Taxing Wealth and Capital Income when Returns are Heterogeneous

Guvenen, Kambourov, Kuruscu, Ocampo

April, 2023

Taxing Capital

What is the optimal tax combination on capital income *(flow)* and wealth *(stock)* when returns are heterogeneous?

- Capital income tax: $a_{\text{after-tax}} = a + (1 - \tau_k) \cdot ra$
- Wealth tax: $a_{\text{after-tax}} = (1 - \tau_a) \cdot a + ra$

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- **Our earlier work:** Quantitative analysis of optimal capital income **vs.** wealth tax
  - **Rich OLG model** that matches both
    (i) distribution of returns **(ii)** extreme concentration and tail of wealth distribution
  - **Find:** Large efficiency and welfare gains from wealth tax

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  - **Find:** Large efficiency and welfare gains from wealth tax
- **This paper:** Theoretical analysis of optimal combination of taxes
  - **Analytical** model entrepreneurs and workers
  - **Find:** conditions for **(i)** efficiency gains **(ii)** welfare gains *(ind.+overall)* **(iii)** optimal taxes

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3. **Practical:** Wealth taxation has been used by governments
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4. **Theoretical:** Interesting **new economic mechanisms**. Example next.
   (Allais, 1977; Piketty, 2014; Guvenen, Kambourov, Kuruscu, Ocampo, and Chen, 2023)

Return Heterogeneity: A Simple Example

- One-period model.
- Government taxes to finance $G = \$50M$.
- Two brothers, Fredo and Mike, each with \$1B of wealth.

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- Government taxes to finance $G = \$50M$.
- Two brothers, Fredo and Mike, each with $1B of wealth.
- **Key heterogeneity**: investment/entrepreneurial ability.
  - (Fredo) Low ability: earns $r_f = 0\%$ rate of return.
  - (Mike) High ability: earns $r_m = 20\%$ rate of return.

Capital Income (τ_k) vs. Wealth Tax (τ_a)

	Capital income tax		Wealth tax
	$a_{i,\text{after-tax}} = a_i + (1-\tau_k) r_i a_i$		
	Fredo $(r_f = 0\%)$	Mike $(r_m = 20\%)$	
Wealth	\$1B	\$1B	
Before-tax Income	**0**	**\$200**M	
	$\tau_k = 25\% \left(= \frac{50}{\mathbf{200}}\right)$		
Tax liability			
After-tax return			
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- Market value internalizes future returns, taxing it weakens use it or lose it effect.

Theoretical Results: preview

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3. **Optimal Taxes:** Utilitarian welfare maximizing taxes depend on the pass-through of productivity to wages *(in model given by elasticity of output to capital, α)*

   - If pass-through (α) is sufficiently high $\longrightarrow \tau_a^* > 0$ & $\tau_k^* < 0$
   - If pass-through (α) is sufficiently low $\longrightarrow \tau_a^* < 0$ & $\tau_k^* > 0$
   - If pass-through (α) is in between $\longrightarrow \tau_a^* > 0$ & $\tau_k^* > 0$.

Theoretical Results: extensions

- Corporate sector with no borrowing constraint
- Rents: Return $\neq$ marginal productivity
- Entrepreneurial effort in production
- Perpetual-youth model with stationary wealth distribution

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1. **Model Description**
2. Efficiency gains from wealth taxation
3. Welfare gains from wealth taxation
4. Optimal taxation
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6. Quantitative Analysis

Theoretical Model

Two groups of infinitely-lived agents:

1. Homogenous **workers** (size L)
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- Workers' and entrepreneurs' preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t) \qquad \text{where } \beta < 1.$$

Theoretical Model

- Entrepreneurs' technology:

$$y = (zk)^{\alpha} n^{1-\alpha}$$

  - $z \in \{z_\ell, z_h\}$, where $z_h > z_\ell \geq 0$ with a transition matrix

$$\mathbb{P} = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix} \text{ with } 0 < p < 1.$$

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  - Autocorrelation is critical: $\rho = 2p - 1 > 0 \longleftrightarrow p > 1/2$.
- Aggregate output:
$$Y = \int (zk)^{\alpha} n^{1-\alpha}$$
- Government finances exogenous expenditure G with τ_k and τ_a
  - τ_a on beginning-of-period wealth

Financial Markets & Entrepreneurs' Problem

Financial markets:

- Collateral constraint ($\lambda \geq 1$): $k \leq \lambda a$, where a is entrepreneur's wealth.
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Entrepreneurs' Production Decision:

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$$\Pi^\star(z,a) = \max_{\mathbf{k} \leq \lambda \mathbf{a}, n} (zk)^\alpha n^{1-\alpha} - rk - wn$$

Solution: $$\Pi^\star(z,a) = \underbrace{\pi^\star(z)}_{\text{Excess return above } r} \times a$$

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Entrepreneurs' Dynamic Problem:

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- Letting $R_i \equiv (1-\tau_a) + (1-\tau_k)(r + \pi^\star(z_i))$ for $i \in \{l, h\}$,

  the savings decision (CRS + Log Utility):

$$a' = \beta R_i a \quad \longrightarrow \text{linearity allows aggregation}$$

Equilibrium Values: Aggregation

Lemma: Aggregate output is

$$Y = (ZK)^{\alpha} L^{1-\alpha} \qquad (Z^{\alpha} \text{ is measured TFP})$$

where

$$K \equiv A_h + A_\ell \qquad K = \text{Aggregate capital}$$

$$Z \equiv s_h z_\lambda + (1 - s_h) z_\ell \qquad Z = \text{Wealth-weighted productivity}$$

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Key variables:

- $s_h = \frac{A_h}{K}$: wealth share of high-productivity entrepreneurs.
- $z_\lambda \equiv z_h + (\lambda - 1)(z_h - z_\ell)$: effective productivity of high-productivity entrepreneurs.

Use it or lose it effect increases efficiency if $s_h \uparrow (\longrightarrow Z \uparrow)$

Evolution of Aggregates

$$A_h' = \underbrace{p\beta R_h A_h}_{\text{stayers' savings}} + \underbrace{(1-p)\,\beta R_l A_l}_{\text{switchers' savings}}$$

A_h: High type wealth

$$A_l' = \underbrace{p\beta R_l A_l}_{\text{stayers' savings}} + \underbrace{(1-p)\,\beta R_h A_h}_{\text{switchers' savings}}$$

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$$K' = \beta \underbrace{\left[(1-\tau_a)\,K + (1-\tau_k)\,\alpha\,(ZK)^{\alpha}\,L^{1-\alpha}\right]}_{\text{Agg. after tax returns}}$$

K: Agg. capital/wealth

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    - $(\lambda - 1) A_h < A_l$: low-type entrepreneurs bid down interest rate: $r = \text{MPK}(z_l)$.
    - **Unique steady state** with:
        - return heterogeneity, misallocation of capital, wealth tax $\neq$ capital income tax.
    - **Empirically relevant:** $R_h > R_l$ and $\frac{\text{Debt}}{\text{GDP}} \gg 1.5$ when $\lambda = \lambda^{\star}$.

Debt-GDP

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Steady State: 2 equations 2 unknowns

Using the law of motion for A_l and A_h we obtain two steady state equations:

Steady State K

$$(1-\tau_k)\overbrace{\alpha Z^{\alpha}\left(K/L\right)^{\alpha-1}}^{\text{MPK}}-\tau_a=\frac{1}{\beta}-1.$$

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Steady State Z (depends on only τ_a!)

How τ_k disappears | graph

$$h\left(Z\right)=\left(1-\rho\beta\left(1-\tau_a\right)\right)Z^2-\frac{z_l+z_\lambda}{2}\left(1+\rho-2\rho\beta\left(1-\tau_a\right)\right)Z+z_l z_\lambda\rho\left(1-\beta\left(1-\tau_a\right)\right)=0.$$

- Simple graphical representation and analysis of the steady state!

