Markup Accounting*

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Abstract

We document new facts of markup dispersion across the firm size distribution. Markup differences between firms of similar size accounts for over three-fourths of overall dispersion, calling for mechanisms that affect markups beyond differences in firm productivity and market concentration. To study these mechanisms, we develop an analytical oligopolistic competition model of variable markups that accounts for the observed joint distribution of markups and firm size, including the large mass of small firms with high markups and the presence of large firms with small markups. The equilibrium markup of a firm in the model depends on market structure, the size and demand elasticities of competitors, firm scale, and firm-specific demand elasticity shifters. These shifters are key for accounting for the variation in markups between similarly-sized firms. Absent this source of heterogeneity, the model captures less than half of the observed variation in markups and counterfactually assigns most of the variation to differences between firms of different size. We apply this model to Indian and Colombian manufacturing, and US public firms. The model implies a negative correlation between marginal costs and markups, consistent with standard models of oligopolistic competition, but also a positive correlation between market shares and marginal costs, indicating that large firms are not necessarily the most productive.

JEL: D2, D4, E2, L1, O4.
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1. Introduction

There is substantial variation in markups both between firms of different sizes and among firms of comparable size. We document new facts of the distribution of markups, showing that most of the dispersion in markups is concentrated among firms with similar sizes, with within firm size dispersion accounting for over three-fourths of the standard deviation of markups. In particular, there is a large dispersion in markups among small firms, including firms charging the highest markups in their respective market, as well as dispersion among large firms, including firms charging markups below their market's average. These new facts call for forces beyond those directly linked to the scale of firms in accounting for markups, such as differences in market power coming from market concentration (see, among others, Atkeson and Burstein 2008; Edmond, Midrigan, and Xu 2023) or non-CES demand functions (see, among others, Kimball 1995; Matsuyama 2023; Baqaee, Farhi, and Sangani 2023).

We disentangle the roles of market concentration, firm heterogeneity, and demand factors in explaining markup dispersion by developing an oligopolistic competition model of variable markups with non-CES demand that accounts for the joint distribution of markups and firm sizes. We find a prominent role for demand factors that shift the elasticity of demand faced by firms in accounting for the dispersion of markups, particularly within firm size. Without these factors, the model generates a counterfactually low dispersion driven by differences between firms of different sizes.

We model oligopolistic product-markets with finitely many firms that differ in their marginal costs, as in Atkeson and Burstein (2008). Firms face demand functions with variable elasticity derived from a "*homothetic direct implicit additivity*" type aggregator (Matsuyama 2023) that generalizes the demand function in Kimball (1995) by introducing idiosyncratic demand elasticity shifters. We solve analytically for equilibrium markups that depend jointly on market structure—through firms' market shares and elasticities of demand—and firms' idiosyncratic factors—namely their marginal costs and demand elasticity shifters. Through these factors, we account for small (or otherwise less productive) firms that have high markups as well as large firms that have low markups, both prominent features of the data. Thus, the two dimensions of firm heterogeneity we model break the standard link between markups and productivity and make it possible to account for the markup distribution.[1]

We estimate markups for Indian, US, and Colombian firms, with different methodologies and use our model to decompose markups into the roles of market concentration, firm scale, and idiosyncratic forces shaping demand elasticities. Our main application is to Indian manufacturing data from the *Annual Survey of Industries* (ASI). The Indian data is particularly advantageous as it includes the price and quantity of products being sold, allowing us to overcome biases in markup estimates generated by estimation based only on revenue (Bond, Hashemi, Kaplan, and Zoch 2021) and to define product markets in which firms compete (see Smith and Ocampo 2025 for a discussion of product and industry-based markets). We estimate markups building on the methodology of De Loecker, Goldberg, Khandelwal, and Pavcnik (2016) and Ackerberg, Caves, and Frazer (2015) using data on single-product firms for our estimation of production functions, avoiding well-known input-allocation biases present in multi-product firms.

The role of within firm size dispersion in markups in driving the degree of markup variation in the data results in a prominent role for idiosyncratic demand elasticity shifters when accounting for the distribution of markups. To show this, we exploit the analytical solution of equilibrium markups together with observed changes in markups and market shares to identify the super-elasticity of demand. The super-elasticity is the relevant measure for how much the elasticity of demand (and hence markups)

[1] In theories of variable markups with a single dimension of firm heterogeneity, typically productivity, only high-productivity—low marginal cost—firms can charge low prices that lead to higher scale of production and market shares, both resulting in higher markups.

vary with firms' scale and thus of the degree of idiosyncratic variation in elasticities beyond the forces embedded in the market aggregator and the market structure through concentration.

To better understand the forces at play, we conduct a decomposition of the observed markup and firm size distribution into the roles of market structure, size, and elasticity shifters. We exploit the fact that the model includes as limiting cases the Atkeson and Burstein (2008) case in which markups depend only on market concentration, and the Kimball (1995) case in which markups depend only on the location of firms' output along a common demand function with variable elasticity, and a case new to the literature where both forces are present. These cases differ from our full model in the absence of elasticity shifters. We find models without shifters capture less than half of the observed dispersion in markups and counterfactually assign two-thirds of the variation to between-firm size variation. Moreover, the markups implied by these models have a correlation of less than 0.25 with measured markups even when matching the same market-level markups. We reproduce these results with data for US public firms following the methodology of De Loecker, Eeckhout, and Unger (2020) and data for Colombian manufacturing firms following the methodology of Raval (2023).[2]

Finally, we contrast the resulting joint distribution of markups, marginal costs, and firm size. Markups are on average higher for more productive firms—with lower marginal costs—in line with standard models of variable markups, but this relationship is not strong as there is a lot of variation in marginal costs for a given level of markups. Nevertheless, productive firms do not command a higher market share on average.[3] In fact, market share is on average higher for low productivity firms, despite the large variation in marginal costs among small firms. These facts are reconciled by the role

[2]Although we rely on revenue data in our applications to US and Colombian data, the results over the dispersion of markups remain robust in line with Grassi, De Ridder, and Morzenti (2025).

[3]See Hubmer, Chan, Ozkan, Salgado, and Hong (2025) and Haddara (2026) for evidence of low productivity among large firms in Canadian administrative data.

that demand elasticity shifters play in our model in separating markups from market shares and firm scale, allowing small firms to have high markups and large firms to have low markups.

Accounting for the distribution of markups thus requires the presence of large, unproductive firms and small, high-markup firms in equilibrium. We show how this can be achieved by introducing heterogeneity in the demand firms face into an otherwise standard model of oligopolistic competition. In this way, our results complement standard theories of firm dynamics and imperfect competition with a single-dimension of heterogeneity, in line with results for Chilean exporters as in Blum, Claro, Horstmann, and Rivers (2023), and theories of firm dynamics emphasizing competition in the acquisition and lock-in of customers, as in Hubmer and Nord (2025), Casal (2026), and Haddara (2026).

Our results also bring new margins for understanding the misallocation resulting from the dispersion of markups within firm sizes as opposed to the dispersion of markups across firms of different sizes (as in Edmond, Midrigan, and Xu 2023; Baqaee, Farhi, and Sangani 2023, among others). Large firms are not necessarily the most productive and small but productive firms can be held back from scaling due to particularly low elasticities of demand. This complements a long-standing literature on the optimality of markup dispersion (see, for example, Dixit and Stiglitz 1977; Dhingra and Morrow 2019) by highlighting the role of demand, such as consumer behavior leading to markup heterogeneity as in Albrecht, Phelan, and Pretnar (2023) or non-CES preferences as in Zhelobodko, Kokovin, Parenti, and Thisse (2012).

Our paper also contributes methodologically by providing a unified analytical framework that integrates two of the leading workhorse models to study variable markups, namely those based on oligopolistic competition between firms facing demands with a common constant elasticity (see, among others, Atkeson and Burstein 2008; Edmond, Midrigan, and Xu 2015, 2023; Burstein, Carvalho, and Grassi 2025) and those based on monopolistic competition between firms facing a common demand curve with variable elasticity (see, among others, Kimball 1995; Dotsey and King 2005; Barde 2008; Behrens, Mion, Murata, and Suedekum 2020; Boar and Midrigan 2024; Herreño, Pinardon-Touati, and Thie 2025) We provide analytical formulas for equilibrium elasticities, markups, and aggregate market variables that include these models as special cases.

Our focus on accounting for the within firm size dispersion in markups—which makes up most of the dispersion in the data—also complements the literature on markup dispersion arising from search frictions. Although our model accounts for markup dispersion when market power comes from product differentiation and market concentration, market power can also come from frictions that prevent competition even when firms offer identical products (see, among others, Butters 1977; Varian 1980; Burdett and Judd 1983; Menzio 2024; Chernoff, Head, and Lapham 2024). Our contribution resides in highlighting the importance of sources of markup dispersion among similarly-sized firms. These sources are different from those generated by search frictions that stem from firms charging lower prices selling with a higher probability than firms charging higher prices, leading to markup dispersion linked to firm size.

2. Estimating Product-Level Markups

Our main analysis uses price and quantity data from Indian firms to measure firm markups and market structure for sectors within manufacturing. The data comes from the *Annual Survey of Industries* (ASI). Crucially for our purposes, the data not only contains information on firm sales and expenditure in inputs, but also on prices of inputs and outputs, distinguishing between sales of different products for multi-product firms. We use price data to estimate quantity-based production functions that overcome many of the biases introduced when estimating markups only on revenue data (e.g., Bond, Hashemi, Kaplan, and Zoch 2021). We also use data from the Colombian *Encuesta Anual Manufacturera* (EAM) and US public firms for additional results.

The ASI follows manufacturing establishments, with large establishments surveyed annually and small establishments included on a sampling basis.[4] The complete dataset is for the years 2001 to 2008 and has a total of 340,129 establishment-product-year observations with an average of 24,245 unique firms per year.

Data selection at the product level. We focus on *product markets* where manufacturing firms are most likely to compete and where firms are most likely to share a common production technology. To do this, we define markets at the 5-digit product level according to the ASI Commodity Classification (ASICC). There are 5073 unique product codes between 2001 and 2008. These correspond to products such as Plywood, Cotton Yarn, Shoe Leather, Detergent, and Sugar. To keep our analysis tractable, we focus (when possible) on the two largest 5-digit products in each of the two main 3-digit sectors of each 2-digit industry in India. This procedure leaves us with 22 product markets in 14 3-digit sectors. The market share of the selected 3-digit sectors within their corresponding 2-digit industries ranges from 13% to 74%, while the market share of the top two products within 3-digit sectors ranges from 10% to 83%. On average, each product market contains about 970 establishment–product–year observations, and each 3-digit sector contains 2,865 establishment–product–year observations. For a complete overview, see Appendix A.1.

Production function estimation. We estimate quantity-based production functions at the sector-level assuming that products of the same 3-digit sector are produced using

[4] Size is determined based on the number of workers. Between 2001 and 2003, all establishments with more than 200 workers were included; afterward, establishments with more than 100 workers were included. Small establishments are those not below these thresholds.

the same technology. We estimate these production functions building on the work of De Loecker, Goldberg, Khandelwal, and Pavcnik (2016) and Ackerberg, Caves, and Frazer (2015). A central challenge for the estimation is that we do not observe product-level input allocations for multi-product firms. We avoid this input allocation bias by estimating using only single-product firms for which input allocations are known.[5] To control for unobserved input quality, we employ a control function as in De Loecker et al. (2016). We effectively assume high-quality products command higher prices and must be produced with higher-quality inputs. This creates a mapping from observed output prices to unobserved input prices, allowing us to account for quality differences.

Markup measures. We use our estimates of quantity-based production functions to recover markups for all establishments in the products markets we study. We define markups as the ratio of prices to marginal costs, $\mu \equiv p/\lambda$. We do so by exploiting the optimal input choices that come from the cost minimization problem,

$$C\left(y| \left\{ p_n \right\}_{n=1}^{N}, \left\{ K_h \right\}_{h=1}^{H}\right) = \min_{\{x_n\}_{n=1}^{N}} \sum_{n=1}^{N} p_n \cdot x_n \quad \text{s.t. } \overline{y} \leq zF\left(x_1, \ldots, x_N, K_1, \ldots, K_H\right), \tag{1}$$

where production requires variable inputs $\{x_n\}_{n=1}^{N}$ and fixed inputs $\left\{K_h\right\}_{h=1}^{H}$. The marginal cost of production is captured by the Lagrangian multiplier (λ) associated with the production constraint, as $\lambda = C'(y)$.

Simple manipulation of the first order condition with respect to variable input x_n $\left(p_n = \lambda \partial F/\partial x_n\right)$ results in an expression of the markup in terms of the cost-to-revenue

[5] Our procedure assumes that multi-product firms employ the same technology for each product, an approach that precludes any within-firm production complementarities. To maximize the number of single-product firms within each sector we complement the sample of single-product firms with those that produce 5-digit product identified as co-produced with the products in our main sample. These are products that are often produced together by multi-product firms and are hence likely to share a common production function. We estimate using data from 43,300 establishment-year observations obtaining separate estimates for 14 3-digit sectors. See Appendix A.2 for details.

share of the input, $s_n \equiv p_n x_n / py$, and the output elasticity with respect to x_n, ϵ_n:

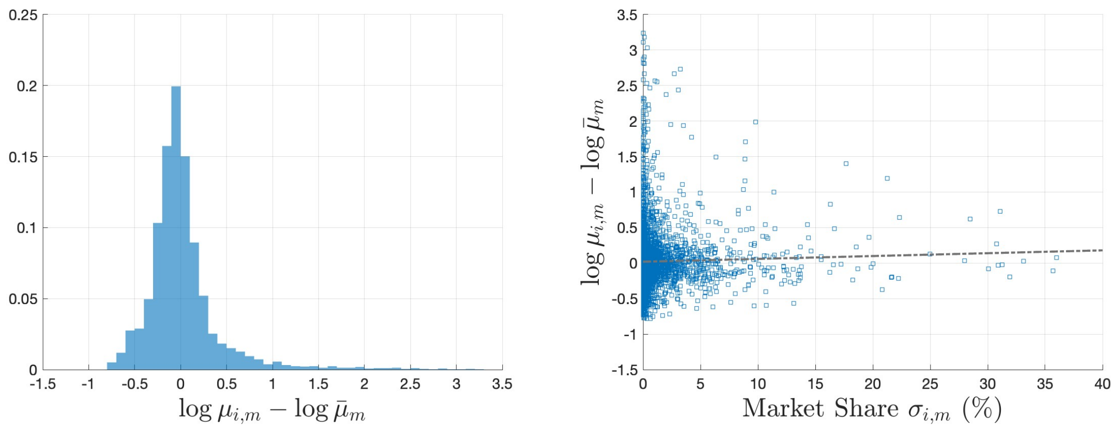
$$\mu = \frac{p}{\lambda} = \frac{py}{p_n x_n}\epsilon_{x_n} = \frac{\epsilon_{x_n}}{s_n}. \tag{2}$$

This condition is entirely compatible with the demand system and the optimal pricing behavior we describe in Section 3. Our data provide us with direct measures of input shares, s_n, and we obtain the elasticity of output with respect to variable inputs, ϵ_{x_n}, from the estimation of quantity-based productions. Together, these estimates allow us to obtain product-level markups for all the firms in our sample.[6]

In additional results using Colombian and US data, we use alternative approaches to measure the elasticity of output with respect to variable inputs ϵ_{x_n} and recover markups as in equation (2). For Colombian data, we follow Raval (2023) and exploit variation in cost shares under the assumption that production functions exhibit constant-returns-to-scale. For the US data, we follow De Loecker, Eeckhout, and Unger (2020). Both of these approaches rely on revenue data, as neither dataset contains information on prices. This introduces well-known biases described in Bond, Hashemi, Kaplan, and Zoch (2021). Nevertheless, we find that our main results regarding the primary role of within firm size groups markup variation and the importance of demand elasticity shifters also hold under these alternative approaches. The robustness of our results is also in line with the findings of Grassi, De Ridder, and Morzenti (2025), who show that revenue-based markup estimates are biased on average but present the same properties in terms of their variability.

FIGURE 1. Markups and Market Share Distribution in Indian Product Markets

(A) Markups Distribution (B) Markups and Market-Share



Notes: Panel A shows the histogram of markups relative to their market's average for the Indian data covering the period 2005–2007. Panel B shows the joint scatter plot of markups and market shares. Markets are defined at the 5-digit ASICC product level.

2.1. Markup dispersion: Between and within firm size

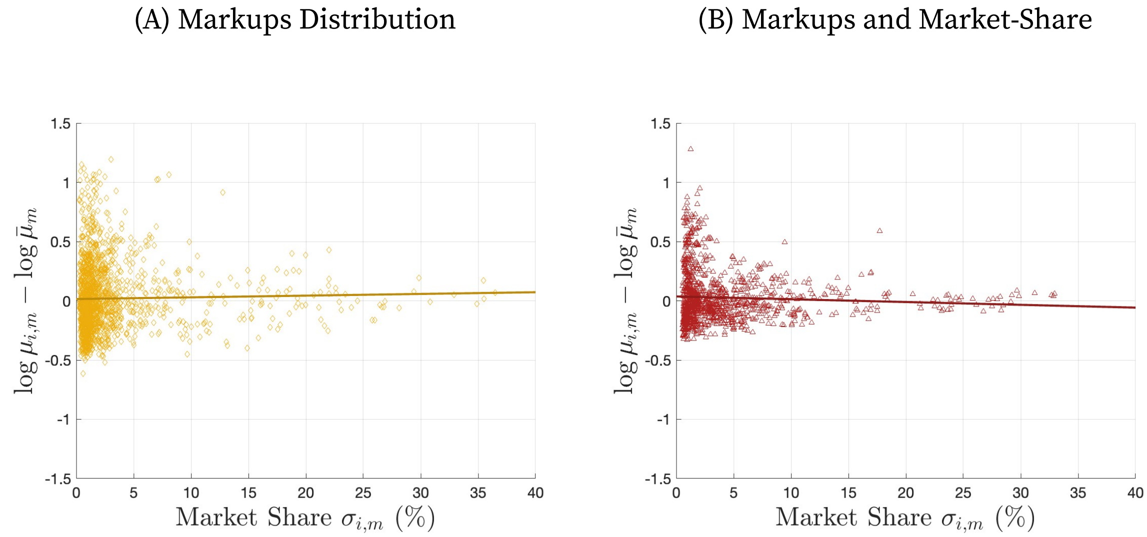
Before turning to our modeling of markup determination, we take a first look at the estimated distribution of markups. Figure 1 shows the distribution of log-markups relative to their market's average, computed as a sales-weighted harmonic mean of product-year markups (Edmond, Midrigan, and Xu 2015; Smith and Ocampo 2025; Thomas Hasenzagl 2023),

$$\mu_m \equiv \left(\sum_{i=1}^{N_m} \frac{\sigma_i^m}{\mu_i^m} \right)^{-1}. \tag{3}$$

The average (sales-weighted) standard deviation of markups across markets is 0.38 log-points.

[6]We assume multi-product firms allocate inputs proportionally to each product's within-firm revenue share, as we do not directly observe input allocations in these firms. For robustness, we also calculated the input allocations generated by the De Loecker et al. (2016) method. We find a correlation close to 0.7 between the revenue shares and the input allocations generated by their model.

FIGURE 2. Markups and Market Share Colombian and US markets



Notes: Panel A shows the joint scatter plot of markups and market shares in Colombia. Markets are defined at the 3-digit ISIC rev 2 classification. Panel B shows the joint scatter plot of markups and market shares in US. Markets are defined at the 3-digit NAICS classification.

Panel B shows that this is a product of both within and between size dispersion in markups.[7] In fact, the figure clearly shows large variation in markups between firms of similar size, particularly among small firms (establishments) that even have the largest measured markups. Conversely, firms with a large market share often have markups below their market's average. This is reflected in the correlation of markups and market shares, which is only 0.06 as we show in Table 1. The same patterns are present in other countries and when measuring markups through other methodologies, as we show in Figure 2 where we plot the distributions of markups and market shares for Colombia and the United States.

Thus, accounting for the markup distribution requires accounting for the presence of a large number of firms with low market shares and high markups, as well as of larger firms with below-average markups. This points to a role for markup differences across the firm size distribution different from the one implied by standard theories

[7]The analysis in Section 3 motivates our use of market sales-shares to measure firm size.

Table 1. Markup Variance Decomposition

	Variance	Share within	Share between	Corr($\log \mu_i$, σ_i)
India	0.196	0.770	0.230	0.061
Colombia	0.059	0.739	0.261	0.030
United States	0.024	0.676	0.324	-0.075

Notes: The variance is computed from the log ratio $\log(\mu_i^m/\mu_m)$. Numbers are averages across markets. To decompose the variance of markups, we divide firms in equally sized bins of 5 percentage point market share. The variance and its decomposition correspond to the average variance and shares across market-year pairs. The correlation between markups and market shares is the pooled correlation of the full sample.

of variable markups and imperfect competition, where firms with large market shares charge high markups, and firms with low market shares charge low markups.

Of course, to fully account for the markup distribution, we must also address markup variation between similarly sized firms. To get an idea of the role this variation plays, we divide observations according to their market share in equally sized bins of 5 percentage points and decompose the variance of log-markups into their within-bin and between-bin components. Table 1 presents the results for our main Indian sample as well as for Colombia and the United States. The variation in markups within bins of similar market share accounts for 77 percent of the total variance of markups (74 percent in Colombia and 67 percent in the United States). Even when we increase the number of bins, so that each covers observations with market shares within 1 percentage point, within-bin variation still accounts for 56 percent of markup variation.

3. Model Setup

The model economy contains M product markets. Each market contains finitely many firms, each producing a differentiated good of a product type. These firms are monopolists in their good variety and compete oligopolistically either *à la* Cournot or *à la* Bertrand as in Atkeson and Burstein (2008). The goods of each market are aggregated according to a Homothetic Direct Implicit Additivity type aggregator that results in residual demand curves with variable elasticity (Matsuyama 2023, Sect. 7). A single representative consumer demands the final consumption good (the aggregate of product output). We present the main features of the model in the main text and provide all the proofs in Appendix C.

3.1. Final good production

Producers of final consumption behave competitively and operate a CES technology. Their problem is

$$\min_{\{Y_m\}} \sum_{m=1}^{M} P_m Y_m \qquad \text{s.t. } Y \leq \left(\sum_{m=1}^{M} \alpha_m Y_m^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}, \tag{4}$$

where $\sum_m \alpha_m = 1$ and $\gamma \geq 1$. The demand for the output of market m satisfies

$$\frac{P_m}{P} = \alpha_m \left(\frac{Y_m}{Y} \right)^{-\frac{1}{\gamma}}, \tag{5}$$

where P is a price index and satisfies

$$P = \left(\sum_{m=1}^{M} \alpha_m^{\gamma} (P_m)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}. \tag{6}$$

3.2. Market good production and demand for individual firms

There are N_m firms operating in each product market. The market's output, Y_m, is an aggregate of differentiated goods produced by these firms, $\left\{ y_i^m \right\}$. The technology of competitive market aggregators is implicitly defined by

$$1 = \sum_{i=1}^{N_m} \Upsilon_i \left(\frac{y_i^m}{Y_m} \right). \tag{7}$$

This is a more general form than the Kimball (1995) aggregator used in Baqaee, Farhi, and Sangani (2023), Edmond, Midrigan, and Xu (2023), among others, in that it allows for firm-specific functions $\Upsilon_i(\cdot)$. This distinction is crucial for our objectives, as it allows us to introduce idiosyncratic demand elasticity shifters that affect the elasticity of demand of any given good. These shifters allow relatively small firms to charge high markups and relatively large firms to charge low markups, introducing a new force determining markups that interacts with market concentration in equilibrium.

