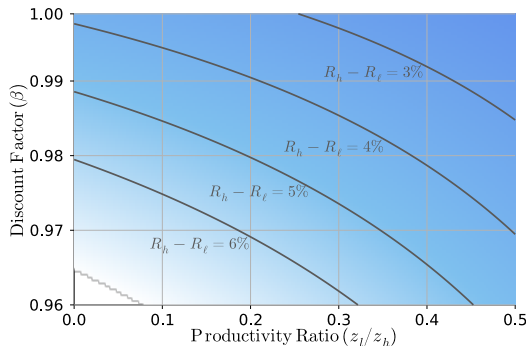


## Upper Bound on Wealth Tax $\bar{\tau}_a$





## Dispersion of Returns in Equilibrium, $R_h - R_\ell$

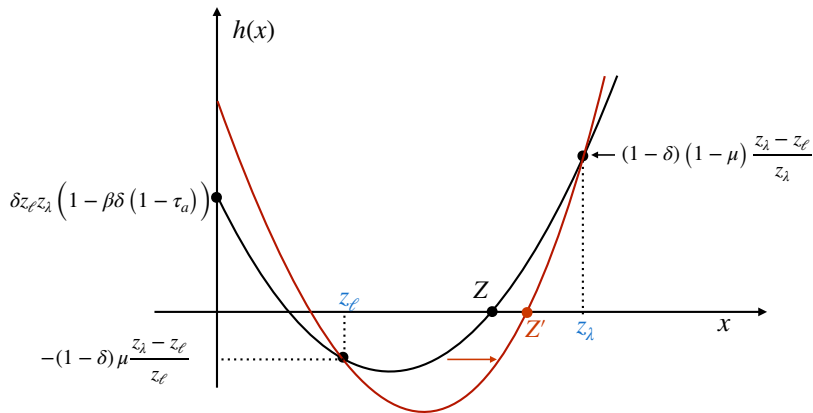


**Note:** The figure reports the value return dispersion in steady state for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_\ell/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_K = 25\%$ , and  $\alpha = 0.4$ .

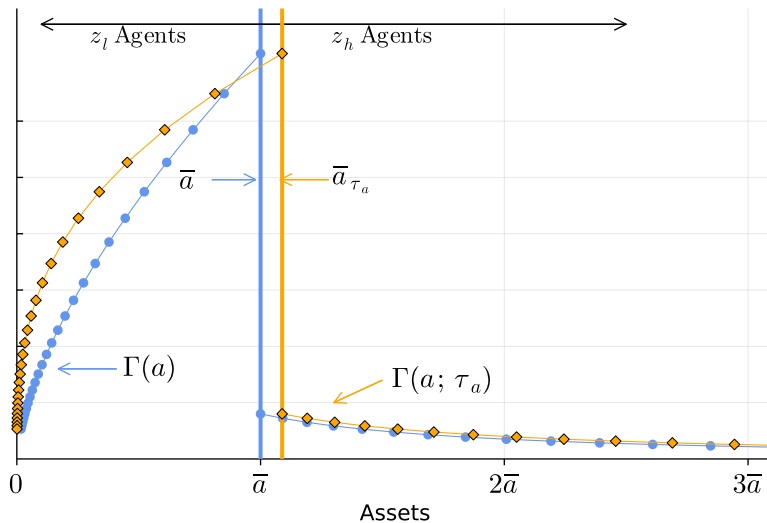
# What happens to $Z$ if $\tau_a \uparrow$ ?

Back to eff. gain

$$\frac{dh(x)}{d\tau_a} = \beta\delta^2 (1 - \tau_a) (x - z_\ell) (z_\lambda - x) < 0 \text{ iff } z_\ell < x < z_\lambda$$



# Stationary wealth distribution and wealth taxes

[back](#)

# Welfare Gains

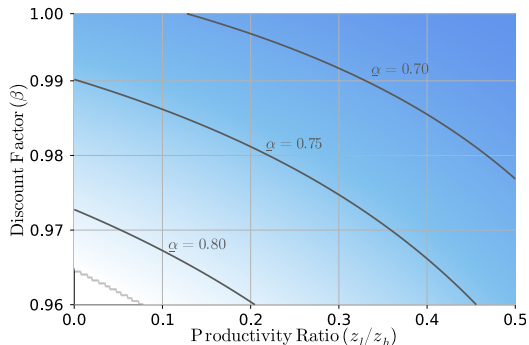
### Proposition:

