Self-Employment and Development*

Juan Herreño
UC San Diego

Sergio Ocampo
Western University

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Abstract

We evaluate the aggregate effects of development policies in environments where subsistence entrepreneurship is prevalent. We extend a standard macro development model to include unemployment risk, which becomes a key driver of selection into self-employment. The model is consistent with the joint distribution of earnings and occupations and with the reaction of wages to labor demand shocks in the data. We study the aggregate effects of various development policies, such as micro-finance and targeted cash transfers, on aggregate output, the real wage, and total factor productivity. We show the sign and magnitude of the effects depend crucially on the design and implementation of these policies, and that the macro effects can deviate substantially from the small micro effects estimated in experimental designs that increase the supply of microloans.

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*Herreño: juanherreno@ucsd.edu; Web: https://sites.google.com/view/juanherreno. Ocampo: socampod@uwo.ca; Web: https://sites.google.com/site/sergiocampod/.
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The livelihoods of the world's poor are riddled with subsistence concerns. These challenges often push them to become self-employed, starting businesses that do not grow and generate little value added (Banerjee and Duflo, 2011). These individuals are referred to as *"subsistence entrepreneurs"* (Schoar, 2010). Indeed, self-employment rates are high in developing countries by international standards, particularly among the poor (Gollin, 2008; Poschke, 2013a,b, 2019). In this context, governments and other organizations enact credit expansions, conditional and unconditional transfers, among other policies, aiming to foster growth and reduce poverty.

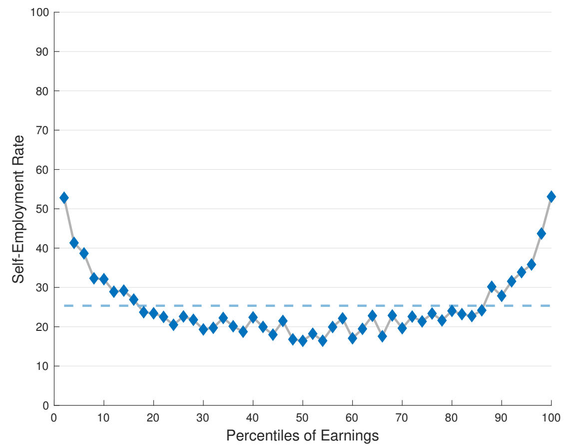
We study the effects of a variety of policies in contexts where subsistence entrepreneurship is prevalent. We do so using an occupational choice model with idiosyncratic income risk, financial frictions and labor frictions. Labor frictions generate unemployment risk for individuals. In equilibrium, self-employment is chosen by high productivity individuals, but also by poor-unemployed individuals who use it as a source of self-insurance. In this way, introducing unemployment risk allows *subsistence* concerns to dominate the selection into self-employment among the poor.

We use existing micro evidence to validate the model. Subsistence concerns are crucial to match the joint distribution of self-employment and earnings. Self-employment rates are highest among the rich and the poor, exhibiting a U-shaped pattern along the earnings distribution as we show in Figure 1. The model also reproduces the response of wages to well-identified labor demand shocks among other non-targeted moments.

We find that policies increasing the availability of credit or providing cash transfers have economically significant effects on aggregate output and productivity, driven by small changes in individual selection into self-employment. We consider policies of similar magnitude and design to those implemented in developing countries, such as those described in Angelucci, Karlan, and Zinman (2015) and Banerjee, Niehaus, and Suri (2019). The model produces small effects on individual-level earnings, consumption, and savings, as those measured in evaluations of micro-finance programs using randomized controlled trials (e.g., Banerjee, Duflo, Glennerster, and Kinnan, 2015; Banerjee, Breza, Duflo, and Kinnan, 2019; Meager, 2019).

Varying the targeting and generosity of policies affects the sign and size of their aggregate effects, even when all designs exhibit small micro effects. For instance, providing loans at market rates increases productivity, while providing (more generous) zero-interest loans decreases it. The difference lies in how policies affect the occupational choices of individuals. Market rate loans are most advantageous to

Figure 1: Self-Employment and Earnings



Note:*The Figure shows the joint distribution of income and self-employment for the U.S. and nine developing economies for which we had access to household surveys (Azerbaijan, Albania, Bulgaria, Brazil, Colombia, Mexico, Peru, Serbia, and South Africa). We compute self-employment rates in 50 equal-sized quantiles of the income distribution. The underlying data comes from household surveys for the developing countries. We report the profile for each individual country in Figure D.2.*

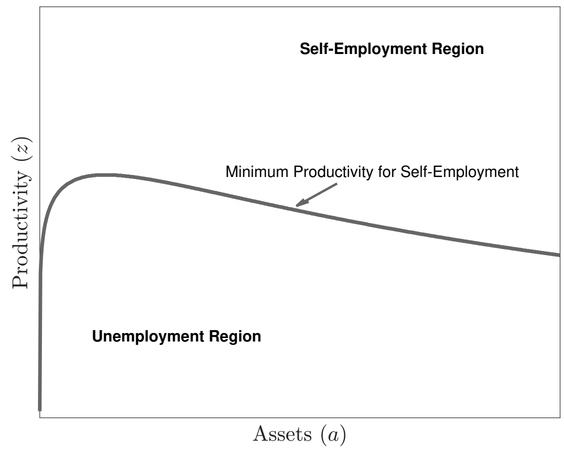
high-productivity individuals with high rates of return, while zero-interest loans benefit low-productivity individuals the most, inducing negative selection into self-employment.

We incorporate unemployment risk into the occupational choice model of Buera, Kaboski, and Shin (2015) in a tractable manner. In the model, agents can be employed, self-employed, or unemployed. Employed and self-employed agents are subject to income fluctuations generated by idiosyncratic productivity shocks. Labor income depends on individual labor productivity. Self-employment income depends on productivity and wealth because of financial frictions. While agents can become self-employed or unemployed at will (by starting their own business or quitting their job), they can only become employed after receiving a job offer, which arrives stochastically. We calibrate the model to match workforce composition and income fluctuations from Mexican longitudinal household data.

The model highlights how the interplay between labor market and financial frictions impacts the decision of individuals to become self-employed. For a risk-averse individual, unemployment becomes intolerable whenever they have low assets and no effective access to credit. To avoid the risk of failing to find a job, poor-unemployed individuals choose to self-insure by becoming self-employed even when they lack the productivity or the assets needed to run a profitable business.[1] In contrast, wealthier

[1]Formally, consider an unemployed individual who derives utility from consumption and satisfies

Figure 2: Occupational Choice: Minimum Productivity for Self-Employment



Note: *The figure presents the occupational choice of individuals. The line depicts the threshold value of productivity* (z) *for each level of assets* (a)*. The agent chooses self-employment if productivity is above the threshold.*

individuals do not face the same trade-offs because they can self-insure by running down their assets while searching for a job. Consequently, wealthy individuals only become self-employed if their productivity is sufficiently high. The same pattern arises for employed individuals, who trade-off their current income against the potential income from operating their own business.

The core mechanism of the model is reflected in the non-monotonicity of the minimum (threshold) productivity required to become self-employed, that we sketch in Figure 2. The horizontal axis of the figure corresponds to the wealth level of an individual, and the vertical axis to their productivity. The solid line shows the minimum threshold of productivity at which an individual with a given wealth level chooses to become self-employed. The threshold decreases for poor-unemployed individuals as they become poorer, so more individuals with low productivity become self-employed. The threshold also decreases as individuals become sufficiently wealthy. However, wealthy individuals become self-employed because doing so generates a higher income, while poor individuals do so as a means of subsistence.

Our model captures non-targeted patterns of self-employment selection and behavior in the Mexican data. The model predicts a U-shaped profile of self-employment rates across the earnings distribution as in Figure 1. Removing unemployment risk causes the pattern to disappear, resulting in an increasing

$\lim_{c \to 0^+} u'(c) = \infty$. There is a probability $p < 1$ of becoming employed and earning w. While unemployed the individual earns $b < w$. If they become self-employed they stop searching for a job, but earn an income $y(a, z)$ that is increasing on their assets a and productivity z.

relationship between income and self-employment rates. The model also captures transition rates across occupations and the auto-correlation of income of occupation-switchers.

Introducing unemployment risk allows the model to match the (non-targeted) response of wages and employment to well-identified labor demand shocks estimated in cross-sectional studies (Breza, Kaur, and Shamdasani, 2021; Muralidharan, Niehaus, and Sukhtankar, 2017). As Breza et al. (2021) show, the response of wages to labor demand shocks is informative about the underlying slack in the labor market. We use this insight to test the validity of our model by replicating the intervention carried out by Breza et al. (2021). In the data, estimates of the response of wages is low in markets where subsistence entrepreneurship is prevalent. Similarly, in our model, subsistence entrepreneurship provides a mass of agents who can accommodate additional labor demand with little upward pressure on wages. A specification of our model without subsistence concerns, in which there is no slack, implies a response of wages to labor demand shocks that is an order of magnitude higher.

We turn to the study the macroeconomic effects of development policies. We start with an expansion of credit availability that matches the size of loans provided by Compartamos Banco in Mexico studied by Angelucci et al. (2015). This policy is not only relevant because of the role of micro-credit in loosening the collateral constraints faced by the self-employed, but also because it provides us with an additional source of validation for our model. The model's response is consistent with the small effects of the program on earnings among the self-employed, which increase almost 1 percent, individual consumption, which barely increases by 0.02 percent, and savings which slightly decrease by 0.4 percent as a response to the additional supply of credit not tied to individuals' collateral.

In contrast with the small micro-effects of the expansion in credit availability, the policy has significant macroeconomic effects. Aggregate output increases, with three fourths of the increase explained by higher total factor productivity (TFP). The increase in TFP is explained by the differential effect of the policy on the occupational choice of low- and high-productivity individuals. More productive individuals disproportionately benefit from additional credit, leading to a reallocation of capital and labor into more productive hands. This, in turn, leads to a small increase in wages, which reduces the profitability of low-productivity self-employment. The end result is a decrease in the self-employment rate of 0.24 percentage points, reinforcing the compositional changes towards more productive self-employed individuals.

Beyond the efficiency and output gains from the increase in credit availability, there are also small welfare effects. Average welfare, measured in consumption equivalent units, goes up almost 0.1 percent. These gains reflect the small but positive changes in individual variables carried by the increase in wages and consumption.

The macro effects of development policies depend crucially on implementation and design variables, even though the individual-level effects of the policies remain small. Meager (2019) shows that there is little heterogeneity in the micro effects of credit expansions in different countries, despite significant heterogeneity in loan terms. We show that this a natural outcome in our model. We modify our credit intervention by increasing its generosity, introducing zero-interest loans as opposed to loans at market rates. We find little difference between the micro effects of this more generous credit intervention and our benchmark exercise.

However, the macro effects of zero-interest loans are vastly different to the benchmark case. TFP falls by almost half a percent after the introduction of zero-interest loans, and the self-employment rate increases. Although, the effects of both interventions are small at the individual level, they change selection into self-employment in opposite directions. Zero-interest loans lower the productivity threshold to become self-employed, with negative aggregate effects.

We also consider the effects of cash transfers that are conditioned on an individual's occupation or income. These policies are used broadly in contexts where subsistence entrepreneurship is prevalent (Banerjee et al., 2019). In particular, we introduce transfers to unemployed agents of a similar magnitude to the increase in earnings generated by the credit expansions we study. Our model highlights the central role of the occupational choices of the unemployed in shaping the aggregate effects of policies. We find these transfers to be very successful in improving selection into self-employment and in increasing productivity and output by first dealing with the subsistence concerns of poor-unemployed individuals.

The design and implementation of the transfers is also crucial for determining their effects. Our implementation of unemployment transfers assumes that the government can perfectly screen unemployed individuals. If governments cannot distinguish between unemployed and self-employed individuals, the positive effects of unemployment transfers disappear. Once transfers reach both unemployed and self-employed individuals, they no longer cause an improvement in the selection to self-employment. Self-employment increases as low-productivity individuals take the transfer but engage in self-employment nonetheless, resulting in a decrease in TFP.

Related Literature

Our work is related to a longstanding strand of literature linking productivity and misallocation to entrepreneurship and self-employment (e.g., Hsieh and Klenow, 2009, 2014; Banerjee and Moll, 2010; Restuccia and Rogerson, 2013; Midrigan and Xu, 2014; Buera, Kaboski, and Shin, 2015). We contribute by showing the conceptual and quantitative importance of *subsistence* concerns behind self-employment in driving macroeconomic aggregates and responses to economic policy.[2]

Selection into self-employment is at the core of our mechanism. Heathcote, Storesletten, and Violante (2008) studied the welfare effects of insurance and risk, while Hombert, Schoar, Sraer, and Thesmar (2020, 2016), Garcia-Cabo and Madera (2019), and Levine and Rubinstein (2018) discussed the role of risk and ability on selection into entrepreneurship and self-employment. We contribute by evaluating the relevance of these margins in influencing the aggregate economy and the response to economic policy when agents face subsistence concerns.

The introduction of unemployment risk is key to generating the subsistence concerns that lead unproductive agents to engage in self-employment. More broadly, Poschke (2019) and Feng, Lagakos, and Rauch (2020) show how labor market frictions are vital to explain cross-country differences in the composition of the workforce. We show that these frictions not only lead to higher self-employment rates in developing countries, but also explain the joint distribution of income and occupations that in turn shape the aggregate response to policy.

We focus on the aggregate impact of development policies in environments where subsistence entrepreneurship is prevalent. In this sense, our work complements that of Buera, Kaboski, and Shin (2020), who highlight the importance of general equilibrium channels and longer time horizons to evaluate the macroeconomic effects of credit-expansions. Similar to Buera et al. (2020), we build on the evidence from the evaluation of policy interventions through RCTs to validate the performance of the model. We also show how cross-sectional moments, like the prevalence of self-employment along the earnings distribution and the response of wages to labor demand shocks, can be used to provide validation of the model's mechanisms.

Finally, our paper speaks to a large literature that studies the efficiency of social programs in developing countries, where informality is high (Meghir, Narita, and

[2]In related studies, Quadrini (2000) and Cagetti and De Nardi (2006, 2009) quantify the extent to which financial frictions distort the scale of firms.

Robin, 2015). Our results indicating that unemployment benefits increase aggregate productivity align with the positive effects of unemployment benefits extensions found by Gerard and Gonzaga (2021) and Britto (2020).

The rest of the paper is organized as follows. First, we describe the model and its calibration in sections 1 and 2. In section 3, we show that the model is consistent with the observed features of self-employment. Finally, in section 4, we use the model to study the effects of cash transfers and credit-deepening reforms.

1 Model

In this section we describe a quantitative model with occupational choices, in which agents face unemployment risk and financial frictions. Consider a small open economy with a continuum of agents facing an international interest rate $r^{\star}$. Time is continuous and goes on forever. Agents are heterogeneous with respect to their productivity (z), their asset holdings (a), and their occupations. Idiosyncratic productivity is the only exogenous state and it follows a Poisson process with arrival rate γ^{z}. Upon arrival, the agent draws a new value for z from a conditional probability distribution $\Pr^{z}(z'|z)$.

There are three occupations: employment (E), unemployment (U), and self-employment (S). Occupations differ on whether or not agents can freely opt into them. Agents can become unemployed or self-employed at will. In contrast, transitions to employment are governed by exogenous processes that capture, in a reduced form, the arrival of job offers to unemployed and self-employed agents. Agents are free to reject job offers and keep their current occupations. Formally, job offers follow Poisson processes with arrival rates γ^{U} for the unemployed and γ^{S} for the self-employed. Finally, employed agents are subject to job-destruction shocks that force them into unemployment. These shocks also follow a Poisson process and arrive at a rate γ^{E}.

Agents have limited access to credit markets. Employed and unemployed agents face a borrowing limit $\underline{a} \leq 0$. Self-employed agents can borrow capital for production of final goods, but they face a collateral constraint as a function on their assets: $k \leq \lambda a$. This reduced-form collateral constraint captures information frictions and commitment problems. See Cagetti and De Nardi (2006) and Buera, Kaboski, and Shin (2011), among others, for micro-foundations.

In Appendix A we present a version of the model without unemployment risk. The contrast between these models highlights the role of unemployment risk in generating self-employed agents with low assets and productivity, as we show in section 2.

1.1 The Agents' Problem

The agents' problem depends on their occupation. We discuss each occupation in turn.

Employment Agents receive an income of $w\epsilon(z)$ while employed, where w is the aggregate wage rate and $\epsilon(z)$ are the agent's effective units of labor, which are a function of their productivity. In practice, we assume that $\epsilon(z) = z^{\eta}$, where η captures the relevance of productivity for the earnings of employed workers.[3] Employed agents are subject to job-destruction shocks with arrival rate γ^E that force them into unemployment and productivity shocks that arrive at a rate γ^z. The value of an employed agent is the solution to the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} \rho V^E(a,z) &= \max_c u(c) + V_a^E(a,z)\,\dot{a} + \gamma^E\left(V^U(a,z) - V^E(a,z)\right) \\ &\qquad + \gamma^z \int \left(V^E(a,z') - V^E(a,z)\right) d\mathrm{Pr}^z(z'|z) \\ \text{s.t.} &\qquad \dot{a} = w\epsilon(z) + ra - c, \qquad a \geq \underline{a}. \end{aligned} \tag{1}$$

Additionally, the value of an employed agent must satisfy:

$$V^E(a,z) \geq \max\left\{V^U(a,z), V^S(a,z)\right\}, \tag{2}$$

where V^U and V^S are the values of unemployed and self-employed agents respectively, which we define below in equations (3) and (6). The inequality in (2) captures the occupational choice of an employed agent. Because agents are free to become unemployed or self-employed at any time, the value of agents choosing to remain employed must be at least that of the alternative occupations.

[3] This is a conservative assumption, as it reduces the gains from reallocation of individuals between employment and self-employment (i.e., there are no individuals who are unproductive as self-employed but highly productive as employed). We also explored results with independent labor and entrepreneurial productivity processes, which are accessible at `https://ocamp020.github.io/HO_Self_Employment.pdf`.