Although most of our results hold for a general functions Υ_i, we later parameterize them for estimation. We follow the literature by focusing on two-parameter functional forms so that $\Upsilon_i(x) = \Upsilon\left(x; \nu_i^m, \theta_m\right)$, but we let the parameters $\left\{\nu_i^m\right\}$ vary by firm, allowing us to capture idiosyncratic differences in the elasticity of demand of each differentiated good. The remaining parameter, $\{\theta_m\}$, is constant for each market. We return to these parameters in Section 4.

The problem of an aggregator is to minimize expenditure $\sum_i p_i^m y_i^m$ subject to (7) and a desired level of output Y_m, where p_i^m and y_i^m are the price and quantity of good i in market m. Not all goods are necessarily demanded in equilibrium. The solution of this problem implies a demand for variety i characterized by

$$\frac{p_i^m}{P_m} = \frac{\Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)}{\sum_j \Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\frac{y_j^m}{Y_m}}, \tag{8}$$

where P_m is market m' ideal price index, that is, $P_m Y_m = \sum_i p_i^m y_i^m$. We provide details for these derivations in Appendix C.1.

The homotheticity of the demand system imposes restrictions on how the market's output and price react to changes in firms' output and prices. These restrictions shape the behavior of equilibrium market shares. We highlight these implications in the following lemma (see also Matsuyama 2023, Sect. 5.2).

LEMMA 1. ***(Market Shares)*** *The change in market output Y_m to firm i's output and of market price P_m to firm i's price satisfy*

$$\frac{\partial Y_m}{\partial y_i^m} = \frac{p_i^m}{P_m} \qquad \textit{and} \qquad \frac{\partial P_m}{\partial p_i^m} = \frac{y_i^m}{Y_m}. \tag{9}$$

So, the market share of firm i in market m, σ_i^m, gives the elasticity of market output and market price to changes in the firm's output and price, respectively,

$$\sigma_i^m \equiv \frac{p_i^m y_i^m}{P_m Y_m} = \frac{\Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}{\sum_j \Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\frac{y_j^m}{Y_m}} = \frac{y_i^m}{Y_m}\frac{\partial Y_m}{\partial y_i^m} = \frac{p_i^m}{P_m}\frac{\partial P_m}{\partial p_i^m}. \tag{10}$$

3.3. Firm's problem: Profit maximization

The problem of firm i in market m is to maximize its profits, taking as given the demand schedule for its goods in (8) and the actions of other firms. The firms problem is

$$\max \; p_i^m y_i^m - C_i\left(y_i^m\right) \qquad \text{s.t. (8), (7), and (5);} \tag{11}$$

where we take as given the firm's optimal input demand choices so that the cost of production is described by an increasing function C_i, as discussed in Section 2. The firm maximizes over its price p_i^m if competition is *à la* Bertrand, or over its quantity y_i^m if competition is *à la* Cournot. In either case, the firm optimally chooses to set a markup μ_i^m over its marginal cost so that

$$p_i^m = \underbrace{\frac{1}{1-\frac{1}{\eta_i^m}}}_{\text{Markup: } \mu_i^m} C_i'\left(y_i^m\right), \tag{12}$$

where $\eta_i^m \equiv -\left(\frac{\partial \log p_i^m}{\partial \log y_i^m}\right)^{-1}$ is the elasticity of demand and μ_i^m the markup.

The role of the demand system and the nature of competition between firms are therefore captured by the firms' elasticities of demand, $\left\{\eta_i^m\right\}$. When a firm changes its output or price, it affects is own demand, the aggregate market output, and the demand faced by other firms. Crucially, firms internalize their effects on the demand faced by other firms and how these changes affect their own elasticity in return. These effects correspond to *(i)* the firm's own-elasticity (taking aggregates and the behavior of other firms as given), *(ii)* the granular effect of the firm on market output Y_m, and *(iii)* the substitution effects across firms that depend on their elasticities.

For instance, under Cournot competition, where firms take each other's output as given, we can write the elasticity of demand using (5) and (8) as[8]

$$\left(\eta_i^m\right)^{-1} = \underbrace{\left(-\frac{\partial \log \Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)}{\partial \log y_i^m}\right)}_{\text{Own-Elasticity}} + \underbrace{\left(-\frac{\partial \log \left(\frac{Y_m}{Y}\right)^{-\frac{1}{\gamma}}}{\partial \log y_i^m}\right)}_{\text{Granular Effect}} - \underbrace{\left(\frac{\partial \log \sum_j \Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\frac{y_i^m}{Y_m}}{\partial \log y_i^m}\right)}_{\text{Substitution Effects}}. \tag{13}$$

The elasticity of demand under Bertrand competition can be similarly expressed as a function of three terms capturing the firm's own elasticity, its granular and substitution effects, see equation (C.33). In both cases, the effect on the firm's demand comes from

[8] We assume throughout that firms internalize the effect of their actions on their market's output and price, but not on the aggregate output and price, Y, P. This is the same assumption used in Atkeson and Burstein (2008) where there is a continuum of markets, or in Grassi (2017) and Burstein, Carvalho, and Grassi (2025) where they have finitely many markets.

two sources. First, a change in market output changes the level of demand; larger firms can take more advantage of this effect, as they have a larger impact on sectoral output. Second, a change in market output changes the elasticity of demand of all firms. If the aggregator Υ_i is concave, a higher relative output implies a lower elasticity of demand, and firms facing lower elasticity charge higher markups.

Under monopolistic competition, only the first term emerges as the firm has no effect on the aggregate market output, and therefore no effect on other firms. It is useful to then define the *own elasticity of demand*—the elasticity of demand for the variety produced by firm i holding the aggregate market output constant—as

$$\varepsilon_i^m \equiv \left(-y_i^m \frac{\partial \log \Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)}{\partial y_i^m} \right)^{-1} = \left(-\frac{\Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)}{\frac{y_i^m}{Y_m}\Upsilon_i''\left(\frac{y_i^m}{Y_m}\right)} \right)^{-1}. \tag{14}$$

This is the first term in (13) and corresponds to the measure of elasticity in Kimball (1995), Klenow and Willis (2016), Edmond, Midrigan, and Xu (2023), and many others.

The rate at which the firm's own-elasticity of demand changes plays a crucial role in the estimation of the model in Section 4. This rate depends on the super-elasticity of demand that measures how much the elasticity changes with output, in this way capturing the shape of the function Υ_i. The super-elasticity of demand is

$$\xi_i^m \equiv -\frac{p_i^m}{P_m} \frac{\partial \log \varepsilon_i^m}{\partial\left(\frac{p_i^m}{P_m}\right)} = 1 + \varepsilon_i^m + \varepsilon_i^m \frac{\frac{y_i^m}{Y_m}\Upsilon_i'''\left(\frac{y_i^m}{Y_m}\right)}{\Upsilon_i''\left(\frac{y_i^m}{Y_m}\right)}. \tag{15}$$

3.4. Equilibrium markups

We solve analytically for the firms' elasticity of demand and, therefore, their markups. The result is a characterization of the firm's elasticity, η_i^m, as a sales-weighted average of the elasticity of market output, γ, and the elasticity of demand for the firm's variety inside the market. This *variety elasticity* is itself a function of the firm's own-elasticity, ε_i^m, and the own-elasticities of its competitors. The form of competition (Bertrand or Cournot) determines how competitors affect the firm's elasticity of demand.

Proposition 1. ***(Elasticities)*** *The elasticity of firm i in market m satisfies*

$$\textbf{\textit{Cournot}} \quad \frac{1}{\eta_i^m} = \underbrace{\frac{1}{\gamma}}_{\text{Market Elasticity}} \sigma_i^m + \underbrace{\left(\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right) + E_\sigma\left[\frac{1}{\varepsilon_j^m}\middle| j \neq i\right]\sigma_i^m\right)}_{\text{Variety Elasticity}}\left(1-\sigma_i^m\right) \; ; \tag{16}$$

$$\textbf{\textit{Bertrand}} \quad \eta_i^m = \underbrace{\gamma}_{\text{Market Elasticity}} \sigma_i^m + \underbrace{\varepsilon_i^m \frac{E_\sigma\left[\varepsilon_j^m | j \neq i\right]}{E_\sigma\left[\varepsilon_j^m\right]}}_{\text{Variety Elasticity}}\left(1-\sigma_i^m\right) \; ; \tag{17}$$

where $\sigma_i^m = p_i^m y_i^m / P_m Y_m$ *is firm i's market share,* ε_i^m *is its own-elasticity as in (14),* $E_\sigma\left[x_j\right] = \sum_j^{N_m} x_j \sigma_j^m$ *is the average with respect to expenditure in market m, and* $E_\sigma\left[x_j | j \neq i\right] = \sum_{j \neq i} x_j \frac{\sigma_j^m}{1-\sigma_i^m}$ *is the average with respect to firm i's competitors.*

The characterization of the elasticity in Proposition 1 makes clear the role of market concentration. As a firm increases its market share, its elasticity reflects more the market's elasticity and less the elasticity of the firm's variety. When firms are granular, they care more about the demand for their market's aggregate output the larger they are. Similarly, larger firms are less affected by the elasticity of their own variety and weight more the elasticity of their competitors, as that becomes the main source of changes in their demand. Crucially, the Proposition also makes it clear that even though firms care about the elasticity of their competitors, these only matter through a sufficient statistic defined as an appropriate average of the competitors' elasticities.

Proposition 1 leads to analytical expressions for the firms' markups in terms of market shares and the set of own-elasticities of demand, $\left\{\sigma_i^m, \varepsilon_i^m\right\}$. We summarize the results in Proposition 2. Its proof follows from manipulating the elasticities in Proposition 1 along with equation (12).

Proposition 2. ***(Markups)*** *The equilibrium markups under Cournot competition are:*

$$\textbf{\textit{Cournot Markup}} \quad \frac{1}{\mu_i^m} = \frac{\gamma-1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\varepsilon_i^m}\right)\left(1-\sigma_i^m\right) + \sigma_i^m\left(\frac{1}{\varepsilon_i^m} - E_\sigma\left[\frac{1}{\varepsilon_j^m}\right]\right) \; ; \tag{18}$$

$$\textbf{\textit{Bertrand Markup}} \quad \frac{1}{\mu_i^m} = 1 - \frac{1}{\gamma\sigma_i^m + \varepsilon_i^m\left[1 - \frac{\varepsilon_i^m}{E_\sigma\left[\varepsilon_j^m\right]}\sigma_i^m\right]} \; ; \tag{19}$$

where ε_i^m is firm i's own-elasticity of demand as in (14), $\sigma_i^m \equiv p_i^m y_i^m / P_m Y_m$ its market share, and $E_\sigma\left[x_j\right] = \sum_{j=1}^{N_m} x_j \sigma_j^m$ is the average with respect to expenditure in market m, in this case giving an harmonic mean of the elasticities, ε_j^m.

Remark The Bertrand and Cournot markups converge to the monopolistic competition markups $\left(\mu_i^m = \frac{\varepsilon_i^m}{\varepsilon_i^m - 1}\right)$ as σ_i^m tends to 0, where elasticities vary according to (14). Equivalently, they converge to the oligopolistically competitive markups of Atkeson and Burstein (2008) when the own-demand elasticity is constant and common across firms, $\varepsilon_i^m = \varepsilon^m$.

3.5. Aggregation: Market level variables

The average markup in market m, μ_m, is defined as the ratio between the market's price P_m and the market's marginal cost λ_m, $\mu_m \equiv P_m/\lambda_m$.[9] Because the market aggregators (7) have constant-returns-to-scale, each market's marginal cost is equal to the output-

[9] Equivalently, Edmond, Midrigan, and Xu (2023) define it as the ratio of the market's revenue $P_m Y_m$ and the market's total cost, which, under constant returns-to-scale, is $\sum_i \lambda_i^m y_i^m$.

weighted average of the individual marginal costs, that we label $\lambda_i^m \equiv C_i'\left(y_i^m\right)$,

$$\lambda_m = \sum_{i=1}^{N_m} \lambda_i^m \frac{y_i^m}{Y_m}. \tag{20}$$

Then, the market's markup is obtained as a sales-weighted harmonic mean of individual markups, as in Edmond, Midrigan, and Xu (2015, 2023), Smith and Ocampo (2025), and Thomas Hasenzagl (2023),

$$\mu_m = \left[\sum_{i=1}^{N_m} \lambda_i^m \frac{y_i^m}{P_m Y_m}\right]^{-1} = \left[\sum_{i=1}^{N_m} \frac{1}{\mu_i^m} \sigma_i^m\right]^{-1} = \left[\sum_{i=1}^{N_m} \left(1 - \frac{1}{\eta_i^m}\right) \sigma_i^m\right]^{-1} = \frac{1}{1 - \frac{1}{\eta_m}}. \tag{21}$$

This is the same expression as for the individual markup μ_i^m in (12), but replacing firm i's elasticity η_i^m with the corresponding market elasticity,

$$\eta_m \equiv \left[\sum_{i=1}^{N_m} \frac{1}{\eta_i^m} \sigma_i^m\right]^{-1}. \tag{22}$$

We can therefore express market markups in terms of the underlying own-elasticities and market shares using the results in Proposition 1.

Proposition 3. ***(Market Elasticities and Markups)*** *Market elasticities satisfy*

$$\textbf{\textit{Cournot}} \qquad \frac{1}{\eta_m} = HHI_m \frac{1}{\gamma} + (1 - HHI_m)\, E_\sigma\left[\frac{1}{\varepsilon_i^m}\right] - 2Cov_\sigma\left(\sigma_i^m, \frac{1}{\varepsilon_i^m}\right); \tag{23}$$

$$\textbf{\textit{Bertrand}} \qquad \frac{1}{\eta_m} = E_\sigma\left[\frac{1}{\varepsilon_i^m}\right] + \frac{HHI_m}{E_\sigma\left[\varepsilon_i^m\right]} - \gamma\left(HHI_m E_\sigma\left[\left(\frac{1}{\varepsilon_i^m}\right)^2\right] + Cov_\sigma\left(\sigma_i^m, \left(\frac{1}{\varepsilon_i^m}\right)^2\right)\right) + \Omega \tag{24}$$

where $HHI_m = \sum_i \left(\sigma_i^m\right)^2$ *is the Herfindahl-Hirschman index,* $E_\sigma\left[x_j\right] = \sum_{j=1}^{N_m} x_j \sigma_j^m$ *is the* *expectation with respect to sales shares and* $Cov_\sigma\left(x_j, y_j\right) = \sum_{j=1}^{N_m}\left(x_j\right)\left(y_j - E_\sigma\left[y_j\right]\right)\sigma_j^m$ *is the covariance with respect to sales in the market. Finally,* $\Omega \equiv \sum_i^{N_m}\frac{\sigma_i^m}{\varepsilon_i^m}\left[\sum_{k=2}^{\infty}\left(\sigma_i^m\left(\frac{\varepsilon_i^m}{E_\sigma\left[\varepsilon_j^m\right]} - \frac{\gamma}{\varepsilon_i^m}\right)\right)^k\right]$ *contains higher order moments.*

The average markup satisfies

$$\textbf{Cournot}\quad \frac{1}{\mu_m} = \underbrace{\left(1-\frac{1}{\gamma}\right)}_{\textit{Monopoly Markup}} + \underbrace{\left(\frac{1}{\gamma} - E_\sigma\left[\frac{1}{\varepsilon_i^m}\right]\right)(1-HHI_m)}_{\textit{Concentration}} + \underbrace{2Cov_\sigma\left(\sigma_i^m, \frac{1}{\varepsilon_i^m}\right)}_{\textit{Distribution}}; \tag{25}$$

$$\textbf{Bertrand}\quad \frac{1}{\mu_m} = \underbrace{\left(1-\frac{1}{\gamma}\right)}_{\textit{Monopoly Markup}} + \underbrace{\left(\frac{1}{\gamma} - E_\sigma\left[\frac{1}{\varepsilon_i^m}\right]\right) + \frac{HHI_m}{\gamma}\left(E_\sigma\left[\left(\frac{\gamma}{\varepsilon_i^m}\right)^2\right] - \frac{\gamma}{E_\sigma\left[\varepsilon_i^m\right]}\right)}_{\textit{Concentration}}$$

$$+ \underbrace{Cov_\sigma\left(\sigma_i^m, \left(\frac{1}{\varepsilon_i^m}\right)^2\right)}_{\textit{Distribution}} - \Omega\,. \tag{26}$$

These results show how market markups come from three sources. This is more easily seen under Cournot competition. First, there is a monopoly markup. That is, the markup that would arise if the market were a monopoly. Second, the markup increases with concentration (as measured by the Herfindahl-Hirschman Index) when the average elasticity of varieties in the market is higher than the market's demand reflecting varieties being more substitutable among themselves than products, $E_\sigma\left[1/\varepsilon_i^m\right] < 1/\gamma$. This is the same force as in Edmond, Midrigan, and Xu (2015). Finally, the dispersion in elasticities introduces a third source of markups coming from the covariance between market shares and the own-elasticity of varieties in the market. This source reduces average markups when sales are concentrated in firms with a low ε_i^m. This is because these large firms care more about the elasticity of the market's demand (Y_m) relative to smaller firms. In this way, it is the small (niche) firms who increase average markups when their varieties are less elastic, whereas larger firms respond more to the market's elasticity.

Under Bertrand competition, the higher order moments of the joint distribution of market shares and elasticities also matter, as captured by the term Ω. These moments capture how concentration interacts with the elasticities of demand and are most relevant in highly concentrated markets, as they depend on higher-order concentration indexes of the type developed in Hannah and Kay (1977). We present more details on these higher order terms in equation C.46. Crucially, these terms are present even when there are no differences in elasticities between firms, as shown in Grassi (2017, Proposition 4), something we verify in Corollary A1.

Average productivity under common constant-return-to-scale technology. We have left the properties of the firms' production technology unspecified so far, showing that aggregation of marginal costs and markups only depends on the properties of the demand system and of the concentration of sales between firms. However, it is useful to consider the standard case where all firms in a market operate the same constant-returns-to-scale technology and face competitive input markets. This is the setup used in most macroeconomic models (e.g., Atkeson and Burstein 2008; Edmond, Midrigan, and Xu 2015, 2023; Baqaee and Farhi 2020) and by some methods to measure markups based on Foster, Grim, and Haltiwanger (2016) or Raval (2023). Moreover, our results for the estimation of the firms' production function shows that deviations from constant-returns-to-scale are not large, even among Indian manufacturers. The average returns-to-scale among establishments in our data are 0.95, see Appendix A.3.

Under constant-returns-to-scale, we can link differences in marginal costs directly to differences in Hicks-neutral productivity. the marginal costs of the firms in market m are a ratio of a common constant Ψ (which depends on common input prices) and idiosyncratic productivity z_i^m. Then, the aggregate market productivity can be obtained from (20) as an output-weighted harmonic mean of productivities,

$$z_m = \left[\sum_{i=1}^{N_m} \left(z_i^m\right)^{-1} \frac{y_i^m}{Y_m} \right]^{-1}. \tag{27}$$

4. Accounting for the Distribution of Markups

We now account for the distribution of firm markups and firm sizes in Figure 1. We proceed as follows: First, the optimal pricing choices of firms in Proposition 2 allow us to recover own-demand elasticities that match the joint distribution of market shares and markups. This step does not depend on the functional form imposed on the demand aggregator Υ_i. We then estimate demand parameters after imposing a parametric form for Υ_i following Klenow and Willis (2016).[10] Finally, we recover relative output, prices, and marginal costs.

4.1. Recovering own-demand elasticities

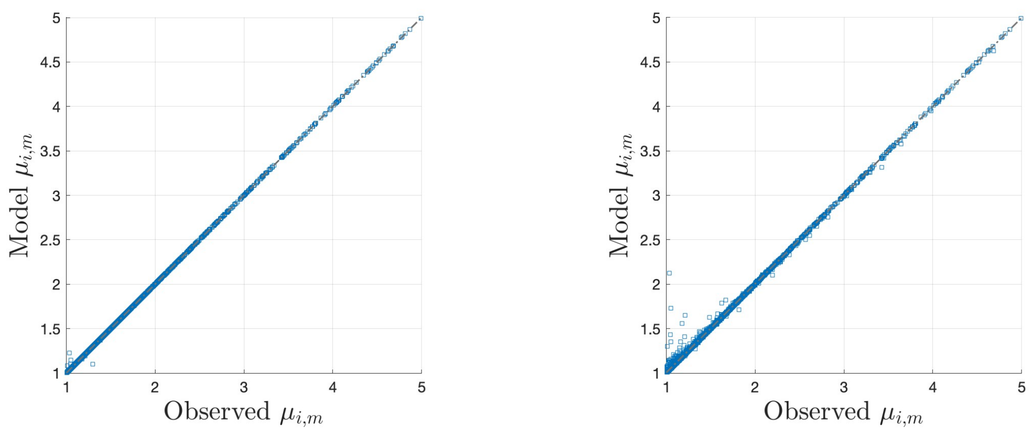
We use the analytical expressions for markups in Proposition 2 to recover the own-elasticities of demand implied by firms' market shares and markups $\left\{\sigma_i^m, \mu_i^m\right\}$. We recover the own-elasticities by minimizing the distance between the observed markups $\left\{\mu_i^m\right\}$ and those implied by the model, this amounts to solving a system of non-linear equations on $\left\{\varepsilon_i^m\right\}$.[11] In doing this we set $\gamma = 1.3$, consistent with the values used in Edmond, Midrigan, and Xu (2023). Figure 3 plots measured and model-implied markups under Bertrand and Cournot competitions. The model matches the distribution of markups and market shares almost exactly.

[10] More generally, we focus on two families of aggregators presented in Dotsey and King (2005) and Klenow and Willis (2016). We show in Appendix D that these families encompass all the functional forms used in the literature.

[11] We show in Appendix E.1 that the system is actually linear under Cournot competition and that it can be solved exactly if $1/\bar{\varepsilon}_i^m > 1/\gamma\, \sigma_i^m$ for all firms, with $1/\bar{\varepsilon}_i^m = 1 - 1/\mu_i^m$. Under Bertrand competition the system is nonlinear but it admits a unique solution and can match markups exactly in many of our samples.

FIGURE 3. Markup Fit in Indian Data (2005–2007)

(A) Bertrand Competition (B) Cournot Competition



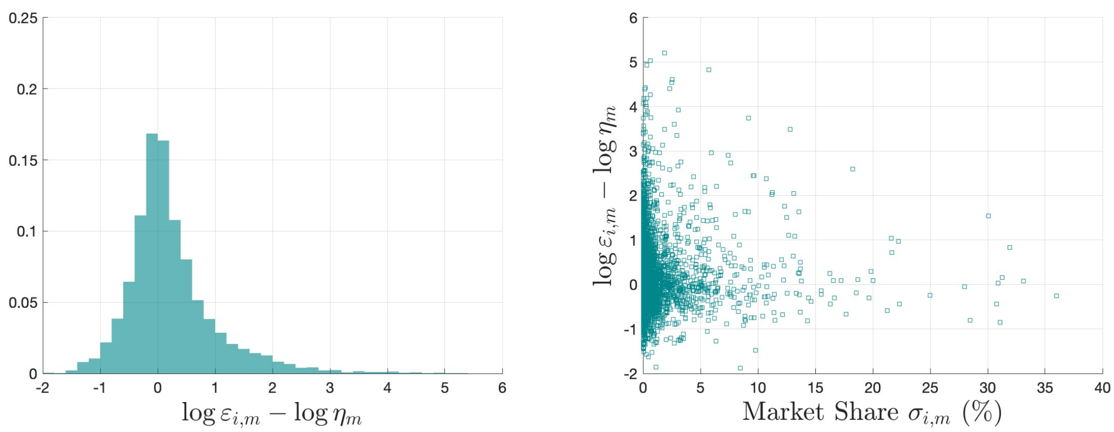
Notes: Panel A shows the scatterplot of measured and model-implied markups under Bertrand competition, obtained by solving for $\{\varepsilon_i^m\}$ in equation 19 given $\{\mu_i^m, \sigma_i^m\}$. Panel A shows the scatterplot under Cournot competition using equation 18. We set $\gamma = 1.3$ in both exercises. A value in the 45 degree line corresponds to a perfect match of the model for a given firm.