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Main Result 1: Efficiency Gains from Wealth Taxation

Proposition:

Graph | $\bar{\tau}_a$ graph

For all $\tau_a < \bar{\tau}_a$ ($\longleftrightarrow \lambda < \lambda^*$), a marginal increase in τ_a **increases steady state Z iff** entrepreneurial productivity is autocorrelated, $\rho > 0$ ($p > 1/2$)

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Note: An increase in τ_a increases productivity, but result says nothing about aggregates!

- Results for aggregates depend on government's budget *(more on this in 2 slides)*

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Proposition:

Graph $\bar{\tau}_a$ graph

For all $\tau_a < \bar{\tau}_a$ ($\longleftrightarrow \lambda < \lambda^*$), a marginal increase in τ_a **increases steady state Z iff** entrepreneurial productivity is autocorrelated, $\rho > 0$ ($p > 1/2$)

Note: An increase in τ_a increases productivity, but result says nothing about aggregates!

- Results for aggregates depend on government's budget *(more on this in 2 slides)*

Why does productivity increase?

- It must be that wealth concentration increases: $s_h \uparrow$ ($Z \uparrow = s_h z_\lambda + (1 - s_h) z_\ell$)
- Wealth shares depend (only) on returns: how do wealth taxes affect returns?

Lemma (Use-it-or-Lose-it):

For all $\tau_a < \overline{\tau}_a$, a marginal increase in wealth taxes increases entrepreneurial returns that are above the wealth-weighted average return and vice versa.

That is, for any z, ${}^{dR(z)}/_{d\tau_a} \geq 0$ if and only if $z \geq Z = (s_h z_\lambda + (1 - s_h) z_\ell)$ and $\rho > 0$.

Taxes and returns: The use-it-or-lose-it effect

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Implications:

1. Dispersion of after-tax returns rises with τ_a:

G.E.

$$\frac{dR_\ell}{d\tau_a} = \underbrace{\left(\frac{z_\ell - Z}{Z}\right)}_{\text{use-it-lose-it}<0} \underbrace{-\left(\frac{1}{\beta} - (1 - \tau_a)\right)\frac{z_\ell}{Z^2}\frac{dZ}{d\tau_a}}_{\text{G.E. effect}<0} < \mathbf{0}$$

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2. Ave. and log-ave. returns decrease with τ_a

Government Budget and Aggregate Variables

Government budget:

$$G = \tau_k \alpha Y + \tau_a K.$$

Assumption: G is a constant fraction $\theta\alpha$ of aggregate output: $G = \theta\alpha Y$.

- In what follows, τ_k adjusts in the background when $\tau_a \uparrow$

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Lemma: For all $\tau_a < \overline{\tau}_a$, a marginal increase in $\boldsymbol{\tau_a}$

- **Increases** capital (K), output (Y), wage (w), h-type wealth (A_h), and **G iff $\rho > 0$**
    - **Key:** Higher $\alpha \longrightarrow$ Larger response of K, Y, w
    - $A_\ell = (1 - s_h) K \downarrow$ iff $\alpha z_\lambda < Z$ and $\rho > 0$

Outline

1. **Model Description**
2. **Efficiency gains from wealth taxation**
3. **Welfare gains from wealth taxation**
4. Optimal taxation
5. Extensions
6. Quantitative Analysis

Welfare gains (across steady states)

$\text{CE}_{1,i}$ measure for agents of type i ($i \in \{$ *workers* , *ℓow prod.* , *high prod.* $\}$):

- (a, i) in **B**enchmark economy v.s.
  (a, i) in **C**ounterfactual economy with higher τ_a (lower τ_k)

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- Welfare gains (**C**$\succ$**B**) if

$$\frac{\log\left(1+\text{CE}_{1,i}\right)}{1-\beta} = V^{\text{C}}(a,i) - V^{\text{B}}(a,i) > 0$$

Note: independent of a because $V(a,i) = m_i + \frac{1}{1-\beta}\log(a)\ i \in \{l,h\}$.

CE_1 Details

Welfare gains (across steady states)

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- Utilitarian welfare CE_1 depends on population shares n_i's:

$$\log\left(1 + \mathrm{CE}_1\right) = \sum_i n_i \log\left(1 + \mathrm{CE}_1(., i)\right)$$

Welfare gains (across steady states)

$\mathrm{CE}_{1,i}$ measure for agents of type i ($i \in \{$ *workers* , *low prod.* , *high prod.* $\}$):

- (a, i) in **B**enchmark economy v.s.
  (a, i) in **C**ounterfactual economy with higher τ_a (lower τ_k)
- Welfare gains (**C**$\succ$**B**) if

$$\frac{\log\left(1+\mathrm{CE}_{1,i}\right)}{1-\beta} = V^{C}(a,i) - V^{B}(a,i) > 0$$

  Note: independent of a because $V(a,i) = m_i + \frac{1}{1-\beta}\log(a)$ $i \in \{l, h\}$.

  CE_1 Details

- Utilitarian welfare CE_1 depends on population shares n_i's:

$$\log\left(1+\mathrm{CE}_1\right) = \sum_i n_i \log\left(1+\mathrm{CE}_1(.,i)\right)$$

- CE_1 does not account for changes in distribution of wealth.
  - Alternative measure CE_2 takes into account changes in wealth levels.

  CE_2 Details

Main Result 2: Welfare gains by type

Proposition:

For all $\tau_a < \overline{\tau}_a$, a marginally higher $\boldsymbol{\tau_a}$ changes welfare as follows **iff** $\boldsymbol{\rho > 0}$

- Workers: Higher $\text{CE}_{1,w} > 0$
- High-type entrepreneurs: Higher $\text{CE}_{1,h} > 0$ iff $R_h - R_\ell < \kappa_R(\beta, \rho)$ κ_R
  - Taking wealth accumulation into account: $\text{CE}_{2,h} > 0$ always.
- Low-type entrepreneurs: Lower $\text{CE}_{1,l} < 0$
  - Taking wealth accumulation into account: $\text{CE}_{2,l} < 0$ if $\alpha z_\lambda < Z$.
- Lower average welfare of entrepreneurs: $\text{CE}_{1,\mathbf{E}} < 0$.

Outline

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Optimal Taxation

Government chooses (τ_a, τ_k) to maximize the utilitarian social welfare CE_1 (or CE_2)

Key trade-off:

1. Higher wages (depends on α) v.s.
2. Lower (LOG) average return (higher return dispersion + negative GE effect)

& changes in $\{A_l, A_h\}$ if CE_2 is the objective.

Main Result 3: Optimal Taxes

Graph | α thresholds

Proposition: There exists a unique optimal tax combination $(\tau_a^\star, \tau_k^\star)$ that maximizes CE_1. An interior optimum $(\tau_a^\star < \bar{\tau}_a)$ is the solution to:

$$\overbrace{n_w}^{\text{Share of Workers}} \underbrace{\xi_w}_{Z\text{-Elasticity of Wages}(=\alpha/(1-\alpha))} + \frac{1-n_w}{1-\beta} \underbrace{\left(\frac{\xi_{R_\ell} + \xi_{R_h}}{2}\right)}_{\text{Av. } Z\text{-Elasticity of Returns}<0} = 0$$

where $\xi_x \equiv \frac{d \log x}{d \log Z}$ is the elasticity of variable x with respect to Z.

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$$\tau_a^\star \in \left[1 - \frac{1}{\beta}, 0\right) \quad \text{and } \tau_k^\star > \theta \qquad \text{if } \alpha < \underline{\alpha}$$

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$$\begin{array}{lll} \tau_a^\star \in \left[1-\frac{1}{\beta}, 0\right) & \text{and } \tau_k^\star > \theta & \text{if } \alpha < \underline{\alpha} \\ \tau_a^\star \in \left[0, \frac{\theta(1-\beta)}{\beta(1-\theta)}\right] & \text{and } \tau_k^\star \in [0,\theta] & \text{if } \underline{\alpha} \leq \alpha \leq \bar{\alpha} \\ \tau_a^\star > \frac{\theta(1-\beta)}{\beta(1-\theta)} & \text{and } \tau_k^\star < 0 & \text{if } \alpha > \bar{\alpha} \end{array}$$

Remark: Opt. τ_a^* is independent of G but $\overline{\alpha}$ increases with G.