Unemployment Agents receive an income of b and get job offers at a rate γ^U while unemployed.[4] We assume that $b < \min_z \epsilon(z)$, so that employment is always preferable to unemployment. Unemployed agents also receive productivity shocks that arrive at a rate γ^z. The value of an unemployed agent is the solution to the following HJB equation:

$$\begin{aligned} \rho V^U(a,z) &= \max_c u(c) + V_a^U(a,z)\dot{a} + \gamma^U V^E(a,z) \\ &\quad + \gamma^z \int \left(V^U(a,z') - V^U(a,z)\right) d\Pr^z(z'|z) \\ \text{s.t.} &\quad \dot{a} = b + ra - c, \qquad a \geq \underline{a}. \end{aligned} \tag{3}$$

Additionally, the value of an unemployed agent must satisfy:

$$V^U(a,z) \geq V^S(a,z). \tag{4}$$

The inequality in (4) captures the occupational choice of an unemployed agent. Because agents are free to become self-employed at any time, the value of agents choosing to remain unemployed must be at least that of being self-employed.

Self-employment Agents engage in the production of final goods which they sell in a competitive market while self-employed. Production combines capital and (efficiency units of) labor through a technology that depends on the agents' productivity. The production technology is captured by a function:

$$f(z,k,n) = z\left(k^\alpha n^{1-\alpha}\right)^\nu, \tag{5}$$

where $\alpha \in (0,1)$ and $\nu \leq 1$. In general, agents are prevented from operating at their optimal scale because of the collateral constraint they face. Profits $\pi(a,z)$ depend on both their assets and their productivity. These profits constitute the income of self-employed agents.[5]

Self-employed agents get job offers at a rate γ^S and productivity shocks at a rate γ^z. Agents are free to reject job offers, and so will only accept offers if the value of employment is higher than the value of self-employment given their current state (a,z).

[4] Unemployment income b can be interpreted as home production, or as transfers from family members or government agencies. In section 4.2 we take the latter view and examine what happens if b increases.

[5] The dependence of self-employment income on assets implies that individual-level earnings across occupations are not perfectly correlated.

The value of an employed agent is the solution to the following HJB equation:

$$\begin{aligned}\rho V^{S}(a,z) &= \max_{c}\, u(c) + V_{a}^{S}(a,z)\,\dot{a} + \gamma^{S}\max\left\{V^{E}(a,z) - V^{S}(a,z),0\right\} \\ &\quad +\gamma^{z}\int\left(V^{S}(a,z') - V^{S}(a,z)\right)d\mathrm{Pr}^{z}(z'|z) \\ \text{s.t.} &\quad \dot{a} = \pi(a,z) + ra - c, \qquad a \geq \underline{a}, \\ &\quad \pi(a,z) = \max_{k\leq\lambda a\,,\,n\geq 0} f(z,k,n) - wn - (r+\delta)\,k\,.\end{aligned} \tag{6}$$

Finally, the value of a self-employed agent must satisfy:

$$V^{S}(a,z) \geq V^{U}(a,z)\,, \tag{7}$$

which captures the occupational choice self-employed agents in much the same way as (4) captured the choice of the unemployed.

Self-employed agents operate a common technology. Introducing a menu of technologies and installation costs, as in Midrigan and Xu (2014) or Buera, Kaboski, and Shin (2011), would only strengthen our results by making low-productivity self-employment more attractive for low-wealth agents, who would choose inferior technologies associated with low installation costs. A similar logic follows from expanding the model to allow for formal and informal technologies, akin to Meghir, Narita, and Robin (2015).

Savings choice The optimal consumption/savings decision can be found in all occupations from the first-order condition of the agent's problem (see Achdou, Han, Lasry, Lions, and Moll, 2021). Letting $o \in \{E,U,S\}$ denote the occupational state of the agent, we have:

$$c^{o}(a,z) = u'^{-1}\left(V_{a}^{o}(a,z)\right). \tag{8}$$

Occupational choice The occupational choice of agents implicitly defines regions Ω^{o} in the space of assets and productivity $\mathcal{S} \equiv [\underline{a},\infty) \times \mathbb{R}_{+}$ in which each occupation $o \in \{E,U,S\}$ prevails over its alternatives. These regions are characterized by equations (2), (4), or (7) respectively holding with strict inequalities. For employed agents

$$\Omega^{E} = \left\{(a,z) \in \mathcal{S} \,|\, V^{E}(a,z) > V^{S}(a,z)\right\}, \tag{9}$$

for unemployed agents

$$\Omega^U = \left\{ (a,z) \in \mathcal{S} \,|\, V^U(a,z) > V^S(a,z) \right\} \tag{10}$$

and for self-employed agents $\Omega^S = \mathcal{S} \setminus \Omega^U$.

The shape of the occupational regions Ω^o plays a central role in determining the composition of self-employment, aggregate productivity, output and wages, as we sketched in Figure 2. We show numerically in Section 3 that Ω^E and Ω^U are characterized by minimum (threshold) values of productivity required for an agent to become self-employed coming from either employment or unemployment. For the case of employed agents, the productivity threshold decreases monotonically with assets, reflecting the fact that self-employment income increases with assets as the collateral constraint loosens. In contrast, for unemployed agents, the productivity threshold is non-monotone in assets. In fact, the minimum productivity required for self-employment tends to zero as the agent's asset holdings decrease towards $\underline{a}$. This is because unemployment becomes intolerable for poor agents who cannot self-insure using their savings. This behavior gives rise to low-productivity self-employed agents who populate the low end of the earnings distribution.

Reforms that affect the availability of credit to the self-employed or that insure unemployed agents from having low consumption levels have a direct effect on the minimum productivity required for self-employment. We show in Section 4 that the occupational choices of agents with low-assets are highly sensitive to the environment they face. By preventing low-productivity agents from becoming self-employed, it is possible to increase aggregate productivity and output, potentially increasing welfare.

1.2 Labor market

There is a perfectly competitive labor market for efficiency units of labor. The supply of labor is the integral over the efficiency units of employed agents:

$$N^S = \int \epsilon(z) dG^E \,, \tag{11}$$

where G^E is the distribution of employed agents in the economy.

The demand for labor comes from the production activities of the self-employed.

Total labor demand is thus given by:

$$N^D = \int n^\star (a,z)\, dG^S \,, \tag{12}$$

where G^S is the distribution of self-employed agents in the economy and $n^\star (a,z)$ is the optimal labor demand from a self-employed agent with assets a and productivity z.

1.3 Aggregate output and productivity

All output is produced by the self-employed. Thus, aggregate output in the economy is

$$Y \equiv \int z \left(k^\star(a,z)^\alpha n^\star(a,z)^{1-\alpha}\right)^\nu dG^S = Z \left(M^S\right)^{1-\nu} \left(K^\alpha N^{(1-\alpha)}\right)^\nu , \tag{13}$$

with aggregate inputs $K \equiv \int k^\star (a,z)\, dG^S$ and N as in (12). $M^S \equiv \int dG^S$ is the mass of self-employed agents, which plays a role when the production technology in (5) exhibits decreasing returns to scale ($\nu < 1$).

Aggregate (average) productivity or TFP is:

$$Z \equiv \left[\frac{1}{M^S} \int \left(z \cdot \frac{1}{\tilde{\tau}(a,z)} \right)^{\frac{1}{1-\nu}} dG^S \right]^{1-\nu} , \tag{14}$$

where $\tilde{\tau}$ is defined as in Hsieh and Klenow (2009) to be a firm-specific wedge that captures the extent of the distortions generated by the collateral constraint.[6] The aggregation procedure is standard and follows Hopenhayn (2014). TFP is affected by changes in the selection into self-employment, through changes in the distribution G^S, and changes in financial frictions faced by self-employed agents, through the wedges $\tilde{\tau}$. Policies that lead more productive agents to become self-employed, that increase the asset holdings of self-employed agents, or that loosen collateral constraints increase productivity through their effects on G^S and $\tilde{\tau}$.

[6]Formally, $\tilde{\tau}$ is such that $k(a,z) = \left[\left(\frac{\nu\alpha}{\tilde{\tau}(a,z)(R+\delta)} \right)^{1-\nu(1-\alpha)} \left(\frac{\nu(1-\alpha)}{w} \right)^{\nu(1-\alpha)} z \right]^{\frac{1}{1-\nu}}$ for each (a,z).

1.4 Equilibrium

We focus on the stationary equilibrium of the economy. Appendix B describes the computational implementation of the solution.

A stationary equilibrium for this economy is a set of value functions $\{V^o\}_{o\in\{E,U,S\}}$, along with optimal consumption functions $\{c^o\}_{o\in\{E,U,S\}}$, capital and labor demand from self-employed $\{k^\star, n^\star\}$, prices $\{r, w\}$, a distribution of agents for each occupation $\{G^o\}_{o\in\{E,U,S\}}$, such that, given an international interest rate $r^\star$ and exogenous processes for job offers, job-destruction, and productivity shocks:

1. Value functions $\{V^o\}_{o\in\{E,U,S\}}$ solve the system of HJB variational inequalities:

$$0 = \max\left\{\rho V^E - \max_c u\left(c\right) - V_a^E \cdot \left(w\epsilon(z) + ra - c\right) - \frac{\mathbb{E}\left[dV^E\right]}{dt}\,,\, V^E - V^S\right\}, \tag{15}$$

$$0 = \max\left\{\rho V^U - \max_c u\left(c\right) - V_a^U \cdot \left(b + ra - c\right) - \frac{\mathbb{E}\left[dV^U\right]}{dt}\,,\, V^U - V^S\right\}, \tag{16}$$

$$0 = \max\left\{\rho V^S - \max_c u\left(c\right) - V_a^S \cdot \left(\pi + ra - c\right) - \frac{\mathbb{E}\left[dV^S\right]}{dt}\,,\, V^S - V^U\right\}. \tag{17}$$

   These variational inequalities capture jointly the dynamic problem of the agent (equations 1, 3, 6) and their occupational choice (equations 2, 4, 7). As a consequence of the agents' occupational choice the value of unemployment and self-employment are equated in equilibrium: $V^U = V^S$.
2. Consumption functions $\{c^o\}_{o\in\{E,U,S\}}$ (and thus asset accumulation functions) are consistent with the agent's optimization as in equation (8).
3. Capital and labor demand $\{k^\star, n^\star\}$ solve the profit-maximization problem of the self-employed:

$$\{k^\star, n^\star\} = \underset{k\leq\lambda a\,,\, n\geq 0}{\text{argmax}}\left\{z\left(k^\alpha n^{1-\alpha}\right)^\nu - wn - \left(r+\delta\right)k\right\}. \tag{18}$$

4. The interest rate r is given by the international interest rate $r^\star$.
5. The wage w is such that the labor market clears: $N^S = N^D$, with labor supply as in (11), and labor demand as in (12).
6. The distributions of agents for each occupation $\{G^o\}_{o\in\{E,U,S\}}$ are stationary and their densities $\{g^o\}_{o\in\{E,U,S\}}$ solve the following system of Kolmogorov-Forward

Equations:

$$
\begin{aligned}
0 =& -\frac{\partial}{\partial a}\left[\dot{a} g^{E}(a, z)\right]-\left(\gamma^{E}+\gamma^{z}\right) g^{E}(a, z) \qquad (19)\\
& +\gamma^{z} \int \operatorname{Pr}^{z}\left(z \mid z^{\prime}\right) g^{E}\left(a, z^{\prime}\right) d z^{\prime}+\gamma^{U} g^{U}(a, z)+\gamma^{S} g^{S}(a, z) \mathbb{1}_{\left\{(a, z) \in \Omega^{E}\right\}} \\
0 =& -\frac{\partial}{\partial a}\left[\dot{a} g^{U}(a, z)\right]-\left(\gamma^{U}+\gamma^{z}\right) g^{U}(a, z) \qquad (20)\\
& +\gamma^{z} \int \operatorname{Pr}^{z}\left(z \mid z^{\prime}\right) g^{U}\left(a, z^{\prime}\right) d z^{\prime}+\gamma^{E} g^{E}(a, z), \\
0 =& -\frac{\partial}{\partial a}\left[\dot{a} g^{S}(a, z)\right]-\left(\gamma^{S} \mathbb{1}_{\left\{(a, z) \in \Omega^{E}\right\}}+\gamma^{z}\right) g^{S}(a, z) \qquad (21)\\
& +\gamma^{z} \int \operatorname{Pr}^{z}\left(z \mid z^{\prime}\right) g^{S}\left(a, z^{\prime}\right) d z^{\prime}+\gamma^{E} g^{E}(a, z) \mathbb{1}_{\left\{(a, z) \notin \Omega^{U}\right\}},
\end{aligned}
$$

where (19) holds for $(a, z) \in \Omega^E$, (20) holds for $(a, z) \in \Omega^U$, and (21) holds for $(a, z) \notin \Omega^U$. Moreover, the distributions $\{G^o\}_{o\in\{E,U,S\}}$ are such that: $1 = \int dG^E + \int dG^U + \int dG^S$. So $\int dG^o$ gives the mass of agents in occupation $o \in \{E, U, S\}$. The full characterization of the stationary distributions requires more attention to the transitions of agents across occupations, we discuss this in Appendix B.

2 Model calibration

We calibrate the model in two steps. First, we calibrate a group of parameters externally, with values taken from the literature or chosen independently of the model's equilibrium outcomes. We then choose a second group of parameters to match targeted moments of the earnings distribution, workforce composition, and transition rates across occupations. We use aggregate and micro data from Mexico to calibrate the second group of parameters. This high-quality data allow us to explore the composition of the workforce, as well as individual transitions in and out of self-employment. Before discussing the parameter values, we define the parameterization of the productivity process and the discretization of assets.

Productivity process We discretize the process for productivity (z) so that the conditional probability distribution $\Pr^z(z'|z)$ is characterized by a stochastic matrix of dimensions $n_z \times n_z$. We use the method proposed in Rouwenhorst (1995) to discretize an AR(1) processes for $\log(z)$. This reduces the number of parameters to choose from $n_z(n_z - 1)$ to just the standard deviation and persistence of the process, σ_z, and ρ_z. We

	Externally Calibrated Parameters	
	Parameter	Value
$r^\star$	International Interest Rate	0.0075
ρ	Discount Factor	0.0125
σ	CRRA Parameter	2
α	Technology - Capital Share	0.3
δ	Capital Depreciation	0.0125
ν	Technology - Decreasing Returns	0.85
λ	Equity Constraint	1.42
γ^ϵ	Labor Efficiency - Arrival Rate	1
γ^z	Productivity - Arrival Rate	1

	Internally Calibrated Parameters	
	Parameter	Value
b	Unemployment Income	$w \cdot 10^{-5}$
γ^E	Job Destruction Arrival Rate	0.20
γ^U	Job Offer Arrival Rate - U	0.80
γ^S	Job Offer Arrival Rate - S	0.50
$\overline{\epsilon}$	Labor Efficiency- Base Value	0.10
η	Labor Efficiency - Shifter	3.10
$\overline{z}$	Productivity - Base Value	1.00
σ_z	Productivity - Variance	0.12
ρ_z	Productivity - Persistence	0.17

(a) Model Parameters

	Parameter	Value
$\underline{a}$	Borrowing Constraint	10^{-5}
$\bar{a}$	Asset Barrier	200
η_a	Asset Grid Curvature	2
n_a	Asset Grid Size	120
n_ϵ	Productivity Grid Size	11

(b) Computational Parameters

Table 1: Parameters

also set $n_z = 11$. We experimented with finer grids and verified that our results do not depend on the particular size we chose.

Employment income We assume that $\epsilon(z) = z^\eta$, where η captures the relevance of productivity for the earnings of employed workers. A low value of η implies lower variation in the income of employed agents.[7]

Asset grid We use a curved grid with $n_a = 120$ nodes. Curvature ensures a higher density for low levels of assets. The limits of the grid are given by the borrowing constraint $(\underline{a})$ and an asset barrier $(\overline{a})$. The asset grid is given by:

$$a_i = \underline{a} + \left(\frac{i-1}{n_a - 1}\right)^{\eta_a} (\overline{a} - \underline{a}) \quad \text{for } i \in \{1, \dots, n_a\}. \tag{22}$$

The values of all computational parameters are presented in Table 1b.

Externally calibrated parameters We set a number of parameters to values common in the literature or to match features of the Mexican economy; we summarize them in Table

[7]The reader might be tempted to conclude that $\text{var}(\log w\epsilon(z)) = \eta^2 \text{var}(\log z)$, however this relationship does not hold with equality due to endogenous selection in and out of employment.

Occupational Rates	Data	Model
Unemployment	4.4	4.1
Self-employment	26.7	26.2
Employment	69.1	69.7

Income Moments	Data	Model
std(y_t^S)	0.86	0.86
std(y_t^E)	0.54	0.58
corr(y_t^S, y_{t+1}^S)	0.59	0.59
corr(y_t^E, y_{t+1}^E)	0.60	0.58

Table 2: Targeted Moments

Note: *The table specifies targeted moments for internally calibrated parameters. We only target two out of the three occupational rates, as together they imply the third. We report all three occupational rates for completeness.*

1a. The discount factor is taken from Moll (2014) to match a 5 percent annual discount rate. We set the annual interest rate at 3 percent, to target an annual gap to the discount factor of 0.02 as in Itskhoki and Moll (2019). The degree of decreasing returns (ν) is taken from Midrigan and Xu (2014). The curvature of the utility function (σ) is set to 2, and the power of capital in the production technology (α) is set to 0.3, consistent with standard values used in the literature. We set the collateral constraint parameter (λ) to match a debt-to-asset ratio of 42 percent, consistent with observed debt-to-asset ratios for large firms in Mexico, gathered from Compustat. We set the depreciation rate of capital to an annual rate of 5 percent. Finally, we set unemployment income to $w \cdot 10^{-5}$, a positive but small value, to avoid unemployed agents having zero consumption.

Internally calibrated parameters We jointly choose the values for 6 parameters, σ_z, ρ_z, γ^U, γ^E, γ^S, and η, to target 6 moments. These are the unemployment rate, the self-employment rate, the volatility of log-income for employed and self-employed agents, and the correlation of individual-level income for agents who stay employed and who stay self-employed in consecutive quarters ($corr(y_t^o, y_{t+1}^o)$ for $o \in S, E$).

Although these six parameters affect all six moments, loosely speaking the γ^o parameters are more important for mean occupational rates as they affect transitions between occupations. The remaining parameters, η, σ_z, and ρ_z, are more important for income moments as they affect the productivity process. We report the values of the parameters in Table 1a, while the value of the targeted moments in the data and the model is shown in Table 2. We match all moments closely, including income moments for the employed and self-employed, despite having a single productivity shifter.

The data we use to construct the targeted moments comes from the *Encuesta Nacional de Ocupación y Empleo* (ENOE), a household survey administered by the

National Institute of Statistics and Geography (INEGI) in Mexico.[8] The ENOE includes a rotating panel of responding households who participate in the survey for up to five quarters. We restrict our attention to men aged 23 to 65 who are heads of households and who live in one of Mexico's ten largest municipalities.[9] We analyze data from 2005.Q1 to 2019.Q4. We define as self-employed an individual who reports working in their own business. In Appendix D.1 we document the comparison of average occupational rates for our samples versus the overall Mexican population, and versus the U.S. population as a benchmark.