There is substantial variation in own-elasticities of demand, in line with the large variation in markups that generate them, see Figure 4. A log-difference of 2 implies an elasticity that is 7.4 times the market average, this goes up to 20 times for a log-difference of 3. This large dispersion in the firms' own-elasticity of demand is present across the firm size distribution, as Figure 4B clearly shows. Even among large firms, that tend to have lower elasticities, some exhibit higher-than-average elasticities ($\log \varepsilon_i^m - \log \eta_m \approx 2$). The dispersion is much larger among small firms. In particular, there are many small firms with lower-than-average elasticities, which is necessary to match the observed dispersion in markups and the presence of high-markup firms with small market shares.

On a technical note, although the value of the elasticities of demand does not depend on the functional form of Υ_i, it does impose conditions on it. In particular, it must be that the demand firms face has a variable super-elasticity of demand in order to be compatible with the distribution of market shares and markups. A constant super-

FIGURE 4. Distribution of Elasticities (2005–2007)

(A) Own-Elasticity Distribution (B) Elasticity and Market-Share



Notes: Panel A shows the histogram of own-elasticity of demand relative to their market's average for the Indian data covering the period 2005–2007. Panel B shows the joint scatter plot of own-elasticities and market shares. Markets are defined at the 5-digit ASICC product level.

elasticity of demand implies a one-to-one link between elasticities and market shares, which is generically rejected by the data. See Proposition A3 in Appendix E.2

4.2. Estimation of demand parameters

We now estimate the shape of the demand faced by firms, captured by the functions Υ_i, using variation in firms' size and elasticity over time. The characteristic feature of homothetic demand aggregators with variable elasticity is that the elasticity varies with firm size (measured by the firms' relative output, y_i^m/Y_m). We use the analytical characterization of equilibrium markups to extend this relationship to one between a change in the firm's observed market share and the firm's elasticity of demand that does not depend on the functional form of Υ. Instead, the change is captured by the super-elasticity of demand, ξ_i^m, the relevant measure of how much the own-elasticity of demand changes as the firm changes in size. We present the details of the derivation in Appendix E.2.

Proposition 4. *The equilibrium relationship between changes in a firm's market share* $\left(\sigma_i^m\right)$ *and its own-elasticity of demand* $\left(\varepsilon_i^m\right)$ *is:*

$$d \log \varepsilon_i^m = -\left(\frac{\xi_i^m}{\varepsilon_i^m}\right)\left(\frac{\varepsilon_i^m}{1+\varepsilon_i^m}\right) d \log \sigma_i^m, \tag{28}$$

where ε_i^m *is the own-elasticity of demand of firm* i *in market* m*, defined as in* (14) *and* ξ_i^m *is its super-elasticity of demand, defined as in* (15).

We approximate this result in the data by regressing the (log) change of the own-elasticities we recovered above on the (log) changes of market shares we observe.

$$\Delta \log \varepsilon_i^{st} = \hat{\beta} \cdot \Delta \log \sigma_i^{st}, \tag{29}$$

and estimate the aggregator by indirect inference, minimizing the distance between $\hat{\beta}$ and the coefficient implied by Proposition 4:

$$\min \left|\hat{\beta} - \beta\left(\Upsilon\right)\right|. \tag{30}$$

We show in Appendix E.2 that minimizing this objective is equivalent to minimizing the sales-share-weighted difference of changes in elasticities because

$$\left|\hat{\beta} - \beta\left(\Upsilon\right)\right| \propto \left|\sum \left(\Delta \log \varepsilon_i^{mt} - \left(\frac{\xi_i^{mt}}{\varepsilon_i^{mt}}\right)\left(\frac{\varepsilon_i^{mt}}{1+\varepsilon_i^{mt}}\right) \Delta \log \sigma_i^{mt}\right) \cdot \Delta \log \sigma_i^{mt}\right|. \tag{31}$$

That is, the estimation weights more changes the in elasticity of firms with the largest changes in market shares. Changes in elasticity are exactly what is needed to identify super-elasticity ξ and the changes of firms whose market share changed the most are the most informative.

This process takes advantage of the panel dimension of the data. The introduction of firm-specific demand shifters makes it unfeasible to estimate the shape of the aggregator solely from cross-sectional data, as in Baqaee, Farhi, and Sangani (2023) or Edmond, Midrigan, and Xu (2023). Instead, changes in elasticities and market shares provide the relevant variation to estimate demand parameters.

Functional form for demand. We estimate Υ_i for the leading functional form of the literature following Klenow and Willis (2016).[12] This functional form is parameterized by a pair of parameters so that $\Upsilon_i(x) = \Upsilon\left(x;\, \nu_i^m, \theta_m\right)$. The first parameter, ν_i^m, directly affects the elasticity of demand and we make it firm-specific; this is our elasticity of demand shifter. The second parameter, θ_m, controls the shape of the function and is constant within a market. When $\theta = 0$ there is no variation in elasticities, $\varepsilon_i^m = \nu_i^m$, recovering the constant-elasticity-of-demand aggregator with firm-specific elasticities.

The functional form determines how demand depends on the firm's (relative) output. The elasticity and the super-elasticity of demand for firm i in market m become

$$\varepsilon_i^m = \nu_i^m \cdot \left(\frac{y_i^m}{Y_m}\right)^{-\frac{\theta_m}{\nu_i^m}} \quad \text{and} \quad \xi_i^m = \theta_m \cdot \left(\frac{y_i^m}{Y_m}\right)^{-\frac{\theta_m}{\nu_i^m}}. \tag{32}$$

The estimation takes advantage of the fact that the ratio of the super-elasticity to the elasticity in Proposition 4 and equation (31) is constant for a given firm,

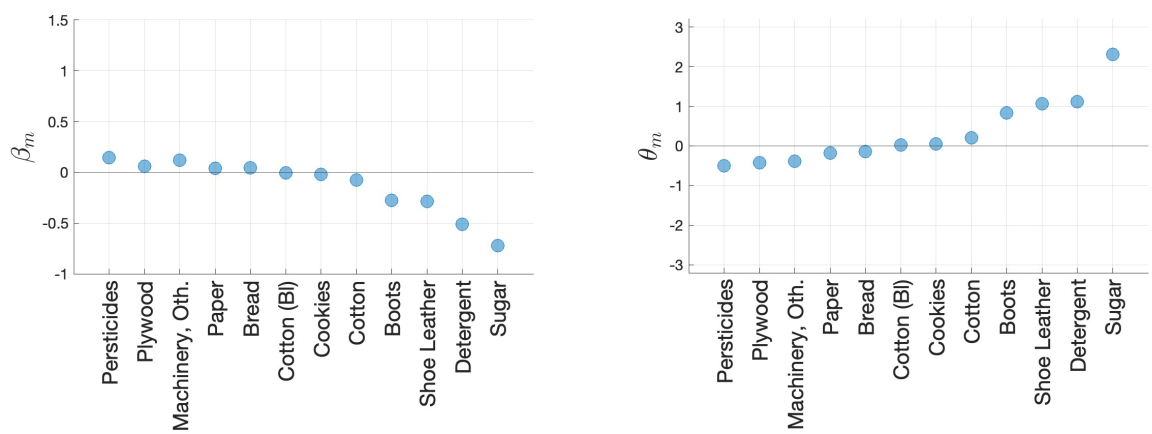
$$\frac{\xi_i^m}{\varepsilon_i^m} = \frac{\theta_m}{\nu_i^m}. \tag{33}$$

Given a value for θ_m, we recover the value of the idiosyncratic shifters, $\left\{\nu_i^m\right\}$, by

[12] Papers like Edmond, Midrigan, and Xu (2023), Boar and Midrigan (2024), and Hubmer and Restrepo (2021) use the functional form of Klenow and Willis (2016). We also consider the functional form adopted by Dotsey and King (2005). We show in appendix D that Barde (2008), Levin, López-Salido, Nelson, and Yun (2008), Darracq Pariès and Loublier (2010), and Kurozumi and Zandweghe (2020) use functions that are equivalent to those in Dotsey and King (2005).

FIGURE 5. Demand Parameters – Bertrand Competition – Selected Markets (2005–2007)

(A) $\hat{\beta}_m$ by Market (B) θ_m by Market



Notes: The left panel shows the $\hat{\beta}$'s estimated from the data using equation (29) and the market-specific β implied by the Klenow and Willis aggregator in Appendix E. The right panel shows the market-specific θ estimated that generate the β that minimize the distance with respect to the observed $\hat{\beta}$. The figure presents the ten larger industries by 1992.

matching the observed market shares of firms. Equation (32) implies that

$$\frac{y_i^m}{Y_m} = \left(\frac{\varepsilon_i^m}{\nu_i^m}\right)^{-\frac{\nu_i^m}{\theta_m}}, \tag{34}$$

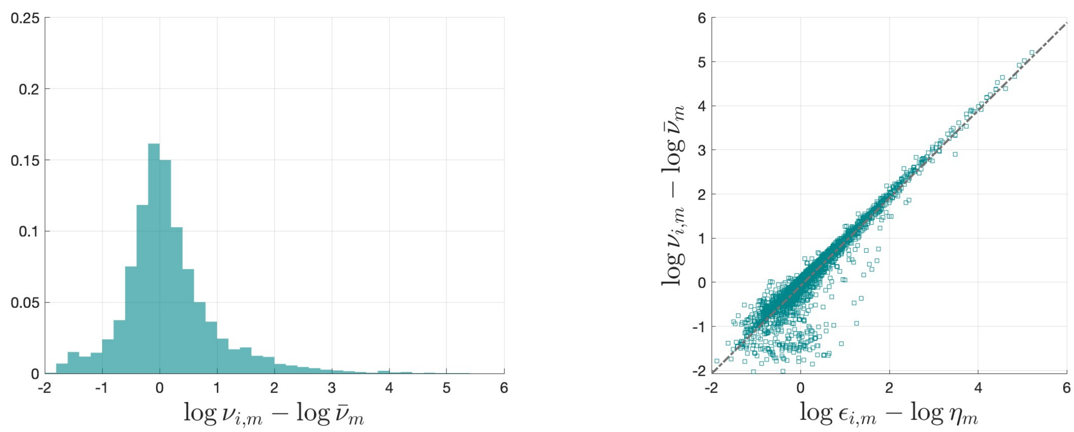
which allows us to evaluate σ_i^m directly as in equation (10) from Lemma 1. Matching the market shares amounts to solving another system of non-linear equations.[13]

Demand estimates. Figure 5 presents the estimates of market-specific parameters $\left\{\hat{\beta}_m\right\}$ in Panel A and $\{\theta_m\}$ in Panel B. The model matches the estimated $\left\{\hat{\beta}_m\right\}$ precisely as we show in Figure B.1 in the Appendix. Most markets have firms with relatively stable

[13]The same process applies for alternative functional forms for Υ as that in Dotsey and King (2005). In this case, the relative output is expressed in terms of parameters as $y_i^m/Y_m = (1+\theta_m)\nu_i^m/\theta_m(\nu_i^m - \varepsilon_i^m)$. Then, the values of $\left\{\nu_i^m\right\}$ are recovered by matching the firms' market shares, $\left\{\sigma_i^m\right\}$, for a given value of θ_m. Once relative outputs are known it is possible to compute the ratio of the super-elasticity to the elasticity of demand as $\xi_i^m/\varepsilon_i^m = -\frac{\theta_m}{1+\theta_m}/\left(\frac{y_i^m}{Y_m} - \frac{\theta_m}{1+\theta_m}\right)$.

FIGURE 6. Elasticity Shifter (ν) and Relationship with Elasticities

(A) ν Distribution (B) ν and Elasticities



Notes: Panel A shows the histogram of demand elasticity shifters relative to their market's average for the Indian data covering the period 2005–2007. Panel B shows the joint scatter plot of own-elasticities and demand elasticity shifters. Markets are defined at the 5-digit ASICC product level.

elasticities, reflected in low values of both parameters. However, the elasticities in these markets tend to increase when firms increase in size, resulting in negative estimates for θ_m, corresponding to varieties being gross-complements from the point of view of the aggregator Υ.

Figure 6 presents the implied distribution of firm-specific elasticity shifters $\left\{\nu_i^m\right\}$ in Panel A and how it correlates with the distribution of own-elasticities of demand $\left\{\varepsilon_i^m\right\}$ in Panel B. These shifters are such that we exactly recover the observed market share of each firm. We find large variation in the distribution of shifters of comparable magnitude to the variation in markups (Figure 1A) and own-elasticities (Figure 4A). This reflects a limited role for differences in firm size to affect the elasticity of demand and, in turn, markups, as evidenced in the strong correlation between ν and ε in Figure 6Bc We expand on this result in Section 5, where we show that idiosyncratic demand elasticity shifters are the key ingredient in accounting for the distribution of markups, particularly the high dispersion in markups among firms with low market shares.

4.3. Recovering relative output, prices, and marginal costs

A byproduct of the estimation of the demand parameters is the set relative outputs, $\left\{ y_i^m/Y_m \right\}$, consistent with firms' choices in the model as in (34). Along with these relative outputs, we also recover relative prices and marginal costs (normalized by the sector's price level) using the measured market shares and markups as[14]

$$\frac{p_i^m}{P_m} = \sigma_i^m \frac{Y_m}{y_i^m} \quad \text{and} \quad \frac{\lambda_i^m}{P_m} = \frac{\sigma_i^m}{\mu_i^m} \frac{Y_m}{y_i^m}. \tag{35}$$

Finally, we express firms' marginal cost (relative to the their market's marginal cost)

$$\frac{\lambda_i^m}{\lambda_m} = \frac{\frac{\sigma_i^m}{\mu_i^m}}{\frac{1}{N_m} \sum_j \frac{\sigma_j^m}{\mu_j^m}} \left(\frac{y_i^m}{Y_m} \right)^{-1}. \tag{36}$$

We find marginal costs are lower for high-markup firms, in line with the standard predictions of variable markup models (either coming from oligopolistic competition or variable elasticity of demand), as we show in Figure 7A. However, there is significant variation in marginal costs among firms with the same markups, particularly for firms with markups between 40 percent below and 65 percent above the market's average markup that make up the large majority of firms.[15]

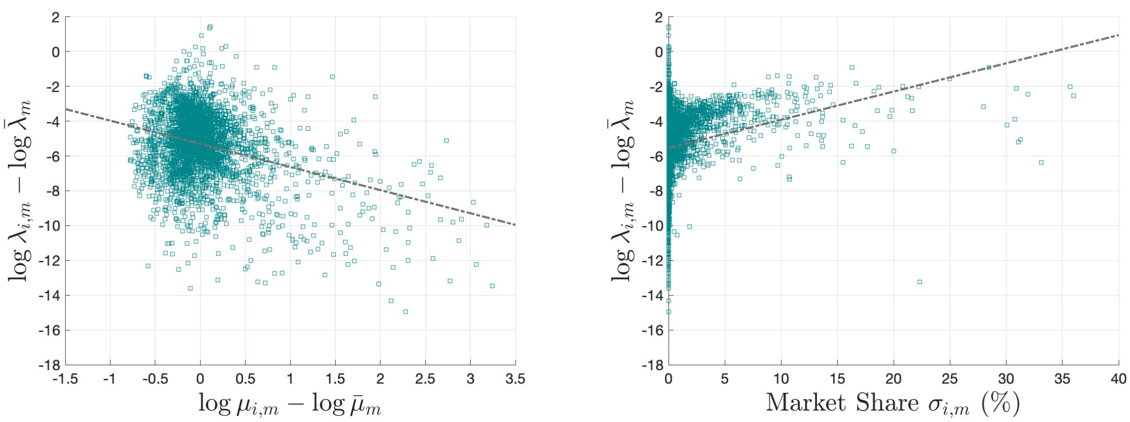
By contrast, larger firms tend to have higher (and not lower!) marginal costs, as we show in Figure 7B. Although there is a lot of variation in marginal costs between small firms with near-zero market share, the correlation we find is present even among larger firms (some of which charge low markups despite their size). This fact opens

[14] Alternatively, we can obtain relative marginal costs as $\lambda_i^m/\lambda_j^m = {}^{p_i^m/P_m}/_{p_j^m/P_m} \cdot \mu_j^m/\mu_i^m$, and report marginal costs relative to, for instance, the lowest cost firm.

[15] Similarly, firms with lower marginal costs tend to produce more, as captured by higher relative output, but there is a lot of variation in the marginal costs given the size of the firm. Marginal costs do correlate strongly with relative prices despite the discrepancies between marginal costs and markups. We show these additional results in Figure B.2 in the Appendix.

FIGURE 7. Marginal Costs and Relationship with Markups and Market Shares

(A) λ and Markups (B) λ and Market-Share



Notes: Panel A shows the scatter plot of firm log-marginal costs relative to their market's marginal cost and log-markups relative to their market's average. Panel B shows the scatter plot of firm log-marginal costs relative to their market share. Marginal costs are computed as in (36).

up an additional source of misallocation that interacts with the standard mechanism highlighted in the literature, where the most productive firms are simultaneously the largest and charge the highest markups, implying gains from reallocation economic activity towards them (Baqaee and Farhi 2020; Baqaee, Farhi, and Sangani 2023). We find that it is not the case that all large firms are productive, nor do they all charge high-markup. This pushes misallocation in opposite directions as there is a mass of highly-productive firms among the small firms of a market, but there are also unproductive firms whose size is kept in check because of their high markups.

5. Market Structure, Firm Scale, and Demand Factors

Our previous results show that accounting for the distribution of markups involves large variation in firms' demand elasticities. These elasticities depend endogenously on the market structure, the elasticities and sizes of competitors, the the size of firm (both in terms of its market share and its scale of production), and additional idiosyncratic factors that capture differences in elasticities between firms of the same size as shown in Proposition 1. This characterization of elasticities (and markups) merges and extends the two main theories of variable markups. Markups depend on market structure and on the elasticity and size of competitors because of strategic interactions embedded in oligopoly competition between granular firms, and depend directly on the scale of production because firms face demands with variable elasticity. We add to these mechanisms by introducing idiosyncratic demand elasticity shifters ν that differentiate the elasticity of demand curves faced by firms.

To understand the role of these mechanisms we now solve our model under three alternative specifications that differ in the form of competition between firms and the properties of demand. Specifically, we consider a model of oligopolistic competition under a common demand with constant elasticity, a model of monopolistic competition under a common variable elasticity of demand curve, and a model of oligopolistic competition under a common n variable elasticity of demand curve. All these differ from our full model in the absence of idiosyncratic demand elasticity shifters, setting $\nu_i^m = \nu_m$ in all of them.

Oligopolistic competition with constant elasticity of demand. Under these conditions, the alternative markups $\left\{\tilde{\mu}_i^m\right\}$ depend only on market concentration through market shares as in Atkeson and Burstein (2008), so that

$$\frac{1}{\tilde{\mu}_i^m} = \frac{\gamma - 1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\tilde{\varepsilon}_m}\right)\left(1 - \sigma_i^m\right), \tag{37}$$

where $\tilde{\varepsilon}_m = \nu_m$ is the (common) elasticity of demand in market m. We set $\tilde{\varepsilon}_m$ so that average markups in each market match the the average measured markup. In this we way we make markups and elasticities are comparable across models. We do this by choosing ν_m to solve $\sum_i \left(\mu_i^m\right)^{-1} \sigma_i^m = \sum_i \left(\tilde{\mu}_i^m\right)^{-1} \sigma_i^m$.

Monopolistic competition with variable elasticity of demand. Under these conditions, the alternative markups $\left\{\tilde{\mu}_i^m\right\}$ depend only on the firms' own-elasticity that varies depending on the firm's scale of production as in Kimball (1995),

$$\frac{1}{\tilde{\mu}_i^m} = 1 - \frac{1}{\varepsilon_i^m}, \qquad \text{where } \varepsilon_i^m = \nu_m \left(\frac{y_i^m}{Y_m}\right)^{\frac{-\theta_m}{\nu_m}} \tag{38}$$

as in Klenow and Willis (2016). We set the demand parameters θ_m and ν_m and the relative outputs in each market $\{\frac{y_i}{Y}\}$ by minimizing the distance between the market shares implied by this demand following Lemma 1,

$$\tilde{\sigma}_i^m = \frac{\frac{y_i}{Y} \exp\left[\frac{1}{\theta_m}\left(1 - \left(\frac{y_i}{Y}\right)^{\frac{\theta_m}{\nu_m}}\right)\right]}{\sum_j \frac{y_j}{Y} \exp\left[\frac{1}{\theta_m}\left(1 - \left(\frac{y_j}{Y}\right)^{\frac{\theta_m}{\nu_m}}\right)\right]}. \tag{39}$$

Oligopolistic competition with variable elasticity of demand. Under these conditions, the alternative markups $\left\{\tilde{\mu}_i^m\right\}$ depend on both market concentration and on own elasticities in the same way as in Proposition 2. In the case of Bertrand competition this results in

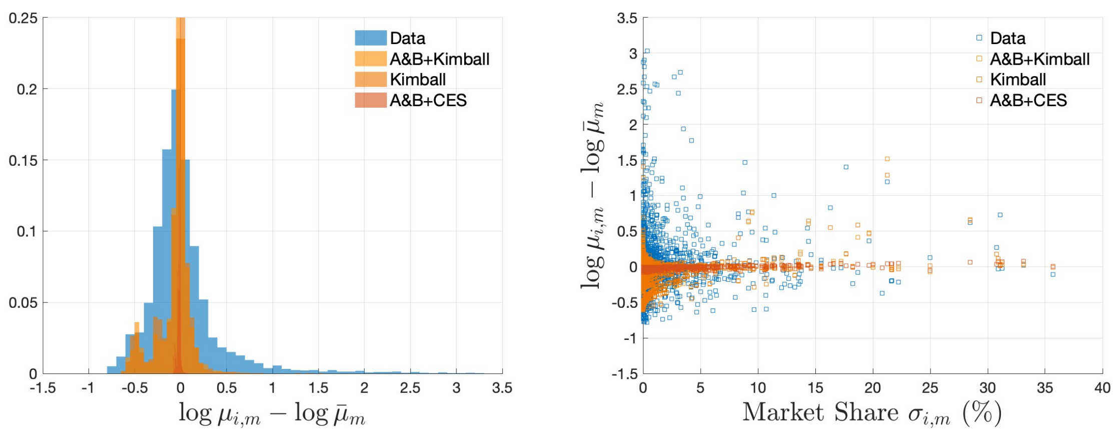
$$\tilde{\mu}_i^m = 1 - \left(\gamma \sigma_i^m + \varepsilon_i^m \left(1 - \frac{\varepsilon_i^m \sigma_i^m}{E_\sigma\left[\varepsilon_i^m\right]}\right)\right)^{-1}. \tag{40}$$

We set the own-elasticities and obtain the required parameters in the same manner as we did in the previous case.