[▶  \$\alpha\$  Thresholds](#)

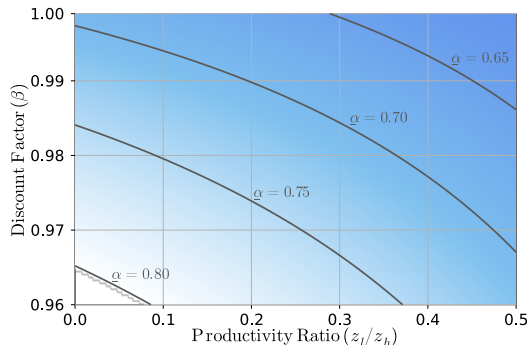
For all  $\tau_a < \bar{\tau}_a$ , a higher  $\tau_a$  changes welfare as follows:

- ▶ Workers: Higher welfare:  $\frac{dV_{workers}}{d\tau_a} > 0$
- ▶ High-z entrepreneurs: Higher welfare  $\left(\frac{dV_h(\bar{a})}{d\tau_a} > 0\right)$  because  $\xi_Z^K + \frac{1}{1-\beta\delta}\xi_Z^{R_h} > 0$
- ▶ Low-z entrepreneurs: Lower welfare  $\left(\frac{dV_\ell(\bar{a})}{d\tau_a} < 0\right)$  iff  $\xi_Z^K + \frac{1}{1-\beta\delta}\xi_Z^{R_\ell} < 0$ ;  $\alpha < \underline{\alpha}_\ell$
- ▶ Entrepreneurs: Lower average welfare iff  $\xi_Z^K + \frac{1}{1-\beta\delta} \left(\mu\xi_Z^{R_h} + (1-\mu)\xi_Z^{R_\ell}\right) < 0$ ;  $\alpha < \underline{\alpha}_E$

Low-Productivity Entrepreneurs:  $dV_\ell/d\tau_a > 0$



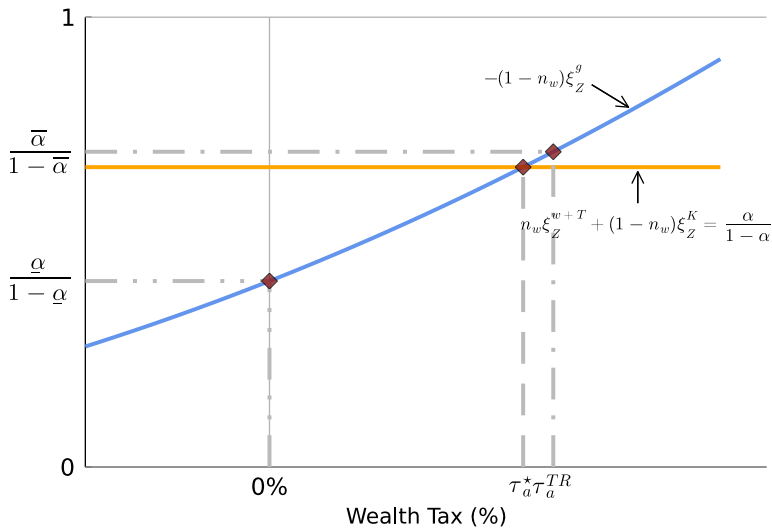
Average Entrepreneur:  $dV_E/d\tau_a > 0$



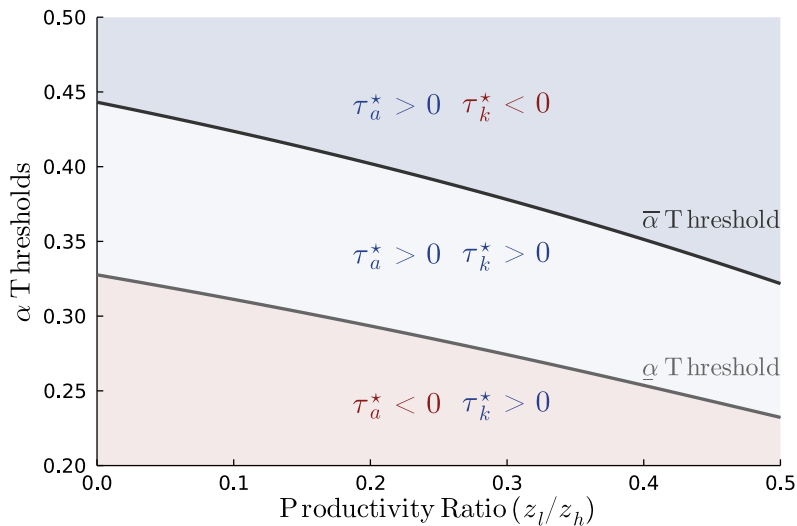
**Note:** The figures report the threshold value of  $\alpha$  above which entrepreneurial welfare increases after an increase in wealth taxes for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_\ell/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_k = 25\%$ , and  $\alpha = 0.4$ .