One key result is that the calibration implies a lower job-offer arrival rate for the self-employed (γ^S), compared to the arrival rate for the unemployed (γ^U). Lower job-arrival rates from self-employment are consistent with cross-sectional findings by Jackson (2020), who indicated that engaging in gig-economy jobs in the U.S. reduces the rate at which individuals find salaried employment. In Appendices D.2 and D.3, we provide a battery of reduced-form correlations documenting that individuals in Mexico who have second earners in their household or receive remittances from abroad (variables that indicate looser financial constraints) transition into self-employment less often, even when their self-reported job finding activities are similar to individuals who do not receive remittances and do not have a second earner in their household. We also document that individuals who engage in self-employment transition less into salaried work than their unemployed counterparts. We take this evidence as suggestive of potential negative effects of self-employment for some agents who engage in it out of necessity.

3 Model performance and validation

We now turn to the model's ability to match salient features of self-employment in developing economies. As mentioned above, the model is able to match the targeted moments (Table 2). More importantly, it performs well on a wide range of non-targeted dimensions, like the prevalence of self-employment across the income distribution, the relative response of wages to employment after a well-identified shock to labor demand, transition rates across occupations, and individual-level income changes after an occupation change.

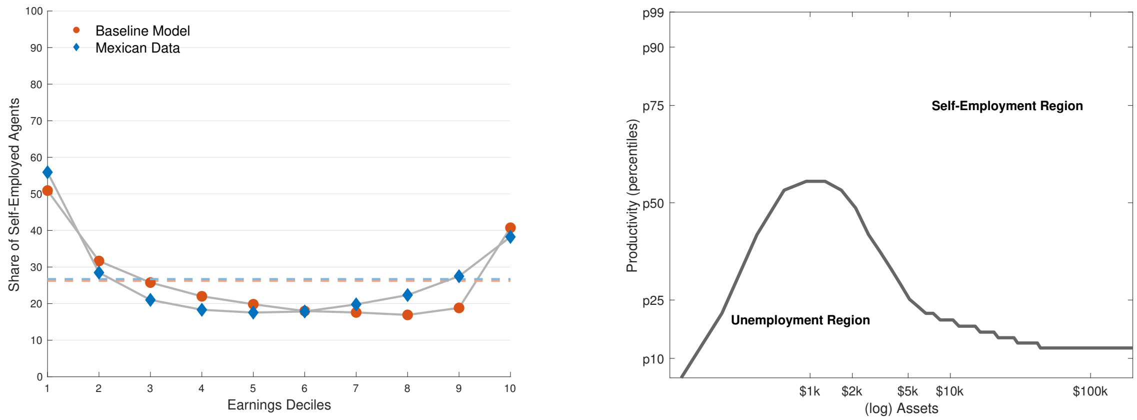
[8] See http://en.www.inegi.org.mx/proyectos/enchogares/historicas/enoe/.

[9] Our results are robust to including both men and women instead.

Figure 3: Model Performance

(a) Self-Employment Rate by Decile of Earnings

(b) Occupational Choice



Note: *The left panel reports the share of self-employed agents for each decile of the earnings distribution. The red circles correspond to the baseline model with unemployment risk. Blue diamonds show the data equivalent for Mexico. The right panel reports productivity thresholds characterizing the occupational choice of unemployed agents. The line depicts the threshold value of productivity* (z) *for each level of assets* (a). *The agent chooses self-employment if her productivity is above the threshold.*

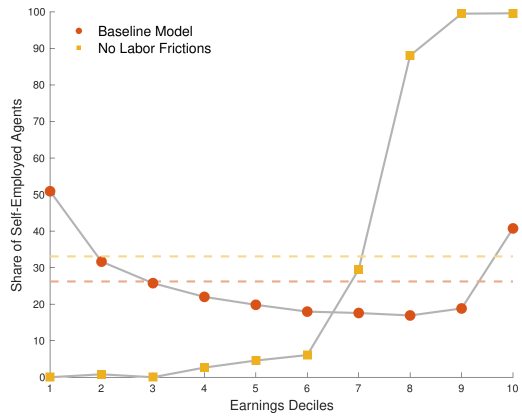
Self-employment across the income distribution The model matches self-employment rates across the income distribution, as in Figure 1. The model produces low-earning self-employed agents in equilibrium, capturing the joint distribution of occupational rates and earnings.

Figure 3a plots the self-employment rate for individuals in each decile of the earnings distribution, both in the data and the model.[10] The model captures the U-shape of self-employment rates thanks to the selection into self-employment from unemployment (Figure 3b). As in Figure 2, poor unemployed agents become self-employed even when their productivity is low, thus generating a mass of low-earning self-employed agents.

In contrast, when we abstract from unemployment risk we cannot generate the u-shaped pattern between income and self-employment. Figure 4 shows the pattern of self-employment across the earnings distribution for the model without unemployment risk presented in Appendix A. Without unemployment, labor income constitutes a lower bound for earnings and self-employment concentrates among high earners exclusively.

[10]To compute the data series in Figure 3a, we first run a regression of the form $\log(earn_{i,t}) = \alpha + \gamma_t + \beta X_{i,t} + \eta_{i,t}$, where $earn_{i,t}$ corresponds to the earnings of individual i at time t, and X is a vector of individual-level controls. We rank $\hat{\eta}_{i,t}$ and classify them in bins of 2 percent of the sample, then compute the self-employment rate in each bin. The pattern we report is robust when we use raw earnings instead of controlling for observables.

Figure 4: Self-Employment Rate by Decile of Earnings: Role of Labor Frictions



Note: *The figure reports the share of self-employed agents for each decile of the earnings distribution. The red circles correspond to the baseline model with unemployment risk. The yellow squares correspond to the alternative model without unemployment risk.*

Response to labor demand shocks We compare the response to an exogenous increase in labor demand in models with and without unemployment risk. We contrast the implications of these models against cross-sectional evidence that identifies the response of employment and wages to this type of shock. The shocks to labor demand are identified in the data from interventions administered at the local labor market level, and are therefore informative about the general equilibrium effects at the core of our mechanism, making them good evidence to validate model specifications.[11]

The empirical evidence comes from a randomized controlled intervention carried out by Breza, Kaur, and Shamdasani (2021) and from the implementation of the NREGS, a National Job Guarantee Scheme in India (Muralidharan, Niehaus, and Sukhtankar, 2017). Both studies estimate the effects of additional labor demand coming from either the researchers' intervention or the government's, both offering employment at market wages. Both studies show that the reaction of wages to the shock is significantly smaller than the reaction of employed work. We refer to this fact as an *"elasticity of wage to labor demand"* caused by a labor demand shock ($^{\Delta\% w}/_{\Delta\% N^d}$) lower than 1.[12]

To reproduce the response in workforce composition and wages to an increase in labor demand, we modify the model by introducing government demand for labor n^{gov},

[11]The small open economy assumption we make implies that local interventions do not change the real interest rate in untreated regions.

[12]This is the ratio of the elasticity of wages to the shock, over the elasticity of employment to the shock.

then solve for the new market-clearing wage.[13] The total labor demand is

$$N^d = \int n^\star(a,z)dG^S + n^{gov}. \tag{23}$$

Government labor demand amounts to 11.5 percent of the baseline demand for efficiency units of labor, as in the intervention by Breza et al. (2021).

In response to an increase in labor demand by the government, the model implies a reallocation of the workforce from self-employment into both higher employment and unemployment. This response is in line with the results of Imbert and Papp (2015), Breza, Kaur, and Shamdasani (2021) and Muralidharan, Niehaus, and Sukhtankar (2017). As Imbert and Papp (2015) found, there is a strong crowding-out effect on private employment (in this case on self-employment). Private demand for labor decreases by 9.35 percent (self-employment decreases by 1.25 percentage points). However, the overall demand for labor increases (by about 2.15 percent), as does the unemployment share of the workforce (by 0.3 percentage points). These movements are qualitatively in line with the findings of Breza, Kaur, and Shamdasani (2021). The decrease in self-employment comes mostly from the low-productivity self-employed agents (see figure C.1 in Appendix C).

Crucially, the increase in labor demand is not accompanied by a sizeable increase in wages. The market wage (per efficiency-unit of labor) increases by 0.35 percent, implying an relative elasticity of wage to labor after a labor demand shock $(\Delta\%w/\Delta\%N^d)$ of 0.16. This is consistent with the experimental evidence referenced above, which establishes a low response of wages relative to increases in labor demand.

The presence of unemployment risk and low-earning self-employed agents plays a crucial role in generating the low response of wages to the increase in labor demand. In contrast, the increase in wages required to meet the additional demand in the model without unemployment risk is an order of magnitude larger than what is implied by the experimental evidence and our baseline model, producing an elasticity of wage to labor demand of 1.6. As with the joint distribution of self-employment and earnings, the data reject the model without unemployment risk in policy-relevant dimensions.

Transition rates between occupations The model does a good job capturing the transition rates between occupations observed in the data, as shown in Table 3. We

[13] In the spirit of the experimental intervention cited above and the implementation of the NREGS program, we do not introduce taxes to finance the government's labor demand.

Occupational Transition Rates								
	Data	Model		Data	Model		Data	Model
$U \to U$	27.4	29.3	$S \to U$	1.9	4.6	$E \to U$	3.1	2.5
$U \to S$	14.6	23.6	$S \to S$	76.8	62.2	$E \to S$	8.1	12.8
$U \to E$	58.0	47.1	$S \to E$	21.3	33.1	$E \to E$	88.8	84.7

Income Moments					
	Data	Model		Data	Model
$\text{corr}(y_t^E, y_{t+1}^S)$	0.43	0.39	$\text{corr}(y_t^S, y_{t+1}^E)$	0.43	0.34

Table 3: Untargeted Moments

Note: *The table specifies untargeted moments. All transition rates are quarterly and reported in percentage points. The income moments are auto-correlations of income conditional on occupational switching in consecutive quarters.*

compute transition rates in the same way in the data and in the model, based on the occupation of agents at the end of each quarter.

All transitions have the right order of magnitude even though none of them was targeted directly in our calibration. However, the model does not capture all transition patterns. In particular, the model overstates the transition rate from unemployment to self-employment (23 percent in the model versus 15 percent in the data). In both the data and the model, the low unemployment rate is explained by high transition rates out of unemployment, particularly into self-employment.

Income of occupational switchers The model captures the correlation of income for switchers observed in the data. This moment is particularly relevant for the gains or losses following a reallocation of agents across occupations. Again, these are untargeted moments. The model implies slightly lower correlations than those from the data despite the common productivity shifter for both occupations, which is explained by the role of assets in determining self-employment income. The correlation between current and future income of individuals who switch from employment to self-employment is 0.39 versus 0.43 in the data. The correlation of switchers from self-employment to employment is 0.34 versus 0.43 in the data (Table 3).

4 Policy analysis

We turn to the analysis of policies targeting the self-employed. Our analysis has two distinct, but related, purposes. The first one is to evaluate the ability of the model to rationalize a key and widely studied result in the development economics literature: the small (micro) effects of an expansion in the availability of credit for the self-employed in Randomized Controlled Trials (RCTs). The second one is to understand whether the small cross-sectional estimated response from RCTs implies that aggregate (macro) effects are also small. Finally, we use the model to study whether differences in targeting or design of development policies affect these aggregate effects.

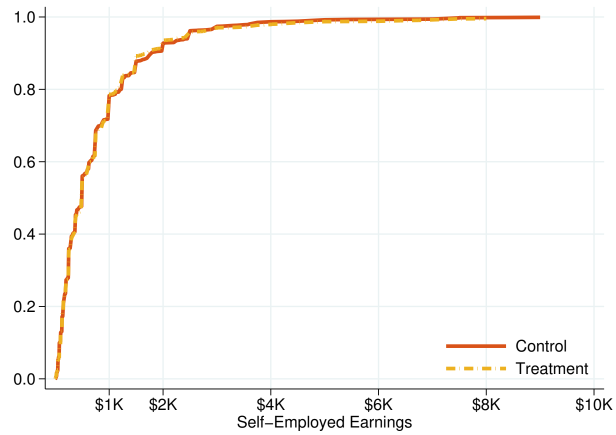
We find that the model is consistent with the micro effects of expansions in credit, which gives us confidence on its usefulness as a laboratory to understand the macro effects of various policy-designs. In particular, we expand access to credit replicating the size of intervention in Angelucci et al. (2015) and find negligible effects on earnings, firm size, and other individual variables, consistent with the findings of the RCT. Using our model we analyze the effects on total output and aggregate productivity of a wide implementation of the program and find substantial increases. Finally, we vary the design of the policy, first by increasing its generosity by reducing the interest rate charged for the loans, and then by replacing loans for transfers targeted to the unemployed or the non-employed. We find that all these variations in policy design generate very similar and small effects at the micro level. However, the aggregates effects vary starkly across designs. Low-interest loans and transfers to the non-employed generate sizeable productivity losses, while transfers to the unemployed generate productivity gains.

4.1 Micro and macro effects of increases in credit availability

We start this section by showing that our model is able to replicate the small micro effects of an RCT that increased the availability of credit to self-employed individuals in Mexico, even when we do not target these statistics directly. Our exercise provides self-employed agents access to a loan of the same size as the loans in the intervention in Angelucci et al. (2015). Formally, we modify the collateral constraint of the self-employed to be:

$$k \leq \lambda \cdot a + \phi, \tag{24}$$

Figure 5: Change in Self-Employment Earnings in Angelucci et al. (2015)



Note: *The figure reports the CDF of the earnings of the self-employed for the control (red) and treatment (yellow) groups of the randomized control trial studied in Angelucci et al. (2015).*

and we set the value of ϕ to be consistent wit the average size of loans in Angelucci et al. (2015), about \$540 dollars per quarter.[14] Consistent with the RCT results, we find negligible gains in self-employment revenue, employment, and production in local economies that had increased availability of micro-credit compared to local economies that did not receive an extension of micro-credit. Our results are also consistent with Meager (2019), who finds that after an extension of micro-credit "the impact on household business and consumption variables is unlikely to be transformative and may be negligible" in a meta study of RCTs from seven different countries.

In order to provide context, Figure 5 shows the CDF of self-employment earnings in treated and control local economies in Angelucci et al. (2015). The two CDFs are almost indistinguishable form each other, meaning that the earnings gains of self-employed individuals caused by an expansion of credit access are negligible throughout the distribution.

Figure 6a shows the model analogue to Figure 5. An increase of the availability of credit produces negligible changes in the distribution of individual-level earnings of the self-employed in the model. The average change in income in general equilibrium, where we allow for adjustment in wages in local economies, is roughly 18 dollars.[15]

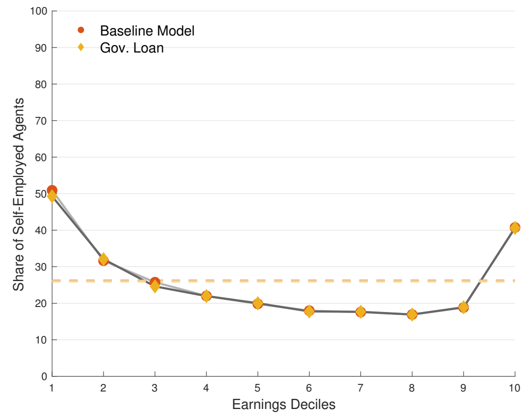
[14]This policy is qualitatively different form a reduction in financial frictions, as captured by an increase in λ. Even though both changes loosen the financial constraint they do so in different ways for poor agents. We explore the consequences of financial reforms that increase λ in Appendix C.2.

[15]In Figure C.4, in Appendix C, we show that this also holds in partial equilibrium, where wages do not adjust. The reason is that, in the model, wage adjustment is small in response to shocks that shift labor demand in the presence of slack in the labor market, in line with evidence from Breza et al. (2021) and the

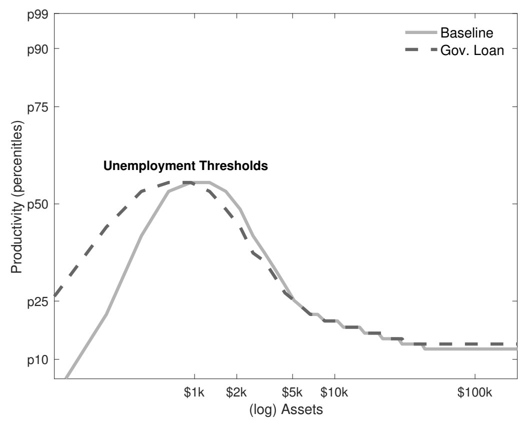
Figure 6: Model Response to Increase in Credit

(a) CDF of Self-Employed Earnings

(b) Self-Employment Rate



(c) Occupational Choice



Note: *Panel (a) reports the CDF of the earnings of the self-employed in the model. The continuous red line corresponds to the baseline model. The dashed yellow line corresponds to the model with expanded credit access in general equilibrium. Panel (b) reports the share of self-employed agents for each decile of the earnings distribution.The red circles correspond to the baseline model. The yellow squares correspond to the model with expanded credit access in general equilibrium. Panel (c) reports productivity thresholds characterizing the occupational choice of unemployed agents for the baseline model (continuous light gray line) and the model with credit deepening (dashed dark gray line). Lines depict the threshold value of productivity* (z) *for each level of assets* (a)*. The agent chooses self-employment if productivity is above the threshold.*

Furthermore, the model predicts small changes in the self-employment rate across the income distribution, Figure 6b.

Table 4 reports the effects of the increase in the availability of credit at the aggregate level. The differential effect of the policy on the occupational choice of low- and high-productivity agents generates a change in the selection into self-employment which plays a key role in shaping the aggregate effects of the policy. In the aggregate, there is a reallocation away from self-employment, although this effect is small. More

results in Section 3.

Variable	Δ	Variable	$\%\Delta$	Variable	$\%\Delta$
Employment	0.08	TFP	0.15	% Output	0.20
Unemployment	0.16	Income(E)	0.04	Consumption	0.02
Self-Employment	-0.24	Income(SE)	0.95	Assets	-0.40

Table 4: Model Response to Increase in Credit

Note: *The table presents changes in variables with respect to the baseline model after the expansion of access to credit. All numbers are in percentage points.*

credit supply could in principle lead to higher self-employment rates as more individuals may decide to start businesses. However, only the more productive agents can successfully take advantage of the new funds. In fact, there is a reallocation of capital and labor into the hands of more productive agents as they increase their production scale. The increase in labor demand from more productive self-employed individuals generates a small increase in wages (0.06 percent) which reduces the profitability of low-productivity self-employment, in turn triggering the reallocation away from self-employment.