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Extensions

- Corporate sector that faces no borrowing constraint Details
  - If $z_\ell < z_C < z_h$, then low-productivity agents invest in the corporate sector.
- Rents: Return $\neq$ marginal productivity. Details
  - Introduce zero-sum return wedges so that $R_h <> R_\ell$.
  - Efficiency gains from $\tau_a \uparrow$ if $\rho > 0$ **and** $R_h > R_\ell$.
  - Efficiency gains from $\tau_a \uparrow$ if $\rho < 0$ **and** $R_h < R_\ell$.
- Entrepreneurial effort in production: Details
  - With GHH preferences, aggregate entrepreneurial effort increases with wealth tax.
- Perpetual youth and stationary distribution of agents: Details
  - $\mathrm{CE}_{2,h} > \mathrm{CE}_{1,h} > 0$ always.

Conclusions

Increasing τ_a (& reducing τ_k):

- Reallocates capital: less productive $\rightarrow$ more productive agents.
  - Higher TFP, output, and wages;
  - Higher dispersion in returns and wealth **iff $\rho > 0$**.

- Workers gain
- Entrepreneurs: High-productivity gain*, low-productivity lose*.

Optimal tax combination: depends on elasticity of output with respect to capital.

Thanks!

Extra

Entrepreneur’s Problem

Financial Markets & Entrepreneurs' Production Problem

Entrepreneurs' Production Decision:

$$\Pi^{\star}(z,a) = \max_{\mathbf{k} \leq \lambda \mathbf{a}, n} (zk)^{\alpha} n^{1-\alpha} - rk - wn.$$

Financial Markets & Entrepreneurs' Production Problem

Entrepreneurs' Production Decision:

$$\textbf{Solution:} \quad \Pi^{\star}(z,a) = \underbrace{\pi^{\star}(z)}_{\text{Excess return above } r} \times a$$

$$\pi^{\star}(z) = \begin{cases} (MPK(z) - r)\,\lambda & \text{if } MPK(z) > r \\ 0 & \text{otherwise.} \end{cases} \qquad k^{\star}(z) \begin{cases} = \lambda a & \text{if } MPK(z) > r \\ \in [0, \lambda a] & \text{if } MPK(z) = r \\ = 0 & \text{if } MPK(z) < r \end{cases}$$

- $(\lambda - 1)\,a$: amount of external funds used by type-z if $MPK(z) > r$.

back

Entrepreneur's Consumption-Saving Problem

$$V(a,z) = \max_{c,a'} \log(c) + \beta \sum_{z'} \mathbb{P}(z' \mid z) V(a', z')$$

$$\text{s.t. } c + a' = \underbrace{(1-\tau_a)a + (1-\tau_k)(r + \pi^\star(z))a}_{\text{After-tax wealth}}.$$

- Letting $R_i \equiv (1-\tau_a) + (1-\tau_k)(r + \pi^\star(z_i)) \quad$ for $i \in \{l, h\}$,

  the savings decision (CRS + Log Utility):

$$a' = \beta R_i a \qquad \longrightarrow \text{linearity allows aggregation}$$

back

Equilibrium

Unstable equilibrium

Back to Eq.

1. Can there be a steady state with $(\lambda - 1)A_h > A_\ell$? **NO.** In that case $R_h = R_\ell$,

$$\frac{A'_h}{A'_\ell} = \frac{pA_h + (1-p)A_\ell}{(1-p)A_h + pA_\ell} = \frac{A_h}{A_\ell},$$

   which implies that $A_h = A_\ell$. But then $(\lambda - 1)A_h > A_\ell$ is violated since $\lambda < 2$.

Unstable equilibrium

Back to Eq.

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2. Can there be a steady state with $(\lambda-1)A_h < A_\ell$? If the answer is yes, then we are already focusing on that SS and that SS implies that $\lambda < \lambda^*$.

Unstable equilibrium

Back to Eq.

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2. Can there be a steady state with $(\lambda - 1) A_h < A_\ell$? If the answer is yes, then we are already focusing on that SS and that SS implies that $\lambda < \lambda^*$.

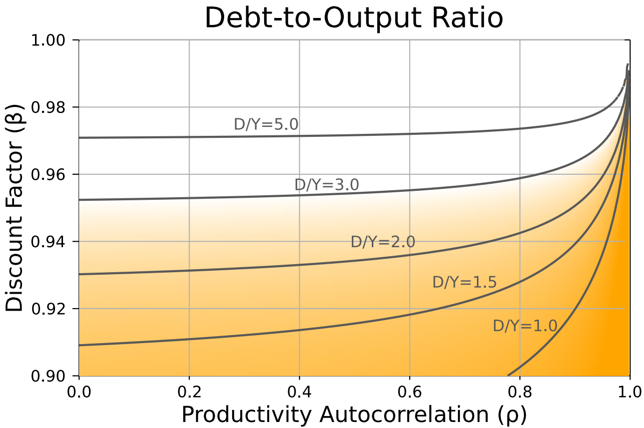
3. If $(\lambda - 1) A_h > A_\ell$ in the transition, then $A_h > A_\ell$ since $\lambda < 2$ and

$$\frac{A_h'}{A_\ell'} = \frac{pA_h + (1-p) A_\ell}{(1-p) A_h + pA_\ell} < \frac{A_h}{A_\ell}.$$

   Then at some point, we will have $(\lambda - 1) A_h < A_\ell$ and we will be in the heterogenous-return case. If this converges to a a steady state, it is the one with $\lambda < \lambda^*$.

Is λ^* too restrictive?

Back to Eq.

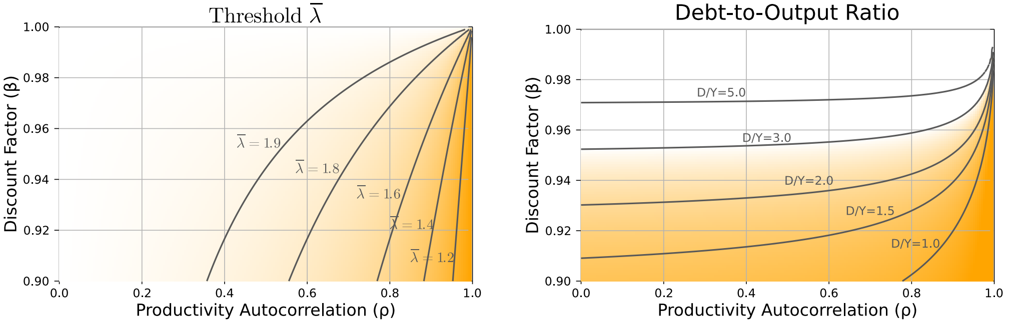


Debt-to-output ratio when $\lambda = \lambda^\star$ computed as ${}^{(\lambda^\star-1)A_h}/_{Y}$.

Is λ^* too restrictive?

Back to Eq.

Figure 1: Conditions for Steady State with Heterogeneous Returns



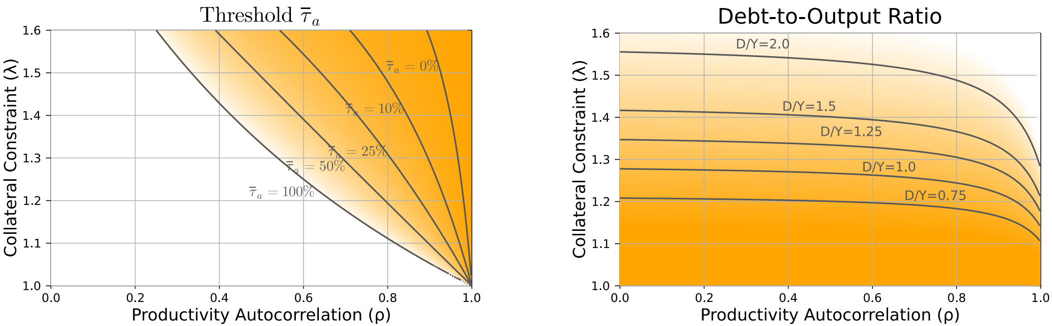
$z_\ell = 0$, $z_h = 2$, $\tau_k = 25\%$, and $\alpha = 0.4$.