FIGURE 8. Markups and Market Share Distribution Across Models (2005–2007)

(A) Markups Distribution

(B) Markups and Market-Share



Notes: The left panel shows the histogram of markups relative to their market's average for the Indian data (blue bars) and the markups implied by the Atkeson and Burstein (2008) Bertrand markups taking as given the market shares of firms and adjusting the (common) elasticity of demand of each industry to match the market's average markup. The right panel shows shows the histogram the implied distribution of own-elasticities, ε_i^m, required to match the distribution of markups under Cournot competition and Cobb-Douglas demand for market output, $\gamma = 1.3$. The average elasticity in each market corresponds to the harmonic sales-weighted average of the individual elasticities.

Markups across models. Figure 8 reproduces Figure 1 overlaying the distribution of markups under the alternative models without demand elasticity shifters. Panel A makes clear that variation in market shares alone is not enough to capture the observed dispersion in firm markups. Market concentration alone is not enough to drive significant variation in markups, even when it generates differences in markup levels.

As our results in Figure 4 hinted at, the dispersion in markups is tied to significant dispersion in demand elasticities. We see this again here in the ability of the two alternative models with variable elasticity of demand to generate much more variation in markups (with or without strategic interactions between firms). Nevertheless, these models still fall short in generating the degree of variation in markups seen in the data. They generate only one fourth of the variance of markups.

Panel B contrasts the joint distributions of markups and market shares across models. Once again, variation in the elasticity of demand is required to generate markup dispersion, but the constraint imposed by the common demand curve proves binding when accounting for the large dispersion in markups among small firms. In particular, the alternative models cannot explain small firms that charge high markups as well as the presence of large firms with below-average markups.

The patterns we describe above come together in the ability of alternative models to account for markup dispersion between firms of similar sizes. As we show in Table 2, the models without idiosyncratic demand elasticity shifters account for only a fraction of the variation in markups. Moreover, they counterfactually predict that at least two-thirds of the variation comes from differences between firms of different sizes, while these account for only 23 percent of the variance of markups in the data. This is further reflected in the implied correlation between markups and market shares (a common measure of market power), which is only 0.06 in the data but is no less than 0.20 in the alternative models.

Taken together, our results provide evidence for factors that affect the elasticity of demand of firms playing a central role in accounting for the distribution of markups, in line with Blum et al. (2023). The results in Section 4 and the first two rows of Table 2 make this clear. This has direct implications for how we think of misallocation as large firms are not necessarily charging high-markups and are often less productive than smaller firms competing in narrowly-defined product markets. It also questions the link between market shares and market power implied by standard theories of imperfect competition. It also highlights the importance of mechanisms that endogenize the distribution of demand factors such as the presence of lock-in innovations that operate by capturing demand instead of raising productivity as in Casal (2026), customer acquisition and customer capital as in Haddara (2026), among many others.

TABLE 2. Markup Variance Decomposition India (2001-2008)

	Variance	Share within	Share between	Corr(log μ_i, σ_i)
Data	0.196	0.770	0.230	0.061
Full Model	0.195	0.772	0.228	0.061
Atkeson & Burstein	0.001	0.175	0.825	0.337
Kimball	0.048	0.338	0.662	0.217
A&B + Kimball	0.047	0.388	0.612	0.211

Notes: The table presents statistics for measured markups and market shares along with statistic from four models: our full model described in Section 3, a model of oligopolistic competition with constant elasticity of demand, a model of monopolistic competition with a common demand with variable elasticity, and a model of oligopolistic competition with variable elasticity of demand. The variance is computed from the log ratio $\log(\mu_i^m/\mu_m)$. Numbers are averages across markets. To decompose the variance of markups, we divide firms in equally sized bins of 5 percentage point market share. The variance and its decomposition correspond to the average variance and shares across market-year pairs. The correlation between markups and market shares is the pooled correlation of the full sample.

6. Conclusions

We document that most markup dispersion arises among firms of similar size, a fact that cannot be explained by productivity differences or market concentration alone. To account for this pattern, we develop an analytically tractable model of oligopolistic competition with variable markups and non-CES demand that merges two of the main models of variable markups and extends them by introducing firm-specific demand elasticity shifters. Quantitatively, these shifters are essential to matching the joint distribution of markups and market shares observed in Indian manufacturing data and in complementary evidence from the United States and Colombia. The model reconciles the presence of small, high-markup firms and large, low-markup firms, and implies a weak link between firm size, productivity, and market power, highlighting demand heterogeneity as a key driver of markup dispersion.

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Appendix A. Estimation of Production Functions and Markups

A.1. The Indian Annual Survey of Industries

TABLE A.1. Product Classification Codes

Sector			Product		
Code	**Name**	**Sales Share in Ind.**	**Code**	**Name**	**Sales Share in Sector**
131	Sugar & Molasses	0.43	13103	Refined Sugar	0.76
134	Bakery Products	0.24	13401	Biscuits & Cookies	0.83
			13402	Bread, Buns & Croissants	0.10
344	Pesticides & Insecticides	0.13	34422	Pesticides	0.59
			34423	Insecticides	0.23
363	Soaps & Detergents	0.61	36304	Detergent Powder	0.20
			36307	Toilet Soap	0.34
443	Leather Footwear	0.68	44302	Boots	0.50
			44306	Leather Shoe Uppers	0.25
511	Wood & Wood Products	0.29	51105	Timber & Wooden Planks	0.22
			51125	Plywood	0.48
551	Printing & Writing Paper	0.54	55102	Printing & Writing Paper	0.76
632	Cotton Yarn & Fiber	0.43	63216	Bleached Cotton Yarn	0.30
			63221	Unbleached Cotton Yarn	0.27
642	Man-Made Yarn & Fiber	0.74	64222	Polyester Yarn	0.21
732	Aluminum & Alloys	0.44	73208	Aluminum Sections, Plates, etc.	0.32
769	Misc. Non-Electrical Machinery	0.34	76904	Boiler Accessories	0.29
			76926	Other Machinery	0.34
772	Electrical Motors & Transformers	0.22	77232	Transformer	0.28
782	Audio/Video Apparatus	0.47	78234	Printed Circuit Boards (PCB)	0.06
			78256	Color TV Set	0.61
821	Motor Vehicles & Parts	0.53	82189	Other Motor Vehicles & Parts	0.08

Note: The sector shares in the left panel are calculated with respect to the total revenue in the 2-digit industry across all the sample (2001-2008). The product shares are calculated with respect to the total revenue in the 3-digit sector across all the sample.

A.2. Co-Production Methodology

We identify co-production patterns using multi-product firms to maximize the number of available single-product firms in 3-digit sectors. Specifically, we consider as part of a 3-digit sector, the single-product firms directly producing in it, and the single-product firms producing in 3-digit sectors identified as co-production sectors based on multi-product firm data. Formally, we identify the 3-digit sectors where we find the highest number of single-product firms. Then, for each 3-digit sector k, we select the multi-product firms that produce at least one good in k. For each mult-iproduct firm m selected, we identify the set of 3-digit sectors k' different from k in which they produce. We then aggregate and calculate the number of multi-product firms that co-produce in k and k'. For each sector k, the top two k' sectors most frequently co-produced with k are retained. Finally, we combine single-product firms producing in k and in the top two k' sectors and estimate the production function for this group, assuming that k and the selected k' use similar technologies.

Example of Sector and Co-production Patterns. To illustrate our methodology, consider the following example from the **Food** industry. In this example, sector 113 (Fish) is one of the top 2 sectors in the *Animal Based Food Products* industry. Within this sector, the two products that generate the most revenue are *Shrimps* and *Frozen Fish*. To increase the number of single-product firms available for production function estimation, we consider co-production patterns. Based on multi-product firm data, we identify the most frequently co-produced sectors: 116 (Preparation used for animal feeding) and 112 (Meat). Single-product firms producing in these co-production sectors are pooled together with those in sector 113, assuming they use similar technologies. This subset of firms is used to estimate production function parameters of *Fish* sector. Our final unit of analysis to solve our model is the subset of 5-digit products.

Industry: 11. Animal Based Food Products	
Main Sector	**Main Products**
113. Fish	11314. Shrimps
	11316. Frozen Fish
Co-production Sectors	
116. Preparation used for animal feeding	
112. Meat	

A.3. Production Function Estimation

We estimate the quantity-based production function

$$q_{ijt} = f(m_{ijt}, l_{ijt}, k_{ijt}) + \omega_{ijt} + \zeta_{ijt}\,, \tag{A.1}$$

where q_{ijt} denotes product j *log-quantities* produced by establishment i at year t; m_{ijt}, l_{ijt}, and k_{ijt} denote product-specific *physical* intermediate inputs, labor and capital, respectively; ω_{ijt} denotes a productivity term; and ζ_{ijt} is measurement error.

Estimating a quantity-based production function uses separate price and quantity information, removing output-price bias, present in revenue-based production functions. However, simultaneity and selection biases remain because productivity is unobserved and firms' input choices depend on productivity. Two further issues arise. First, input allocations across products within multi-product firms are unobserved. This matters, for instance, for firms using differentiated inputs for differentiated products so that input prices vary depending on the product they are used for. Although we have data on the number of workers and some input quantities, physical units of capital are not observed. We therefore use input expenditures for all inputs following De Loecker et al. (2016).

In general, without product-level input data for a variable input x, the cost-share in equation (2) has to be computed as

$$\mu_{ijt} = \epsilon_{x,ijt} \times \frac{P_{ijt}Q_{ijt}}{\rho_{x,ijt}R_{x,it}}, \tag{A.2}$$

where $\rho_{x,ijt}$ is the share of input x expenditures attributable to product j for establishment i in period t, and $R_{x,it}$ are total expenditures in input x. Not observing product-level inputs also means that there are no product-level input prices available, which makes the estimation of the elasticity $\epsilon_{x,ijt}$ challenging.

The relationship between product- and firm-level input demand is then

$$x_{ijt} = \rho_{x,ijt} + r_{x,it} - w_{x,ijt}, \tag{A.3}$$

where $\hat{w}_{x,ijt}$ denotes the deviation of the unobserved (log) firm–product-specific variable input price from the (log) industry-wide variable input price index used to deflate input expenditures.

Consequently, equation A.1 can be written as:

$$\begin{aligned} q_{ijt} = {} & f\left(r_{l,it}, r_{m,it}, r_{k,it}\right) + \omega_{ijt} + \zeta_{ijt} \\ & + A\left(\rho_{ijt}, r_{l,it}, r_{m,it}, r_{k,it}\right) + B\left(w_{ijt}, \rho_{ijt}, r_{l,it}, r_{m,it}, r_{k,it}\right), \end{aligned} \tag{A.4}$$

where we use expenditures $r_{x,it}$ to redefine the production function in terms of observables, and functions A and B are unobservable due to the absence of information about ρ_{ijt} and w_{ijt}, the vectors of (log) input expenditure shares and price deviations by product. The bias emerges from the correlation between the unobservable productivity ω and A and B via the deflated input expenditures.

We follow De Loecker et al. (2016) to address these issues. First, we control for the simultaneity bias by employing a control function as in Olley and Pakes (1996) and Levinsohn and Petrin (2003). We assume that the intermediate input demand is a monotone function in productivity, while labor and capital are chosen before the firm observes the productivity realization ω. Under this assumption, we can invert a control function for productivity that depends on input demands.

To address the unobserved input allocation issue, we limit estimation to single-product firms for which input allocation is known, that is, $\rho_{x,ijt} = 1$. For single-product firms, equation A.4 reduces to:

$$q_{ijt} = f\left(r_{l,it}, r_{m,it}, r_{k,it}\right) + B\left(w_{ijt}, r_{l,it}, r_{m,it}, r_{k,it}\right) + \omega_{ijt} + \zeta_{ijt}. \tag{A.5}$$

We assume multi-product firms use the same technologies in the manufacturing of each product as as single-product firms that make that product. This means production technology is independent across product lines within a firm.

Finally, we have to correct for the term related to the firm-specific input prices $B(\cdot)$. To address this, we rely on a control function that exploits the theoretical relation between input prices, input quality, and output quality. We follow De Loecker et al. (2016) and argue that high-quality products produced using high-quality inputs command higher final prices. This theoretical connection establishes a positive relationship between unobserved product-specific input prices and observed product-specific output prices. In particular, we use a non-parametric control function using output prices and market shares to approximate for this relation. We complement this control function with state-level fixed effects.

For estimation, we measure the expenditure in labor services, $r_{l,it}$, using the wage bill and the expenditure in intermediate inputs, $r_{m,it}$, using the intermediate input total expenditure. Expenditure in capital services, $r_{k,it}$, is measured using the perpetual inventory method. All inputs are deflated using a 3-digit sector-year specific deflator.

We implement the estimation of A.4 following Wooldridge's 2009 method. We use first-order lags to instrument for intermediate input expenditure, output prices, and market shares. We assume that labor and capital are predetermined and are not correlated with productivity. We assume $f(\cdot)$ is Cobb-Douglas.

After estimating output elasticities at the 3-digit sector level using only single-product firms, we assign these elasticities to each product line in the sector for multi-product firms. We then assume multi-product firms allocate inputs in proportion to each product's revenue share within the firm as in Blum et al. (2023), since direct input allocations are unobserved.

For robustness, we also compute input allocations following De Loecker et al. (2016). Their approach assumes common productivity across products within the same establishment, which allows for the construction of a system of equations that determines input allocations. Using this method, we find a correlation of approximately 0.7 between revenue shares and the allocations generated by their model. We retain item-level revenue shares as our baseline because, although highly correlated, the De Loecker et al. (2016) routine often produces input allocations near the boundaries, which distort the resulting markup distributions leading to negative or implausibly large markups.

A.4. Revenue production functions: Cost-share approach

In additional exercises we use data from the Colombian manufacturing survey and the US Census of manufactures. These datasets do not report prices and quantities separately and thus force us to estimate revenue-based production functions. We do this following the cost-share approach This method relies on the cost-minimization (2) and imposes constant-returns-to-scale on variable inputs to recover the elasticity of output with respect to variable inputs from their cost-shares. We follow the exposition

TABLE A.2. Sector-Level Production Function Betas and Returns to Scale

Sector Code	**Sector Name**	β_M	β_K	β_L	**RTS**
131	Sugar & Molasses	0.85	0.01	0.11	0.97
134	Bakery Products	1.05	0.06	-0.12	0.99
344	Pesticides & Insecticides	0.63	0.20	0.01	0.84
363	Soaps & Detergents	0.66	0.17	0.30	1.13
443	Leather Footwear	0.83	-0.05	0.16	0.94
511	Wood & Wood Products	0.82	0.02	0.09	0.93
551	Printing & Writing Paper	0.90	0.07	-0.03	0.94
632	Cotton Yarn & Fiber	1.06	-0.06	-0.02	0.97
642	Man-Made Yarn & Fiber	0.59	0.01	0.05	0.65
732	Aluminum & Alloys	0.88	0.01	0.09	0.97
769	Misc. Non-Electrical Machinery	0.75	0.11	0.21	1.07
772	Electrical Motors & Transformers	0.26	0.16	0.45	0.87
782	Audio/Video Apparatus	0.82	-0.09	0.18	0.91
821	Motor Vehicles & Parts	0.69	-0.12	0.58	1.15

Note: Only the beta coefficients for material (β_M), capital (β_K), and labor (β_L) are shown. The "RTS" column provides an empirical estimate of returns to scale for each sector. The estimates correspond to the results described in section 2, for a Cobb-Douglas production function.

of Raval (2023) who builds on Foster, Grim, and Haltiwanger (2016). This is the approach followed, for instance, by Edmond, Midrigan, and Xu (2023).

The key assumption is that firms operating constant-returns-to-scale technologies optimize over the level of all their inputs on average, so that the following expression holds in the cross-section of firms for each variable input x_h:

$$E\left[P_n X_{n,i}\right] = \epsilon_n E\left[\lambda_i Y_i\right] . \tag{A.6}$$

This is weaker than demanding that each firm's first order condition holds, and allows for firms to have adjustment costs, or time-to-build costs, as long as they average in the aggregate. This method also allows for non-neutral technology as opposed to only Hicks-neutral technology. In practice this amounts to separating firms into groups that (are expected to) differ in their elasticities, for instance because of differences in productivity (Raval 2023). However, this condition is clearly violated if not all inputs are perfectly flexible (as is the case for capital and labor) and there is an industry-wide shock.

Equation (A.6) coupled with the assumption of constant-returns-to-scale on the inputs over which the firm optimizes lets us recover the quantity $\lambda_i Y_i$ as the total cost of the firm. So, if there are N flexible inputs as in (1) we get the elasticity of output with respect to the n^{th} input as the input's cost-share:

$$\epsilon_n = \frac{E\left[P_n X_{n,i}\right]}{E\left[\sum_n^N P_n X_{n,i}\right]} . \tag{A.7}$$

Implementing (A.7) requires assuming that capital and labor are flexible on average and that the production function has constant returns to scale in all the observable inputs.

Remark This method also applies to cases where data on combined inputs is available. Say that one observes only the total expenditure in two inputs (labeled 1 and 2), then we can sum equation (A.6) applied to each input to obtain:

$$E\left[P_1 X_{1,i} + P_2 X_{2,i}\right] = (\epsilon_1 + \epsilon_2)\, E\left[\lambda_i Y_i\right] . \tag{A.8}$$

In this way, we get the sum of elasticities from the combined cost-share of the inputs:

$$\epsilon_1 + \epsilon_2 = \frac{E\left[P_1 X_{1,i} + P_2 X_{2,i}\right]}{E\left[\sum_n^N P_n X_{n,i}\right]} . \tag{A.9}$$

This is precisely the value needed to recover markups. Applying (2) to the two inputs and summing we can write:

$$\mu\left(\rho_1 + \rho_2\right) = \epsilon_1 + \epsilon_2 . \tag{A.10}$$

In practice this is useful when only data on cost-of-goods-sold is available, combining the costs of several inputs.

Assumptions and output-price bias Relying on cost shares to measure markups requires strong assumptions, namely constant-returns-to-scale and uniform output elasticities across firms. Moreover, cost shares only contain firms' revenues and expenditures. An alternative approach that avoids these strong assumptions is to estimate the output elasticity to variable inputs using a production function estimation approach as in De Loecker and Warzynski (2012).

However, the estimation of production functions when firm-level output without price data is also problematic, as it is only feasible to estimate revenue-based production functions instead of quantity-based production functions. Elasticity and markup estimates coming from revenue-based production functions suffer from "output-price bias" as shown in Bond et al. (2021). The bias arises because the price-setting behavior of firms in non-competitive markets make it impossible to establish a connection between revenue- and quantity-based production functions.

It is possible to counteract the output-price bias when prices are observable. Furthermore, under specific conditions, it becomes feasible to consistently derive markups from revenue data provided there is independent variation in variables associated with markups (Kirov, Mengano, and Traina 2022).

A.5. Markup Measures from Production

We consider several approaches to measure markups from firm level data. These approaches exploit the relationship between the firm's cost minimization problem and their markups, defined as the ratio of the price to the marginal cost of the firm, $\mu = p/\lambda$.

The cost minimization problem of the firm is:

$$C\left(y|\left\{p_n\right\}_{n=1}^{N},\{K_m\}_{m=1}^{M}\right) = \min_{\{x_n\}_{n=1}^{N}} \sum_{n=1}^{N} p_n \cdot x_n \quad \text{s.t. } \overline{y} \leq zF\left(x_1,\ldots,x_N,K_1,\ldots,K_M\right), \tag{A.11}$$

where production requires variable inputs $\{x_n\}_{n=1}^{N}$ and fixed inputs $\{K_m\}_{m=1}^{M}$. The marginal cost of production is captured by the Lagrangian multiplier (λ) of the production constraint. In equilibrium it holds that $\lambda = C'(y)$.

The first order condition with respect to variable input x_n is:

$$p_n = \lambda \frac{\partial F}{\partial x_n}. \tag{A.12}$$

Simple manipulation of this condition makes it possible to express the markup in terms of the cost-to-revenue share of input x_n and the elasticity of output with respect to the variable input x_n, ϵ_n:

$$\mu = \frac{p}{\lambda} = \frac{py}{p_n x_n}\epsilon_n = \frac{\epsilon_n}{s_n}. \tag{A.13}$$

Crucially, this condition is entirely compatible with the demand system and the optimal pricing behavior of firms in Section 3. Moreover, data on the cost-to-revenue share of inputs ($s_n \equiv p_n x_n / py$) is readily available from manufacturing surveys, economic censuses, or public firm data like compustat.

Appendix B. Additional Results

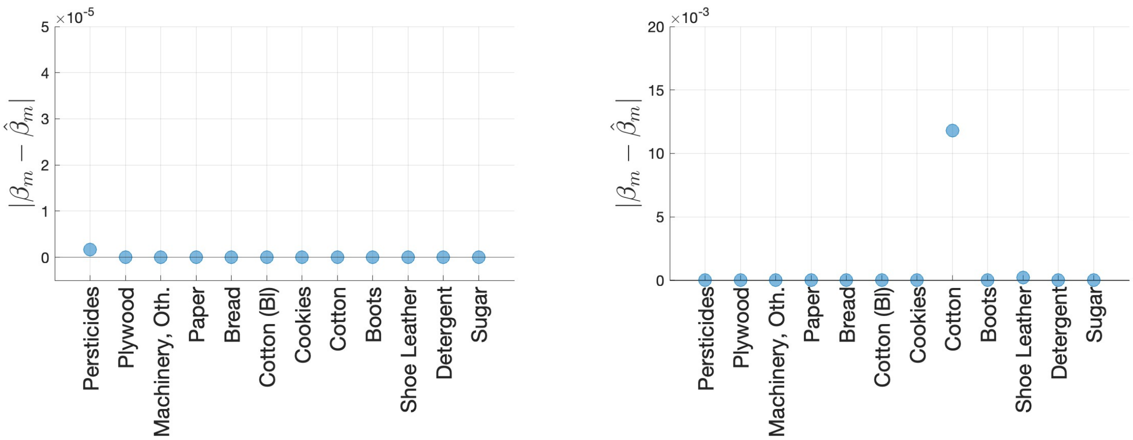
TABLE B.1. Variance decomposition of observed centered (log) markup: Within vs Between Shares

Sample	Std. Dev.	Share Within	Share Between
1	0.434	0.579	0.421
2	0.333	0.550	0.450
Overall	0.383	0.564	0.436

Notes: The standard deviation is computed from the log ratio $\log(\mu_i/\mu_m)$. Subsample and Overall are computed with market share weights. we divide firms in equally sized bins of 1 percentage point market share.