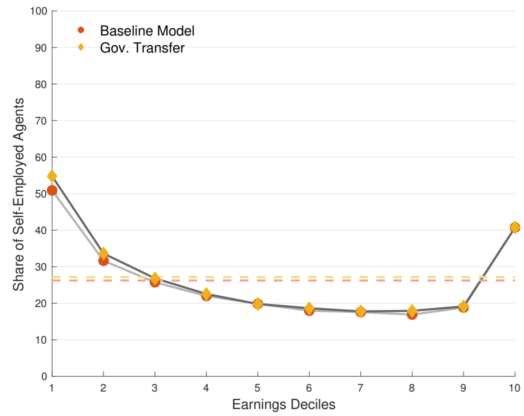
The increased availability of credit has meaningful effects on macroeconomic aggregates. TFP, measured as in (14), increases by 0.15 percent. Both the change in the composition of the self-employed (skewed towards more productive agents) and the loosening of their collateral constraints result in higher aggregate productivity. Output increases by 0.2 percent, driven mainly by the increase in TFP and the increase in scale of production of the self-employed. Finally, higher availability of credit reduces the need to self-finance, which decreases aggregate assets by 0.4 percent. These results highlight the importance of the aggregation exercise, showing that positive aggregate effects can be consistent with small effects at the micro level.

Finally, we find that the expansion of credit availability has positive welfare effects. We measure welfare in consumption-equivalent units, defined as the percentage increase in consumption that would make an agent indifferent between our baseline economy (B) and the economy where the policy has been implemented (P). Formally, the consumption equivalent welfare measure for an agent in occupation o with a assets

Figure 7: Model Response to Introduction of Zero-Interest Loans

(a) CDF of Self-Employed Earnings

(b) Self-Employment Rate



Note: *Panel (a) reports the CDF of the earnings of the self-employed in the model. The continuous red line corresponds to the baseline model. The dashed yellow line corresponds to the model with zero-interest loans in general equilibrium. Panel (b) reports the share of self-employed agents for each decile of the earnings distribution.The red circles correspond to the baseline model. The yellow squares correspond to the model with zero-interest loans in general equilibrium.*

and a productivity of z is characterized by:[16]

$$1 + \mathrm{CE}^o(a, z) = \left(\frac{V_P^o(a, z)}{V_B^o(a, z)} \right)^{\frac{1}{1-\sigma}}. \tag{25}$$

The average welfare change in the economy is of 0.07 percent. Gains are broad based and only slightly higher among the unemployed, who gain on average the equivalent to 0.13 percent of lifetime consumption.

4.2 Micro and macro effects of policy design

Generosity of funds to the self-employed Changes in implementation variables of development policies, like the generosity of interest rate subsidies, maintain the small and negligible micro effects, but can flip the sign of the macro effects. We implement an increase in the availability of credit of the same magnitude as in the previous exercise, but increase the generosity of the program by setting a zero interest rate for these loans. We do this in the spirit of Meager (2019) who reports wide variation in the generosity of micro-finance programs.

[16]The functional form of the utility implies that changing the consumption plan of an agent by a fixed proportion γ changes the value of the agent by a factor of $\gamma^{1-\sigma}$.

Variable	Δ	Variable	$\%\Delta$	Variable	$\%\Delta$
Employment	-0.24	TFP	-0.45	% Output	0.24
Unemployment	-0.72	Income(E)	0.50	Consumption	0.15
Self-Employment	0.96	Income(SE)	-2.40	Assets	-1.11

Table 5: Model Response to Introduction of Zero-Interest Loans

Note: *The table presents changes in variables with respect to the baseline model after the introduction of zero-interest loans. All numbers are in percentage points.*

The effects of introducing zero-interest loans at the micro-level are remarkably similar to those of regular loans explored above. Figure 7 shows the CDF of self-employed earnings and the joint distribution of income and occupations in our baseline economy and the economy with the expansion in zero-interest loans. In both cases the distributions are nearly indistinguishable regardless of treatment status. Moreover, the effects of this policy intervention on these variables are almost identical to those presented in Figure 6, consistent with the results in Meager (2019).

Although it is difficult to tell apart the micro effects of these two types of loans, the macro effects are almost the opposite, see Tables 4 and 5. Zero-interest loans result in an increase in the self-employment rate. TFP goes down by almost half a percentage point, and the average self-employment income goes down by 2.4 percent, while the income of those with wage employment increases by 0.5 percent, driven by an increase of 0.72 percent in the wage. As a result of the availability of cheap credit, aggregate private assets in the economy go down more than 1 percent.

The decline in TFP is triggered by two forces. First, a change in occupational choices generates an inflow of low-productivity self-employed individuals. Unlike interest-bearing loans, zero-interest loans are particularly advantageous to low-productivity self-employed individuals who have low rates of return on their entrepreneurial activities. As a result, more low-productivity agents choose to become self-employed, reducing aggregate productivity. Second, the higher availability of funds partly crowds-out private assets. This increases the magnitude of the distortions faced by agents (captured by $\tilde{\tau}$ in equation 14) in turn reducing aggregate productivity.

Although TFP decreases, output increases 0.24 percent. This increase is due to two forces that counteract the decline in productivity. First, the availability of cheap credit increases the overall amount of capital used for production. Second, the increase in the self-employment rate increases measured output as a consequence of the decreasing-returns-to-scale at the micro-level, see (13). Welfare also increases, on average, by 0.68

percent. The higher welfare gain relative to the previous policy comes as no surprise given the higher generosity and higher take-up of zero-interest loans.

The exercise shows how small changes in the design of policy can substantially change the aggregate effects without significant changes in the micro effects on individuals. The key driver of these differences in our model is the endogenous occupational choice of agents. When driven by subsistence concerns, agents respond even to small incentives, leading to changes in the aggregate effects of policies.

Direct transfers We go beyond studying the aggregate implications of reforms that increase the availability of credit, and ask what the model has to tell about the aggregate implications of more general transfer policies. The key mechanism of the model is the endogenous response of occupational choices to changes in outside options, and the relationship of these choices with production, saving, and borrowing decisions. For that reason we study transfers that alter these margins of occupational choice in a direct way, and consider the effects of transfers to the unemployed, and transfers to those who are not employed. In Appendix C.4 we also consider a broader policy that provides means-tested transfers to the entire population.

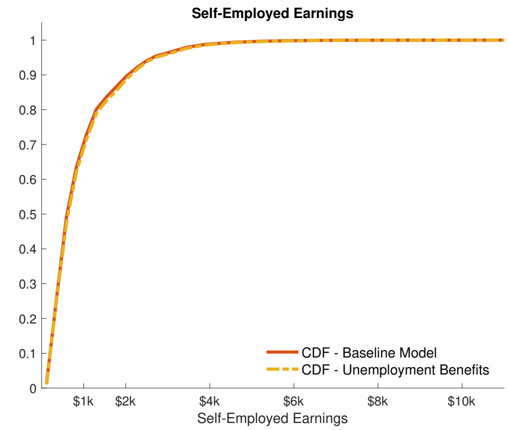
Policies that improve the outside option of unemployed individuals, like transfers to the unemployed, increase productivity, since they improve selection into self-employment. We show however that details on the screening ability of the government are key determinants of the aggregate effects: when the government cannot distinguish between unemployed and self-employed individuals, the same transfer has negative effects on aggregate productivity. These effects are even stronger when expanding transfers to all individuals, as we show in Appendix C.4.

We first introduce a policy that gives transfers to unemployed agents, raising their income to $b + \tilde{b}$. Our policy grants a transfer of $18 dollars, or just over 10% of the lowest wage among the employed in the model. The magnitude of the transfer matches the average increase in self-employment earnings of the micro-finance policy implemented in Section 4.1 above. Benefits do not expire and are financed via labor income taxes.

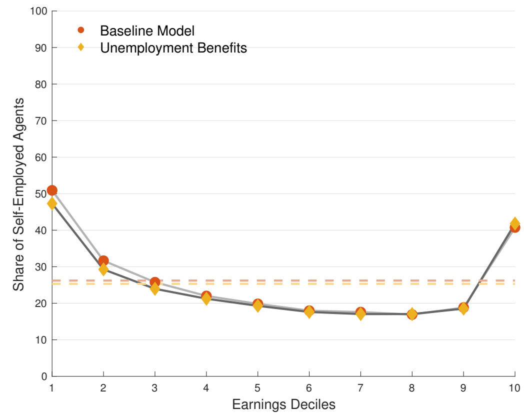
Comparing the CDFs of self-employment earnings across economies with and without transfers to the unemployed, yields very small effects, consistent with our analysis of expansions to credit. Figure 8b shows a small impact among low-earning individuals for whom the self-employment rate drops slightly. This drop is explained by the change in the occupational choices of poor-unemployed agents. The transfer

Figure 8: Model Response to Transfers to the Unemployed

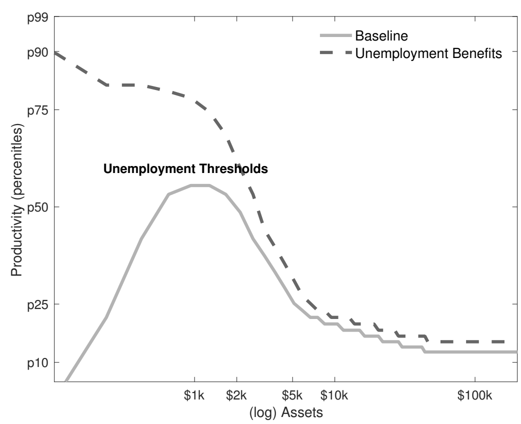
(a) CDF of Self-Employed Earnings



(b) Self-Employment Rate



(c) Occupational Choice



Note: *Panel (a) reports the CDF of the earnings of the self-employed in the model. The continuous red line corresponds to the baseline model. The dashed yellow line corresponds to the model with transfers to the unemployed in general equilibrium. Panel (b) reports the share of self-employed agents for each decile of the earnings distribution.The red circles correspond to the baseline model. The yellow squares correspond to the model with transfers to the unemployed in general equilibrium. Panel (c) reports productivity thresholds characterizing the occupational choice of unemployed agents for the baseline model (continuous light gray line) and the model with credit deepening (dashed dark gray line). Lines depict the threshold value of productivity* (z) *for each level of assets* (a). *The agent chooses self-employment if productivity is above the threshold.*

effectively provides insurance for poor individuals, preventing those with low-productivity from entering self-employment. Figure 8c makes this clear by showing the change in the productivity threshold characterizing the unemployed agents' occupational choice—the minimum productivity level at which they become self-employed. We return below to this change in selection into self-employment as it plays a central role in explaining the aggregate effects of the policy.

Providing transfers to the unemployed results in an increase in aggregate productivity of 0.42 percent, and an increase in output of 0.25 percent (see Table 6).

Variable	Δ	Variable	$\%\Delta$	Variable	$\%\Delta$
Employment	0.06	TFP	0.42	% Output	0.25
Unemployment	0.85	Income(E)	-0.40	Consumption	-0.26
Self-Employment	-0.90	Income(SE)	3.70	Assets	-0.52

Table 6: Model Response to Transfers to the Unemployed

Note: *The table presents changes in variables with respect to the baseline model after the introduction of transfers to the unemployed. All numbers are in percentage points.*

These changes are significant given the magnitude of the transfer. The total increase in output is 2.6 times the cost of transfers. The gains in productivity and output are explained by better selection into self-employment highlighted above. Even though the decreases of 0.9 percentage points in self-employment drives output down (see equation 13), the increase in the average productivity of the self-employed more than compensates for the decrease. These effects are accentuated by the change in the wage rate, which decreases to clear the labor market. Lower wages increase entrepreneurial income, but do so disproportionately for the most productive agents, increasing their scale.

These results highlight how safety net programs can play a role in increasing productivity in developing countries by affecting the occupation choices of individuals. These programs not only allow individuals to search for jobs longer (Acemoglu and Shimer, 1999, 2000; Chetty, 2008); they also prevent unproductive agents from engaging in entrepreneurial activities for which they are ill-suited. This in an additional rationale for these programs, beyond their insurance role against labor income fluctuations.[17]

Despite the potential for productivity gains coming from transfer programs, we show now that these gains are sensitive to the design of the programs. We do this by lifting the implicit assumption that the institution in charge of introducing transfers to the unemployed, can in fact, distinguish people's occupation. This is admittedly a stark assumption, since there would be no incentives for the self-employed to reveal their status, and the institution in charge of introducing the transfers may have limited capacity for screening.

We consider an alternative scenario in which the transfers go to every individual who is not a wage worker, effectively assuming zero screening capacities, and

[17]More generous safety nets that provide insurance against productivity risk may also improve productivity by spurring entrepreneurship. See the recent work by Robinson (2020) and evidence from France in Hombert, Schoar, Sraer, and Thesmar (2020, 2016).

individuals reacting strongly to the incentive to take the transfer. The micro effects of this alternative policy are small and very similar to the ones before (see Figure C.5 in Appendix C.3), but the aggregate effects flip sign relative to the previous policy of transfers to the unemployed (Tables 6 and C.1).

When transfers are also reaching the self-employed, self-employment rates increase 0.36 percentage points, and productivity goes down 0.32 percent. These effects are even stronger if transfers are expanded to cover all (poor) individuals irrespective of occupation (Appendix C.4). The reason is twofold. First, the transfer damages selection into self-employment, as low-productivity individuals take the transfer and engage in self-employment at the same time. Second, there is a strong disincentive to save. Assets go down 1.9 percent when the transfers are available to the self-employed and go down even more when transfers are available to all individuals regardless of occupation. In this way, changes in the implementation of policy can do away with the desired effects, just as in the expansions of credit studied above.

5 Concluding Remarks

Self-employment can reflect entrepreneurial drive leading to innovation and growth. But it can also reflect subsistence concerns of individuals looking for an income source when no alternatives are available. This is painfully evident across developing economies characterized by high self-employment rates concentrated among the poor.

We show that the features of self-employment in developing economies are consistent with its role as self-insurance for poor individuals. Consequently, policies that supplement this insurance role, like providing unemployment income, are effective at preventing low-productivity individuals from becoming self-employed. This in turn has important general equilibrium implications, tied to increases in productivity and employment. On the contrary, policies aiming to promote entrepreneurship can have unintended consequences by making self-employment more attractive for low-productivity individuals, resulting in lower aggregate productivity. The challenging material conditions of the poor mean that even small incentives have a large impact on their occupational choices.

We argue that incorporating labor market frictions alongside financial frictions is critical to correctly capture the aggregate dynamics of developing economies and their responses to policy. Doing so not only improves the fit of models to salient features of developing economies, but it also brings macro-development models in line with micro-estimates of the identified response of the economy to shocks (e.g., the response of wages to labor demand shocks). The key mechanism lies in incorporating the view that subsistence concerns dominate selection into self-employment among the poor.

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Appendices for Online Publication

Table of Contents

A Model without unemployment risk

In this section we describe a simplified model without unemployment risk. We lay out the model in general terms allowing for a labor efficiency component independent (ϵ) of the agent's productivity (z). In the calibration we set the variance of ϵ to zero for comparability with our baseline model, results with both sources of variation are available upon request. As before, an agent is free to become self-employed at any point, but now the agent is also free to become employed in response to changes in her labor efficiency, productivity or assets. There is no change in the profit maximization problem of the self-employed, or in the definition of total supply and demand for labor.

A.1 Agent's Problem

Because agents can change instantly across occupations, the occupational choice problem reduces to maximizing instantaneous income. The value of an agent is then:

$$\begin{aligned}\rho V(a,z,\epsilon) = \max_{c}\ & u(c) + V_a(a,z,\epsilon)\,\dot{a} + \\ & \gamma^{\epsilon}\int V(a,z,\epsilon')\,d\mathrm{Pr}^{\epsilon}(\epsilon'|\epsilon) + \gamma^{z}\int V(a,z',\epsilon)\,d\mathrm{Pr}^{z}(z'|z) \\ & \text{s.t. } \dot{a} = \max\{w\varepsilon(\epsilon,z), \pi(a,z)\} + ra - c \qquad a \geq \underline{a}\end{aligned} \tag{A.1}$$

with $\pi(a,z)$ given as in equation (18), and $\varepsilon(\epsilon,z) = \epsilon + \eta z$.

The optimal consumption decision is found as in Section 1, following equation (8). Finally agents are employed if $w\varepsilon(\epsilon,z) \geq \pi(a,z)$ and as self-employed otherwise.

A.2 Equilibrium

An stationary equilibrium for this economy is a value function $\{V\}$, along with an optimal consumption function $\{c\}$, labor and capital demand from self-employed $\{n,k\}$, prices $\{r,w\}$ and a distribution of agents $\{G\}$ such that:

1. The value function satisfies (A.1).
2. Consumption and asset accumulation are given by equation (8).
3. Capital and labor demand solve the self-employed's profit maximization problem.
4. Labor market clears: $N^S = N^D$, satisfying equations (11) and (12).
5. The interest rate is given by the international interest rate $r^{\star}$.
6. The distribution of agents is stationary and solves:

$$\begin{aligned}0 = & -\frac{\partial}{\partial a}\left[\dot{a}g(a,z,\epsilon)\right] - \gamma^{z}\int \mathrm{Pr}^{z}(z'|z)\,g(a,z,\epsilon)\,dz' + \gamma^{z}\int \mathrm{Pr}^{z}\left(z|z'\right)g(a,z',\epsilon)\,dz' \\ & - \gamma^{\epsilon}\int \mathrm{Pr}^{\epsilon}(\epsilon'|\epsilon)\,g(a,z,\epsilon)\,d\epsilon' + \gamma^{\epsilon}\int \mathrm{Pr}^{\epsilon}\left(\epsilon|\epsilon'\right)g(a,z,\epsilon')\,d\epsilon'\end{aligned} \tag{A.2}$$

B Computational appendix

B.1 Solution to HJB equations

The model is solved using an implicit finite difference method as the one shown in Achdou, Han, Lasry, Lions, and Moll (2021). The occupational choice is solved through a splitting method, solving first for an auxiliary value $\tilde{V}$, the value that applies if the agent continues in the same occupation, and then solving for the occupational choice. We present a more general model that allows for separate labor efficiency and productivity processes.