# Optimal Taxes

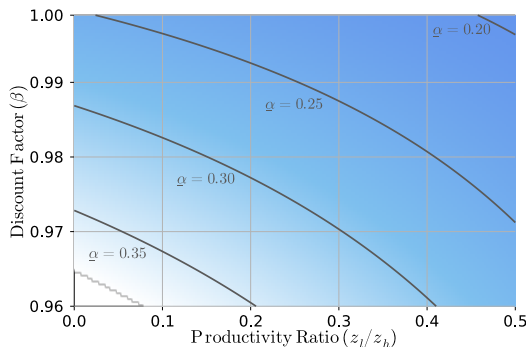
# Optimal Tax and $\underline{\alpha}$ and $\bar{\alpha}$ Thresholds



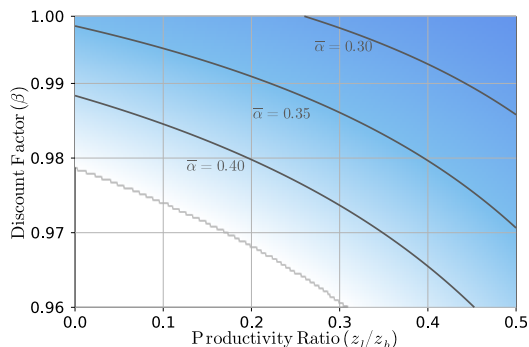




Lower Threshold  $\underline{\alpha}$  for  $\tau_a^* > 0$

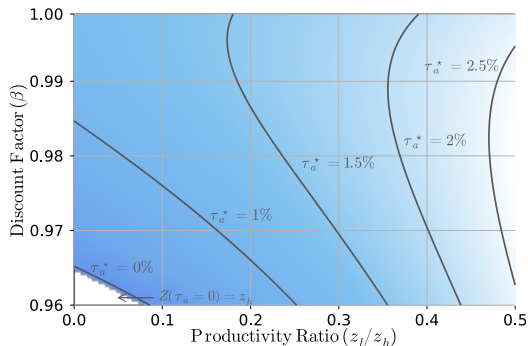


Upper Threshold  $\bar{\alpha}$  for  $\tau_k^* < 0$



**Note:** The figures report the threshold value of  $\alpha$  for the optimal wealth taxes to be positive (left) and capital income taxes to be positive (right) for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_l/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_k = 25\%$ , and  $\alpha = 0.4$ .

Optimal Wealth Tax  $\tau_a^*$



**Note:** The figure reports the value of the optimal wealth tax for combinations of the discount factor ( $\beta$ ) and productivity dispersion ( $z_l/z_h$ ). We set the remaining parameters as follows:  $\delta = 49/50$ ,  $\beta\delta = 0.96$ ,  $\mu = 0.10$ ,  $z_h = 1$ ,  $\tau_k = 25\%$ , and  $\alpha = 0.4$ .

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# Extensions

- ▶ Technology:  $Y_c = (z_c K_c)^\alpha L_c^{1-\alpha}$ 
  - No financial constraints!
- ▶ Corporate sector imposes lower bound on  $r$ :

$$r \geq \alpha z_c \left( \frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}}.$$

**Interesting case:**  $z_\ell < z_c < z_h$

- ▶ Corporate sector and high-productivity entrepreneurs produce
- ▶ Low-productivity entrepreneurs lend all of their funds.
- ▶ No real changes in the aggregates of the economy!  $z_c$  takes the place of  $z_\ell$

$$Y = (ZK)^\alpha L^{1-\alpha}$$

but now  $Z = s_h z_\lambda + s_l z_c$ , where  $z_\lambda = z_h + (\lambda - 1)(z_h - z_c)$ .