Debt-to-output ratio when $\lambda = \lambda^\star$ computed as $(\lambda^\star - 1) A_h / Y$

Bound on τ_a

Back to Eq. Back to Eff.

Figure 2: Conditions for Steady State with Heterogeneous Returns



$z_\ell = 0$, $z_h = 2$, $\tau_k = 25\%$, and $\alpha = 0.4$.

Debt-to-output ratio with $\tau_a = 0$ (benchmark) computed as $(\lambda^\star - 1)A_h/Y$

Steady State: 2 equations 2 unknowns

Back to ss Back to Eff.

SteadyState K:

$$(1-\tau_a)+(1-\tau_k)\overbrace{\alpha Z^{\alpha}\left({}^{K}\!/_{L}\right)^{\alpha-1}}^{\text{Marginal Product } K}=\frac{1}{\beta}$$

Steady State R:

$$R_i=(1-\tau_a)+(1-\tau_k)\overbrace{\alpha\left({}^{ZK}\!/_{L}\right)^{\alpha-1}}^{\text{Marginal Product } ZK} z_i \qquad \text{Equilibrium R}$$

$$R_i=(1-\tau_a)+(1-\tau_k)\,\alpha Z^{\alpha}\left({}^{K}\!/_{L}\right)^{\alpha-1}\frac{z_i}{Z} \qquad \text{Change to MPK}$$

$$R_i=(1-\tau_a)+\left(\frac{1}{\beta}-(1-\tau_a)\right)\frac{z_i}{Z} \qquad \text{Steady State}$$

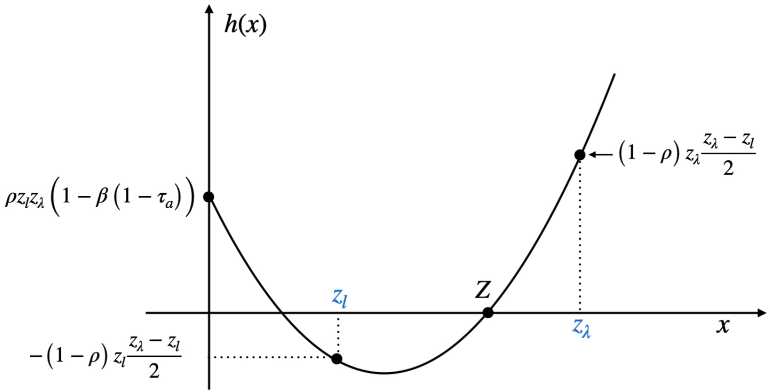
Key: Steady state K adjusts to maintain constant (after-tax) MPK:

$$(1-\tau_k)\,\text{MPK}=\frac{1}{\beta}-(1-\tau_a)$$

As in NGM τ_k affects level of K but not long run (after-tax) MPK $\left({}^{1}\!/_{\beta}-1+\tau_a\right)$.

Existence and Uniqueness of Steady State (when $\rho > 0$)

Back to ss



- $Z = s_h\, z_\lambda + (1 - s_h)\, z_\ell$ so $z_\ell \leq Z \leq z_\lambda$
- $R_h > R_\ell$ if and only if $Z < z_h \longrightarrow$ Characterization of bound $\lambda^\star$ so that $Z(\lambda^\star) = z_h$

Welfare Gains

Welfare gains (with changes in wealth)

Back to CE_1

$\mathbf{CE}_{2,i}$ measure $(i \in \{w, l, h\})$**:**

- Evaluate welfare gain at average wealth levels for each economy.
- (A_i^{B}, i) in the **B**enchmark economy v.s. (A_i^{C}, i) in the **C**ounterfactual economy.
- Welfare gains (**C**$\succ$**B**) if

$$\frac{\log(1 + \mathrm{CE}_{2,i})}{1-\beta} = V^{\mathrm{C}}(A_i^{\mathrm{C}}, i) - V^{\mathrm{B}}(A_i^{\mathrm{B}}, i) > 0 \qquad i \in \{w, l, h\}$$

  - Relation to CE_1:

$$\log(1 + \mathrm{CE}_{2,i}) = \log(1 + \mathrm{CE}_{1,i}) + \log\left(A_i^C / A_i^B\right)$$

Welfare gains: Functional Forms

Back to CE_1

- **Workers:** Value depends only on wages

$$\log\left(1+\mathsf{CE}_{1,w}\right)=\log {}^{w_a}/_{w_k}$$

Welfare gains: Functional Forms

Back to CE_1

- **Workers:** Value depends only on wages

$$\log\left(1+\mathrm{CE}_{1,w}\right)=\log {}^{w_a}/_{w_k}$$

- **Entrepreneurs:** Value depends on assets and returns $V(a,i)=m_i\left(R_h,R_\ell\right)+\frac{\log(a)}{1-\beta}$

$$\log\left(1+\mathrm{CE}_{1,i}\right)=\frac{1}{(1-\beta)(1-\beta\rho)}\left[(1-\beta)\underbrace{\log\frac{R_{a,i}}{R_{k,i}}}_{\text{Own Return}}+\beta(1-p)\left(\underbrace{\log\frac{R_{a,l}}{R_{k,l}}+\log\frac{R_{a,h}}{R_{k,h}}}_{\text{Average (log) Returns}}\right)\right]$$

Welfare gains: Functional Forms

Back to CE_1

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- **Entrepreneurs:** Value depends on assets and returns $V(a,i)=m_i\left(R_h,R_\ell\right)+\frac{\log(a)}{1-\beta}$

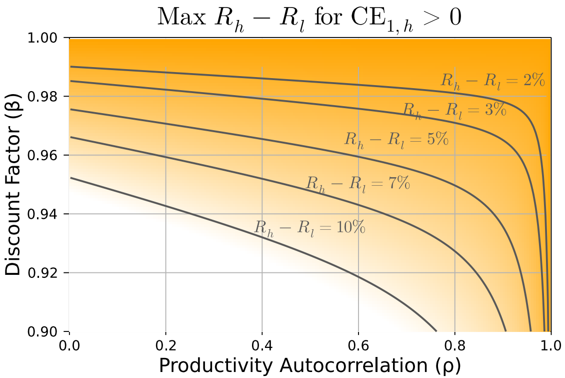
$$\log\left(1+\text{CE}_{1,i}\right)=\frac{1}{(1-\beta)(1-\beta\rho)}\left[(1-\beta)\underbrace{\log\frac{R_{a,i}}{R_{k,i}}}_{\text{Own Return}}+\beta(1-p)\left(\underbrace{\log\frac{R_{a,l}}{R_{k,l}}+\log\frac{R_{a,h}}{R_{k,h}}}_{\text{Average (log) Returns}}\right)\right]$$

  - Total entrepreneurial value:

$$\log\left(1+\text{CE}_1^{\text{e}}\right)\equiv\sum_{i\in\{h,l\}}\frac{1}{2}\log\left(1+\text{CE}_{1,i}\right)=\frac{1}{1-\beta}\left(\log\frac{R_{a,l}}{R_{k,l}}+\log\frac{R_{a,h}}{R_{k,h}}\right)$$