Consider grids over assets, entrepreneurial ability and labor efficiency:

$$\vec{a} = [a_1, \ldots, a_{n_a}] \qquad \vec{z} = [z_1, \ldots, z_{n_z}] \qquad \vec{\epsilon} = [\epsilon_1, \ldots, \epsilon_{n_\epsilon}]$$

with n_a, n_z and n_ϵ elements respectively, and constant distance between grid points of Δa, Δz and $\Delta\epsilon$. Let i denote the index of the asset dimension, j of the entrepreneurial ability, and k of the labor efficiency.

For notational convenience we will treat all value functions as depending on all three states, it is understood that V^U and V^S do not vary across ϵ. Denote $V^o_{ijk} = V^o(a_i, z_j, \epsilon_k)$ and let the backward and forward difference of the value function approximate the derivative:

$$V^o_a(a_i, z_j, \epsilon_k) \approx \frac{V^o_{i+1,jk} - V^o_{ijk}}{\Delta a} = \partial_a V^o_{ijk,F} \qquad V^o_a(a_i, z_j, \epsilon_k) \approx \frac{V^o_{ijk} - V^o_{i-1,jk}}{\Delta a} = \partial_a V^o_{ijk,B}\,.$$

The problem to solve is:

$$\begin{aligned}\rho\tilde{V}^E_{ijk} &= u\left(c^E_{ijk}\right) + \partial_a V^E_{ijk} \cdot \left(y^E_{ijk} + r a_i - c^E_{ijk}\right) + \gamma^E \left(V^U_{ij} - \tilde{V}^E_{ijk}\right) \\ &\quad + \gamma^z \sum_{j'=1}^{n_z} \left(V^E_{ij'k} - \tilde{V}^E_{ijk}\right) \Pr^z(z_{j'}|z_j) + \gamma^\epsilon \sum_{k'=1}^{n_\epsilon} \left(V^E_{ijk'} - \tilde{V}^E_{ijk}\right) \Pr^\epsilon(\epsilon_{k'}|\epsilon_k)\end{aligned} \tag{B.1}$$

$$\begin{aligned}\rho\tilde{V}^U_{ij} &= u\left(c^U_{ij}\right) + \partial_a V^U_{ij} \cdot \left(y^U_{ij} + r a_i - c^U_{ij}\right) + \gamma^U \sum_{k'=1}^{n_\epsilon} \left(V^E_{ijk'} - \tilde{V}^U_{ij}\right) \Pr^U(\epsilon_{k'}) \mathbb{1}_{\left\{V^E_{ijk'} > V^U_{ij}\right\}} \\ &\quad + \gamma^z \sum_{j'=1}^{n_z} \left(V^U_{ij'} - \tilde{V}^U_{ij}\right) \Pr^z(z_{j'}|z_j)\end{aligned} \tag{B.2}$$

$$\begin{aligned}\rho\tilde{V}^S_{ij} &= u\left(c^S_{ij}\right) + \partial_a V^S_{ij} \cdot \left(y^S_{ij} + r a_i - c^S_{ij}\right) \\ &\quad + \gamma^S \sum_{k'=1}^{n_\epsilon} \left(V^E_{ijk'} - \tilde{V}^S_{ij}\right) \Pr^S(\epsilon_{k'}) \mathbb{1}_{\left\{V^E_{ijk'} > V^S_{ij}\right\}} + \gamma^z \sum_{j'=1}^{n_z} \left(V^S_{ij'} - \tilde{V}^S_{ij}\right) \Pr^z(z_{j'}|z_j)\end{aligned} \tag{B.3}$$

together with:

$$V_{ijk}^{E} = \max\left\{\tilde{V}_{ij,k}^{E}, \tilde{V}_{ij}^{U}, \tilde{V}_{ij}^{S}\right\} \tag{B.4}$$

$$V_{ij}^{U} = \max\left\{\tilde{V}_{ij}^{U}, \tilde{V}_{ij}^{S}\right\} \tag{B.5}$$

$$V_{ij}^{S} = \max\left\{\tilde{V}_{ij}^{U}, \tilde{V}_{ij}^{S}\right\} \tag{B.6}$$

The implicit method solves the following equation on $\tilde{V}_{ijk}^{o,n+1}$ given a value for $V_{ijk}^{o,n}$. For employment the equation is:

$$\begin{aligned}\frac{\tilde{V}_{ijk}^{E,n+1} - \tilde{V}_{ijk}^{E,n}}{\Delta} + \rho\tilde{V}_{ijk}^{E,n+1} = u\left(c_{ijk}^{E,n}\right) &+ \partial_a\tilde{V}_{ijk}^{E,n+1}\cdot s_{ijk}^{E,n} + \gamma^{E}\left(\tilde{V}_{ij}^{U,n+1} - \tilde{V}_{ijk}^{E,n+1}\right) \\ &+ \gamma^{z}\sum_{j'=1}^{n_z}\left(\tilde{V}_{ij'k}^{E,n+1} - \tilde{V}_{ijk}^{E,n+1}\right)\Pr^{z}\left(z_{j'}|z_j\right) \\ &+ \gamma^{\epsilon}\sum_{k'=1}^{n_\epsilon}\left(\tilde{V}_{ijk'}^{E,n+1} - \tilde{V}_{ijk}^{E,n+1}\right)\Pr^{\epsilon}\left(\epsilon_{k'}|\epsilon_k\right)\end{aligned} \tag{B.7}$$

For $o \in \{U, S\}$ the equation is:

$$\begin{aligned}\frac{\tilde{V}_{ij}^{o,n+1} - \tilde{V}_{ij}^{o,n}}{\Delta} + \rho\tilde{V}_{ij}^{o,n+1} = u\left(c_{ij}^{o,n}\right) + \partial_a\tilde{V}_{ij}^{o,n+1}\cdot s_{ij}^{o,n} &+ \sum_{k'=1}^{n_\epsilon}\tilde{\gamma}_{ijk'}^{o,n}\left(\tilde{V}_{ijk'}^{E,n+1} - \tilde{V}_{ij}^{o,n+1}\right)\Pr^{o}\left(\epsilon_{k'}\right) \\ &+ \gamma^{z}\sum_{j'=1}^{n_z}\left(\tilde{V}_{ij'}^{o,n+1} - \tilde{V}_{ij}^{o,n+1}\right)\Pr^{z}\left(z_{j'}|z_j\right)\end{aligned} \tag{B.8}$$

Note that the (known) value at iteration n is used to compute consumption, and the drift of the assets, which we will call savings for convenience:

$$s_{ijk}^{o,n} = y_{ijk}^{o} + ra_i - c_{ijk}^{o,n} \qquad \text{where} \qquad c_{ijk}^{o,n} = u^{'-1}\left(\partial_a V_{ijk}^{o,n}\right)$$

It is also used to define if the agent is willing to change after a job offer. We have:

$$\tilde{\gamma}_{ijk}^{U,n} = \gamma^{U}\mathbb{1}_{\left\{V_{ijk'}^{E,n} > V_{ij}^{U,n}\right\}} \qquad \tilde{\gamma}_{ijk}^{S,n} = \gamma^{S}\mathbb{1}_{\left\{V_{ijk'}^{E,n} > V_{ij}^{S,n}\right\}}$$

Next it is necessary to determine whether to use the forward or backward approximation to the first derivatives of the value function. We follow the "upwind scheme" presented in Achdou et al. (2021).

Since consumption can be defined with the backward or forward difference approximation we get:

$$s_{ijk,B}^{o,n} = y_{ijk}^{o} + ra_i - u^{'-1}\left(\partial_a V_{ijk,B}^{o,n}\right) \qquad s_{ijk,F}^{o,n} = y_{ijk}^{o} + ra_i - u^{'-1}\left(\partial_a V_{ijk,F}^{o,n}\right)$$

The idea is to use the backward difference when the implied drift is negative, and the forward difference when the drift is positive. Yet there are cases for which $s^{o,n}_{ijk,F} < 0 < s^{o,n}_{ijk,B}$, in these cases we set savings equal to zero, so the derivative is not used, in any case the FOC of the problem gives the exact derivate of the value function as: $\partial_a \overline{V}^{o,n}_{ijk} = u'\left(y_{jk} + ra_i\right)$.[18]

Consumption is then:

$$c^{o,n}_{ijk} = u'^{-1}\left(\partial_a V^{o,n}_{ijk,B}\right) \mathbb{1}_{\left\{s^{o,n}_{ijk,B}<0\right\}} + u'^{-1}\left(\partial_a V^{o,n}_{ijk,F}\right) \mathbb{1}_{\left\{s^{o,n}_{ijk,F}>0\right\}} + \left(y^{o}_{ijk} + ra_i\right) \mathbb{1}_{\left\{s^{o,n}_{ijk,F}<0<s^{o,n}_{ijk,B}\right\}},$$

and the drift term for assets is replaced by:

$$\begin{aligned}
\partial_a \tilde{V}^{o,n+1}_{ijk} \cdot s^{o,n}_{ijk} &= \partial_a \tilde{V}^{o,n+1}_{ijk,B} \left[s^{o,n}_{ijk,B}\right]^{-} + \partial_a \tilde{V}^{o,n+1}_{ijk,F} \left[s^{o,n}_{ijk,F}\right]^{+} \\
&= \frac{\tilde{V}^{o,n+1}_{ijk} - \tilde{V}^{o,n+1}_{i-1,jk}}{\Delta a} \left[s^{o,n}_{ijk,B}\right]^{-} + \frac{\tilde{V}^{o,n+1}_{i+1,jk} - \tilde{V}^{o,n+1}_{ijk}}{\Delta a} \left[s^{o,n}_{ijk,F}\right]^{+}
\end{aligned}$$

Grouping terms we get the following expression for employment:

$$\begin{aligned}
\frac{\tilde{V}^{E,n+1}_{ijk} - \tilde{V}^{E,n}_{ijk}}{\Delta} + \rho \tilde{V}^{E,n+1}_{ijk} = u\left(c^{E,n}_{ijk}\right) &+ \gamma^{E} \tilde{V}^{U,n+1}_{ij} \\
&+ x^{E}_{ijk} \tilde{V}^{E,n+1}_{ijk} + x^{E-}_{ijk} \tilde{V}^{E,n+1}_{i-1,jk} + x^{E+}_{ijk} \tilde{V}^{E,n+1}_{i+1,jk} \\
&+ \gamma^{z} \sum_{j'=1}^{n_z} \Pr^{z}\left(z_{j'}|z_j\right) \tilde{V}^{E,n+1}_{ij'k} + \gamma^{\epsilon} \sum_{k'=1}^{n_\epsilon} \Pr^{\epsilon}\left(\epsilon_{k'}|\epsilon_k\right) \tilde{V}^{E,n+1}_{ijk'}
\end{aligned}$$

where

$$\begin{aligned}
x^{E}_{ijk} &= \frac{\left[s^{E,n}_{ijk,B}\right]^{-}}{\Delta a} - \frac{\left[s^{E,n}_{ijk,F}\right]^{+}}{\Delta a} - \gamma^{E} - \gamma^{z} - \gamma^{\epsilon} \\
x^{E-}_{ijk} &= -\frac{\left[s^{E,n}_{ijk,B}\right]^{-}}{\Delta a} \\
x^{E+}_{ijk} &= \frac{\left[s^{E,n}_{ijk,F}\right]^{+}}{\Delta a}
\end{aligned}$$

[18]Additional care is needed because of the non-convexities introduced by the occupational choice of agents. It is possible that both $s^{o,n}_{ijk,B} < 0$ and that $s^{o,n}_{ijk,F} > 0$ for the same state. In this case we take the drift that provides the highest change in value by comparing $u\left(c^{o,n}_{ijk,B}\right) + \partial_a V^{o,n}_{ijk,B} \cdot s^{o,n}_{ijk,B}$ with $u\left(c^{o,n}_{ijk,F}\right) + \partial_a V^{o,n}_{ijk,F} \cdot s^{o,n}_{ijk,F}$. We omit this from the notation for readability.

For unemployment and self-employment:

$$\frac{\tilde{V}_{ij}^{o,n+1}-\tilde{V}_{ij}^{o,n}}{\Delta}+\rho\tilde{V}_{ij}^{o,n+1}=u\left(c_{ij}^{o,n}\right)+\sum_{k'=1}^{n_\epsilon}\tilde{\gamma}_{ijk'}^{o,n}\Pr{}^{o}\left(\epsilon_{k'}\right)\tilde{V}_{ijk'}^{E,n+1}$$
$$+x_{ij}^{o}\tilde{V}_{ij}^{o,n+1}+x_{ij}^{o-}\tilde{V}_{i-1,j}^{o,n+1}+x_{ij}^{o+}\tilde{V}_{i+1,j}^{o,n+1}$$
$$+\gamma^{z}\sum_{j'=1}^{n_z}\Pr{}^{z}\left(z_{j'}|z_j\right)\tilde{V}_{ij'}^{o,n+1}$$

where

$$x_{ij}^{o}=\frac{\left[s_{ij,B}^{o,n}\right]^{-}}{\Delta a}-\frac{\left[s_{ij,F}^{o,n}\right]^{+}}{\Delta a}-\sum_{k'=1}^{n_\epsilon}\tilde{\gamma}_{ijk'}^{o,n}\Pr{}^{o}\left(\epsilon_{k'}\right)-\gamma^{z}$$
$$x_{ij}^{o-}=-\frac{\left[s_{ij,B}^{o,n}\right]^{-}}{\Delta a}$$
$$x_{ij}^{o+}=\frac{\left[s_{ij,F}^{o,n}\right]^{+}}{\Delta a}$$

B.1.1 Boundary Conditions

A final loose end before writing up the linear system in matrix form is what to do with the boundaries of the different grids. At the lower boundary of the asset grid the agent is subject to a no-borrowing constraint. Hence it has to be the case that the agent does not try to borrow. The drift has to be non-negative at that point, which implies that $\tilde{x}_{1jk}^{o-}=0$ for all (j,k). At the upper boundary a similar constraint can be imposed, so that $\tilde{x}_{n_a jk}^{o+}=0$. This should arise naturally if the upper boundary is high enough. Notice that imposing these restrictions implies that V_{0j}^{n+1} and $V_{n_a+1,j}^{n+1}$ are not part of the system.

B.1.2 System Solution

The equations above describe a system of $n_a\times n_z\left(2+n_\epsilon\right)$ equations, its best to define the value function a stack of three value functions, one for each occupation:

$$V=\left[V^{U};V^{S};V^{E}\right]^{T}$$
$$V^{o}=\text{vec}\left[V_{ijk}^{o}\right]$$

The system is:

$$\frac{1}{\Delta}\left(V^{n+1}-V^{n}\right)+\rho V^{n+1}=u^{n}+A^{n}V^{n+1}$$

where $u^{n}=\left[u^{U,n};u^{S,n};u^{E,n}\right]$ and $u^{o,n}=\text{vec}\left[u\left(c_{ijk}^{o,n}\right)\right]$ with consumption computed as explained above.

Matrix A^n is given by:

$$A^n = B^n + C + D$$

$$B^n = \begin{bmatrix} B^n_{UU} & \mathbf{0} & B^n_{UE} \\ \mathbf{0} & B^n_{SS} & B^n_{SE} \\ B_{EU} & \mathbf{0} & B^n_{EE} \end{bmatrix} \qquad C = \begin{bmatrix} \tilde{C} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tilde{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \tilde{C}_E \end{bmatrix} \qquad D = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \tilde{D} \end{bmatrix}$$

The matrices B^n_{oo} are sparse and they only contain elements in the diagonal, upper diagonal and lower diagonal. Consider $X_o = \left[\tilde{x}^o_{ijk}\right]$, $X^-_o = \left[\tilde{x}^{o,-}_{ijk}\right]$ and $X^+_o = \left[\tilde{x}^{o,+}_{ijk}\right]$, all three dimensional matrix that contain the coefficients $\tilde{x}$ (note that $\tilde{x}$ is already adjusted for the boundaries). Then we have: $\text{diag}\left(B^n_{oo}\right) = \text{vec}\left(X_o\right)$, $\text{diag}^+\left(B^n_{oo}\right) = \text{vec}\left(X^+_o\right)$ and $\text{diag}^-\left(B^n_{oo}\right) = \text{vec}\left(X^-_o\right)$, where the upper diagonal and lower diagonal are adjusted not to include the zero terms of the boundaries.

The matrices $B^n_{oo'}$ depend on the type of transition. For the transition from employment to unemployment we have:

$$B_{EU} = \gamma^E \begin{bmatrix} I_{n_a \cdot n_z} \\ \vdots \\ I_{n_a \cdot n_z} \end{bmatrix}$$

so that B_{EU} is of size $n_a \cdot n_z \cdot n_\epsilon \times n_a \cdot n_z$. For the transition from unemployment and self-employment to employment:

$$B^n_{oE} = \gamma^o \left[\begin{array}{ccc} \text{Pr}^o\left(\epsilon_1\right) \text{diag}\left(\text{vec}\left(\mathbb{1}_{\left\{V^E_{ij1} > V^o_{ij}\right\}}\right)\right) & \cdots & \text{Pr}^o\left(\epsilon_{n_\epsilon}\right) \text{diag}\left(\text{vec}\left(\mathbb{1}_{\left\{V^E_{ijn_\epsilon} > V^o_{ij}\right\}}\right)\right) \end{array} \right]$$

where we abuse notation by letting $\text{diag}\left(\cdot\right)$ give a diagonal matrix when it is evaluated in a vector.

Matrices $\tilde{C}$ and $\tilde{D}$ are also sparse and they are independent of the iteration. Their construction takes advantage of the fact that the elements of $\tilde{C}$ only vary with j and the elements of $\tilde{D}$ only vary with k. We first construct $\tilde{C} = \gamma^z \text{Pr}^z \otimes I_{n_a}$ and $\tilde{C}_E = \gamma_z I_{n_\epsilon} \otimes \tilde{C}$. Finally, $\tilde{D} = \gamma^\epsilon \text{Pr}^\epsilon \otimes I_{n_a \cdot n_z}$.

This problem can now be expressed as:

$$T^n V^{n+1} = t^n$$

where:

$$T^n = \left(\frac{1}{\Delta} + \rho\right) I_{n_a n_z (2+n_\epsilon)} - A^n \qquad t^n = u^n + \frac{1}{\Delta} V^n$$

B.1.3 Algorithm

1. Compute matrices C and D. These matrices do not change with equilibrium prices or iterations.
2. Take as given w.
3. Solve for earnings in each state: y^o_{ijk} for each combination of (a, z, ϵ) and

occupation. These values don't change with iterations.