FIGURE B.1. Demand Estimation Residuals – Selected Products (2005–2007)

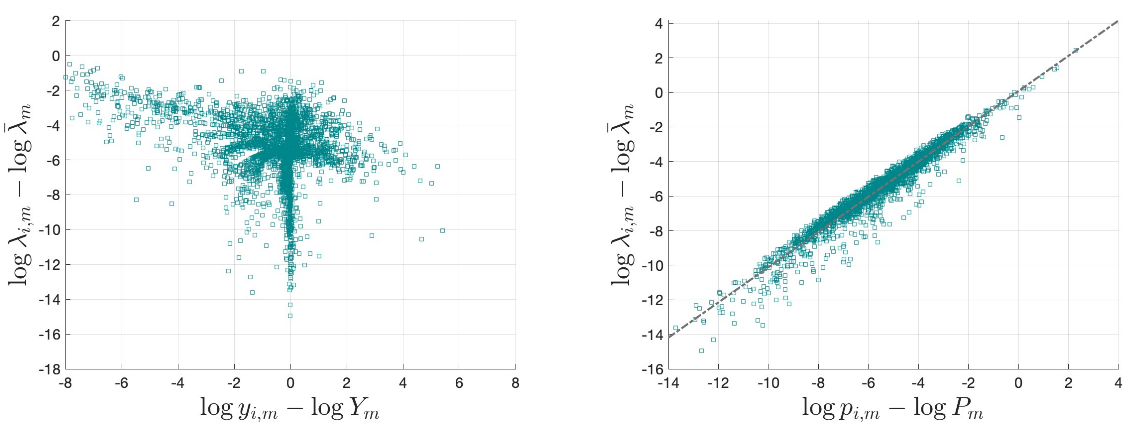
(A) Bertrand Competition | (B) Cournot Competition



Notes: The figures plot the residuals of the demand estimation captured by the absolute difference between the coefficient β in equation (29) and its model counterpart in Proposition 4. Panel A does so under Bertrand competition and panel B under Cournot competition. We impose the Klenow and Willis aggregator in both cases.

FIGURE B.2. Marginal Costs (λ) and Relationship with Output and Prices

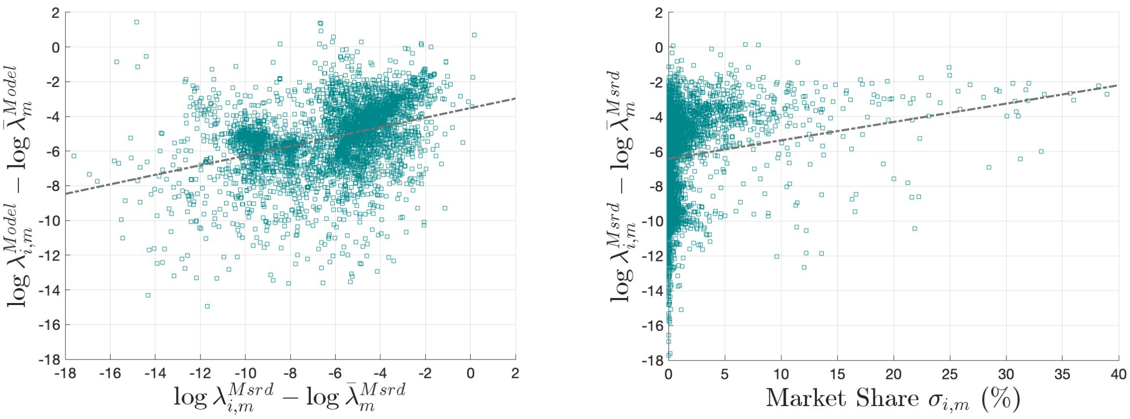
(A) λ and Output y_i/y_m | (B) λ and Prices p_i/p_m



Notes: Panel A shows the scatter plot of firm log-marginal costs relative to their market's marginal cost and log-markups relative to their market's average. Panel B shows the scatter plot of firm log-marginal costs relative to their market share. Marginal costs are computed as in (36), relative output $\frac{y_i}{Y}$ obtained as in (34), and relative prices as in (35).

FIGURE B.3. Marginal Costs Fit and Relationship with Market Shares (2005-2007)

(A) Measured λ and Model λ (B) Measured λ and Market-Share



Notes: Panel A shows the scatter of the model's marginal costs λ from (36) and the measured marginal costs, recovered as the ratio between observed output prices and the estimated markup from Section 2. Panel B shows the scatter of the measured marginal costs and market shares.

TABLE B.2. Variance decomposition of Kimball centered (log) markup: Within vs Between Shares

Sample	Std. Dev.	Share Within	Share Between
1	0.170	0.084	0.916
2	0.091	0.082	0.918
Overall	0.130	0.083	0.917

Notes: The standard deviation is computed from the log ratio $\log(\mu_i/\mu_m)$. Subsample and Overall are computed with market share weights.we divide firms in equally sized bins of 1 percentage point market share.

TABLE B.3. Variance decomposition of Atkenson & Burstein-Kimball markup: Within vs Between Shares

Sample	Std. Dev.	Share Within	Share Between
1	0.160	0.109	0.891
2	0.091	0.111	0.889
Overall	0.126	0.110	0.890

Notes: The standard deviation is computed from the log ratio $\log(\mu_i/\mu_m)$. Subsample and Overall are computed with market share weights. we divide firms in equally sized bins of 1 percentage point market share.

TABLE B.4. Markup Variance Decomposition US

Markup	**Variance**	**Share between**	**Share within**	**Corr**($\log\mu$, σ)
Data	0.024	0.676	0.324	-0.075
Full Model	0.024	0.676	0.324	-0.075
Atkeson & Burstein	0.000	0.033	0.967	0.628
Kimball	0.002	0.143	0.857	-0.018
AB + Kimball	0.006	0.226	0.774	-0.061

Notes: The standard deviation is computed from the log ratio $\log(\mu_i/\mu_m)$. We divide firms in equally sized bins of 5 percentage point market share. he variance and its decomposition correspond to the average variance and shares across market-year pairs. The correlation between markups and market shares is the pooled correlation of the full sample.

Table B.5. Markup Variance Decomposition Colombia

Markup	**Variance**	**Share between**	**Share within**	**Corr**(log μ, σ**)**
Data	0.059	0.739	0.261	0.030
Full Model	0.059	0.739	0.261	0.030
Atkeson & Burstein	0.001	0.042	0.958	0.608
Kimball	0.006	0.154	0.846	0.213
AB + Kimball	0.006	0.223	0.777	0.183

Notes: The standard deviation is computed from the log ratio $\log(\mu_i/\mu_m)$. We divide firms in equally sized bins of 5 percentage point market share. he variance and its decomposition correspond to the average variance and shares across market-year pairs. The correlation between markups and market shares is the pooled correlation of the full sample.

Appendix C. Model Appendix

C.1. Oligopolistic Competition Setup

The aggregate good in a market m is produced using N differentiated inputs, each produced by a monopolist. Market aggregators behave competitively and aggregate intermediate inputs operating a technology implicitly defined by the Kimball (1995) aggregator. The problem of the good-m producer is:

$$\min_{y_i^m} \sum_{i=1}^{N_m} p_i^m y_i^m \qquad \text{s.t.} \quad 1 = \sum_{i=1}^{N_m} \Upsilon\left(\frac{y_i^m}{Y_m}\right), \tag{C.1}$$

where y_i^m is the quantity of variety i purchased. The first order conditions are

$$p_i^m = \Upsilon'\left(\frac{y_i^m}{Y_m}\right)\frac{\Lambda_m}{Y_m} \tag{C.2}$$

where Λ_m is the Lagrange multiplier associated with the constraint.

To arrive to the demand for variety i in (8) we need to solve for the Lagrange Multiplier, Λ_m. To do this, multiply both sides of (C.2) by y_i^m and sum across varieties to obtain

$$\Lambda_m = \frac{\sum_i p_i^m y_i^m}{\sum_i \Upsilon'\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}. \tag{C.3}$$

Then, define the price of the sectoral good, P_m so that it satisfies

$$P_m \cdot Y_m = \sum_{i=1}^{N_m} p_i^m \cdot y_i^m. \tag{C.4}$$

Finally, replace Λ_m into (C.2) and divide both sides by P_m to obtain (8):

$$\frac{p_i^m}{P_m} = \Upsilon'\left(\frac{y_i^m}{Y_m}\right)\frac{\Lambda_m}{P_m Y_m} = \Upsilon'\left(\frac{y_i^m}{Y_m}\right)\frac{\sum_i \frac{p_i^m y_i^m}{P_m Y_m}}{\sum_i \Upsilon'\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}} = \frac{\Upsilon'\left(\frac{y_i^m}{Y_m}\right)}{\sum_i \Upsilon'\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}. \tag{C.5}$$

Having derived the demand for variety i we can further characterize the P_m following Boar and Midrigan (2024). It is useful to define the auxiliary variable D_m,

$$D_m = \sum_{i=1}^{N_m} \Upsilon'\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}. \tag{C.6}$$

Then, the sectoral price is obtained implicitly as:

$$P_m = \frac{\sum_i p_i^m y_i^m}{Y_m} = \frac{\Lambda_m \cdot D_m}{Y_m}. \tag{C.7}$$

This allow us to characterize P_m by inverting (C.2) to get

$$\frac{y_i^m}{Y_m} = \left(\Upsilon'\right)^{-1}\left(Y_m \frac{p_i^m}{\Lambda_m}\right) = \left(\Upsilon'\right)^{-1}\left(D_m \frac{p_i^m}{P_m}\right). \tag{C.8}$$

Replacing on the definition of the price index gives

$$P_m = \sum_{i=1}^{N} p_i^m \frac{y_i^m}{Y_m} = \sum_{i=1}^{N} p_i^m \left(\Upsilon'\right)^{-1}\left(D_m \frac{p_i^m}{P_m}\right). \tag{C.9}$$

Then, from the definition of the aggregator (7) we get

$$1 = \sum_{i=1}^{N_m} \Upsilon\left(\left(\Upsilon'\right)^{-1}\left(D_m \frac{p_i^m}{P_m}\right)\right). \tag{C.10}$$

This delivers a pair of equations that jointly determine P_m and D_m given prices.

C.2. Proofs for Model Results

Lemma A1. ***(Market Shares)*** *The change in market output Y_m to firm i's output and of market price P_m to firm i's price satisfy*

$$\frac{\partial Y_m}{\partial y_i^m} = \frac{p_i^m}{P_m} \qquad \text{and} \qquad \frac{\partial P_m}{\partial p_i^m} = \frac{y_i^m}{Y_m}. \tag{C.11}$$

So, the market share of firm i in market m gives the elasticity of market output and market price to changes in the firm's output and price, respectively,

$$\sigma_i^m \equiv \frac{p_i^m y_i^m}{P_m Y_m} = \frac{\Upsilon_i'\left(\frac{y_i^s}{Y_s}\right)\frac{y_i^s}{Y_s}}{\sum_j \Upsilon_j'\left(\frac{y_j^s}{Y_s}\right)\frac{y_j^s}{Y_s}} = \frac{y_i^m}{Y_m}\frac{\partial Y_m}{\partial y_i^m} = \frac{p_i^m}{P_m}\frac{\partial P_m}{\partial p_i^m}. \tag{C.12}$$

Proof. Equation (10) comes from the definition of market shares and the (inverse) demand for good i in (8).

We now prove that $\partial Y_m/\partial y_i^m = p_i^m/P_m$. From the aggregation technology (7), we obtain by totally differentiating,

$$0 = \sum_{j=1}^{N_m} \Upsilon'\left(\frac{y_j^m}{Y_m}\right)\left(\frac{1}{Y_m} d y_j^m - \frac{y_j^m}{Y_m^2} dY_m\right). \tag{C.13}$$

So that the derivative with respect to firm i's output, y_i^m is

$$\frac{dY_m}{d y_i^m} = \frac{\Upsilon'\left(\frac{y_i^m}{Y_m}\right)}{\sum_{i=1}^{N} \Upsilon'\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}} = \frac{p_i^m}{P_m}. \tag{C.14}$$

We now prove that $\partial P_m/\partial p_i^m = y_i^m/Y_m$. The proof works in three steps. First define an auxiliary variable $D_m \equiv \sum_{h=1}^{N_m} \Upsilon'_h \left(y_h^m/Y_m\right) \cdot y_h^m/Y_m$, this will make the remaining steps more tractable. Second establish restrictions on the changes in total demand from a change in prices. For this consider equations (7) and (8) to get

$$1 = \sum_{j=1}^{N_m} \Upsilon\left(\left(\Upsilon'\right)^{-1}\left(D_m \frac{p_j^m}{P_m}\right)\right). \tag{C.15}$$

Differentiate with respect to p_i^m we get

$$0 = \sum_{j=1}^{N_m} \frac{\Upsilon'\left(\frac{y_j^m}{Y_m}\right)}{\Upsilon''\left(\frac{y_j^m}{Y_m}\right)} \left[\frac{\partial}{\partial p_i^m} D_m \frac{p_j^m}{P_m}\right]. \tag{C.16}$$

Finally, from the definition of the price index we get

$$1 = \sum_{j=1}^{N_m} \left(\Upsilon'\right)^{-1}\left(D_m \frac{p_j^m}{P_m}\right) \frac{p_j^m}{P_m}. \tag{C.17}$$

Differentiate with respect to p_i^m we get

$$0 = \sum_{j=1}^{N_m} \left[\frac{p_j^m}{P_m} \frac{\partial}{\partial p_i^m} \left(\Upsilon'\right)^{-1}\left(D_m \frac{p_j^m}{P_m}\right) + \frac{y_j^m}{Y_m} \frac{\partial}{\partial p_i^m} \frac{p_j^m}{P_m}\right] \tag{C.18}$$

$$0 = \sum_{j=1}^{N_m} \frac{\frac{p_j^m}{P_m}}{\Upsilon''\left(\frac{y_j^m}{Y_m}\right)} \left[\frac{\partial}{\partial p_i^m} D_m \frac{p_j^m}{P_m}\right] - \sum_{j=1}^{N_m} \frac{y_j^m}{Y_m} \frac{p_j^m}{P_m} \frac{1}{P_m} \frac{\partial P_m}{\partial p_i^m} + \frac{y_i^m}{Y_m} \frac{1}{P_m}$$

$$0 = \underbrace{\frac{1}{D_m} \sum_{j=1}^{N_m} \frac{\Upsilon'\left(\frac{y_j^m}{Y_m}\right)}{\Upsilon''\left(\frac{y_j^m}{Y_m}\right)} \left[\frac{\partial}{\partial p_i^m} D_m \frac{p_j^m}{P_m}\right]}_{=0 \text{ by (C.16)}} - \frac{1}{P_m} \left(\underbrace{\left(\sum_{j=1}^{N_m} \sigma_j^m\right)}_{=1} \frac{\partial P_m}{\partial p_i^m} - \frac{y_i^m}{Y_m}\right)$$

$$0 = -\frac{\partial P_m}{\partial p_i^m} + \frac{y_i^m}{Y_m},$$

which gives the result. See Matsuyama (2023) for a general proof for homothetic demand. □

Proposition A1. ***(Elasticities)*** *The elasticity of firm i in market m satisfies*

$$\textbf{\textit{Cournot}} \quad \frac{1}{\eta_i^m} = \underbrace{\frac{1}{\gamma}}_{\textit{Market Elasticity}} \sigma_i^m + \underbrace{\left(\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right) + E_\sigma\left[\frac{1}{\varepsilon_j^m}\middle| j \neq i\right]\sigma_i^m\right)\left(1-\sigma_i^m\right)}_{\textit{Variety Elasticity}} \; ; \tag{C.19}$$

$$\textbf{\textit{Bertrand}} \quad \eta_i^m = \underbrace{\gamma}_{\textit{Market Elasticity}} \sigma_i^m + \underbrace{\varepsilon_i^m \frac{E_\sigma\left[\varepsilon_j^m | j \neq i\right]}{E_\sigma\left[\varepsilon_j^m\right]}}_{\textit{Variety Elasticity}}\left(1-\sigma_i^m\right) \; ; \tag{C.20}$$

where $\sigma_i^m = p_i^m y_i^m / P_m Y_m$ *is firm i's market share,* ε_i^m *is its own-elasticity as in (14),* $E_\sigma\left[x_j\right] = \sum_j^{N_m} x_j \sigma_j^m$ *is the average with respect to expenditure in market m, and* $E_\sigma\left[x_j | j \neq i\right] = \sum_{j \neq i} x_j \frac{\sigma_j^m}{1-\sigma_i^m}$ *is the average with respect to firm i's competitors.*

Proof. We start by tackling Cournot competition. We take as given the output of competitors. The total (or average) elasticity of demand is

$$\left(\eta_i^m\right)^{-1} = -y_i^m \frac{\partial \log p_i^m}{\partial y_i^m} = -y_i \frac{\partial \log\left(\frac{\Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)\alpha_m\left(\frac{Y_m}{Y}\right)^{-\frac{1}{\gamma}} P}{\sum_j \Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}\right)}{\partial y_i} \tag{C.21}$$

So that we can express the elasticity in terms of three terms and solve using Lemma 1 and equation (14).

$$\frac{1}{\eta_i^m} = -y_i^m\left[\frac{\partial \log \Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)}{\partial y_i^m} + \frac{\partial \log \alpha_m\left(\frac{Y_m}{Y}\right)^{-\frac{1}{\gamma}}}{\partial y_i} - \frac{\partial \log \sum_j \Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\frac{y_j^m}{Y_m}}{\partial y_i^m}\right] \tag{C.22}$$

$$= -y_i^m\left[\frac{\Upsilon_i''\left(\frac{y_i^m}{Y_m}\right)}{\Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)}\frac{\partial\left(\frac{y_i^m}{Y_m}\right)}{\partial y_i^m} - \frac{1}{\gamma}\frac{1}{Y_m}\frac{\partial Y_m}{\partial y_i^m}\right] \tag{C.23}$$

$$+ \frac{y_i^m}{\sum_j \Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\frac{y_j^m}{Y_m}}\left(\frac{\partial \Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}{\partial y_i^m} + \sum_{j \neq i}\frac{\partial \Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\frac{y_{m_j}}{Y_m}}{\partial y_i^m}\right)$$

$$= -\frac{\Upsilon_i''\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}{\Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)}\left(1 - \frac{y_i^m}{Y_m}\frac{\partial Y_m}{\partial y_i^m}\right) + \frac{1}{\gamma}\frac{y_i^m}{Y_m}\frac{\partial Y_m}{\partial y_i^m} \tag{C.24}$$

$$+\frac{\frac{y_i^m}{Y_m}}{\sum_j \Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\frac{y_j^m}{Y_m}}\left(\Upsilon_i''\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}+\Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)\right)\left(1-\frac{y_i^m}{Y_m}\frac{\partial Y_m}{\partial y_i^m}\right)$$

$$-\frac{\frac{y_i^m}{Y_m}}{\sum_j \Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\frac{y_j^m}{Y_m}}\sum_{j\neq i}\left(\Upsilon_j''\left(\frac{y_j^m}{Y_m}\right)\frac{y_j^m}{Y_m}+\Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\right)\left(\frac{y_j^m}{Y_m}\frac{\partial Y_m}{\partial y_i^m}\right)$$

At this point we suppress the arguments of Υ and we take advantage of Lemma 1 that gives us $\frac{\partial Y_m}{\partial y_i^m}=\frac{p_i^m}{P_m}$ to group terms and obtain market shares σ_i^m.

$$\frac{1}{\eta_i^m}=-\frac{\Upsilon_i''\cdot\frac{y_i^m}{Y_m}}{\Upsilon_i'}\left(1-\sigma_i^m\right)+\frac{1}{\gamma}\sigma_i^m+\frac{\frac{y_i^m}{Y_m}}{\sum_j \Upsilon_j'\cdot\frac{y_j^m}{Y_m}}\left(\Upsilon_i''\cdot\frac{y_i^m}{Y_m}+\Upsilon_i'\right)\left(1-\sigma_i^m\right) \tag{C.25}$$

$$-\frac{1}{\sum_j \Upsilon_j'\cdot\frac{y_j^m}{Y_m}}\sum_{j\neq i}\left(\Upsilon_j''\cdot\frac{y_j^m}{Y_m}+\Upsilon_j'\right)\left(\frac{y_j^m}{Y_m}\sigma_i^m\right)$$

$$=-\frac{\Upsilon_i''\cdot\frac{y_i^m}{Y_m}}{\Upsilon_i'}\left(1-\sigma_i^m\right)+\frac{1}{\gamma}\sigma_i^m+\frac{\Upsilon_i'\cdot\frac{y_i^m}{Y_m}}{\sum_j \Upsilon_j'\cdot\frac{y_j^m}{Y_m}}\left(\frac{\Upsilon_i''\cdot\frac{y_i^m}{Y_m}}{\Upsilon_i'}+1\right)\left(1-\sigma_i^m\right) \tag{C.26}$$

$$-\sum_{j\neq i}\left(\frac{\Upsilon_j''\cdot\frac{y_j^m}{Y_m}}{\Upsilon_j'}+1\right)\frac{\Upsilon_j'\cdot\frac{y_j^m}{Y_m}}{\sum_j \Upsilon_j'\cdot\frac{y_j^m}{Y_m}}\sigma_i^m$$

$$=\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right)+\frac{1}{\gamma}\sigma_i^m+\sigma_i^m\left(1-\frac{1}{\varepsilon_i^m}\right)\left(1-\sigma_i^m\right)-\sum_{j\neq i}\left(1-\frac{1}{\varepsilon_j^m}\right)\sigma_j^m\sigma_i^m, \tag{C.27}$$

where we use the definition of the own-elasticity ε_i^m in (14) and equation (10) from Lemma 1. The result is obtained after some manipulation:

$$\frac{1}{\eta_i^m}=\frac{1}{\gamma}\sigma_i^m+\left[\frac{1}{\varepsilon_i^m}+\left(1-\frac{1}{\varepsilon_i^m}\right)\cdot\sigma_i^m-\sigma_i^m\sum_{j\neq i}\left(1-\frac{1}{\varepsilon_j^m}\right)\cdot\frac{\sigma_j^m}{1-\sigma_i^m}\right]\left(1-\sigma_i^m\right) \tag{C.28}$$

$$=\frac{1}{\gamma}\sigma_i^m+\left[\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right)+\sigma_i^m\sum_{j\neq i}\frac{1}{\varepsilon_j^m}\cdot\frac{\sigma_j^m}{1-\sigma_i^m}\right]\left(1-\sigma_i^m\right)$$

$$=\frac{1}{\gamma}\sigma_i^m+\left[\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right)+E_\sigma\left[\frac{1}{\varepsilon_j^m}\middle| j\neq i\right]\sigma_i^m\right]\left(1-\sigma_i^m\right),$$

where

$$E_\sigma\left[\frac{1}{\varepsilon_j^m}\middle| j\neq i\right]\equiv\sum_{j\neq i}\frac{1}{\varepsilon_j^m}\frac{\sigma_j^m}{1-\sigma_i^m}. \tag{C.29}$$

We can also write the elasticity as

$$\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right)+\frac{1}{\eta_{-i}^m}\sigma_i^m=\sum_{j\neq i}\frac{\sigma_j^m}{1-\sigma_i^m}\left[\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right)+\sigma_i^m\frac{1}{\varepsilon_j^m}\right]. \tag{C.30}$$

So that

$$\frac{1}{\eta_i^m}=\sigma_i^m\frac{1}{\gamma}+\left(1-\sigma_i^m\right)\sum_{j\neq i}\frac{\sigma_j^m}{1-\sigma_i^m}\left[\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right)+\sigma_i^m\frac{1}{\varepsilon_j^m}\right]. \tag{C.31}$$

We now tackle Bertrand competition. We take as given the prices of competitors. The total (or average) elasticity of demand is

$$\eta_i^m=-p_i^m\frac{\partial\log y_i^m}{\partial p_i^m}=-p_i^m\frac{\partial\log\left(Y_m\cdot\left(\Upsilon_i'\right)^{-1}\left(\left(\sum_j\Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\cdot\frac{y_j^m}{Y_m}\right)\frac{p_i^m}{P_m}\right)\right)}{\partial p_i^m}. \tag{C.32}$$

This allows us to break down the elasticity into three terms

$$\eta_i^m=\underbrace{-p_i^m\frac{\partial\log Y_m}{\partial p_i^m}}_{\text{Granular Effect}}\underbrace{-\frac{p_i^m\sum_j\Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\cdot\frac{y_j^m}{Y_m}}{\Upsilon_i''\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}\frac{\partial p_i^m/P_m}{\partial p_i^m}}_{\text{Own-Elasticity}}\underbrace{-\frac{p_i^m\frac{p_i^m}{P_m}}{\Upsilon_i''\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}\frac{\partial\sum_j\Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\cdot\frac{y_j^m}{Y_m}}{\partial p_i^m}}_{\text{Substitution Effects}}. \tag{C.33}$$

We proceed by tackling each effect separately.

$$\underbrace{-p_i^m\frac{\partial\log Y_m}{\partial p_i^m}}_{\text{Granular Effect}}=-p_i^m\frac{\partial\log Y\left(\frac{1}{\alpha_m}\frac{P_m}{P}\right)^{-\gamma}}{\partial p_i^m}=\gamma\frac{p_i^m}{P_m}\frac{\partial P_m}{\partial p_i^m}=\gamma\sigma_i^m \tag{C.34}$$

where the last step follows from Lemma 1.