Return Dispersion for Welfare Gains of High-Type Entrepreneurs

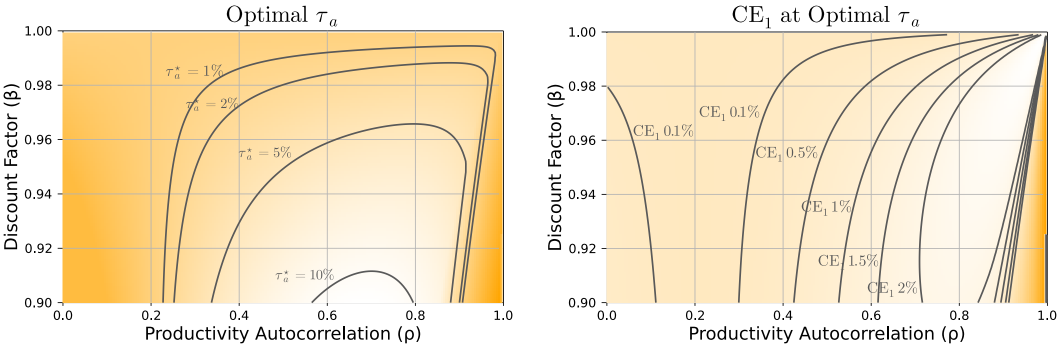
Back to CE_1



Optimal Taxes

Optimal Wealth Taxes and Welfare Gain

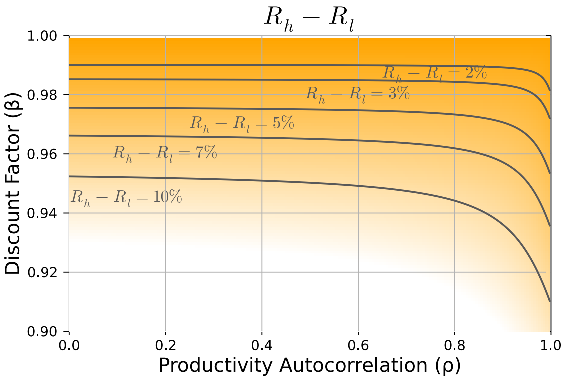
α-thresholds Back to opt. tax



$z_\ell = 0$, $z_h = 2$, $\theta = 25\%$, and $\lambda = 1.3$.

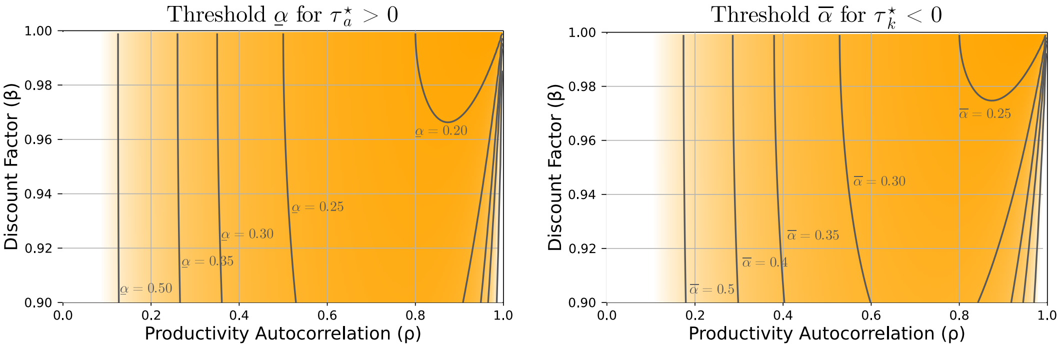
Return dispersion $R_h - R_\ell$:

Back to α-thresholds



α-thresholds for Optimal Wealth Taxes

Back to opt. tax

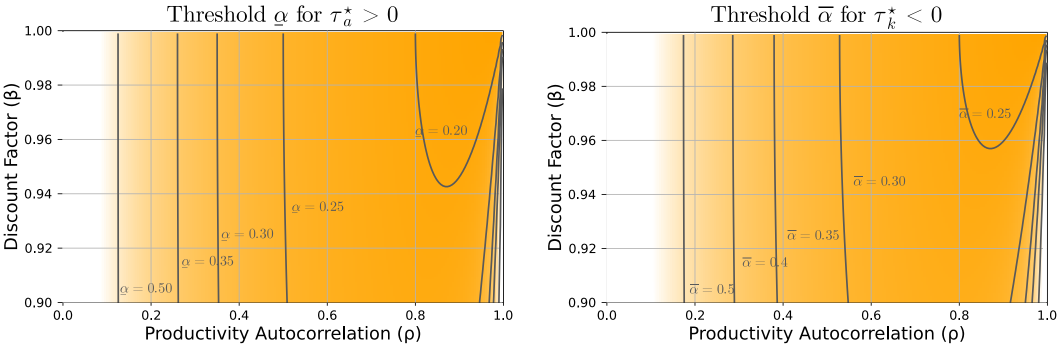


$z_\ell = 0$, $z_h = 2$, $\lambda = 1.3$, and $\theta = 25\%$.

Alt. Parameters | $R_h - R_l$ | Opt. Tax and Welfare Gains

α-thresholds for Optimal Wealth Taxes (alternative parameters)

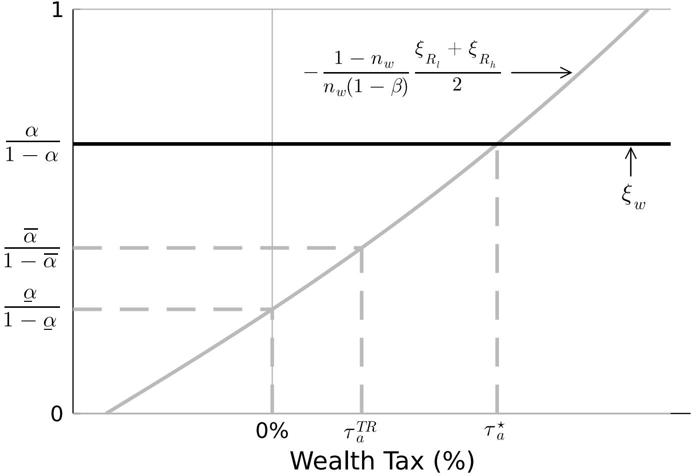
Back to opt. tax



$z_\ell = 0.5$, $z_h = 1.5$, $\lambda = 1.2$, and $\theta = 25\%$.

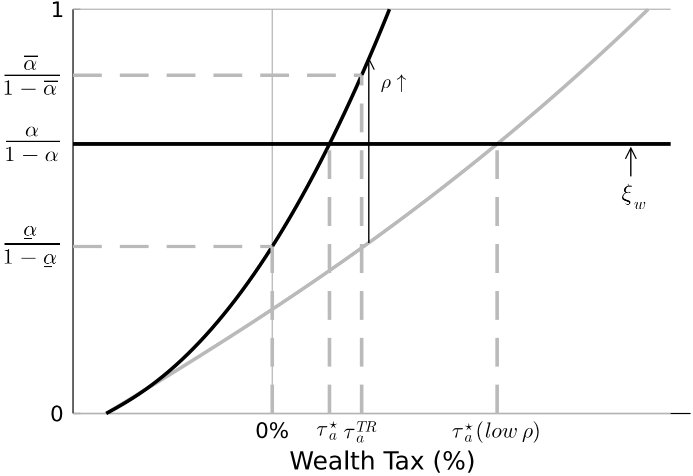
Optimal Wealth Taxes and α Thresholds

Back to opt. tax



Optimal Wealth Taxes and α Thresholds

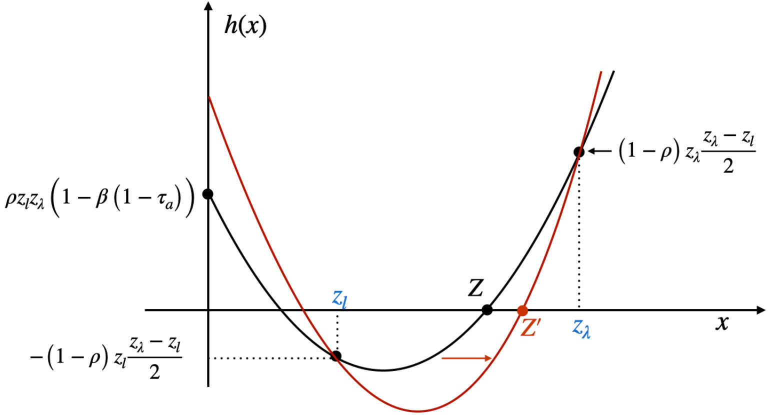
Back to opt. tax



What happens to Z if $\tau_a \uparrow$?

Back to eff. gain

$$\frac{dh(x)}{d\tau_a} = \rho\,(x - z_l)\,(z_\lambda - x) < 0 \text{ iff } \rho > 0 \text{ and } z_l < x < z_\lambda$$



Extensions

Extension: Corporate sector

Back to extensions

- Corporate sector produces final goods using CRS technology:

$$Y_c = (z_c K_c)^{\alpha} L_c^{1-\alpha}$$

  - No financial constraints!