4. Guess a value for V^n, a $n_a n_z (2 + n_\epsilon)$ vector. It is easier to store it as three separate matrices of dimensions $n_a \times n_z$, $n_a \times n_z$ and $n_a \times n_\epsilon \times n_z$.
   (a) We find it better to find the initial condition by solving for a fixed point of the problem without occupational choice (this same algorithm without the last step).
5. Compute the backward and forward drift: $s^{o,n}_{ijk,B}$ and $s^{o,n}_{ijk,F}$ for $i = \{2, \ldots, n_a\}$ and $i = \{1, \ldots, n_a - 1\}$ respectively, and all (j, k, o).

$$s^{o,n}_{ijk,B} = y^o_{jk} + ra_i - u^{'-1}\left(\partial_a V^{o,n}_{ijk,B}\right) \qquad s^{o,n}_{ijk,F} = y^o_{jk} + ra_i - u^{'-1}\left(\partial_a V^{o,n}_{ijk,F}\right)$$

   These values are stored in six matrices (two per occupation, one with backward drift and the other one with forward drift).
6. For all (i, j, k, o) compute consumption as:

$$c^{o,n}_{ijk} = u^{'-1}\left(\partial_a V^{o,n}_{ijk,B}\right) 1_{s^{o,n}_{ijk,B}<0} + u^{'-1}\left(\partial_a V^{o,n}_{ijk,F}\right) 1_{s^{o,n}_{ijk,F}>0} + \left(y^o_{jk} + ra_i\right) 1_{s^{o,n}_{ijk,F}<0<s^{o,n}_{ijk,B}}$$

   These values are stored in three matrices of dimensions $n_a \times n_z$, $n_a \times n_z$ and $n_a \times n_\epsilon \times n_z$.
7. Compute the utility vector as: $u^n = \left[u^{U,n}; u^{S,n}; u^{E,n}\right]$ and $u^{o,n} = \text{vec}\left[u\left(c^{o,n}_{ijk}\right)\right]$.
8. Compute the adjusted shock arrival rates:

$$\tilde{\gamma}^{U,n}_{ijk} = \gamma^U 1_{V^{E,n}_{ijk} > V^{U,n}_{ijk}} \qquad \tilde{\gamma}^{S,n}_{ijk} = \gamma^S 1_{V^{E,n}_{ijk} > V^{U,n}_{ijk}} \qquad \tilde{\gamma}^{E,n}_{ijk} = \gamma^E$$

9. Compute the matrices $X_o = \left[\tilde{x}^o_{ijk}\right]$, $X^-_o = \left[\tilde{x}^{o,-}_{ijk}\right]$ and $X^+_o = \left[\tilde{x}^{o,+}_{ijk}\right]$.
10. Compute matrix $B^n = \begin{bmatrix} B^n_{UU} & \mathbf{0} & B^n_{UE} \\ \mathbf{0} & B^n_{SS} & B^n_{SE} \\ B^n_{EU} & \mathbf{0} & B^n_{EE} \end{bmatrix}$, where $\text{diag}\left(B^n_{oo}\right) = \text{vec}\left(X_o\right)$, $\text{diag}^+\left(B^n_{oo}\right) = \text{vec}\left(X^+_o\right)$ and $\text{diag}^-\left(B^n_{oo}\right) = \text{vec}\left(X^-_o\right)$, where the upper diagonal and lower diagonal are adjusted not to include the zero terms of the boundaries. The matrices $B^n_{oo'}$ are defined above.
11. Compute the matrix $A^n = B^n + C + D$.
12. Compute the matrix T and vector t:

$$T^n = \left(\frac{1}{\Delta} + \rho\right) I_{3n_a n_\epsilon n_z} - A^n \qquad t^n = u^n + \frac{1}{\Delta} V^n$$

13. Compute $V^{n+1/2}$ as:

$$V^{n+1/2} = \left(T^n\right)^{-1} t^n$$

   (a) We use the Biconjugate gradients stabilized (l) method, preconditioned with LU Factorization. See Matlab functions "ilu" and "bicgstabl."
14. Divide the vector $V^{n+1/2}$ into three matrices of $n_a \times n_z$, $n_a \times n_z$ and $n_a \times n_\epsilon \times n_z$: $V^{U,n+1/2}$, $V^{S,n+1/2}$, and $V^{E,n+1/2}$.

15. Compute $V^{U,n+1}$, $V^{S,n+1}$, and $V^{E,n+1}$ as follows:

$$\begin{aligned}
V_{ijk}^{U,n+1} &= \max\left\{V_{ijk}^{U,n+1/2}, \tilde{V}_{ijk}^{S}\right\} \\
V_{ijk}^{S,n+1} &= \max\left\{V_{ijk}^{U,n+1/2}, V_{ijk}^{S,n+1/2}\right\} \\
V_{ijk}^{E,n+1} &= \max\left\{V_{ijk}^{U,n+1/2}, \tilde{V}_{ijk}^{S}, V_{ijk}^{E,n+1/2}\right\}
\end{aligned}$$

(a) Define the following matrices as indicators of the occupation choice: $\left[\tilde{\chi}_{ijk}^{oo'}\right]$

$$\tilde{\chi}_{ij}^{US} = \begin{cases} 1 & \text{if } V_{ij}^{U,n+1} = \tilde{V}_{ij}^{S} \\ 0 & \text{otw} \end{cases} \qquad \tilde{\chi}_{ij}^{SU} = \begin{cases} 1 & \text{if } V_{ij}^{S,n+1} = V_{ij}^{U,n+1/2} \\ 0 & \text{otw} \end{cases}$$

$$\tilde{\chi}_{ijk}^{EU} = \begin{cases} 1 & \text{if } V_{ijk}^{E,n+1} = V_{ij}^{U,n+1/2} \\ 0 & \text{otw} \end{cases} \qquad \tilde{\chi}_{ijk}^{ES} = \begin{cases} 1 & \text{if } V_{ijk}^{E,n+1} = \tilde{V}_{ij}^{S} \\ 0 & \text{otw} \end{cases}$$

These functions are 1 if the agent changes occupations at (i, j, k).

(b) Define now the vectors $\chi^{oo'} = \text{vec}\left(\tilde{\chi}^{oo'}\right)$ to be used later. χ is a vector of length $n_a n_z (2 + n_\epsilon)$.

B.2 Solution to KFE equations

Before solving the KFE the transition matrix A has to be modified to include the endogenous transitions between unemployment and self-employment. For this we use the indicators χ constructed as part of the value function iteration.

Now, consider a transition matrix P:

$$P = \begin{bmatrix} P^{UU} & P^{US} & A^{UE} \\ P^{SU} & P^{SS} & A^{SE} \\ P^{EU} & P^{ES} & A^{EE} \end{bmatrix}$$

note that since there are not endogenous transitions to employment the last column of matrices are just as in matrix A. The other columns are modified only if there are endogenous transitions. Note that each matrix $P^{oo'}$ is of size $n_a n_\epsilon n_z \times n_a n_\epsilon n_z$.

1. Make all matrices $P^{oo'} = A^{oo'}$ and $P^{oo} = A^{oo}$.
2. For matrix P make zero any (column) entry related to an endogenous transition, since these states are not reached. For all m and q in $\{1, \ldots, n_a n_\epsilon n_z\}$:

$$P_{mq}^{*U} = 0 \qquad \text{if } \chi^{US}(q) = 1$$

$$P_{mq}^{*S} = 0 \qquad \text{if } \chi^{SU}(q) = 1$$

$$P^{*E}_{mq} = 0 \qquad \text{if } \chi^{EU}(q) = 1 \quad \text{or} \quad \chi^{ES}(q) = 1$$

where $* \in \{U, S, E\}$.

3. For matrix P adjust entries to take into account endogenous transitions coming from other occupation o into occupation $o^{'}$. This implies moving the columns of P^{o*} that were set to 0 because of transitions into $P^{*o^{'}}$. For all m and q in $\{1, \ldots, n_a n_\epsilon n_z\}$:

$$P^{*S}_{m,q-l_q} = P^{*S}_{m,q-l_q} + A^{*U}_{mq} \qquad \text{if } \chi^{US}(q) = 1$$
$$P^{*U}_{mq} = P^{*U}_{mq} + A^{*S}_{mq} \qquad \text{if } \chi^{SU}(q) = 1$$
$$P^{*U}_{mq} = P^{*U}_{mq} + A^{*E}_{mq} \qquad \text{if } \chi^{EU}(q) = 1$$
$$P^{*S}_{m,q-l_q} = P^{*S}_{m,q-l_q} + A^{*E}_{mq} \qquad \text{if } \chi^{ES}(q) = 1$$

where l_q maps the index of the agent after paying the l_k units of adjustment cost.

4. Finally as explained in Moll's example for stopping time (multiple assets with adjustment costs) the diagonal elements with transitions have to be adjusted:

$$P^{UU}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{US}(m) = 1$$

$$P^{SS}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{SU}(m) = 1$$

$$P^{EE}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{EU}(m) = 1 \quad \text{or} \quad \chi^{ES}(m) = 1$$

Moll says: "To see why the $^{-1}/_{\Delta}$ term shows up, consider the time-discretized process for g:

$$\dot{g}_t = P^T g_t \longrightarrow g_{t+\Delta t} = (\Delta P + I)^T g_t$$

where I is the identity matrix. The transition matrix $\tilde{P} = \Delta P + I$ needs to have all entries in the adjustment region $\tilde{C}_{mm} = 0$ and hence $\Delta P + I = 0$. Without the adjustment, the matrix P is singular.

The system to solve is:

$$P^T g = 0$$

A simple way to solve the system is to make one of the elements of g to be equal to an arbitrary number, and replace such row of P^T by a row of zeros with a one in the diagonal. Call this matrix $\tilde{P}^T$ and let $\tilde{\iota} = [0, \ldots, 0, 0.1, 0, \ldots, 0]^T$ then solve for:

$$\tilde{g} = \left[\tilde{P}^T\right]^{-1} \tilde{\iota}$$

Normalize $\tilde{g}$ so that it sums to 1: $\tilde{g} = {}^{\tilde{g}}/_{\text{sum}(\tilde{g})}$. Finally define g as:

$$g_i = \frac{\tilde{g}_i}{\Delta a_i}$$

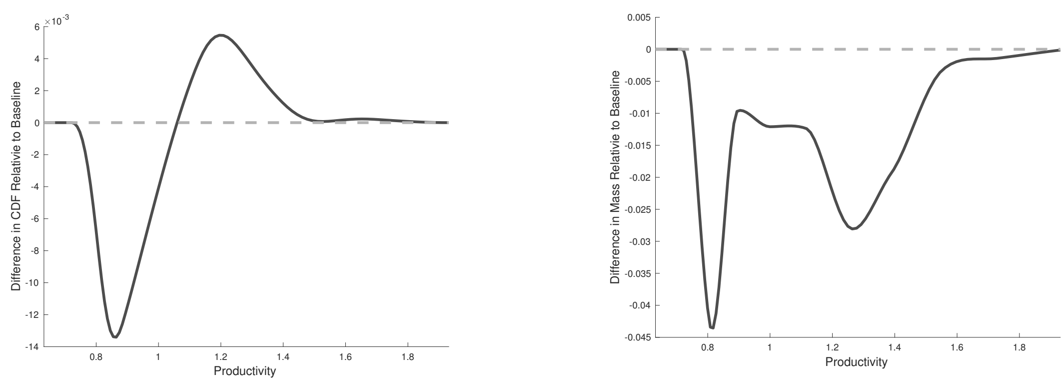
C Model: additional graphs and tables

C.1 Job-guarantee programs

We use the model to simulate the effects of a labor demand shock as described in Section 3. The model implies a reallocation of the workforce from self-employment into both higher employment and unemployment. The decrease in self-employment comes mostly from low-productivity agents; because of this, the productivity distribution improves relative to the baseline (in the first-order stochastic dominance sense). Figure C.1 makes this clear by showing the changes in the distribution of productivity among the self-employed. The mass of self-employment decreases throughout, Figure C.1b, but the decrease is concentrated at the bottom of the distribution.

Figure C.1: Productivity Changes after Labor Demand Shock

(a) Difference of Cumulative Distributions (b) Difference of Mass per $z-$type



Note: *The figures show changes in the distribution of productivity (z) between the equilibrium of the baseline model and the model with government labor demand. The left panel presents the difference in the CDF of productivity among the self-employed relative to the baseline model. Negative values indicate that the new distribution first order stochastically dominates the baseline distribution. The right panel presents the difference in the mass of self-employed agents for each $z-$type. Differences are due to changes in the distribution and overall mass of self-employed agents.*

C.2 Financial Sector Reform

We study the role of the collateral constraint in the model by increasing the value of λ. This change is qualitatively different from the policies we explore in Section 4, where we simulate the effects of expanding the credit limit of the self-employed by a fixed amount. Although both types of exercise loosen financial constraints, an increase in λ maintains the dependence of credit on the individual's asset holdings. First, we study the effects of a moderate reform that loosens the collateral constraint of the self-employed. Second, we compare these results with those following a complete elimination of credit frictions.

Credit Deepening We introduce a partial loosening of self-employed agents' collateral constraints. The new collateral constraint is

$$k \leq (\lambda + \lambda_{cd}) \cdot a, \tag{C.1}$$

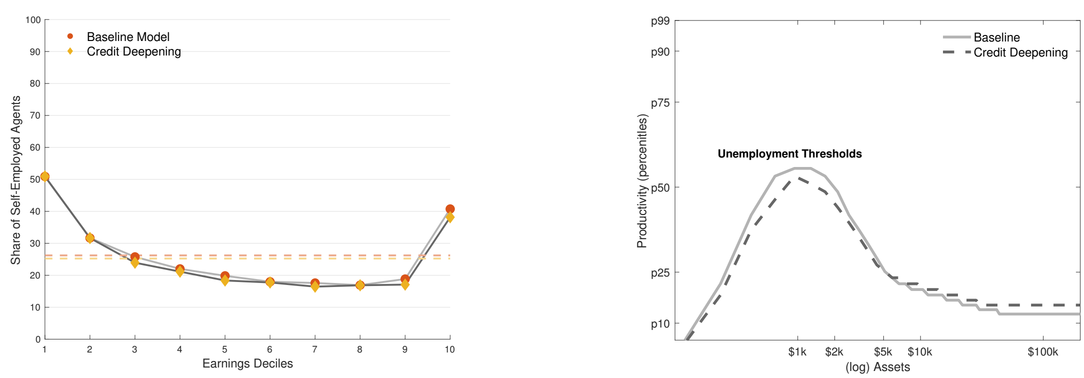
where λ_{cd} is set to make the constraint 20 percent less stringent than in the baseline.

We find that the effects of the reform are muted due to the reaction of prices in general equilibrium. In partial equilibrium, the loosening of the collateral constraint induces individuals to enter into self-employment.With fixed prices, the reform increases self-employment by 3.72 percentage points. The change is mostly matched by a decrease in employment of 3.34 percentage points. However, most of these occupational changes dissipatewhen the wage rate increases in order to clear the labor market. Figure C.2 shows that there are small changes in the distribution of the self-employed and the occupational choices of agents. The main differences are a slight increase in productivity threshold for wealthy individuals (explained by the increase in wages) and a slight decrease for poor individuals (explained by the availability of credit).

The final result of the reform is a decrease in the self-employment rate of around 0.98 percentage points, and an increase of the unemployment rate of 0.54 percentage points. These results are reminiscent of the channels highlighted by Feng, Lagakos, and Rauch (2020). As economies develop, unemployment is created as a byproduct. Because capital can be assigned more efficiently to high-productivity entrepreneurs, the reform increases TFP in 0.9 percent and is the main driver behind an increase in aggregate output of 1.1 percent.

Figure C.2: Model Response to Credit Deepening

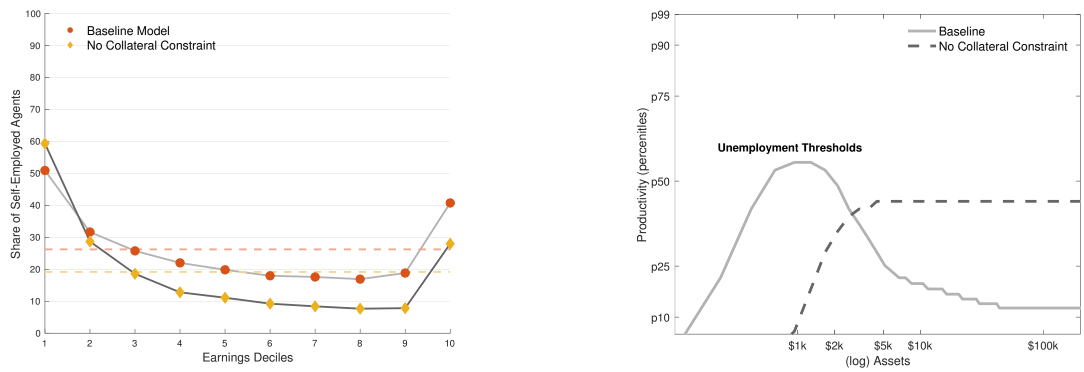
(a) General Equilibrium (b) Occupational Choice



Note: *The left panel reports the share of self-employed agents for each decile of the earnings distribution. The red circles correspond to the baseline model. The yellow squares correspond to the model with a credit-deepening reform. The right panel reports productivity thresholds characterizing the occupational choice of unemployed agents for the baseline model (continuous light gray line) and the model with credit deepening (dashed dark gray line). Lines depict the threshold value of productivity (z) for each level of assets (a). The agent chooses self-employment if productivity is above the threshold.*

Figure C.3: Model without Collateral Constraints

(a) Self-Employment Rate (b) Occupational Choice



Note: *Panel (a) reports the share of self-employed agents for each decile of the earnings distribution. The red circles correspond to the baseline model. The yellow squares correspond to the model without collateral constraints. Panel (b) reports productivity thresholds characterizing the occupational choice of unemployed agents for the baseline model (continuous light gray line) and the model without collateral constraint (dashed dark gray line). The agent chooses self-employment if productivity is above the threshold.*

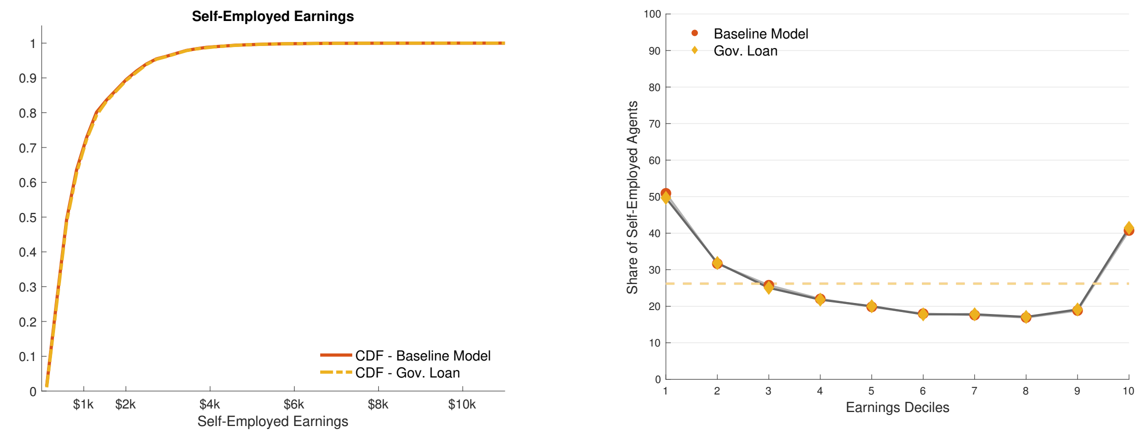
Elimination of Collateral Constraints ($\lambda_{cd} \to \infty$) Financial reform, even when taken to the limit of eliminating collateral constraints, does not solve the subsistence concerns that lead poor individuals to choose self-employment. Figure C.3b shows that for sufficiently rich agents, the reform makes selection into self-employment flat with respect to wealth. Nevertheless, the model still predicts an increasing profile of selection for sufficiently poor agents who face have subsistence concerns while unemployed. The elimination of collateral constraints does not do away with the liquidity constraints of unemployment agents, nor the labor frictions that prevent individuals from becoming employed. In this way, the u-shape pattern of self-employment along the earnings distribution is preserved as seen in Figure C.3a.