Now to the own-elasticity. We solve for this only partially as it interacts with the substitution effects term.

$$\underbrace{-\frac{p_i^m\sum_j\Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\cdot\frac{y_j^m}{Y_m}}{\Upsilon_i''\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}\frac{\partial p_i^m/P_m}{\partial p_i^m}}_{\text{Own-Elasticity}}=-\frac{\frac{p_i^m}{P_m}\sum_j\Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\cdot\frac{y_j^m}{Y_m}}{\Upsilon_i''\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}\left(1-\frac{p_i^m}{P_m}\frac{\partial P_m}{\partial p_i^m}\right) \tag{C.35}$$

$$= -\frac{\Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)}{\Upsilon_i''\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}\left(1-\sigma_i^m\right) \;=\; \varepsilon_i^m\left(1-\sigma_i^m\right)$$

Now the substitution term. As in Lemma 1 we define $D_m \equiv \sum_h \Upsilon_h'\left(y_h^m/Y_m\right)\cdot y_h^m/Y_m$ and consider first its derivative with respect to the price of good i. We start from (C.16)

$$0 = \sum_{j=1}^{N_m}\frac{\Upsilon'\left(\frac{y_j^m}{Y_m}\right)}{\Upsilon''\left(\frac{y_j^m}{Y_m}\right)}\left[\frac{\partial}{\partial p_i^m}D_m\frac{p_j^m}{P_m}\right] \tag{C.36}$$

$$= \sum_{j=1}^{N_m}\frac{\Upsilon'\left(\frac{y_j^m}{Y_m}\right)}{\Upsilon''\left(\frac{y_j^m}{Y_m}\right)}\left[\frac{\partial D_m}{\partial p_i^m}\frac{p_j^m}{P_m}+\frac{D_m}{P_m^2}\left(P_m\frac{\partial p_j^m}{\partial p_i^m}-p_j^m\frac{\partial P_m}{\partial p_i^m}\right)\right]$$

$$= \sum_{j=1}^{N_m}\frac{\Upsilon'\left(\frac{y_j^m}{Y_m}\right)}{\Upsilon''\left(\frac{y_j^m}{Y_m}\right)\frac{y_j^m}{Y_m}}\frac{y_j^m}{Y_m}\left[\frac{1}{D_m}\frac{\partial D_m}{\partial p_i^m}p_j^m-\frac{p_j^m}{p_i^m}\sigma_i^m\right]+\frac{\Upsilon'\left(\frac{y_i^m}{Y_m}\right)}{\Upsilon''\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}\frac{y_i^m}{Y_m}$$

$$= -\sum_{j=1}^{N_m}\varepsilon_j^m\sigma_j^m\left[\frac{P_m}{D_m}\frac{\partial D_m}{\partial p_i^m}-\frac{P_m}{p_i^m}\sigma_i^m\right]-\varepsilon_i^m\frac{y_i^m}{Y_m}$$

$$= -\left(\sum_{j=1}^{N_m}\varepsilon_j^m\sigma_j^m\right)\frac{P_m}{D_m}\frac{\partial D_m}{\partial p_i^m}+\left[\left(\sum_{j=1}^{N_m}\varepsilon_j^m\sigma_j^m\right)-\varepsilon_i^m\right]\frac{y_i^m}{Y_m}$$

This lets us get the derivative of D_m

$$\frac{\partial D_m}{\partial p_i^m} = \left(1-\frac{\varepsilon_i^m}{\sum_j\varepsilon_j^m\sigma_j^m}\right)\sigma_i^m\frac{D_m}{p_i^m}. \tag{C.37}$$

Now we turn to the substitution effects

$$-\underbrace{\frac{p_i^m\frac{p_i^m}{P_m}}{\Upsilon_i''\frac{y_i^m}{Y_m}}\frac{\partial D_m}{\partial p_i^m}}_{\text{Substitution Effects}} = -\frac{\Upsilon_i'}{\Upsilon_i''\frac{y_i^m}{Y_m}}\left(1-\frac{\varepsilon_i^m}{\sum_j\varepsilon_j^m\sigma_j^m}\right)\sigma_i^m\frac{\frac{p_i^m}{P_m}D_m}{\Upsilon_i'} = \varepsilon_i^m\left(1-\frac{\varepsilon_i^m}{\sum_j\varepsilon_j^m\sigma_j^m}\right)\sigma_i^m$$

Finally, we get the elasticity

$$\eta_i^m = \gamma\sigma_i^m+\varepsilon_i^m\left(1-\sigma_i^m\right)+\varepsilon_i^m\left(1-\frac{\varepsilon_i^m}{\sum_j\varepsilon_j^m\sigma_j^m}\right)\sigma_i^m. \tag{C.38}$$

We can also write down as

$$\eta_i^m = \gamma\sigma_i^m + \varepsilon_i^m\left(1 - \frac{\varepsilon_i^m\sigma_i^m}{\sum_j \varepsilon_j^m\sigma_j^m}\right) = \gamma\sigma_i^m + \varepsilon_i^m \frac{E_\sigma\left[\varepsilon_j^m | j \neq i\right]}{E_\sigma\left[\varepsilon_j^m\right]}\left(1 - \sigma_i^m\right).$$

□

Proposition A2. ***(Market Elasticities and Markups)*** *Market elasticities satisfy*

$$\textbf{Cournot} \qquad \frac{1}{\eta_m} = HHI_m\frac{1}{\gamma} + (1 - HHI_m)\,E_\sigma\left[\frac{1}{\varepsilon_i^m}\right] - 2Cov_\sigma\left(\sigma_i^m, \frac{1}{\varepsilon_i^m}\right); \tag{C.39}$$

$$\begin{aligned}\textbf{Bertrand} \qquad \frac{1}{\eta_m} = {} & E_\sigma\left[\frac{1}{\varepsilon_i^m}\right] + \frac{HHI_m}{E_\sigma\left[\varepsilon_i^m\right]} - \\ & \gamma\left(HHI_m E_\sigma\left[\left(\frac{1}{\varepsilon_i^m}\right)^2\right] + Cov_\sigma\left(\sigma_i^m, \left(\frac{1}{\varepsilon_i^m}\right)^2\right)\right) + \Omega\end{aligned} \tag{C.40}$$

where $HHI_m = \sum_i\left(\sigma_i^m\right)^2$ *is the Herfindahl-Hirschman index,* $E_\sigma\left[x_j\right] = \sum_{j=1}^{N_m} x_j\,\sigma_j^m$ *is the expectation with respect to sales shares, and* $Cov_\sigma\left(x_j, y_j\right) = \sum_{j=1}^{N_m}\left(x_j\right)\left(y_j - E_\sigma\left[y_j\right]\right)\sigma_j^m$ *is the covariance with respect to sales in the market. Finally,* $\Omega \equiv \sum_i^{N_m}\frac{\sigma_i^m}{\varepsilon_i^m}\left[\sum_{k=2}^{\infty}\left(\sigma_i^m\left(\frac{\varepsilon_i^m}{E_\sigma\left[\varepsilon_j^m\right]} - \frac{\gamma}{\varepsilon_i^m}\right)\right)^k\right]$ *contains higher order moments.*

The average markup satisfy

$$\textbf{Cournot} \quad \frac{1}{\mu_m} = \underbrace{\left(1 - \frac{1}{\gamma}\right)}_{\textit{Monopoly Markup}} + \underbrace{\left(\frac{1}{\gamma} - E_\sigma\left[\frac{1}{\varepsilon_i^m}\right]\right)(1 - HHI_m)}_{\textit{Concentration}} + \underbrace{2Cov_\sigma\left(\sigma_i^m, \frac{1}{\varepsilon_i^m}\right)}_{\textit{Distribution}}; \tag{C.41}$$

$$\begin{aligned}\textbf{Bertrand} \quad \frac{1}{\mu_m} = {} & \underbrace{\left(1 - \frac{1}{\gamma}\right)}_{\textit{Monopoly Markup}} + \underbrace{\left(\frac{1}{\gamma} - E_\sigma\left[\frac{1}{\varepsilon_i^m}\right]\right) + \frac{HHI_m}{\gamma}\left(E_\sigma\left[\left(\frac{\gamma}{\varepsilon_i^m}\right)^2\right] - \frac{\gamma}{E_\sigma\left[\varepsilon_i^m\right]}\right)}_{\textit{Concentration}} \\ & + \underbrace{Cov_\sigma\left(\sigma_i^m, \left(\frac{1}{\varepsilon_i^m}\right)^2\right)}_{\textit{Distribution}} - \Omega\,.\end{aligned} \tag{C.42}$$

Proof. We start with Cournot competition. The market elasticity is

$$\frac{1}{\eta_m} = \sum_{i=1}^{N_m}\left(\frac{1}{\gamma}\sigma_i^m + \left[\frac{1}{\varepsilon_i^m}\left(1 - \sigma_i^m\right) + \sigma_i^m\sum_{j \neq i}\frac{1}{\varepsilon_j^m}\cdot\frac{\sigma_j^m}{1 - \sigma_i^m}\right]\left(1 - \sigma_i^m\right)\right)\sigma_i^m \tag{C.43}$$

$$= \frac{1}{\gamma}\text{HHI} + \sum_{i=1}^{N_m} \left(\frac{1}{\varepsilon_i^m} \left(1 - 2\sigma_i^m\right) + \sigma_i^m \left(\sum_{j=1}^{N_m} \frac{1}{\varepsilon_j^m} \cdot \sigma_j^m \right) \right) \sigma_i^m$$

$$= \frac{1}{\gamma}\text{HHI} + \sum_{i=1}^{N_m} \frac{1}{\varepsilon_i^m} \left(1 - \sigma_i^m\right) \sigma_i^m - \sum_{i=1}^{N_m} \sigma_i^m \left(\frac{1}{\varepsilon_i^m} - E_\sigma \left[\frac{1}{\varepsilon_i^m} \right] \right) \sigma_i^m$$

$$= \frac{1}{\gamma}\text{HHI} + \sum_{i=1}^{N_m} \left[\frac{1}{\varepsilon_i^m} \left(1 - \sigma_i^m\right) + \sigma_i^m \left(\frac{1}{\varepsilon_i^m} - E_\sigma \left[\frac{1}{\varepsilon_i^m} \right] \right) \right] \sigma_i^m - 2\text{Cov}_\sigma \left(\sigma_i^m, \frac{1}{\varepsilon_i^m} \right)$$

$$= \text{HHI}\frac{1}{\gamma} + (1 - \text{HHI})\, E_\sigma \left[\frac{1}{\varepsilon_i^m} \right] - 2\text{Cov}_\sigma \left(\sigma_i^m, \frac{1}{\varepsilon_i^m} \right)$$

The markup follows from grouping terms in the expression $1/\mu_m = 1 - 1/\eta_m$.

For Bertrand competition we first express firm i's effective elasticity as

$$\eta_i^m = \gamma\sigma_i^m + \varepsilon_i^m \left(1 - \sigma_i^m\right) + \varepsilon_i^m \left(1 - \frac{\varepsilon_i^m}{\sum_j \varepsilon_j^m \sigma_j^m} \right) \sigma_i^m = \varepsilon_i^m \left(1 - \sigma_i^m \left(\frac{\varepsilon_i^m}{E_\sigma \left[\varepsilon_j^m \right]} - \frac{\gamma}{\varepsilon_i^m} \right) \right) \tag{C.44}$$

Average elasticity of the market is

$$\frac{1}{\eta_m} = \sum_i^{N_m} \frac{\sigma_i^m}{\varepsilon_i^m} \frac{1}{1 - \sigma_i^m \left(\frac{\varepsilon_i^m}{E_\sigma\left[\varepsilon_j^m\right]} - \frac{\gamma}{\varepsilon_i^m} \right)} \tag{C.45}$$

$$= \sum_i^{N_m} \frac{\sigma_i^m}{\varepsilon_i^m} \sum_{k=0}^{\infty} \left(\sigma_i^m \left(\frac{\varepsilon_i^m}{E_\sigma \left[\varepsilon_j^m \right]} - \frac{\gamma}{\varepsilon_i^m} \right) \right)^k$$

$$= \sum_i^{N_m} \frac{\sigma_i^m}{\varepsilon_i^m} \left[1 + \left(\frac{\sigma_i^m \varepsilon_i^m}{E_\sigma \left[\varepsilon_j^m \right]} - \gamma \frac{\sigma_i^m}{\varepsilon_i^m} \right) + \sum_{k=2}^{\infty} \left(\sigma_i^m \left(\frac{\varepsilon_i^m}{E_\sigma \left[\varepsilon_j^m \right]} - \frac{\gamma}{\varepsilon_i^m} \right) \right)^k \right]$$

$$= E_\sigma \left[\frac{1}{\varepsilon_i^m} \right] + \frac{\text{HHI}_m}{E_\sigma \left[\varepsilon_i^m \right]} - \gamma E_\sigma \left[\sigma_i^m \left(\frac{1}{\varepsilon_i^m} \right)^2 \right] + \Omega$$

$$= E_\sigma \left[\frac{1}{\varepsilon_i^m} \right] + \frac{\text{HHI}_m}{E_\sigma \left[\varepsilon_i^m \right]} - \gamma \left(\text{HHI}_m E_\sigma \left[\left(\frac{1}{\varepsilon_i^m} \right)^2 \right] + \text{Cov}_\sigma \left(\sigma_i^m, \left(\frac{1}{\varepsilon_i^m} \right)^2 \right) \right) + \Omega$$

where $\Omega \equiv \sum_i^{N_m} \frac{\sigma_i^m}{\varepsilon_i^m} \left[\sum_{k=2}^{\infty} \left(\sigma_i^m \left(\frac{\varepsilon_i^m}{E_\sigma\left[\varepsilon_j^m\right]} - \frac{\gamma}{\varepsilon_i^m} \right) \right)^k \right]$ contains higher order moments and $\text{Cov}_\sigma \left(\sigma_i^m, X_i^m \right) = E_\sigma \left[\sigma_i^m X_i^m \right] - E_\sigma \left[\sigma_i^m \right] E_\sigma \left[X_i^m \right] = E_\sigma \left[\sigma_i^m X_i^m \right] - \text{HHI}_m E_\sigma \left[X_i^m \right]$.

The third-order higher order terms are

$$\Omega_2 = \gamma^2 \left(\mathrm{HK}_m(3)^3 E_\sigma \left[\left(\frac{1}{\varepsilon_i^m} \right)^3 \right] + \mathrm{Cov}_\sigma \left((\sigma_i^m)^2, \left(\frac{1}{\varepsilon_i^m} \right)^3 \right) \right) \tag{C.46}$$
$$- 2 \frac{\gamma}{E_\sigma \left[\varepsilon_i^m \right]} \left(\mathrm{HK}_m(3)^3 E_\sigma \left[\left(\frac{1}{\varepsilon_i^m} \right) \right] + \mathrm{Cov}_\sigma \left((\sigma_i^m)^2, \left(\frac{1}{\varepsilon_i^m} \right) \right) \right)$$
$$+ \frac{1}{\left(E_\sigma \left[\varepsilon_i^m \right] \right)^2} \left(\mathrm{HK}_m(3)^3 E_\sigma \left[\varepsilon_i^m \right] + \mathrm{Cov}_\sigma \left((\sigma_i^m)^2, \varepsilon_i^m \right) \right) ;$$

where $\mathrm{HK}_m(k) \equiv \left(\sum_i^{N_m} \left(\sigma_i^m \right)^k \right)^{\frac{1}{k}}$ is the Hannah and Kay index of order k. □

Corollary A1. ***(Markups with Constant Elasticity)*** *If all firms have the same common own-elasticity, $\varepsilon_i^m = \varepsilon_m$, markups are*

Cournot
$$\frac{1}{\mu_m} = \left(1 - \frac{1}{\gamma} \right) + \left(\frac{1}{\gamma} - \frac{1}{\varepsilon_m} \right) (1 - HHI_m) ; \tag{C.47}$$

Bertrand
$$\frac{1}{\mu_m} = \frac{\varepsilon - 1}{\varepsilon} - \frac{1}{\varepsilon} \sum_{k=2}^{\infty} \left(1 - \frac{\gamma}{\varepsilon} \right)^{k-1} (HK_m(k))^k ; \tag{C.48}$$

where $HHI_m = \sum_i \left(\sigma_i^m \right)^2$ is the Herfindahl-Hirschman index and $HK_m(k) \equiv \left(\sum_i^{N_m} \left(\sigma_i^m \right)^k \right)^{\frac{1}{k}}$ is the Hannah and Kay index of order k. This corresponds to Grassi (2017, Proposition 4).

Proof. The proof for Cournot competition is immediate as the covariance term is zero with common elasticities. For Bertrand we express the markup in terms of he Hannah and Kay (1977) concentration index, $\mathrm{HK}_m(k) \equiv \left(\sum_i^{N_m} \left(\sigma_i^m \right)^k \right)^{\frac{1}{k}}$,

$$\frac{1}{\mu_m} = 1 - \frac{1}{\varepsilon} \sum_i^{N_m} \frac{\sigma_i^m}{1 - \sigma_i^m \left(1 - \frac{\gamma}{\varepsilon}\right)} = 1 - \frac{1}{\varepsilon} \sum_i^{N_m} \sigma_i^m \sum_{k=0}^{\infty} \left(\sigma_i^m \left(1 - \frac{\gamma}{\varepsilon} \right) \right)^k$$
$$= 1 - \frac{1}{\varepsilon} \sum_{k=0}^{\infty} \left(1 - \frac{\gamma}{\varepsilon} \right)^k \sum_i^{N_m} (\sigma_i^m)^{k+1} = \frac{\varepsilon - 1}{\varepsilon} - \frac{1}{\varepsilon} \sum_{k=2}^{\infty} \left(1 - \frac{\gamma}{\varepsilon} \right)^{k-1} \left(\sum_i^{N_m} (\sigma_i^m)^k \right)$$
$$= \frac{\varepsilon - 1}{\varepsilon} - \frac{1}{\varepsilon} \sum_{k=2}^{\infty} \left(1 - \frac{\gamma}{\varepsilon} \right)^{k-1} (\mathrm{HK}_m(k))^k .$$

□

Appendix D. Functional forms for Kimball's aggregator

Several functional forms for the Kimball aggregator have been employed in the literature. The main functional forms are those introduced by Dotsey and King (2005) and Klenow and Willis (2016). We show how other functional forms are equivalent to the Dotsey and King (2005) aggregator after a suitable change of variables. Tables D.1 and D.2 summarize these results.

We also propose a new functional form that implies a constant super elasticity of demand. We show how this has direct implications for the relationship between markups and market shares in equilibrium. This relationship turns out to be one-to-one, so that knowing either markups or market shares implies the other. The data rejects this relationship.

TABLE D.1. Kimball Aggregators I

Function	CES	D&K(2005)	K&W(2016)
$\Upsilon(x)$	$x^{\frac{\epsilon-1}{\epsilon}}$	$\frac{1}{(1+\theta)\rho}[(1+\theta)x-\theta]^{\rho}-\left[1+\frac{1}{(1+\theta)\rho}\right]$	$1+(\epsilon-1)exp\left(\frac{1}{\theta}\right)\theta^{\frac{\epsilon}{\theta}-1}\left[\Gamma\left(\frac{\epsilon}{\theta},\frac{1}{\theta}\right)-\Gamma\left(\frac{\epsilon}{\theta},\frac{x^{\frac{\theta}{\epsilon}}}{\theta}\right)\right]$
$\Upsilon'(x)$	$\frac{\epsilon-1}{\epsilon}x^{-\frac{1}{\epsilon}}$	$\left[(1+\theta)x-\theta\right]^{\rho-1}$	$\frac{\epsilon-1}{\epsilon}exp\left(\frac{1-x^{\frac{\theta}{\epsilon}}}{\theta}\right)$
$\Upsilon''(x)$	$\frac{1-\epsilon}{\epsilon^2}x^{-\frac{(1+\epsilon)}{\epsilon}}$	$(\rho-1)(1+\theta)\left[(1+\theta)x-\theta\right]^{\rho-2}$	$\frac{1-\epsilon}{\epsilon^2}exp\left(\frac{1-x^{\frac{\theta}{\epsilon}}}{\theta}\right)x^{\frac{\theta-\epsilon}{\epsilon}}$
$[\Upsilon']^{-1}(z)$	$\left(\frac{\epsilon}{\epsilon-1}z\right)^{-\epsilon}$	$\frac{z^{\frac{1}{\rho-1}}+\theta}{(1+\theta)}$	$\left[1-\theta\left(log\left(\frac{\epsilon}{\epsilon-1}\right)+log(z)\right)\right]^{\frac{\epsilon}{\theta}}$
$\left([\Upsilon']^{-1}\right)'(x)$	$\frac{-\epsilon^2}{\epsilon-1}x^{\frac{1+\epsilon}{\epsilon}}$	$\frac{1}{(\rho-1)(1+\theta)}[(1+\theta)x-\theta]^{2-\rho}$	$\frac{\epsilon^2}{(1-\epsilon)exp\left(\frac{1-x^{\frac{\theta}{\epsilon}}}{\theta}\right)x^{\frac{\theta-\epsilon}{\epsilon}}}$
$\varepsilon(x)=-\frac{1}{x}\frac{\Upsilon'(x)}{\Upsilon''(x)}$	ϵ	$\frac{1}{1-\rho}\left[1-\frac{\theta}{1+\theta}x^{-1}\right]$	$\epsilon\cdot x^{-\frac{\theta}{\epsilon}}$
Change of parameters		$\epsilon=\frac{1}{1-\rho}$	
(Simplified) Elasticity of demand	ϵ	$\epsilon\left[1-\frac{\theta}{1+\theta}x^{-1}\right]$	$\epsilon\cdot x^{-\frac{\theta}{\epsilon}}$
Super-elasticity of demand (ξ)	0	$-\frac{\epsilon\theta}{1+\theta}x^{-1}$	$\theta\cdot x^{-\frac{\theta}{\epsilon}}$

D.1. Equivalence results

In this Section we highlight 4 additional Kimball aggregators used in the literature which are equivalent to the Dotsey and King (2005) aggregator.

Barde (2008) Aggregator. The functional form for the aggregator is:

$$\Upsilon(x) = \frac{(\tilde{\theta}x - (\tilde{\theta} - 1))^{\rho}}{\tilde{\theta}} - \frac{(1 - \tilde{\theta})^{\rho}}{\tilde{\theta}} \tag{D.1}$$

where $\rho = \frac{\epsilon - 1}{\epsilon}$, ϵ is the elasticity of substitution between varieties, $\tilde{\theta}$ controls the curvature of the function. If $\tilde{\theta} = 1$, this specification becomes the CES aggregator.