- ▶ Introduce wedge for returns above/below productivity:

$$R_i = (1 - \tau_a) + (1 - \tau_k) \underbrace{(1 + \omega_i)}_{\text{Return Wedge}} \alpha (ZK/L)^{\alpha-1} z_i$$

- ▶ Zero-sum condition on wedges:  $\omega_l z_\ell A_\ell + \omega_h z_h A_h = 0$
- ▶ Characterization of eq. in terms of “effective productivity”  $\tilde{z}_i = (1 + \omega_i) z_i$

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### Proposition:

For all  $\tau_a < \bar{\tau}_a$ , a marginal increase in wealth taxes ( $\tau_a$ ) increases  $Z$ ,  $\frac{dZ}{d\tau_a} > 0$ , **iff**

1.  $\rho > 0$  and  $R_h > R_\ell \longrightarrow$  Same mechanism as before
2.  $\rho < 0$  and  $R_h < R \longrightarrow$  Reallocates wealth to the true high types next period



► Entrepreneurial production:

$$y = (zk)^\alpha e^\gamma n^{1-\alpha-\gamma} \longrightarrow e: \text{effort}$$

- Production functions is CRS  $\longrightarrow$  Aggregation

► Entrepreneurial preferences:

$$u(c, e) = \log(c - \psi e) \quad \psi > 0$$

- GHH preferences with no income effects  $\longrightarrow$  Aggregation
- $\psi$  plays an important role: Cost of effort in consumption units

Problem is isomorphic to having preferences

$$u(\hat{c}) = \log \hat{c} \quad \text{where } \hat{c} = c - \psi e$$

and modifying entrepreneurial problem to:

$$\hat{\pi}(z, k) = \max_{n, e} y - wn - rk - \underbrace{\frac{\psi}{1 - \tau_k}}_{\text{Effective cost of effort}} e$$

- ▶ Solution is just as before (linear policy functions  $a'$ ,  $n$ , **and**  $e$ )
- ▶ **Key:** Effective cost of effort depends on capital income tax  $\tau_k$ !
  - Effort affects entrepreneurial income
  - Income subject to capital income taxes but not to **book value** wealth taxes

- ▶ Aggregate effort:

$$E = \left( \frac{(1 - \tau_k) \gamma}{\psi} \right)^{\frac{1}{1-\gamma}} (ZK)^{\frac{\alpha}{1-\gamma}} L^{\frac{1-\alpha-\gamma}{1-\gamma}}$$

- Comparative statics:  $K \uparrow$ ,  $Z \uparrow$ , and  $\tau_k \downarrow$

- ▶ New wedge from capital income taxes on aggregate output and wages!
- ▶ Effort affects marginal product of capital  $\rightarrow$  Affects  $K_{ss}$

### A neutrality result:

- ▶ **No changes to steady state productivity!**
- ▶ Steady state capital adjusts in background to satisfy:

$$(1 - \tau_k) \text{MPK} - \tau_a = \frac{1}{\beta\delta} - 1$$

### Results:

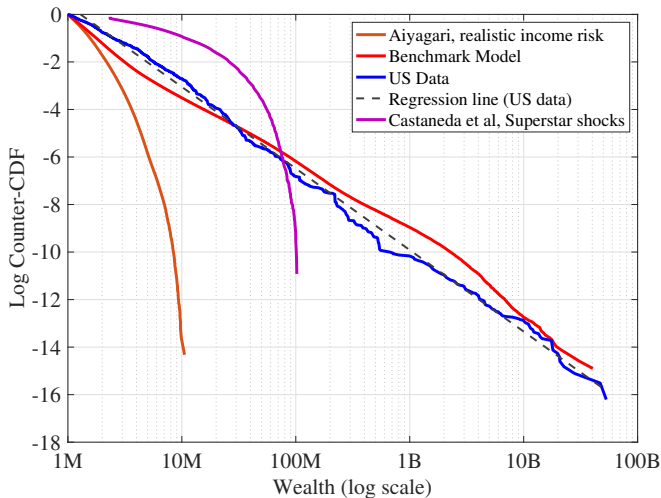
1. Efficiency gains from wealth taxation remain
2. Effect on aggregates is stronger if capital income taxes go down

■ **Effort increases with wealth taxes:**

$$E = \left( \frac{(1 - \tau_k) \gamma}{\psi} \right)^{\frac{1}{1-\gamma}} (ZK)^{\frac{\alpha}{1-\gamma}} L^{\frac{1-\alpha-\gamma}{1-\gamma}}$$

3. Optimal taxes: **higher wealth tax** and **lower capital income tax**

# Pareto Tail of Wealth Distribution: Model vs. Data

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**Note:** Both axes are in natural logs.

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