In the aggregate, financial sector reforms are beneficial to productivity and generate unemployment as a byproduct, as more individuals move from low-productivity self-employment activities into frictional labor markets. The self-employment rate goes down by 7 percentage points and unemployment goes up by 4.5 percentage points. Output increases by 32 percent, and TFP by 11 percent. In order to clear the labor market, the wage rate goes up by 19 percent.

The effects of financial reforms illustrate the importance of general equilibrium analysis. While the partial equilibrium effects of the reform point to a large increase in self-employment, the general equilibrium analysis shows those conclusions would be premature. As firms have better access to credit, productive firms grow and increase the overall productivity of the economy. This invariably leads to higher wages, which in turn affect agents' occupational choices. The net effect of these reforms is likely to be a better selection into self-employment, with fewer but more successful self-employed agents.

Figure C.4: Model Response to Increase in Credit in Partial Equilibrium

(a) CDF of Self-Employed Earnings (b) Self-Employment Rate



Note: *The left panel reports the CDF of the earnings of the self-employed in the model. The continuous red line corresponds to the baseline model. The dashed yellow line corresponds to the model with expanded credit access in partial equilibrium. The right panel reports the share of self-employed agents for each decile of the earnings distribution.The red circles correspond to the baseline model. The yellow squares correspond to the model with expanded credit access in partial equilibrium.*

C.3 Policy Analysis: Additional Results

We present here additional results for the analysis of micro-credit and transfers to unemployed and non-employed individuals.

Figure C.4 is the partial equilibrium counterpart of Figure 6. Both figures exhibit the same behavior, mostly slight changes with respect to the baseline model.

Table C.1 and Figure C.5 report the changes in model aggregates and the behavior of agents after the introduction of transfers to the non-employed (i.e., both unemployed and self-employed individuals receive the transfer). Transfers effectively alleviate subsistence concerns for the poorest agents as reflected in the increase in the threshold productivity for them to become self-employed (Figure C.5c). However, the policy also decreases overall savings by 1.9 percentage points. This decrease is concentrated on the unemployed, whose assets go down over 9 percent. The result is a shift of the distribution of agents towards poorer individuals. In the net self-employment increases by 0.36 percentage points, but aggregate productivity (TFP) falls 0.32 percent. The decrease in productivity is partly responding to larger distortions caused by the deaccumulation of assets. Nevertheless, the effects remain small at the micro level. See, for instance, the changes in the distribution of self-employment earnings in Figure C.5a.

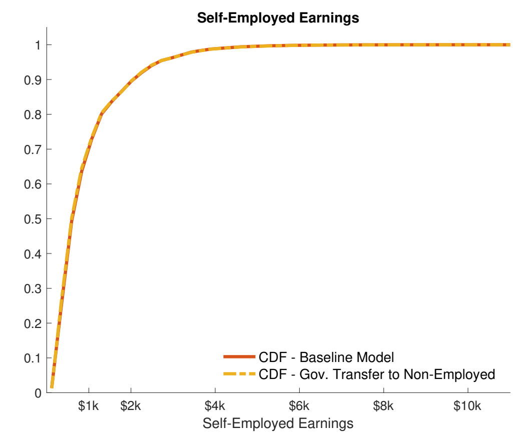
Variable	Δ	Variable	$\%\Delta$	Variable	$\%\Delta$
Employment	-0.22	TFP	-0.32	% Output	-0.04
Unemployment	-0.14	Income(E)	-0.01	Consumption	-0.61
Self-Employment	0.36	Income(SE)	-1.40	Assets	-1.90

Table C.1: Model Response to Transfers to the Non-Employed

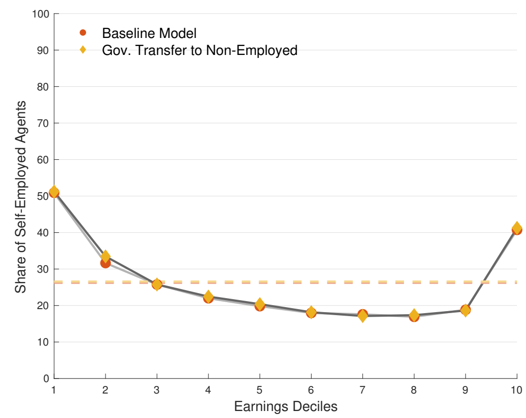
Note: *The table presents changes in variables with respect to the baseline model after the introduction of transfers to the unemployed. All numbers are in percentage points.*

Figure C.5: Model Response to Transfers to the Non-Employed

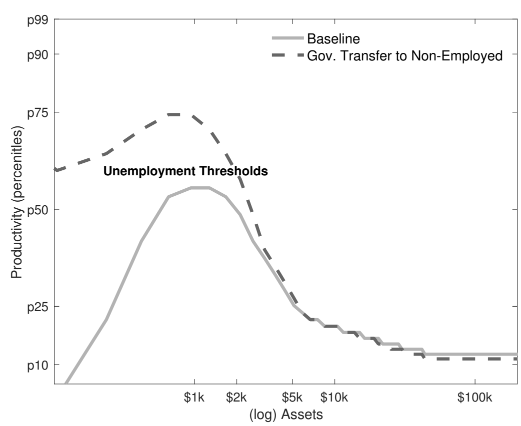
(a) CDF of Self-Employed Earnings



(b) Self-Employment Rate



(c) Occupational Choice



Note: *Panel (a) reports the CDF of the earnings of the self-employed in the model. The continuous red line corresponds to the baseline model. The dashed yellow line corresponds to the model with transfers to the non-employed in general equilibrium. Panel (b) reports the share of self-employed agents for each decile of the earnings distribution.The red circles correspond to the baseline model. The yellow squares correspond to the model with transfers to the non-employed in general equilibrium. Panel (c) reports productivity thresholds characterizing the occupational choice of unemployed agents for the baseline model (continuous light gray line) and the model with credit deepening (dashed dark gray line). Lines depict the threshold value of productivity* (z) *for each level of assets* (a)*. The agent chooses self-employment if productivity is above the threshold.*

C.4 Income-Targeted Cash Transfers

Providing cash transfers to low-income individuals is one of the most ubiquitous policies in the developing world. Transfers in low- and middle-income countries reach 11 percent of their population and amount to roughly 18 percent of the expenditures of recipient households (Banerjee, Niehaus, and Suri, 2019) .

To be consistent with these figures, we provide a transfer targeting the poorest 10% in terms of income in the baseline model. The transfer amounts to 15% of this group's consumption expenditures, or roughly 17% of the mean average labor income in the economy. Importantly, the targeting is independent of the individual's occupation or other variables such as wealth.

Targeted cash transfers in partial equilibrium Unlike transfers targeted to the unemployed, transfers given to the poor, irrespective of their occupation, lead to less productive self-employed agents. Figure C.6a shows that the distribution of productivity worsens (in the FOSD sense) after the introduction of the transfers. This can seem surprising, given the change in the productivity threshold that characterizes the occupational choice of the unemployed (Figure C.6b). Self-employment becomes less attractive for the poorest among the unemployed, but it also becomes more attractive for relatively wealthier agents with low productivity. At the same time, the transfers induce agents to save less. In fact, asset holdings go down 35 percent in the economy—39 percent among the self-employed. Daruich and Fernandez (2020) found similar effects on savings when analyzing the effects of Universal Basic Income. The combination of these two effects leads to a pool of self-employed agents that are less productive and less wealthy.

The consequences of the change in selection and savings are made apparent by the distribution of self-employment across the earnings distribution. Figure C.7a shows that the reduction in self-employment (3.8 percentage points) is concentrated among high earners, with a smaller decline among the bottom 10 percent of earners.

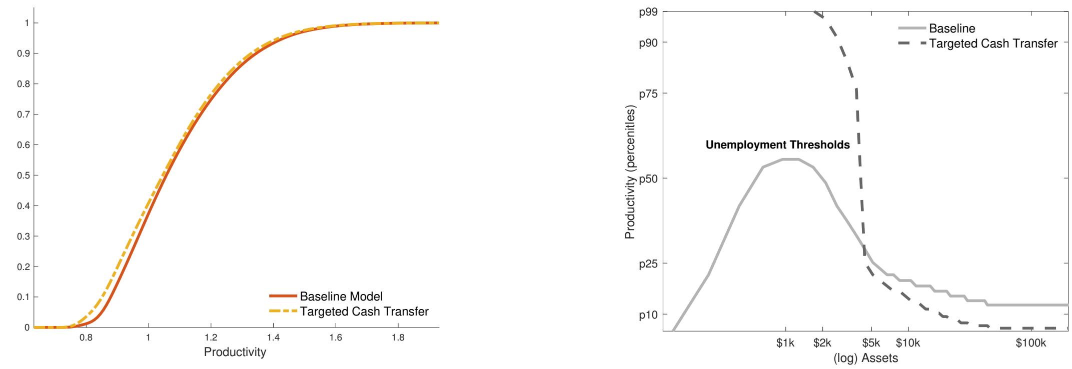
Targeted cash transfers in general equilibrium The fall in self-employment, particularly among top earners, is undone in general equilibrium. Figure C.7b makes this clear. As for the case of transfers to the unemployed, these effects are driven by lower wages and higher labor income taxes required to finance the transfers (after tax wages decrease 3.76 percent). Both changes simultaneously make unemployment/employment less attractive and make self-employment more profitable for highly productive agents.

In equilibrium, the introduction of targeted cash transfers incentivizes self-employment (which goes up 1.15 percentage points), at the expense of salaried employment (which goes down 1.04 percentage points). These changes are consistent with cross-sectional results by Bianchi and Bobba (2013), who compared treated and control regions after the enactment of *Progresa*, a large conditional cash transfer

Figure C.6: Model Response to Targeted Cash Transfers in Partial Equilibrium

(a) CDF of Productivity

(b) Occupational Choice



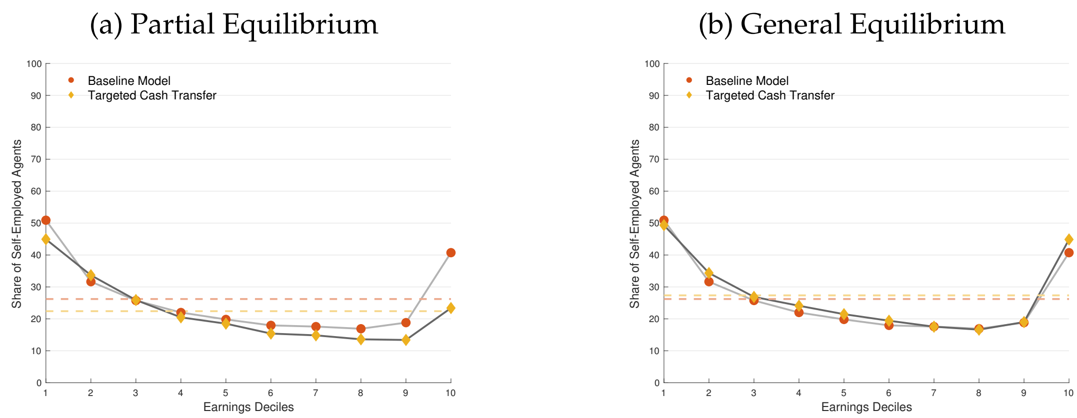
Note: *The left panel reports the CDF of productivity among self-employed agents. The continuous red line corresponds to the baseline model. The dashed yellow line corresponds to the model with targeted cash transfers in partial equilibrium. The right panel reports productivity thresholds characterizing the occupational choice of unemployed agents for the baseline model (continuous light gray line) and the model with targeted cash transfers (dashed dark gray line). Lines depict the threshold value of productivity* (z) *for each level of assets* (a)*. The agent chooses self-employment if productivity is above the threshold.*

program in Mexico.

Despite reversing some of the changes to self-employment, the worsening of the productivity distribution among the self-employed is not undone in general equilibrium. On top of this, the decrease in asset holdings among the self-employed makes them more constrained, further reducing productivity (see equation 14). This is reflected in a fall of aggregate TFP by 1.15 percent.

Thus, in the presence of subsistence self-employment, across-the-board income transfers entail general equilibrium effects that reverse most of the positive effects in partial equilibrium. Once we account for the behavior of wages to clear the labor market, self-employment increases and total factor productivity falls.

Figure C.7: The Self-Employment Rate and Targeted Cash Transfers



Note: *The figure reports the share of self-employed agents for each decile of the earnings distribution. The red circles correspond to the baseline model. The yellow squares in the left panel correspond to the model with targeted cash transfers in partial equilibrium. The yellow squares in the right panel correspond to the model with targeted cash transfers in general equilibrium.*

D Additional Reduced Form Analysis

D.1 Workforce composition

Table D.1 shows the composition of the labor force in our sample and contrasts it with the composition of the whole Mexican economy and the U.S. over the period 2005-2019. Our sample behaves in a similar way to the overall Mexican labor force, while the differences between Mexico and the U.S. are apparent, particularly in the share of self-employed individuals.

Labor Status	Our Sample	General Population	U.S.
Worker	69.1%	57.9%	80.7%
Unemployed	4.4%	3.9%	6.3%
Self Employed	26.7%	38.1%	12.9%

Table D.1: Workforce Composition

Note: *The data for the Mexican general population are taken from the world development indicators (WDI). The data for the U.S. are taken from the current population survey (CPS).*

D.2 Mobility across occupations

In what follows we dig deeper into how the labor market status of an individual affects transitions. To do so, we follow the same strategy as Katz and Krueger (2017) in their study of alternative work arrangements in the U.S.. The first question we ask is whether individuals who are unemployed are more likely to transition into self-employed. To

	(1) SE	(2) SE	(3) SE	(4) SE
U_{t-1}	0.209***	0.209***	0.208***	0.208***
	[0.003]	[0.003]	[0.003]	[0.003]
S_{t-1}	0.717***	0.717***	0.706***	0.706***
	[0.012]	[0.012]	[0.012]	[0.012]
Age			0.002***	0.002***
			[0.000]	[0.000]
Constant	0.080***	0.109***	-0.027	-0.038
	[0.005]	[0.005]	[395.990]	[167.520]
Observations	1033397	1033397	1033397	1033397
Mean Ent	0.285	0.285	0.285	0.285
Schooling Controls	No	No	Yes	Yes
City Fixed Effect	No	No	No	Yes
Time Fixed Effect	No	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes

Table D.2: Transitions to Self-Employment

Note: *The LHS variable is an indicator variable that takes the value of one if the individual is self-employed and zero if the individual is employed. U_{t-1} and S_{t-1} are indicator variables that take the value of 1 if the individual was unemployed or self employed in the previous quarter respectively. Age is the age in years. Standard errors are clustered at the city level. Schooling controls are a set of dummies by education level to control non-parametrically for education. Time fixed effects are at the year-quarter level. The sample consists of individuals who are either employed or self-employed in period t. We run the regressions by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

answer this we focus on the universe of individuals who are either employed or self-employed in period t, and check whether the transitions to self-employment are larger for those agents who were unemployed in the previous period.

This exercise differs from the conditional transition rates reported in Table 3. The regression in this section allows us to control for the observable characteristics of individuals, comparing transition rates across similar individuals in terms of age, education and location, instead of computing transitions among individuals with the same labor market status (e.g. unemployed). We take this evidence as only suggestive of the mechanisms we study.

Table D.2 reports the regression results. The transition rates of unemployed agents to self-employment are 20.9 percentage points higher than those exhibited by observationally comparable agents who had a salaried job. This result holds after controlling for age, education, and after adding time and city fixed-effects. While we are not able to control for all the relevant factors affecting transitions into self-employment, we interpret the higher transition rate from unemployment as suggestive of the role of self-employment as an outside option for individuals who need an income source, as opposed to self-employment representing entrepreneurial activities for which the individual is better suited (relative to working in a salaried job). Our results align with those of Katz and Krueger (2017) for the U.S.. They find that unemployed individuals are more likely to transition to an alternative work arrangement job than agents who are employed.

	(1) E	(2) E	(3) E	(4) E	(5) E
S_{t-1}	-0.268***	-0.268***	-0.255***	-0.254***	-0.340***
	[0.021]	[0.021]	[0.020]	[0.020]	[0.014]
Age			-0.006***	-0.006***	-0.004***
			[0.000]	[0.000]	[0.000]
Second Earner					0.022
					[0.018]
$S_{t-1}\times$ Second Earner					0.024**
					[0.011]
Constant	0.463***	0.417***	0.704***	0.684***	0.589
	[0.015]	[0.014]	[0.103]	[0.111]	[1203.540]
Observations	327250	327250	327250	327250	145945
Mean Emp	0.221	0.221	0.221	0.221	0.221
Schooling Controls	No	No	Yes	Yes	Yes
Time Fixed Effect	No	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes	Yes

Table D.3: Transitions to Employment

Note: *The LHS variable is an indicator variable that takes the value of one if the individual is employed in period t. S_{t-1} and Second Earner are indicator variables that take the value of 1 if the individual was self-employed and if the individual's couple was an income earner in the previous quarter respectively. Age is the age in years. Standard errors are clustered at the city level. Schooling controls are a set of dummies by education level to control non-parametrically for education. Time fixed effects are at the year-quarter level. The sample consists of individuals who were either unemployed or self-employed in period $t-1$. We run the regressions by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

The second question we ask is whether individuals who become self-employed transition less to salaried jobs. The effect of self-employment on the transitions into employment matters to determine how persistent the effects of individual occupational choices are, and how policies that affect those choices affect in turn the labor market. To answer this question we focus on the universe of agents who are either unemployed or self-employed in period $t-1$, and follow them to determine whether or not they become employed. As before this allows us to compare the transition rates of self-employed individuals with comparable unemployed individuals.

Table D.3 presents the regression results. We do in fact find that self-employed individuals are 34 percentage points less likely to transition to employment than comparable unemployed individuals. Even if the actual effect of self-employment is not as large, this estimate indicates that opting into self-employment can have long-lasting implications for an individual, particularly for low-productivity self-employed who are now less likely to abandon this state and move to employment. Jackson (2020) finds similar results studying self-employed workers in the United States.

An important caveat for the results in Table D.3 is that we are not controlling for selection into self-employment on the basis of entrepreneurial ability or preference for self-employment. We also lack the full occupational history of individuals, so we cannot condition on the attachment to self-employment of each individual. However, we can partially address some of these shortcomings by focusing on individuals who were unemployed to begin with (in period $t-2$), and who then either remain unemployed or transition to self-employment (in period $t-1$). This lets us compare individuals that start in a common state, and who then differ on whether or not they transition through self-employment. When we perform this analysis the same result emerges, with individuals who become self-employed being 14.4 percentage points less likely to transition to employment in period t. The regression results can be found in Table D.4.

	(1)	(2)	(3)	(4)	(5)
	E	E	E	E	E
S_{t-1}	-0.066***	-0.069***	-0.084***	-0.086***	-0.144***
	[0.014]	[0.014]	[0.017]	[0.017]	[0.050]
Age			-0.009***	-0.009***	-0.007***
			[0.001]	[0.001]	[0.001]
Second Earner					0.035
					[0.057]
$S_{t-1}\times$ Second Earner					-0.024
					[0.058]
Constant	0.383***	0.456***	0.854***	0.863***	1.327***
	[0.017]	[0.064]	[0.226]	[0.228]	[0.075]
Observations	7320	7320	7320	7320	3205
Mean Emp	0.355	0.355	0.355	0.355	0.355
Schooling Controls	No	No	Yes	Yes	Yes
Time Fixed Effect	No	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes	Yes

Table D.4: Transitions to Employment from Unemployment

Note: *The LHS variable is an indicator variable that takes the value of one if the individual is employed in period t. S_{t-1} and Second Earner are indicator variables that take the value of 1 if the individual was self-employed and if the individual's couple was an income earner in the previous quarter respectively. Age is the age in years. Standard errors are clustered at the city level. Schooling controls are a set of dummies by education level to control non-parametrically for education. Time fixed effects are at the year-quarter level. The sample consists of individuals who were unemployed in period $t-2$, and were not employed in period $t-1$. We run the regressions by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

D.3 Constrained agents transition more into self-employment

We now ask whether the transitions to self-employment we observe are consistent with the idea that individuals who are more resource-constrained are more likely to become self-employed. We focus on this mechanism for two main reasons. First, individuals who choose self-employment out of necessity do so based less on their own preferences for self-employment or entrepreneurial ability,[19] and more on the insurance properties of self-employment, producing a larger share of unproductive small enterprises and

[19] Hurst and Pugsley (2016) proposed non-pecuniary benefits from self-employment to explain the patterns of self-employment in the U.S. This taste for self-employment complements standard arguments for selection into self-employment based on own-perceptions (correct or not) of entrepreneurial ability, or other idiosyncratic traits like a low job-finding rate.

reducing aggregate productivity (Figure 2). Second, the decision to become self-employed out of necessity can be influenced by policy more directly than when the decision is dictated by the preferences or ability of the individual.

We use two variables to proxy for access to additional sources of income: the presence of a second earner in the household (as in Chetty (2008)), and the receipt of remittances from relatives living abroad. We compare transitions from unemployment for constrained and unconstrained individuals. Table D.5 show the results. Individuals with additional sources of income transit less into self-employment.[20]

Turning to the results in Table D.5a, we estimate that individuals in a household with a second earner have a 3.2 percentage point higher probability of transitioning to employment and a 3.9 percentage point lower probability of transitioning to self-employment (a 17 percent decrease, from 22.2 to 18.3 percent) than individuals without a second earner. We also test whether the individual's (self-reported) job search activity changes with the presence of a second earner (Table D.6). Most job-search activities (e.g., examining job postings, looking for a temporary job) are not significantly different for individuals with a second earner. Importantly, there is no difference in whether agents report being making plans to start their own business.

The same results are backed by the exercise using remittances as a proxy for the resources of the individual. Individuals who receive remittances in times of unemployment transition at a lower rate to self-employment (10.8 percent vs 18.8 percent, Table D.5b). We also check whether individuals receiving remittances differ on search activities or the intent to start a business. Receiving remittances is associated with a lower likelihood of using the internet to look for a job, and a lower likelihood of of searching for temporary employment; however, it is also associated with a higher likelihood of asking directly for a job (Table D.7). Of course, we cannot rule out other sources of bias as the receipt of remittances is not created by exogenous variation across individuals.

Finally, we repeat the exercise on the transition rates out of self-employment. We find that less-constrained individuals are more likely to transition out of self-employment. Self-employed agents are on average 2.4 percentage points more likely to transition to employment when they are able to rely on a second earner. Unlike less-constrained self-employed, those constrained cannot devote time/effort to search activities that facilitate the transition to a salaried job. We report these results in column 5 of Tables D.3 and D.4.

[20]The results can also be interpreted as indicating that preferences and ability are not the main drivers of the transition to self-employment. If they were, we should observe that people with external income sources transit more to self-employment, as they could enjoy the non-pecuniary benefits of work independence or try their luck at entrepreneurship, while not worrying (as much) about low income levels.

	(1)	(2)	(3)	(4)
	U→E	U→S	U→U	U→I
Second Earner	0.032***	-0.039**	0.007	-0.000
	[0.010]	[0.018]	[0.015]	[0.000]
Age	-0.008***	0.003***	0.005***	0.000
	[0.000]	[0.000]	[0.000]	[0.000]
Constant	0.835***	0.209	-0.044	-0.001
	[0.301]	[0.326]	[0.098]	[0.002]
Observations	8376	8376	8376	8376
Mean Dep. Variable	0.505	0.222	0.272	0.000104
Schooling Controls	Yes	Yes	Yes	Yes
Time Fixed Effect	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes

(a) Second Earner and Transitions from Unemployment

	(1)	(2)	(3)	(4)	(5)
	U→E	U→S	U→U	U→I	U→S
Remittances	0.058	-0.080***	-0.033	0.055	
	[0.053]	[0.021]	[0.040]	[0.037]	
Age	-0.012***	0.002***	0.002**	0.008***	0.001***
	[0.000]	[0.000]	[0.001]	[0.001]	[0.000]
Latent Remittances					-0.045
					[0.036]
Constant	1.237***	0.147	-0.168***	-0.216	0.177
	[0.262]	[0.202]	[0.050]	[0.227]	[0.114]
Observations	8615	8615	8615	8615	25135
Mean Dep. Variable	0.463	0.188	0.256	0.0932	0.188
Schooling Controls	Yes	Yes	Yes	Yes	Yes
Time Fixed Effect	Yes	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes	Yes

(b) Remittances and Transitions from Unemployment

Table D.5: Transitions from Unemployment

Note: *In both panels, the LHS variable is an indicator variable that takes the value of one if the individual experienced the transition specified in each column. U denotes unemployment, E salaried work, S self-employment, and I inactivity. Second Earner is an indicator variable that takes the value of one if the individual's partner was an income earner in period $t-1$. Remittances is an indicator variable that takes the value of one if the individual reported having received remittances in period $t-1$. Latent Remittances is a dummy equal to one if we observe the individual to ever receive remittances. Standard errors are clustered at the city level. Schooling controls are a set of dummies by education level to control for education in a non parametric manner. Time fixed effects are at the year-quarter level. The sample consists of individuals who were unemployed in period $t-1$. The regressions are run by weighted OLS. *, **, and *** denote significance at the 10%, 5%, and 1% level.*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Asked	Job Post	Public Ag	Temp	SE Plans	Internet	Newspaper	Need to Work	Age
Second Earner	-0.033	-0.009	-0.006	-0.002	0.002	-0.058***	-0.022	0.000	1.564***
	[0.023]	[0.010]	[0.008]	[0.003]	[0.001]	[0.019]	[0.017]	[0.000]	[0.528]
Constant	0.203***	0.021**	0.020**	0.008***	0.001	0.105***	0.069***	0.000	41.063***
	[0.022]	[0.009]	[0.008]	[0.002]	[0.001]	[0.018]	[0.016]	[.]	[0.497]
Observations	11214	11214	11214	11214	11214	11214	11214	11214	11214

Table D.6: Second Earner and Job-Search Activities

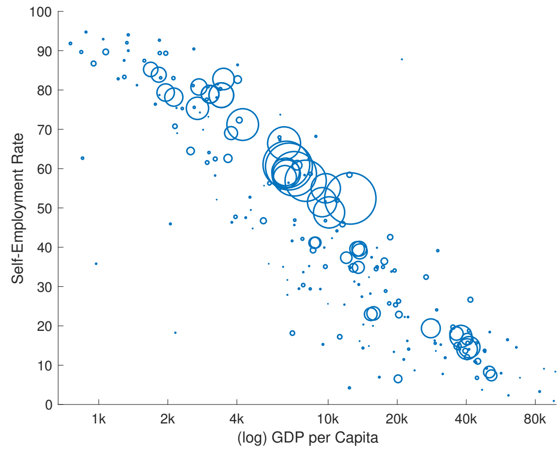
Note: *The LHS variable is an indicator variable that takes the value of one if individual i performed the given activity to search for a job in the previous quarter. The last two columns correspond to weather or not the individual declares to have a need to work, and differences in age. Second Earner is an indicator variable that takes the value of one if the individual's couple was an income earner in period $t-1$. Standard errors are clustered at the city level. All regressions include schooling controls (a set of dummies by education level to control non-parametrically for education), and time fixed effects at the year-quarter level. The sample consists of individuals who were unemployed in period $t-1$. The regressions are run by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

	(1) Asked	(2) Job Posting	(3) Public Ag.	(4) Temp	(5) SE Plans	(6) Internet	(7) Newspaper	(8) Need to Work	(9) Age
Remittances	0.183*	0.023	0.004	-0.007***	-0.004**	-0.035***	0.092	0.045	-0.250
	[0.103]	[0.023]	[0.012]	[0.002]	[0.002]	[0.004]	[0.089]	[0.051]	[2.385]
Constant	0.157***	0.007***	0.008***	0.007***	0.004**	0.035***	0.040***	0.043***	43.766***
	[0.007]	[0.002]	[0.002]	[0.002]	[0.002]	[0.004]	[0.004]	[0.004]	[0.212]
Observations	8200	8200	8200	8200	8200	8200	8200	8200	8200

Table D.7: Remittances and Job Search Activities

Note: *The LHS variable is an indicator variable that takes the value of one if individual i performed the given activity to search for a job in the previous quarter. The last two columns correspond to weather or not the individual declares to have a need to work, and differences in age. Remittances is an indicator variable that takes the value of one if the individual reported having received remittances in period $t-1$. Standard errors are clustered at the city level. All regressions include schooling controls (a set of dummies by education level to control non-parametrically for education), and time fixed effects at the year-quarter level. The sample consists of individuals who were unemployed in period $t-1$. The regressions are run by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

Figure D.1: Self-Employment Rate Across Countries



Note: *The figure shows self-employment rates for countries around the world, as a function of their GDP per capita. The size of each bubble illustrates the population size of each country. Data source: World Development Indicators of the World Bank.*

D.4 Self-employment around the world

In this appendix we report additional results on the prevalence of self-employment around the world. As Figure D.1 shows, self-employment decreases consistently as countries' income rises. Moreover, self-employment tends to be concentrated in the bottom and top of the earnings distribution within each country. We reproduce the exercise behind Figure 1 with data from 10 countries around the world We first run a regression of the form $\log(earn_{i,t}) = \alpha + \gamma_t + \beta X_{i,t} + \eta_{i,t}$, where $earn_{i,t}$ corresponds to the earnings of individual i at time t, and X is a vector of individual-level controls. We rank $\hat{\eta}_{i,t}$ and classify them in bins of 2 percent of the sample, then compute the self-employment rate in each bins. In all cases we use harmonized household surveys.

Figure D.2 presents the results. The u-shape relationship between self-employment and earnings with a skewed presence in the bottom of the earnings distribution is common across countries, with the exception of Azerbaijan.

Finally, there is the concern that misreporting of income correlated with occupation is behind this pattern. To address this, we show in Figure D.3 that the same pattern of concentration at the bottom is present when comparing self-employment across the consumption distribution. Self-employment is concentrated among those who consume less.

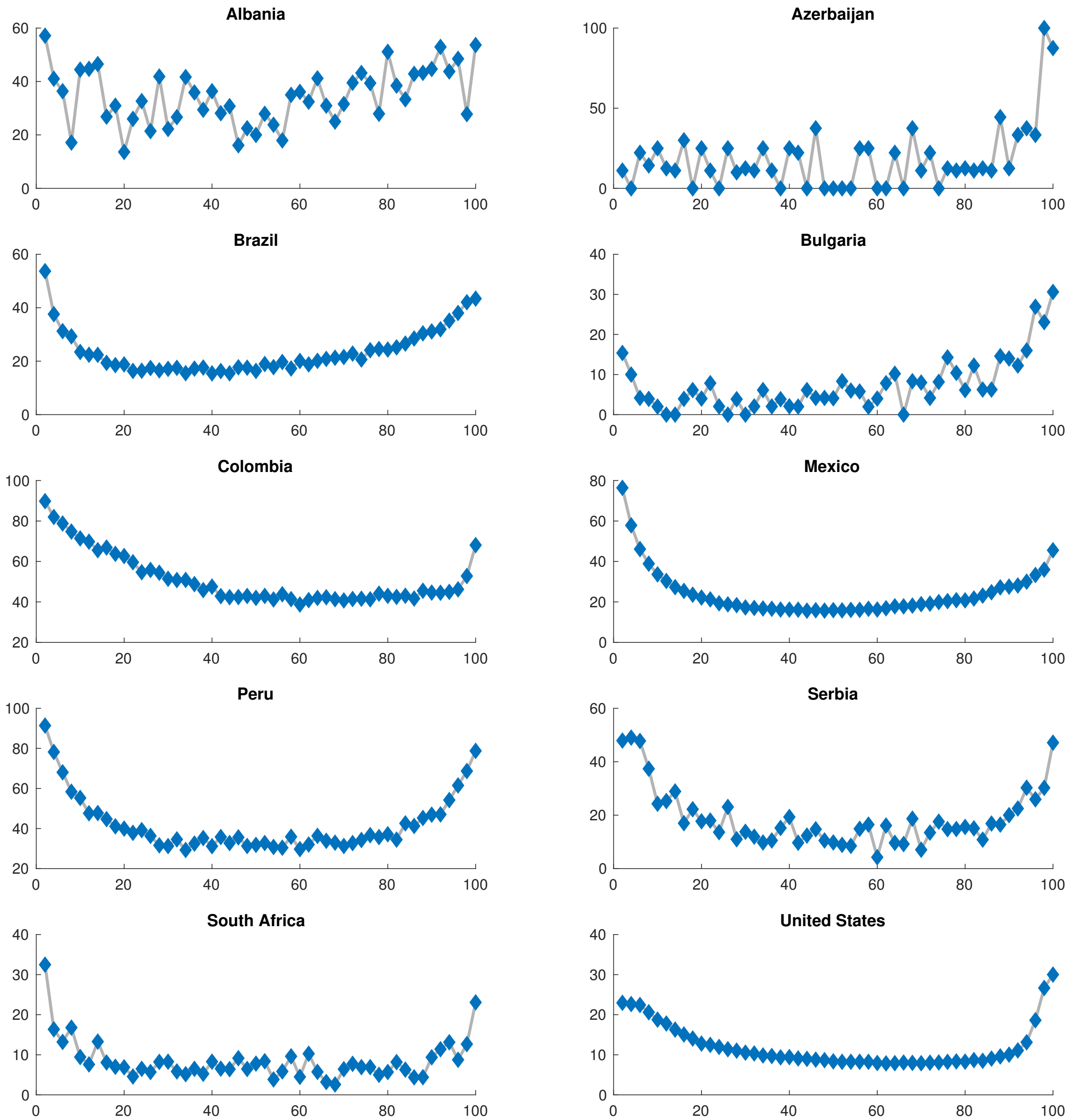
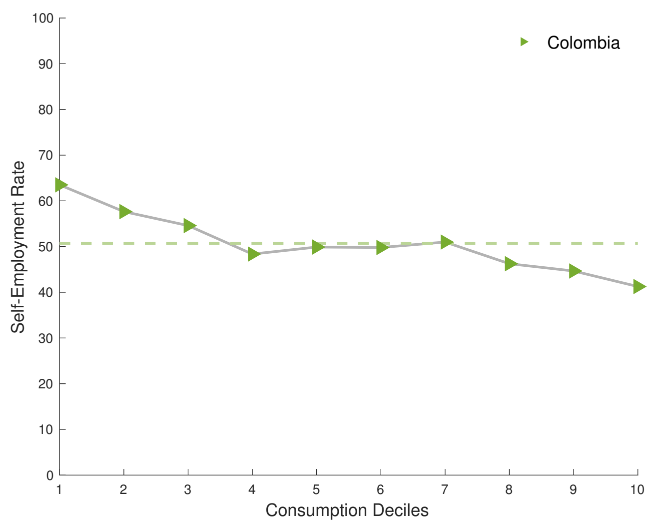


Figure D.2: Self-Employment and Earnings Across Countries

Note: *The figure reports the share of the population classified as self-employed for bins of the earnings distribution for ten countries around the world. Each bin corresponds to two percent of the population. The horizontal dashed lines correspond to the average self-employment rate in each country.*

Figure D.3: SE by Consumption Distribution



Note: *The figure reports the share of the population classified as self-employed for bins of the consumption distribution for Colombia. Each bin corresponds to ten percent of the population. The horizontal dashed lines correspond to the average self-employment rate.*