The first derivative is:

$$\Upsilon'(x) = \rho(\theta x - (\theta - 1))^{\rho - 1}$$

The second derivative is:

$$\Upsilon''(x) = \theta\rho(\rho - 1)(\theta x - (\theta - 1))^{\rho - 2}.$$

The elasticity of demand is:

$$\varepsilon\left(\frac{y_i^s}{Y_s}\right) = -\frac{\Upsilon'\left(\frac{y_i^s}{Y_s}\right)}{\frac{y_i^s}{Y_s}\Upsilon''\left(\frac{y_i^s}{Y_s}\right)} = \frac{1}{1-\rho}\left[1 - \frac{\tilde{\theta} - 1}{\tilde{\theta}}\left(\frac{y_i^s}{Y_s}\right)^{-1}\right]$$

Changing variables to $\epsilon = \frac{1}{1-\rho}$ and $\theta = \tilde{\theta} - 1$ we get the same expression as in Dotsey and King (2005):

$$\varepsilon\left(\frac{y_i^s}{Y_s}\right) = \epsilon\left[1 - \frac{\theta}{1+\theta}\left(\frac{y_i^s}{Y_s}\right)^{-1}\right]$$

Levin et al. (2008) and Lindé and Trabandt (2018) Aggregator. The functional form for the Kimball Aggregator is:

$$\Upsilon(x) = \frac{\phi}{1+\theta}\left[(1+\theta)x - \theta\right]^{\frac{1}{\phi}} - \left[\frac{\phi}{1+\theta} - 1\right] \tag{D.2}$$

where $\phi = \frac{\epsilon(1+\theta)}{\epsilon(1+\theta)-1}$, ϵ is the elasticity of substitution between varieties, and θ controls the curvature of the function. If $\theta = 0$ this specification becomes the CES aggregator.

The first derivative is:

$$\Upsilon'(x) = \left[(1+\theta)x - \theta\right]^{\frac{1-\phi}{\phi}}$$

The second derivative is:

$$\Upsilon''(x) = \left(\frac{1-\phi}{\phi}\right)(1+\theta)\left[(1+\theta)x - \theta\right]^{\frac{1}{\phi} - 2}$$

The elasticity of demand is:

$$\varepsilon\left(\frac{y_i^s}{Y_s}\right) = -\frac{\Upsilon'\left(\frac{y_i^s}{Y_s}\right)}{\frac{y_i^s}{Y_s}\Upsilon''\left(\frac{y_i^s}{Y_s}\right)} = \frac{\phi}{\phi - 1}\left[1 - \frac{\theta}{1+\theta}\left(\frac{y_i^s}{Y_s}\right)^{-1}\right]$$

Changing variables to $\epsilon = \frac{\phi}{\phi - 1}$ we get the same expression as in Dotsey and King (2005):

$$\varepsilon\left(\frac{y_i^s}{Y_s}\right) = \epsilon\left[1 - \frac{\theta}{1+\theta}\left(\frac{y_i^s}{Y_s}\right)^{-1}\right]$$

Darracq Pariès and Loublier (2010) Aggregator. The functional form for the Kimball Aggregator is:

$$\Upsilon(x) = \frac{\tilde{\epsilon}}{\tilde{\epsilon}(1+\theta)-1}\left[(1+\theta)x - \theta\right]^{\frac{\tilde{\epsilon}(1+\theta)-1}{\tilde{\epsilon}(1+\theta)}} - \left[\frac{\tilde{\epsilon}}{\tilde{\epsilon}(1+\theta)-1} - 1\right] \tag{D.3}$$

where $\tilde{\epsilon}$ is the elasticity of substitution between varieties, θ controls the curvature of the function. If $\theta = 0$ this specification becomes the CES aggregator.
The first derivative is:

$$\Upsilon'(x) = \left[(1+\theta)x - \theta\right]^{-\frac{1}{\tilde{\epsilon}(1+\theta)}}$$

The second derivative is:

$$\Upsilon''(x) = -\frac{1}{\tilde{\epsilon}}\left[(1+\theta)x - \theta\right]^{-\left(\frac{1+\tilde{\epsilon}(1+\theta)}{\tilde{\epsilon}(1+\theta)}\right)}$$

The elasticity of demand is:

$$\varepsilon\left(\frac{y_i^s}{Y_s}\right) = -\frac{\Upsilon'\left(\frac{y_i^s}{Y_s}\right)}{\frac{y_i^s}{Y_s}\Upsilon''\left(\frac{y_i^s}{Y_s}\right)} = \tilde{\epsilon}\cdot\left[(1+\theta) - \theta\left(\frac{y_i^s}{Y_s}\right)^{-1}\right]$$

Changing variables to $\epsilon = \tilde{\epsilon}\,(1+\theta)$ we get the same expression as in Dotsey and King (2005):

$$\varepsilon\left(\frac{y_i^s}{Y_s}\right) = \epsilon\left[1 - \frac{\theta}{1+\theta}\left(\frac{y_i^s}{Y_s}\right)^{-1}\right]$$

Kurozumi and Zandweghe (2020) Aggregator. The functional form for the Kimball Aggregator is:

$$\Upsilon(x) = \frac{\phi}{(1+\theta)(\phi-1)}\left[(1+\theta)x - \theta\right]^{\frac{\phi-1}{\phi}} - \left[\frac{\phi}{(1+\theta)(\phi-1)} - 1\right] \tag{D.4}$$

where $\phi = \epsilon(1+\theta)$, ϵ is the elasticity of substitution between varieties, $-(\theta\epsilon)$ controls the curvature of the function. If $\theta = 0$ this specification becomes the CES aggregator.
The first derivative is:

$$\Upsilon'(x) = \left[(1+\theta)x - \theta\right]^{-\frac{1}{\phi}}$$

The second derivative is:

$$\Upsilon''(x) = \left(-\frac{1}{\phi}\right)(1+\theta)\left[(1+\theta)x-\theta\right]^{-\frac{(1+\phi)}{\phi}}$$

The elasticity of demand is:

$$\varepsilon\left(\frac{y_i^s}{Y_s}\right) = -\frac{\Upsilon'\left(\frac{y_i^s}{Y_s}\right)}{\frac{y_i^s}{Y_s}\Upsilon''\left(\frac{y_i^s}{Y_s}\right)} = \phi\left[1-\frac{\theta}{(1+\theta)}\left(\frac{y_i^s}{Y_s}\right)^{-1}\right]$$

Changing variables to $\epsilon = \phi$ we get the same expression as in Dotsey and King (2005):

$$\varepsilon\left(\frac{y_i^s}{Y_s}\right) = \epsilon\left[1-\frac{\theta}{1+\theta}\left(\frac{y_i^s}{Y_s}\right)^{-1}\right]$$

TABLE D.2. Kimball Aggregators II

Function	Barde (2008)	Levin et al (2008)	DP&L(2010)	K&VZ(2020)
$\Upsilon(x)$	$\frac{(\theta x-(\theta-1))^{\rho}}{\theta} - \frac{(1-\theta)^{\rho}}{\theta}$	$\frac{\phi}{1+\theta}\left[(1+\theta)x-\theta\right]^{\frac{1}{\phi}} - \left[\frac{\phi}{1+\theta}-1\right]$	$\frac{\epsilon}{\epsilon(1+\theta)-1}\left[(1+\theta)x-\theta\right]^{\frac{\epsilon(1+\theta)-1}{\epsilon(1+\theta)}} - \left[\frac{\epsilon}{\epsilon(1+\theta)-1}-1\right]$	$\frac{\phi}{(1+\theta)(\phi-1)}\left[(1+\theta)x-\theta\right]^{\frac{\phi-1}{\phi}} - \left[\frac{\phi}{(1+\theta)(\phi-1)}-1\right]$
$\Upsilon'(x)$	$\rho(\theta x-(\theta-1))^{\rho-1}$	$\left[(1+\theta)x-\theta\right]^{\frac{1-\phi}{\phi}}$	$\left[(1+\theta)x-\theta\right]^{-\frac{1}{\epsilon(1+\theta)}}$	$\left[(1+\theta)x-\theta\right]^{-\frac{1}{\phi}}$
$\Upsilon''(x)$	$\theta\rho(\rho-1)(\theta x-(\theta-1))^{\rho-2}$	$\left(\frac{1-\phi}{\phi}\right)(1+\theta)\left[(1+\theta)x-\theta\right]^{\frac{1}{\phi}-2}$	$-\left(\frac{1}{\epsilon}\right)\left[(1+\theta)x-\theta\right]^{-\left(\frac{1+\epsilon(1+\theta)}{\epsilon(1+\theta)}\right)}$	$\left(-\frac{1}{\phi}\right)(1+\theta)\left[(1+\theta)x-\theta\right]^{-\frac{(1+\phi)}{\phi}}$
$[\Upsilon']^{-1}(z)$	$\left[\left(\frac{z}{\rho}\right)^{\frac{1}{\rho-1}}+(\theta-1)\right]\frac{1}{\theta}$	$\frac{z^{\frac{\phi}{1-\phi}}+\theta}{1+\theta}$	$\frac{z^{-\epsilon(1+\theta)}+\theta}{(1+\theta)}$	$\frac{z^{-\phi}+\theta}{1+\theta}$
$\left([\Upsilon']^{-1}\right)'(x)$	$\frac{(\theta x-(\theta-1))^{2-\rho}}{\theta(\rho-1)}$	$\frac{\phi}{(1-\phi)(1+\theta)}[(1+\theta)x-\theta]^{2-\frac{1}{\phi}}$	$-\epsilon\left[(1+\theta)x-\theta)\right]^{\frac{1+\epsilon(1+\theta)}{\epsilon(1+\theta)}}$	$-\frac{\phi[(1+\theta)x-\theta]^{\frac{(1+\phi)}{\phi}}}{(1+\theta)}$
$\varepsilon(x) = -\frac{1}{x}\frac{\Upsilon'(x)}{\Upsilon''(x)}$	$\frac{1}{1-\rho}\left[1-\frac{\theta-1}{\theta}x^{-1}\right]$	$\frac{\phi}{\phi-1}\left[1-\frac{\theta}{1+\theta}x^{-1}\right]$	$\epsilon\left[(1+\theta)-\theta x^{-1}\right]$	$\phi\left[1-\frac{\theta}{1+\theta}x^{-1}\right]$
Change of parameters	$\epsilon = \frac{1}{1-\rho}$	$\epsilon = \frac{\phi}{\phi-1}$	$\epsilon = \epsilon(1+\theta)$	$\epsilon = \phi$
(Simplified) Elasticity of demand	$\epsilon\left[1-\frac{\theta-1}{\theta}x^{-1}\right]$	$\epsilon\left[1-\frac{\theta}{1+\theta}x^{-1}\right]$	$\epsilon\left[1-\frac{\theta}{1+\theta}x^{-1}\right]$	$\epsilon\left[1-\frac{\theta}{1+\theta}x^{-1}\right]$

D.2. Constant super-elasticity of demand (CSE) aggregator

The super-elasticity of demand for an aggregator $\Upsilon(\cdot)$ is

$$\xi_i = -\frac{p_i}{P}\frac{\partial \log \varepsilon_i}{\partial\left(\frac{p_i}{P}\right)} = 1+\varepsilon_i+\varepsilon_i\frac{\frac{y_i}{Y}\Upsilon_i'''\left(\frac{y_i}{Y}\right)}{\Upsilon_i''\left(\frac{y_i}{Y}\right)} = 1-\frac{\Upsilon_i'\left(\frac{y_i}{Y}\right)}{\frac{y_i}{Y}\Upsilon_i''\left(\frac{y_i}{Y}\right)} - \frac{\Upsilon_i'\left(\frac{y_i}{Y}\right)\Upsilon_i'''\left(\frac{y_i}{Y}\right)}{\left(\Upsilon_i''\left(\frac{y_i}{Y}\right)\right)^2}. \tag{D.5}$$

$\Upsilon(\cdot)$ exhibits a constant super-elasticity of θ if it solves the differential equation:

$$\theta = 1-\frac{\Upsilon'\left(\frac{y_i}{Y}\right)}{\frac{y_i}{Y}\Upsilon''\left(\frac{y_i}{Y}\right)} - \frac{\Upsilon'\left(\frac{y_i}{Y}\right)\Upsilon'''\left(\frac{y_i}{Y}\right)}{\left(\Upsilon''\left(\frac{y_i}{Y}\right)\right)^2}. \tag{D.6}$$

The solution is:

$$\Upsilon\left(\frac{y_i}{Y}\right) \propto \Gamma\left[1+\frac{1}{\theta}, \frac{\epsilon_i}{\theta} - \log\frac{y_i}{Y}\right] = \int_{\frac{\epsilon_i}{\theta}-\log\frac{y_i}{Y}}^{\infty} \left(t^{\frac{1}{\theta}} e^{-t}\right) dt\,, \tag{D.7}$$

with derivatives:

$$\Upsilon'\left(\frac{y_i}{Y}\right) \propto e^{-\frac{\epsilon_i}{\theta}} \left(\frac{\epsilon_i}{\theta} - \log\frac{y_i}{Y}\right)^{\frac{1}{\theta}} \tag{D.8}$$

$$\Upsilon''\left(\frac{y_i}{Y}\right) \propto -\frac{e^{-\frac{\epsilon_i}{\theta}} \left(\frac{\epsilon_i}{\theta} - \log\frac{y_i}{Y}\right)^{\frac{1}{\theta}}}{\frac{y_i}{Y}\left(\epsilon_i - \theta\log\frac{y_i}{Y}\right)} \tag{D.9}$$

$$\Upsilon'''\left(\frac{y_i}{Y}\right) \propto -\frac{e^{-\frac{\epsilon_i}{\theta}} \left(\frac{\epsilon_i}{\theta} - \log\frac{y_i}{Y}\right)^{\frac{1}{\theta}}}{\left(\frac{y_i}{Y}\right)^2\left(\epsilon_i - \theta\log\frac{y_i}{Y}\right)^2}\left(1-\theta+\epsilon_i - \theta\log\frac{y_i}{Y}\right) \tag{D.10}$$

which imply an elasticity of demand of:

$$\varepsilon_i = \frac{-\Upsilon'\left(\frac{y_i}{Y}\right)}{\frac{y_i}{Y}\Upsilon''\left(\frac{y_i}{Y}\right)} = \epsilon_i - \theta\log\frac{y_i}{Y}\,. \tag{D.11}$$

Implications for market shares. If the super-elasticity of demand is constant it is possible to recover market shares as a function of elasticities, ε, and the super-elasticity, θ as in the Proposition below. The implication of this is that, having recovered elasticities as in Section 4.1, the estimate is generically not compatible with the observed market shares for any given super-elasticity θ. Under this aggregator we cannot jointly match markup and sales-share data.

PROPOSITION A3. *If the super-elasticity of demand is constant, the market shares satisfy*

$$\sigma_i = \frac{e^{-\frac{\varepsilon_i}{\theta}}\left(\frac{\varepsilon_i}{\theta}\right)^{\frac{1}{\theta}}}{\sum_j e^{-\frac{\varepsilon_j}{\theta}}\left(\frac{\varepsilon_j}{\theta}\right)^{\frac{1}{\theta}}}. \tag{D.12}$$

PROOF. The elasticity satisfies $\varepsilon_i = \epsilon_i - \theta\log\frac{y_i}{Y}$. From this we obtain

$$\frac{\varepsilon_i}{\theta} = \frac{\epsilon_i}{\theta} - \log\frac{y_i}{Y} \qquad \text{and} \qquad e^{-\frac{\epsilon_i}{\theta}} = \frac{e^{-\frac{\varepsilon_i}{\theta}}}{y_i/Y}.$$

Then, we use this along with the first derivative of Υ in equation (D.8) to express market shares as in equation (10) in Lemma 1 and obtain

$$\sigma_i = \frac{\Upsilon'\left(\frac{y_i}{Y}\right)\frac{y_i}{Y}}{\sum_j \Upsilon'\left(\frac{y_j}{Y}\right)\frac{y_j}{Y}} = \frac{e^{-\frac{\epsilon_i}{\theta}}\left(\frac{\epsilon_i}{\theta} - \log\frac{y_i}{Y}\right)^{\frac{1}{\theta}}\frac{y_i}{Y}}{\sum_j e^{-\frac{\epsilon_j}{\theta}}\left(\frac{\epsilon_j}{\theta} - \log\frac{y_j}{Y}\right)^{\frac{1}{\theta}}\frac{y_j}{Y}} = \frac{e^{-\frac{\varepsilon_i}{\theta}}\left(\frac{\varepsilon_i}{\theta}\right)^{\frac{1}{\theta}}}{\sum_j e^{-\frac{\varepsilon_j}{\theta}}\left(\frac{\varepsilon_j}{\theta}\right)^{\frac{1}{\theta}}}. \tag{D.13}$$

$\square$

Appendix E. Estimation Appendix

E.1. Recovering demand elasticities

Cournot Markups. Consider the reciprocal of the markup as expressed in equation (25) of Proposition 2 and the elasticity in (16) in Proposition 1.

$$\frac{1}{\mu_i^m} = 1 - \frac{1}{\varepsilon_i^m} = 1 - \frac{1}{\gamma}\sigma_i^m - \left(\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right) + \frac{1}{\bar{\varepsilon}_{-i}^m}\sigma_i^m\right)\left(1-\sigma_i^m\right). \tag{E.1}$$

Rearranging,

$$\left(\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right) + \frac{1}{\bar{\varepsilon}_{-i}^m}\sigma_i^m\right)\left(1-\sigma_i^m\right) = 1 - \frac{1}{\gamma}\sigma_i^m - \frac{1}{\mu_i^m} \tag{E.2}$$

$$\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right) + \frac{1}{\bar{\varepsilon}_{-i}^m}\sigma_i^m = \frac{1-\frac{1}{\gamma}\sigma_i^m}{1-\sigma_i^m} - \frac{1}{\mu_i^m\left(1-\sigma_i^m\right)} \tag{E.3}$$

$$\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right) + \sum_{j\neq i}\frac{1}{\bar{\varepsilon}_j^m}\sigma_j^m\frac{\sigma_i^m}{1-\sigma_i^m} = \frac{1-\frac{1}{\gamma}\sigma_i^m}{1-\sigma_i^m} - \frac{1}{\mu_i^m\left(1-\sigma_i^m\right)}. \tag{E.4}$$

This is a linear system of equations on the reciprocal of the own-elasticities, $\left\{\varepsilon_i^m\right\}$. In matrix form,

$$\begin{bmatrix} 1-\sigma_1^m & \frac{\sigma_1^m}{1-\sigma_1^m}\sigma_2^m & \cdots & \frac{\sigma_1^m}{1-\sigma_1^m}\sigma_{N_m}^m \\ \frac{\sigma_2^m}{1-\sigma_2^m}\sigma_1^s & 1-\sigma_2^m & \cdots & \frac{\sigma_2^m}{1-\sigma_2^m}\sigma_{N_m}^m \\ \vdots & & \ddots & \\ \frac{\sigma_{N_m}^m}{1-\sigma_{N_m}^m}\sigma_1^m & \frac{\sigma_{N_m}^m}{1-\sigma_{N_m}^m}\sigma_2^m & \cdots & 1-\sigma_{N_m}^m \end{bmatrix} \begin{bmatrix} \frac{1}{\varepsilon_1^m} \\ \frac{1}{\varepsilon_2^m} \\ \vdots \\ \frac{1}{\varepsilon_{N_m}^m} \end{bmatrix} = \begin{bmatrix} \frac{1-\frac{1}{\gamma}\sigma_1^m}{1-\sigma_1^m} - \frac{1}{\mu_1^m(1-\sigma_1^m)} \\ \frac{1-\frac{1}{\gamma}\sigma_2^m}{1-\sigma_2^m} - \frac{1}{\mu_2^m(1-\sigma_2^m)} \\ \vdots \\ \frac{1-\frac{1}{\gamma}\sigma_{N_m}^m}{1-\sigma_{N_m}^m} - \frac{1}{\mu_{N_m}^m\left(1-\sigma_{N_m}^m\right)}. \end{bmatrix} \tag{E.5}$$

The matrix in the left is a stochastic matrix and is generically invertible. However, the system can only be solved exactly if it holds that

$$1 > \frac{1}{\gamma}\sigma_i^m + \frac{1}{\mu_i^m} \tag{E.6}$$

for all firms in each market. This restriction comes from the implications of the model for concentration and markups. Larger firms hold more market power and command higher markups. This relationship has to be respected in the data. An alternative way of understanding this condition is that it is necessary for the the own-elasticities to be positive $\varepsilon_i^m > 0$. This can be seen by expressing the condition in terms of the firms' average elasticity

$$\frac{1}{\bar{\varepsilon}_i^m} > \frac{1}{\gamma}\sigma_i^m. \tag{E.7}$$

Bertrand Markups. We prove that the injectivity of the markup function

$$\mu_i = F_i(\vec{\varepsilon}) \equiv \frac{\sigma_i + \varepsilon_i\left[1 - \frac{\varepsilon_i \sigma_i}{\bar{\varepsilon}}\right]}{\sigma_i + \varepsilon_i\left[1 - \frac{\varepsilon_i \sigma_i}{\bar{\varepsilon}}\right] - 1},$$

where $\bar{\varepsilon} = \sum_{j=1}^{N} \varepsilon_j \sigma_j$. This implies that if, given market shares $\{\sigma_i\}$, there is a vector of elasticities $\{\varepsilon_i\}$ that generates the observed vector of markups $\{\mu_i\}$, then this vector of elasticities is unique. For notational simplicity we omit the sector script, s.

To prove that F is injective we evaluate its Jacobian and show that it is a P-matrix. The result then follows from Theorem 4 in Gale and Nikaido (1965).