- Corporate sector imposes lower bound on r:

$$r \geq \alpha z_c \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} .$$

Interesting case: $z_\ell < z_c < z_h$

- Corporate sector and high-productivity entrepreneurs produce
- Low-productivity entrepreneurs lend all of their funds.
- No real changes in the aggregates of the economy! z_c takes the place of z_ℓ

$$Y = (ZK)^{\alpha} L^{1-\alpha}$$

but now $Z = s_h z_\lambda + s_l \mathbf{z_c}$, where $z_\lambda = z_h + (\lambda - 1)(z_h - \mathbf{z_c})$.

Extension: Rents

Back to extensions

- Introduce wedge for returns above/below productivity:

$$R_i = (1 - \tau_a) + (1 - \tau_k) \underbrace{(1 + \omega_i)}_{\text{Return Wedge}} \alpha \left(ZK/L \right)^{\alpha - 1} z_i$$

- Zero-sum condition on wedges: $\omega_l z_\ell A_\ell + \omega_h z_\lambda A_h = 0$
- Characterization of eq. in terms of "effective productivity" $\tilde{z}_i = (1 + \omega_i) z_i$

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Back to extensions

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Proposition:

For all $\tau_a < \overline{\tau}_a$, a marginal increase in wealth taxes (τ_a) increases Z, $\frac{dZ}{d\tau_a} > 0$, **iff**

1. $\rho > 0$ and $R_h > R_\ell \longrightarrow$ Same mechanism as before
2. $\rho < 0$ and $R_h < R \longrightarrow$ Reallocates wealth to the true high types next period

Extension: Entrepreneurial Effort

Back to extensions

- Entrepreneurial production:

$$y = (zk)^{\alpha} e^{\gamma} n^{1-\alpha-\gamma} \quad \longrightarrow \quad e: \text{ effort}$$

  - Production functions is CRS $\longrightarrow$ Aggregation

- Entrepreneurial preferences:

$$u(c, e) = \log(c - \psi e) \qquad \psi > 0$$

  - GHH preferences with no income effects $\longrightarrow$ Aggregation
  - ψ plays an important role: Cost of effort in consumption units

Extension: Entrepreneurial Effort

Back to extensions

Problem is isomorphic to having preferences

$$u(\hat{c}) = \log \hat{c} \qquad \text{where } \hat{c} = c - \psi e$$

and modifying entrepreneurial problem to:

$$\hat{\pi}(z,k) = \max_{n,e} y - wn - rk - \underbrace{\frac{\psi}{1-\tau_k}}_{\text{Effecive cost of effort}} e$$

- Solution is just as before (linear policy functions a', n, **and e**)
- **Key:** Effective cost of effort depends on capital income tax τ_k!
  - Effort affects entrepreneurial income
  - Income subject to capital income taxes but not to **book value** wealth taxes

Extension: Entrepreneurial Effort

Back to extensions

- Aggregate effort:

$$E = \left(\frac{(1-\tau_k)\,\gamma}{\psi} \right)^{\frac{1}{1-\gamma}} (ZK)^{\frac{\alpha}{1-\gamma}} L^{\frac{1-\alpha-\gamma}{1-\gamma}}$$

  - Comparative statics: $K \uparrow$, $Z \uparrow$, and $\tau_k \downarrow$
- New wedge from capital income taxes on aggregate output and wages!
- Effort affects marginal product of capital $\longrightarrow$ Affects K_{ss}

A neutrality result:

- **No changes to steady state productivity!**
- Steady state capital adjusts in background to satisfy:

$$(1-\tau_k)\,\text{MPK} - \tau_a = \frac{1}{\beta} - 1$$

Extension: Entrepreneurial Effort

Back to extensions

Results:

1. Efficiency gains from wealth taxation remain
2. Effect on aggregates is stronger if capital income taxes go down
   - **Effort increases with wealth taxes** (if $\rho > 0$)**!**
3. Characterization of optimal taxes is similar but
   higher wealth taxes and lower capital incomes taxes are optimal

Extension: Perpetual Youth

Back to extensions

- Baseline model has no stationary distribution

Perpetual youth: Entrepreneurs die with probability $1 - \delta$

- Replaced by new entrepreneur with assets $\overline{a}$ and productivity z_i ($i \in \{h, l\}$)
- $\overline{a}$ endogenous: Average bequest (= average wealth).

Extension: Perpetual Youth

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- $\overline{a}$ endogenous: Average bequest (= average wealth).

Solution:

- Entrepreneur's savings choice: $a' = \beta\delta R(z)\, a$.
- Aggregate law of motion: $A_i' = \beta\delta^2 R_i A_i + (1-\delta)\,\overline{a}$
  - Depends only on R_i!
- Similar characterization of SS and aggregates

Extension: Perpetual Youth

Back to extensions

Effects of wealth taxation:

- Efficiency gains from wealth taxation “always” (bc productivity is persistent)
- Increase return dispersion: $R_\ell \downarrow + R_h \uparrow$

Extension: Perpetual Youth

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Welfare and optimal taxes:

$$\sum_a \left(V_k(a,i) + \frac{\log\left(1 + \mathrm{CE}_{2,i}\right)}{1-\beta\delta} \right) \Gamma_k(a,i) \quad = \quad \sum_a V_a(a,i)\, \Gamma_a(a,i)$$

- Consumption equivalent measure takes into account asset levels!

$$\log\left(1 + \mathrm{CE}_{2,i}\right) = \frac{1-\beta\delta^2}{(1-\delta)(1-\beta\delta)} \log \frac{R_{a,i}}{R_{k,i}} + \log \frac{K_a}{K_k}.$$

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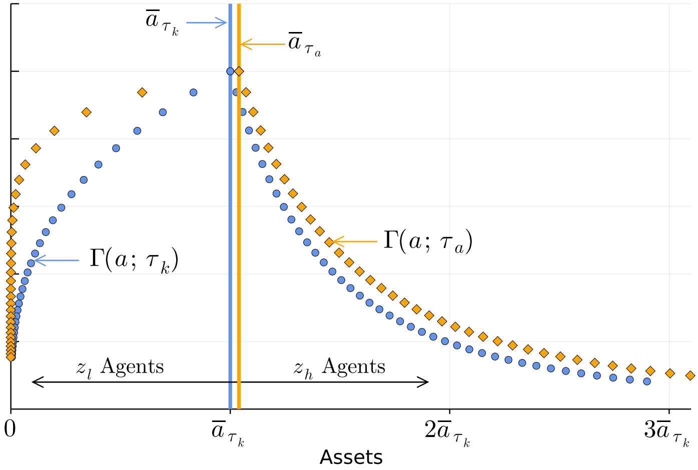
- High-productivity entrepreneurs always benefit from wealth taxes
- Optimal taxes are higher $\longrightarrow$ Include gains of capital accumulation

Extension: Perpetual Youth

Back to extensions



Quantitative Analysis and New Results

Model: Households

- **OLG** demographic structure *(retirement, mortality risk)*.
- **Bequest motive**, inheritance goes to (newborn) offspring.

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Individuals:

- Preferences over consumption, leisure and bequests
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  consumption-savings ‖ labor supply ‖ portfolio choice

- Two exogenous characteristics:

  y_{ih} (labor market productivity) ‖ z_{ih} (entrepreneurial productivity)

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Markets: monopolistic competition $\rightarrow$ **decreasing returns to scale** (μ)

Entrepreneurial Productivity z_{ih}: Key Source of Heterogeneity

Idiosyncratic wage risk:

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Entrepreneurial productivity, z_{ih}, varies

1. permanently across individuals: z_i^p *(imperfectly correlated across generations)*
2. stochastically over the life cycle

$$z_{ih} = f(z_i^p, \mathbb{I}_{ih}) = \begin{cases} \left(z_i^p\right)^{\lambda} & \text{if } \mathbb{I}_{ih} = H \qquad \text{where } \lambda > 1 \\ z_i^p & \text{if } \mathbb{I}_{ih} = L \\ z_{min} & \text{if } \mathbb{I}_{ih} = 0 \end{cases}$$

λ: degree of superstar productivity *(consistent w/ Halvorsen, Hubmer, Ozkan, Salgado, 2021)*.

Government

Government budget balances:

- **Outlays:** Expenditure (G) + Social Security pensions
- **Revenues:** tax on consumption (τ_c), labor income (τ_ℓ), bequests (τ_b) plus:

1. tax on capital income (τ_k), or
2. tax on wealth (τ_a).

Calibration summary

- Bequest motive $\rightarrow$
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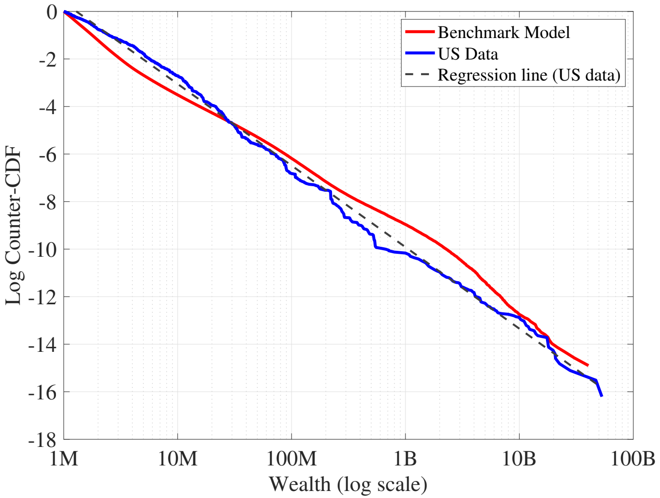
  - Intergenerational correlation of return fixed effect
- Entrepreneurs' collateral constraint $\rightarrow$
  - Business debt plus external funds/GDP

Details Entrepreneurship Intergenerational wealth ranks Dbn. of capital income

Pareto Tail of Wealth Distribution: Model vs. Data



Note: Both axes are in natural logs.

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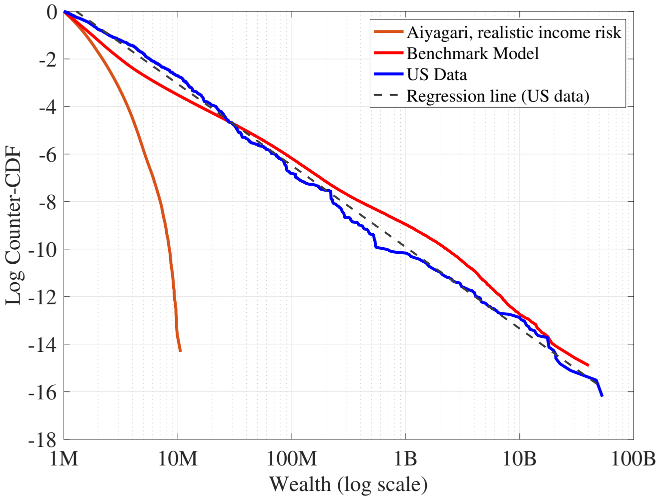

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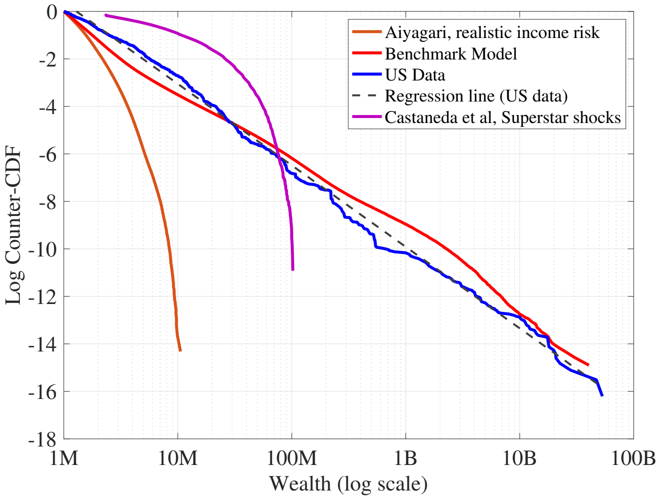

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Performance of the benchmark model: return heterogeneity

Table 1: Distribution of Rates of Return (Untargeted) in the Model and the Data

	Annual Returns			Persistent Component of Returns					
	Std dev	P90-P10	Kurtosis	Std dev	P90-P10	Kurtosis	P90	P99	P99.9
Data (Norway)	8.6	14.2	47.8	6.0	7.7	78.4	4.3	11.6*	23.4*
Data (Norway, bus. own.)	–	–	–	4.8	10.9	14.2	10.1	–	–
Data (US, private firms)	17.7	33.8	8.3	–	–	–	–	–	–
Benchmark Model	8.4	17.1	7.6	4.1	9.2	6.1	5.8	13.9	19.7
L-INEQ Calibration	6.7	13.1	9.2	3.8	9.2	4.3	5.6	11.2	15.8

*Notes: Returns on wealth in percentage points. All cross-sectional returns are value weighted. *The statistics for Norway are for individual returns on wealth (net worth) taken from Fagereng, Guiso, Malacrino, and Pistaferri (2020). The US statistics are from Smith, Zidar, and Zwick (2021) and are for S-corps' returns on investment; they also report statistics for partnerships, which are very similar (std dev of 17.8% and P90-P10 of 27.9). For each individual, the persistent component of returns is calculated following Fagereng et al as the unweighted average of annual, before-tax, returns between ages 25 and 75, after taking out the average return by age.*

Tax Reform

Taxes and welfare:

	τ_k	τ_ℓ	τ_a	ΔWelfare
Benchmark	25%	22.4%	–	–
Tax reform	–	22.4%	1.19%	7.2

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Aggregate variables (% Change):

	K	Q	TFP	L	Y	w	w(net)
Tax reform	16.4	22.6	2.1	1.2	9.2	8.0	8.0

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Key: Tax reform **replaces** τ_k with τ_a. This is $\neq$ from adding wealth taxes.

- Adding wealth taxes reduces welfare by −6% to −9%

Tax Reform: Who Gains? Who Loses?

Average (consumption equivalent) **welfare gain** by age-productivity groups:

	Productivity group (Percentile)					
Age	0-40	40-80	80-90	90-99	99-99.9	99.9+
20	**6.7**	**6.3**	**6.8**	**8.5**	**11.5**	**13.4**
21-34						
35-49						
50-64						
65+						

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21-34	6.3	5.5	5.5	6.5	8.5	9.7
35-49	4.9	3.8	3.3	3.3	3.1	2.8
50-64	2.2	1.5	1.1	0.9	0.4	-0.2
65+	-0.2	-0.3	-0.4	-0.4	-0.7	-1.0

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50-64	2.2	1.5	1.1	0.9	0.4	-0.2
65+	-0.2	-0.3	-0.4	-0.4	-0.7	-1.0

Adjusting pensions turns welfare losses of retirees to gains, ranging from 2.3% to 6.5%.

Tax Reform and Optimal Taxes

Taxes and welfare:

	τ_k	τ_ℓ	τ_a	ΔWelfare
RN Tax reform	–	22.4%	1.19%	7.2
Opt. τ_a				
Opt. τ_k				

Aggregate variables (% Change):

	K	Q	TFP	L	Y	w	w(net)
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	τ_k	τ_ℓ	τ_a	ΔWelfare
RN Tax reform	–	22.4%	1.19%	7.2
Opt. τ_a	–	15.4%	3.03%	8.7
Opt. τ_k				

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	K	Q	TFP	L	Y	w	w(net)
Tax reform	16.4	22.6	2.1	1.2	9.2	8.0	8.0
Optimal τ_a	**2.6**	10.5	3.1	3.3	6.1	2.8	**12.0**
Optimal τ_k							

Tax Reform and Optimal Taxes

Taxes and welfare:

	τ_k	τ_ℓ	τ_a	ΔWelfare
RN Tax reform	–	22.4%	1.19%	7.2
Opt. τ_a	–	15.4%	3.03%	8.7
Opt. τ_k	−13.6%	31.2%	–	5.1

Aggregate variables (% Change):

	K	Q	TFP	L	Y	w	w(net)
Tax reform	16.4	22.6	2.1	1.2	9.2	8.0	8.0
Optimal τ_a	**2.6**	10.5	3.1	3.3	6.1	2.8	**12.0**
Optimal τ_k	**38.6**	46.1	2.2	−1.0	15.7	16.8	**3.6**

Welfare: Levels vs Redistribution

Welfare gain comes from changes in consumption (c) and leisure(ℓ).

- How much comes from changes in the **level** vs **distribution** of c and ℓ?

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	Tax Reform	Opt.τ_a	Opt.τ_k
CE_2 (NB)	7.2	8.7	5.1
Level $(\bar{c}, \bar{\ell})$	8.9		
Dist. (c, ℓ)	-1.5		

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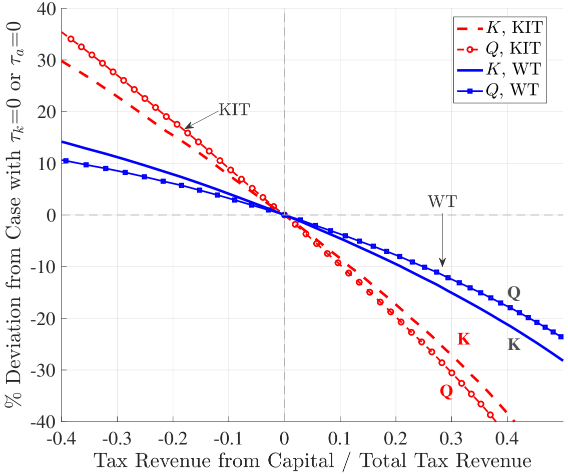
Level $(\bar{c}, \bar{\ell})$	8.9	5.9	14.7
Dist. (c, ℓ)	-1.5	**2.6**	**−8.3**

Mechanisms at Play: K and Q respond differently to taxes



Taking into Account the Transition

- Fix opt. tax level (τ_k or τ_a) and solve transition to new steady state
- Use labor income tax (τ_ℓ) to finance debt from deficits during transition

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	$\boldsymbol{\tau_k}$ Transition	$\boldsymbol{\tau_a}$ Transition
τ_k	–13.6*	0.00
τ_a	0.00	3.03*
τ_ℓ	39.90	17.01
$\overline{CE}_2$ (newborn)	**–8.4** (5.1)	**6.0** (8.7)
$\overline{CE}_2$ (all)	–6.1 (4.5)	3.5 (4.3)

Conclusions from quantitative analysis

Tax reform from τ_k to τ_a: Substantial welfare gains.

Optimal taxes: Welfare gain substantially larger under wealth taxes.

- Capital income taxes (τ_k): small gains that go away with transition .
- Wealth taxes (τ_a): large gains act through reallocation not accumulation.

Quantitative OLG Model

Labor Market Productivity y_{ih}

- Labor market efficiency of household i at age h is

$$\log y_{ih} = \underbrace{\kappa_h}_{\text{life cycle}} + \underbrace{\theta_i}_{\text{permanent}} + \underbrace{\eta_{ih}}_{\text{AR(1)}}$$

- Permanent component θ_i is <u>imperfectly inherited</u> from parents:

$$\theta_i^{child} = \rho_\theta \theta_i^{parent} + \varepsilon_\theta$$

Back to Households

Targeted moments

back

Targeted Moments	Data	Benchmark	Low-Inequality Calibration
Bequest/Wealth	0.012	0.012	0.012
90th percentile of bequest distribution	4.31	4.10	6.60
Intergenerational corr. of return fixed effect	0.10	0.10	0.10
Top 1% wealth share	0.36	0.36	**0.20**†
Self-made billionaires (fraction)	0.54	0.56	0.26
Population share of entrepreneurs in top 1%	0.65	0.68	0.68
Wealth share of entrepreneurs	0.42	0.39	0.34
Business debt plus external funds/GDP	1.52	1.50	1.50

Entrepreneurship in the Model

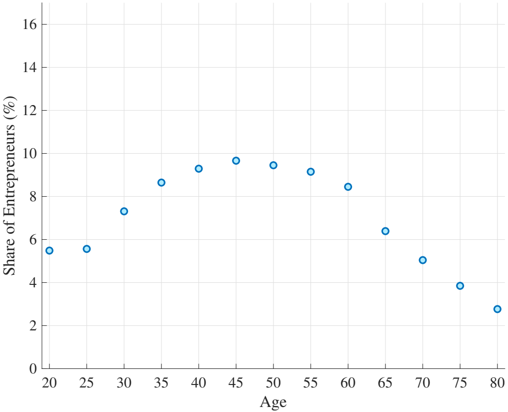
back

- Not all individuals are active entrepreneurs:
  - Only 47% of working-age population have positive productivity.
- 7% of of individuals earn more than half of their income from their business:
  - These entrepreneurs account for 68% (39%) of the top 1% (10%) of wealth holders
  - They hold 40% of aggregate wealth (and 50% within top 1%)
  - Most of them are 35-64 years old (in the model)
- These are in line with SCF:

  Pass-through business owners are ~12% of households, account for 46% of wealth and constitute 70% of top 1% wealth holders.

Fraction of Entrepreneurs over the Life Cycle, Benchmark Model

back



Notes: The figure plots the fraction of entrepreneurs over the life cycle for our baseline economy. All numbers are in percentage points. An entrepreneur is defined as someone who earns more than 50% of their income from their business.

Entrepreneurship over lifecycle is hump-shaped as documented in the data (see, e.g., Kelley, Singer, and Herrington (2011); Liang, Wang, and Lazear JPE, 2018).

Concentration of Capital Income and Wealth in the Model

back

Top x% of Wealth Dbn.	Wealth Share (%)	Capital Income Share (%)	Top x% of Capital Income Dbn.	Capital Income Share (%)
0.1	22.3	32.0	0.1	34.3
0.5	30.5	43.0	0.5	45.7
1	35.1	48.2	1	51.9
10	64.9	73.1	10	78.9
50	96.4	97.0	50	98.1

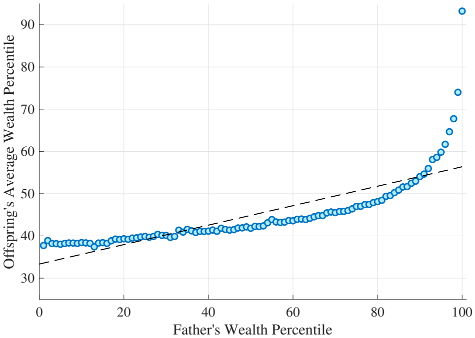
Notes: The table reports wealth and capital income shares for individuals at the top of the wealth distribution (first three columns) and at the top of the capital income distribution (last two columns). All numbers are in percentage points.

- The top 0.1% share by capital income varies between 30% and 41% since 2000 according to Saez and Zucman (QJE, fig 3).
- Smith, Zidar, Zwick (2021, fig A5) report shares sorted by individual components of capital income and the top 1% share for interest, dividend, and capital gains income are all above 60% since 2000

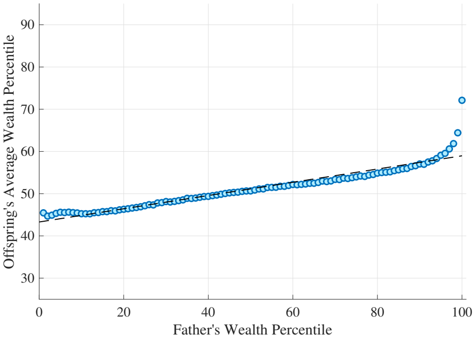
Inequality in the model

Intergenerational Rank Correlation of Wealth

back



(a) Baseline Model



(b) Norway: Fagereng, Guiso, Malacrino, and Pistaferri (2020, Figure 11)

Notes: *The figures show rank-rank plots for the wealth distribution of parents and children.*