We start by examining the first derivatives of F_i. It will be useful to define the auxiliary function

$$g_i(\varepsilon) = \sigma_i + \varepsilon_i\left[1 - \frac{\varepsilon_i \sigma_i}{\bar{\varepsilon}}\right]$$

so that:

$$F_i(\varepsilon) = \frac{g_i(\varepsilon)}{g_i(\varepsilon) - 1} \qquad \text{and} \qquad \frac{\partial F_i}{\partial \varepsilon_j} = \frac{-1}{\left(g_i(\varepsilon) - 1\right)^2} \frac{\partial g_i(\varepsilon)}{\partial \varepsilon_j}$$

We need to determine the value of the derivatives of g_i with respect to ε_i and ε_j:

$$\frac{\partial g_i(\varepsilon)}{\partial \varepsilon_i} = \left[1 - \frac{\varepsilon_i \sigma_i}{\bar{\varepsilon}}\right] - \varepsilon_i \sigma_i \left[\frac{\bar{\varepsilon} - \varepsilon_i \sigma_i}{\bar{\varepsilon}^2}\right] = \left[1 - \frac{\varepsilon_i \sigma_i}{\bar{\varepsilon}}\right]^2$$

$$\frac{\partial g_i(\varepsilon)}{\partial \varepsilon_j} = \frac{\varepsilon_i^2 \sigma_i \sigma_j}{\bar{\varepsilon}^2}$$

We want to show that the Jacobian of F is a P-matrix, which is defined as a matrix having positive determinants for all its principal minors. The principal minor of the Jacobian excludes one of the rows and one of the columns with the same index. Instead of tackling the full Jacobian we note that it is proportional to:

$$\boldsymbol{J}^N \propto -\tilde{\boldsymbol{J}}^N \equiv -\begin{bmatrix} \left[\sum_{j \neq 1} \varepsilon_j \sigma_j\right]^2 & \dots & \varepsilon_1^2 \sigma_1 \sigma_i & \dots & \varepsilon_1^2 \sigma_1 \sigma_N \\ \vdots & \ddots & & & \vdots \\ \varepsilon_i^2 \sigma_i \sigma_1 & & \left[\sum_{j \neq i} \varepsilon_j \sigma_j\right]^2 & & \varepsilon_i^2 \sigma_i \sigma_N \\ \vdots & & & \ddots & \vdots \\ \varepsilon_N^2 \sigma_N \sigma_1 & \dots & \varepsilon_N^2 \sigma_N \sigma_i & \dots & \left[\sum_{j \neq N} \varepsilon_j \sigma_j\right]^2 \end{bmatrix}$$

So it is sufficient to prove that $\tilde{J}$ is a P-matrix. We do this by induction. It is immediate that for $N = 1$ and $N = 2$ all principal minors are (weakly) positive because the diagonal is non-negative. It is also easy to check the determinant directly when $N = 3$. Now we prove formally that $\tilde{\boldsymbol{J}}$ is a P-matrix for $N \geq 3$ under the induction hypothesis that it is a

P-matrix for $N-1$. Without loss of generality, the principal minor of $\tilde{\boldsymbol{J}}$ has the form:

$$\tilde{\boldsymbol{J}}_{PM}^{N} = \begin{bmatrix} \left[\sum_{j\neq 1}\varepsilon_j\sigma_j\right]^2 & \dots & \varepsilon_1^2\sigma_1\sigma_i & \dots & \varepsilon_1^2\sigma_1\sigma_{k-1} & \varepsilon_1^2\sigma_1\sigma_{k+1} & \dots & \varepsilon_1^2\sigma_1\sigma_N \\ \vdots & \ddots & & & & & & \vdots \\ \varepsilon_i^2\sigma_i\sigma_1 & & \left[\sum_{j\neq i}\varepsilon_j\sigma_j\right]^2 & & & & & \varepsilon_i^2\sigma_i\sigma_N \\ \vdots & & & \ddots & & & & \vdots \\ \varepsilon_{k-1}^2\sigma_{k-1}\sigma_1 & & & & \left[\sum_{j\neq k-1}\varepsilon_j\sigma_j\right]^2 & & & \varepsilon_{k-1}^2\sigma_{k-1}\sigma_N \\ \varepsilon_{k+1}^2\sigma_{k+1}\sigma_1 & & & & & \left[\sum_{j\neq k+1}\varepsilon_j\sigma_j\right]^2 & & \varepsilon_{k+1}^2\sigma_{k+1}\sigma_N \\ \vdots & & & & & & \ddots & \vdots \\ \varepsilon_N^2\sigma_N\sigma_1 & \dots & \varepsilon_N^2\sigma_N\sigma_i & \dots & \varepsilon_N^2\sigma_N\sigma_{k-1} & \varepsilon_N^2\sigma_N\sigma_{k+1} & \dots & \left[\sum_{j\neq N_s}\varepsilon_j\sigma_j\right]^2 \end{bmatrix}$$

The matrix $\tilde{\boldsymbol{J}}_{PM}^{N}$ can be written as:

$$\tilde{\boldsymbol{J}}_{PM}^{N} = \tilde{\boldsymbol{J}}_{PM}^{N-1} + \operatorname{diag}\left[\varepsilon_k\sigma_k\left[2\sum_{j\neq i,k}\varepsilon_j\sigma_j + \varepsilon_k\sigma_k\right]\right]$$

where $\operatorname{diag}\left[a_i\right]$ is a diagonal matrix with diagonal entries $\{a_1, \dots, a_{N-1}\}$. The result follows because

$$\left|\tilde{\boldsymbol{J}}_{PM}^{N}\right| \geq \left|\tilde{\boldsymbol{J}}_{PM}^{N-1}\right| + \varepsilon_k\sigma_k\prod_{i\neq k}\left[2\sum_{j\neq i,k}\varepsilon_j\sigma_j + \varepsilon_k\sigma_k\right] \geq \left|\tilde{\boldsymbol{J}}_{PM}^{N-1}\right| \geq 0,$$

where the first inequality follows from the properties of determinants, the second inequality from the fact $\varepsilon_i, \sigma_i \geq 0$ for all i, and the last inequality comes from the induction hypothesis. This completes the proof.

E.2. Identification of demand parameters

We construct the result in Proposition 4 in three steps. We first relate changes in market shares to changes in firm output. Then, we relate changes in the elasticity of demand to changes in firm output. Finally, we bring these two results together to complete the proof of Proposition 4. In what follows we omit the sector script, s.

Market shares and output changes. We derive an equation relating the change in the market share of a firm σ_i when there is an (exogenous) change in firm output y_i, while taking into account the effects on aggregate quantity (Y) and price (P).

We start from the definition of $\sigma_i \equiv p_i y_i/PY$ and group the effect of the change in output $\left(y_i\right)$ on relative price (p_i/P) and quantity (y_i/Y). The effect is obtained from the total differential of σ_i.

$$\begin{aligned}
\sigma_i &= \left(\frac{p_i}{P}\right)\left(\frac{y_i}{Y}\right) && \\
d\sigma_i &= \left(\frac{1}{P}\frac{\partial p_i}{\partial y_i}dy_i - \frac{p_i}{P^2}\frac{\partial P}{\partial p_i}\frac{\partial p_i}{\partial y_i}dy_i\right)\left(\frac{y_i}{Y}\right) + \left(\frac{p_i}{P}\right)\left(\frac{1}{Y}dy_i - \frac{y_i}{Y^2}\frac{\partial Y}{\partial y_i}dy_i\right) && \text{Total Differential} \\
d\sigma_i &= \left(\frac{1}{P} - \frac{p_i}{P^2}\frac{\partial P}{\partial p_i}\right)\left(\frac{y_i}{Y}\right)\frac{\partial p_i}{\partial y_i}dy_i + \left(\frac{p_i}{P}\right)\left(\frac{1}{Y} - \frac{y_i}{Y^2}\frac{\partial Y}{\partial y_i}\right)dy_i && \text{Group } dy_i \text{ terms} \\
d\sigma_i &= \left(\frac{1}{P} - \frac{p_i}{P^2}\frac{y_i}{Y}\right)\left(\frac{y_i}{Y}\right)\frac{\partial p_i}{\partial y_i}dy_i + \left(\frac{p_i}{P}\right)\left(\frac{1}{Y} - \frac{y_i}{Y^2}\frac{p_i}{P}\right)dy_i && \frac{\partial P}{\partial p_i} = \frac{y_i}{Y}, \frac{\partial Y}{\partial y_i} = \frac{p_i}{P} \\
d\sigma_i &= \frac{1}{P}\left(1 - \sigma_i\right)\left(\frac{y_i}{Y}\right)\frac{\partial p_i}{\partial y_i}dy_i + \frac{1}{Y}\left(\frac{p_i}{P}\right)\left(1 - \sigma_i\right)dy_i && \text{Express ratios as } \sigma_i \\
d\sigma_i &= \left(\frac{1}{p_i}\frac{\partial p_i}{\partial y_i} + \frac{1}{y_i}\right)\frac{y_i}{Y}\frac{p_i}{P}\left(1 - \sigma_i\right)dy_i && \text{Factor terms} \\
d\sigma_i &= \left(\frac{1}{p_i}\frac{\partial p_i}{\partial y_i} + \frac{1}{y_i}\right)\sigma_i\left(1 - \sigma_i\right)dy_i && \text{Express ratios as } \sigma_i \\
d\sigma_i &= \left(\frac{y_i}{p_i}\frac{\partial p_i}{\partial y_i} + 1\right)\sigma_i\left(1 - \sigma_i\right)\frac{dy_i}{y_i} && \text{Factor out } \frac{1}{y_i} \\
d\sigma_i &= \left(\frac{1}{\varepsilon_i} + 1\right)\sigma_i\left(1 - \sigma_i\right)d\log y_i && \text{Express as } \varepsilon, d\log y_i
\end{aligned}$$

The final step gives us the equilibrium relationship we wanted, so to a first order approximation we know that the market share of firm i evolves as:

$$\sigma_i = \overline{\sigma}_i + \left(\frac{1}{\varepsilon_i} + 1\right)\sigma_i\left(1 - \sigma_i\right)d\log y_i \tag{E.8}$$

Alternatively we can write the relationship in terms of the change in logs:

$$d\log\sigma_i = \left(\frac{1}{\varepsilon_i} + 1\right)\left(1 - \sigma_i\right)d\log y_i. \tag{E.9}$$

Demand elasticities and output changes. We derive an equation relating the change in the elasticity of a firm ε_i when there is an (exogenous) change in firm output y_i, while taking into account the effects on aggregate quantity (Y) and price (P).

We start from the definition of the elasticity,

$$\varepsilon_i \equiv \left(-\frac{y_i}{p_i}\frac{\partial p_i}{\partial y_i}\right)^{-1} = -\frac{\Upsilon'\left(\frac{y_i}{Y}\right)}{\frac{y_i}{Y}\Upsilon''\left(\frac{y_i}{Y}\right)}, \tag{E.10}$$

and use the first order condition of the aggregator firm to express it in terms of relative output (y_i/Y). The effect is obtained from the total differential of ε_i.

$$\varepsilon_i = -\frac{\Upsilon'\left(\frac{y_i}{Y}\right)}{\frac{y_i}{Y}\Upsilon''\left(\frac{y_i}{Y}\right)}$$

$$d\varepsilon_i = -\left(\frac{\Upsilon''\left(\frac{y_i}{Y}\right)}{\frac{y_i}{Y}\Upsilon''\left(\frac{y_i}{Y}\right)} d\left(\frac{y_i}{Y}\right) - \frac{\Upsilon'\left(\frac{y_i}{Y}\right)}{\left(\frac{y_i}{Y}\right)^2\Upsilon''\left(\frac{y_i}{Y}\right)} d\left(\frac{y_i}{Y}\right) - \frac{\Upsilon'\left(\frac{y_i}{Y}\right)\Upsilon'''\left(\frac{y_i}{Y}\right)}{\frac{y_i}{Y}\left(\Upsilon''\left(\frac{y_i}{Y}\right)\right)^2} d\left(\frac{y_i}{Y}\right)\right)$$

$$d\varepsilon_i = -\left(\left(\frac{y_i}{Y}\right)^{-1} + \varepsilon_i\left(\frac{y_i}{Y}\right)^{-1} + \varepsilon_i\frac{\Upsilon'''\left(\frac{y_i}{Y}\right)}{\Upsilon''\left(\frac{y_i}{Y}\right)}\right) d\left(\frac{y_i}{Y}\right)$$

$$\varepsilon_i = -\left(\left(\frac{y_i}{Y}\right)^{-1} + \varepsilon_i\left(\frac{y_i}{Y}\right)^{-1} + \varepsilon_i\frac{\Upsilon'''\left(\frac{y_i}{Y}\right)}{\Upsilon''\left(\frac{y_i}{Y}\right)}\right)\left(\frac{1}{Y} - \frac{y_i}{Y^2}\frac{\partial Y}{\partial y_i}\right) d y_i$$

$$d\varepsilon_i = -\left(\left(\frac{y_i}{Y}\right)^{-1} + \varepsilon_i\left(\frac{y_i}{Y}\right)^{-1} + \varepsilon_i\frac{\Upsilon'''\left(\frac{y_i}{Y}\right)}{\Upsilon''\left(\frac{y_i}{Y}\right)}\right)\frac{y_i}{Y}\left(1 - \frac{y_i}{Y}\frac{p_i}{Y}\right) d\log y_i$$

$$d\log\varepsilon_i = \frac{-1}{\varepsilon_i}\left(1 + \varepsilon_i + \varepsilon_i\frac{\frac{y_i}{Y}\Upsilon'''\left(\frac{y_i}{Y}\right)}{\Upsilon''\left(\frac{y_i}{Y}\right)}\right)(1 - \sigma_i)\, d\log y_i$$

We can simplify this expression by introducing the super-elasticity of demand (see, Klenow and Willis 2016):

$$\xi_i \equiv -\frac{p_i}{P}\frac{\partial\log\varepsilon_i}{\partial\left(\frac{p_i}{P}\right)} = 1 - \frac{\Upsilon'\left(\frac{y_i}{Y}\right)}{\frac{y_i}{Y}\Upsilon''\left(\frac{y_i}{Y}\right)} - \frac{\Upsilon'\left(\frac{y_i}{Y}\right)\Upsilon'''\left(\frac{y_i}{Y}\right)}{\left(\Upsilon''\left(\frac{y_i}{Y}\right)\right)^2} = 1 + \varepsilon_i + \varepsilon_i\frac{\frac{y_i}{Y}\Upsilon'''\left(\frac{y_i}{Y}\right)}{\Upsilon''\left(\frac{y_i}{Y}\right)}. \tag{E.11}$$

Replacing gives the desired expression:

$$d\log\varepsilon_i = -\frac{\xi_i}{\varepsilon_i}(1 - \sigma_i)\, d\log y_i. \tag{E.12}$$

Demand elasticities and market share changes. We derive a an equation relating the change in the elasticity of a firm ε_i to the change in its market share σ_i.

From the first set of results we get:

$$d \log y_i = \frac{d \log \sigma_i}{\left(\frac{1}{\varepsilon_i} + 1\right)(1 - \sigma_i)}.$$

Replacing in the second equation gives us the desired result:

$$d \log \varepsilon_i = -\left(\frac{\xi_i}{\varepsilon_i}\right)\left(\frac{\varepsilon_i}{1+\varepsilon_i}\right) d \log \sigma_i. \tag{E.13}$$

This expression provides us with an equation that we can take to the data, exploiting the demand elasticities $\{\varepsilon_i\}$ that we recover from data on markups and market shares $\{\mu_i, \sigma_i\}$.

Regression parameters. We estimate the equation in Proposition 4 to obtain

$$\Delta \log \varepsilon_i^{st} = \hat{\beta} \cdot \Delta \log \sigma_i^{st}. \tag{E.14}$$

The population value of the coefficient β is a weighted average of the super-elasticity to elasticity ratio:

$$\beta = \frac{\sum (\Delta \log \varepsilon_i)(\Delta \log \sigma_i)}{\sum (\Delta \log \sigma_i)^2}$$

$$\beta = \frac{\sum \left(-\left(\frac{\xi_i}{\varepsilon_i}\right)\left(\frac{\varepsilon_i}{1+\varepsilon_i}\right) \Delta \log \sigma_i\right)(\Delta \log \sigma_i)}{\sum (\Delta \log \sigma_i)^2}$$

$$\beta = -\sum \left(\frac{\xi_i}{\varepsilon_i}\right)\left(\frac{\varepsilon_i}{1+\varepsilon_i}\right) \frac{(\Delta \log \sigma_i)^2}{\sum (\Delta \log \sigma_i)^2}$$

We can further express this coefficient in terms of the underlying function Υ_i by replacing the super-elasticity

$$\beta = -\sum \left(1 + \frac{\varepsilon_i}{1+\varepsilon_i} \frac{\frac{y_i}{Y}\Upsilon'''\left(\frac{y_i}{Y}\right)}{\Upsilon''\left(\frac{y_i}{Y}\right)}\right) \frac{(\Delta \log \sigma_i)^2}{\sum (\Delta \log \sigma_i)^2} \tag{E.15}$$

However, this does not prove to be useful, as the ratio ξ/ε proves to be more tractable for the leading functional for Υ_i in the literature.

E.3. Estimation for the Klenow & Willis aggregator

We first restate the elasticity and super-elasticity for the Klenow and Willis (2016)aggregator:

$$\varepsilon_i = \epsilon_i \left(\frac{y_i}{Y}\right)^{-\frac{\theta}{\epsilon_i}} \qquad \text{Elasticity}$$

$$\xi_i = \theta \left(\frac{y_i}{Y}\right)^{-\frac{\theta}{\epsilon_i}} \qquad \text{Superelasticity}$$

$$\frac{\xi_i}{\varepsilon_i} = \frac{\theta}{\epsilon_i} \qquad \text{Ratio}$$

With this, the regression coefficient becomes

$$\beta^{KW}\left(\theta, \left\{\epsilon_i, \sigma_i\right\}\right) = \frac{-\sum \left(\frac{\theta}{\epsilon_i}\right)\left(\frac{\varepsilon_i}{1+\varepsilon_i}\right)\left(\Delta \log \sigma_i\right)^2}{\sum \left(\Delta \log \sigma_i\right)^2} = \frac{-\sum \left(\frac{1}{\epsilon_i}\right)\left(\frac{\varepsilon_i}{1+\varepsilon_i}\right)\left(\Delta \log \sigma_i\right)^2}{\sum \left(\Delta \log \sigma_i\right)^2}\theta.$$

Algorithm.

(a) Observe $\left\{\mu_i, \sigma_i\right\}$ for a cross section of N firms in a given year (t) and sector (s)

(b) Use markup equation to back out elasticities $\left\{\varepsilon_i\right\}$. This is independent of Kimball parameters and depends only on the form of competition.

(c) Run regression in data, save $\hat{\beta} = \frac{\sum \Delta \log \varepsilon_i \cdot \Delta \log \sigma_i}{\sum (\Delta \log \sigma_i)^2}$

(d) Guess $\theta \geq 0$ that is common across firms. **Note:** $\theta = 0$ implies $\beta^{KW}(0, \cdot) = 0$.

(e) Solve for parameters $\left\{\epsilon_i\right\}$ so as to match market shares $\left\{\sigma_i\right\}$.

(f) Evaluate $\left|\hat{\beta} - \beta\left(\theta, \left\{\epsilon_i, \sigma_i\right\}\right)\right|$, note that all is observable given θ.

   (i) The objective satisfies:

$$\left|\hat{\beta} - \beta\left(\theta, \left\{\epsilon_i, \sigma_i\right\}\right)\right| \propto \left|\sum \left(\Delta \log \varepsilon_i - \left(\frac{\theta}{\epsilon_i}\right)\left(\frac{\varepsilon_i}{1+\varepsilon_i}\right)\Delta \log \sigma_i\right) \cdot \Delta \log \sigma_i\right|$$

   which can be useful if we need to rescale since $\sum \left(\Delta \log \sigma_i\right)^2$ might be numerically hard to work with. The expression also has a nice interpretation as a weighted difference with weights given by the change in market shares. So we are trying to match closer the change in elasticity of the firms that changed their market shares the most.

(g) Repeat (4)-(6) to minimize distance

E.4. Estimation for the Dotsey & King aggregator

The process is the same, only changing the functional form of the elasticity ans superelasticity in steps (e) and (f).

E.5. Recovering model quantities and prices

Relative quantities and prices.

(a) Record the sales share of each firm for a given sector s.

(b) Solve the system of non-linear equations defined by

$$\sigma_i^s = \frac{\Upsilon'\left(\frac{y_i^s}{Y_s}\right)\frac{y_i^s}{Y_s}}{\sum_j \Upsilon'\left(\frac{y_j^s}{Y_s}\right)\frac{y_j^s}{Y_s}},$$

(i) Change variables to $x_i^s \equiv \Upsilon'\left(\frac{y_i^s}{Y_s}\right)\frac{y_i^s}{Y_s}$. The system is now:

$$\sigma_i^s \sum_j x_j^s = x_j^s,$$

or in matrix form:

$$\Sigma_s \vec{x}_s = \vec{x}_s,$$

where

$$\Sigma_s = \vec{\sigma}_s \otimes 1_{1\times N} = \begin{bmatrix} \sigma_1^s & \cdots & \sigma_1^s & \cdots & \sigma_1^s \\ \vdots & \ddots & & & \\ \sigma_n^s & \cdots & \sigma_n^s & \cdots & \sigma_n^s \\ \vdots & & & \ddots & \\ \sigma_N^s & \cdots & \sigma_N^s & \cdots & \sigma_N^s \end{bmatrix} \quad \text{and} \quad \vec{x}_s = \begin{bmatrix} x_1^s \\ \vdots \\ x_n^s \\ \vdots \\ x_N^s \end{bmatrix}.$$

(ii) Solve the linear problem. The solution is given by the eigenvector of Σ_s associated to its unit eigenvalue.

- The existence of a unit eigenvalue is guaranteed because Σ_s is a stochastic matrix. By the Perron–Frobenius theorem the unit eigenvector is also the largest eigenvalue, and the eigenvector can be chosen to be positive.
- Note that eigenvectors are only unique up to a scalar multiple.

(iii) Transform back to relative output units (y_i^s/Y_s) by solving the following system:

$$\alpha_s x_i^s = \Upsilon'\left(\frac{y_i^s}{Y_s}\right)\frac{y_i^s}{Y_s} \quad \forall_i \qquad \text{and} \qquad 1 = \sum_i \Upsilon\left(\frac{y_i^s}{Y_s}\right),$$

the scale $\alpha_s > 0$ is chosen to ensure that the aggregate output equation is satisfied. The solution of the system depends on the functional form of the Kimball aggregator. We consider two cases:

i. **D&K (2005)**:

$$\Upsilon\left(\frac{y_i^s}{Y_s}\right) = \frac{1}{(1+\theta)\frac{\epsilon-1}{\epsilon}}\left[(1+\theta)\frac{y_i^s}{Y_s}-\theta\right]^{\frac{\epsilon-1}{\epsilon}} - \left[1+\frac{1}{(1+\theta)\frac{\epsilon-1}{\epsilon}}\right]$$

$$\Upsilon'\left(\frac{y_i^s}{Y_s}\right) = \left((1+\theta)\,\frac{y_i^s}{Y_s}-\theta\right)^{\frac{-1}{\epsilon}}$$

ii. **K&W (2016)**:

$$\Upsilon\left(\frac{y_i^s}{Y_s}\right) = 1+(\epsilon-1)\exp\left(\frac{1}{\theta}\right)\theta^{\frac{\epsilon}{\theta}-1}\left[\Gamma\left(\frac{\epsilon}{\theta},\frac{1}{\theta}\right)-\Gamma\left(\frac{\epsilon}{\theta},\frac{(y_i^s/Y)^{\frac{\theta}{\epsilon}}}{\theta}\right)\right]$$

$$\Upsilon'\left(\frac{y_i^s}{Y_s}\right) = \frac{\epsilon-1}{\epsilon}\exp\left(\frac{1-\frac{y_i^s}{Y_s}^{\frac{\theta}{\epsilon}}}{\theta}\right)$$

(c) Obtain relative prices as

$$\frac{p_i^s}{P_s} = \frac{\Upsilon'\left(\frac{y_i^s}{Y_s}\right)}{\sum_j \Upsilon'\left(\frac{y_j^s}{Y_s}\right)\frac{y_j^s}{Y_s}} = \frac{\sigma_i^s}{y_i^s/Y_s}$$

(d) Compute elasticities of demand by firm:

$$\varepsilon\left(\frac{y_i^s}{Y_s}\right) = -\left(\frac{y_i^s}{Y_s}\frac{\Upsilon''\left(\frac{y_i^s}{Y_s}\right)}{\Upsilon'\left(\frac{y_i^s}{Y_s}\right)}\right)^{-1} = \begin{cases}\epsilon\left(1-\frac{\theta}{1+\theta}\left(\frac{y_i^s}{Y_s}\right)^{-1}\right) & If \quad D\&K(2005)\\ \epsilon\left(\frac{y_i^s}{Y_s}\right)^{-\frac{\theta}{\epsilon}} & If \quad K\&W(2016)\end{cases}$$

(e) Compute markups according to type of competition: