THE MACROECONOMICS OF SELF-EMPLOYMENT: OCCUPATIONS OF LAST RESORT*

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Abstract. We document three facts about self-employment in developing economies. First, self-employment is prevalent in the left tail of the income distribution. Second, transitions in-and-out of self-employment are common, with liquidity-constrained agents transitioning more to self-employment. Finally, when salaried work opportunities emerge, self-employment rates go down. Models that predict positive selection into self-employment are at odds with these facts. We augment a workhorse macro-development model with a mechanism supported by the data, generated by interacting unemployment risk and credit frictions. Low wealth, unemployed agents choose self-employment to earn subsistence income, regardless of their entrepreneurial ability. Low job-finding rates from self-employment make subsistence entrepreneurs stay self-employed. As a result, large shares of the labor force own low-productive businesses. Improving the generosity of safety nets in the model increases welfare by 2%. Also, self-employment goes down, salaried work goes up, the unemployment rate rises.

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Introduction

Business ownership is regarded as a key determinant of economic dynamism, innovation, and growth. However, developing countries exhibit large rates of self-employment and they lag precisely on dynamism, innovation, and standards of living of their populations. A large fraction of the self-employment rate is explained by small and unproductive establishments that provide income to workers who cannot get a "good job" in a larger, more productive, and usually formal establishment (Poschke, 2013a,b).

In this paper we document how the combination of high self-employment rates and low aggregate productivity in developing economies is explained in part by the interaction of weak safety nets and dysfunctional financial and labor markets. A low-income individual who has no access to credit or to a safety net has low "tolerance to unemployment". Under these circumstances an individual may opt into self-employment despite having little entrepreneurial abilityThese individuals start small and unproductive businesses, invest and hire little, and their businesses do not grow. As a consequence, the average business quality in developing countries is low, affecting aggregate productivity, wages, and the allocation of inputs of production (Katz and Krueger, 2017). This situation also leads to the coexistence of high self-employment rates and low unemployment rates in developing economies, which hides the true level of slack in the labor market (Breza et al., 2017).

The role of self-employment among low-wealth individuals with low entrepreneurial ability contrasts the traditional role highlighted by the macroeconomic development literature. In most of the literature the self-employed are productive entrepreneurs, whose access to capital is limited due to ill-functioning financial markets.[1] Each role highlights different mechanisms. While the traditional mechanism discourages entry into self-employment, and prevents highly productive individuals to grow larger businesses, our mechanism makes self-employment a more compelling option for low-productivity/low-wealth individuals, increasing the share of lower quality businesses in the economy.

We suggest that policies that are aimed at agents with high entrepreneurial ability (e.g. micro-finance) only deal with a fraction of the distortions that affect developing countries. Furthermore, policies that provide benefits contingent on the creation of businesses can have

[1]Individuals with high productivity might 1) delay their entry, and 2) operate at smaller scales than they would in a counterfactual world with better-functioning financial markets. See Moll (2014) , Buera, Kaboski, and Shin (2011) among others.

unintended consequences for productivity, since they make self-employment more attractive for agents with low entrepreneurial ability in the need of income.[2] On the contrary, policies that allow agents to smooth consumption when facing unemployment risk act by preventing (otherwise constrained) agents form becoming self-employed, effectively allowing for better selection into self-employment.

Using microdata from two large developing countries, Mexico and India, we document the characteristics and behavior of self-employed populations. We document three main facts. First, we find a u-shape relationship of the share of self-employment as a function of the percentile of the income distribution, with a higher concentration among bottom earners in a developing country like Mexico than for a developed country like the United States. Second, we document large transition probabilities between unemployment, wage employment, and self-employment, ruling out the possibility of perfectly segmented labor markets. Third, self-employed agents have lower job-finding rates than comparable individuals who remained unemployed.

The main alternative explanation to our facts is heterogeneity in preferences for self-employment, as in Hurst and Pugsley (2016). This would imply that a large fraction of individuals in developing economies are willing to give up pay in order to have independence in their work. We use panel data from Mexico to test this alternative. If agents have a strong preference for self-employment, we should observe higher transition rates into self-employment among agents who have covered their basic needs.

To capture the extent to which agents are constrained we use two proxies of liquidity: the presence of a dual earner (as in Chetty (2008)), and the reception of remittances from relatives who live abroad. If preferences for self-employment are the driver mechanism in the data, we should observe that households with more liquidity transition to self-employment at higher rates. We find exactly the opposite. These individuals transition at lower rates to self-employment from unemployment, shading doubt to the preference hypothesis as the main driver of the variation in the data. The difference in transition rates is statistically significant and economically relevant. However we are cautious in interpreting this evidence, since there might be unobserved individual characteristics, like higher job-finding rates and assortative matching in household formation, that can explain the variation in the data.

[2]This is similar to the general equilibrium effects of micro-finance on TFP explored in Buera, Kaboski, and Shin (2012).

In order to address selection issues in our data, we need a source of variation that is unrelated with individual characteristics that drive both occupational choices and liquidity measures at the same time. We use the implementation of the National Rural Employment Guarantee Act (NREGA) in India. The NREGA is a program that provides short-term work at market wages. In 2010-2011 53 million people used the program.[3] To test for the effect of the program on labor market outcomes we use a difference-in-difference approach. The method exploits variation in the timing of the implementation of the program across districts.[4] Specifically, we test whether self-employment falls in districts where the program was implemented, compared with districts that were not a part of the program at the time. Notably, the fact that the program is implemented at the district level, lets us analyze the effect of additional salary work on self-employment rates without having identification concerns coming from unobserved individual characteristics, which was the main concern when using the Mexican microdata. We find that self-employment falls when the program is implemented, even after controlling for district fixed effects, time fixed effects, and individual level controls.

The main concern about the interpretation of our results is that NREGA may have introduced job-offers that were superior in some dimension to jobs prevalent in the market. If this is the case people could be moving away from self-employment as a response is an arbitrage decision. However, Breza, Kaur, and Shamdasani (2017) find experimental evidence evidence that support our results. They randomize market-level transitory positive labor demand shocks. They find that self-employment falls as a response to the increase in labor demand, without an increase in the market wage. This result rules out the preference for self-employment hypothesis, and, due to the randomization, rules out the relevance of unobserved individual characterstics to explain the data.

The facts we document provide evidence on what is the main driver of high self employment rates in developing countries. The evidence from Mexico and India support low tolerance for unemployment among households as the main driver of high self-employment rates. Moreover, our results imply that high self-employment and low unemployment rates

[3]For detailed informtion on the NREGA program go to nrega.nic.in.

[4]The NREGA program was rolled over different districts in India from 2006 to 2008, where each district can be thought as a distinct labor market. The difference-in-difference approach has been used by Imbert and Papp (2015) to test whether public employment created by this program crowded out private employment.

are a symptoms of unobserved slack in the labor market instead of being a signal of economic dynamism.

Having established the empirical relevance of this mechanism, we augment a state-of-the art macroeconomic model to test counterfactual policies and the strength of the mechanism in the aggregate. The model includes explicitly dynamic choices, uncertainty about the future, and the general equilibrium effects that are relevant to understand questions in macrodevelopment, as suggested by Moll (2014).

The model is an extension of Moll (2014) allowing for occupational choice and labor search frictions. Agents face uncertainty over labor income and productivity, as well as liquidity (borrowing) constraints. At any point in time an agent can start a business after paying an installation cost, agents can become unemployed at any time, but in order to get a salaried work, individuals in the economy have to receive a job-offer.

We show that the mechanism forcing low-wealth/low-productivity agents into self-employment is present in the model. When unemployment is sufficiently painful, poor unemployed agents move to self-employment even if they lack entrepreneurial ability. We also show how a transfer to agents in the form of unemployment insurance eliminates this mechanism.

What is the aggregate effect of this mechanism on productivity and welfare? It depends on two effects. The first one is the elasticity of the occupation choice of unemployed agents as wealth decreases. Economies with deficient safety nets and tight liquidity constraints will exhibit higher elasticities, and higher self-employment rates in equilibrium. This is because it is too expensive for unemployed households to sustain long spells of job-search, making individuals keen to look for any alternative source of income. The second one is the effect of occupational choice on future job prospects. Transition rates to employment tend to be higher from unemployment than from self-employment, for instance by depreciating job-finding skills, or by preventing individuals to exert search effort. This induces a dynamic effect by lowering the job prospects of agents that become self-employed. The extent to which these transition rates differ determines the effect on the equilibrium allocation of workers.

The rest of the paper proceeds as follows. Section one discusses the role of self-employment as an outside option for unemployed agents. Section two the data and the main facts we will use to discipline the model. Section three presents the model. Section four presents counterfactual exercises with the model used to inspect the mechanism and the main estimation and results. Section five concludes.

1. Self-Employment as an outside option

The core of our results comes from recognizing the role of self-employment as an outside option for unemployed agents. When faced with uninsurable labor income risk an agent can turn to self-employment as a way of generating income. This mechanism changes the focus of self-employment from an activity carried out by wealthy individuals with high entrepreneurial ability, to one carried out by constrained agents facing low income spells. It also provides an extra margin for policies that provide a safety net to agents (such as unemployment insurance) to change the allocations in equilibrium, and therefore affect macroeconomic aggregates.

Agents choose to become self-employed in three cases: When they are sufficiently productive, sufficiently wealthy, or sufficiently poor. The first two cases capture the standard view in the macro-development literature of self-employment coinciding with entrepreneurship.[5] In this view there is a positive selection into self-employment, so that this activity is carried out by agents whose abilities or resources are high. Productivity differentials between developed and developing economies would then spam from barriers to entry to self-employment, or credit market frictions that generate misallocation of resources among self-employed agents.

The third case is relevant for agents who have little to no wealth and are usually excluded from credit markets. In developing countries these agents comprise the majority of the population. Faced with no income possibilities an agent might choose to become self-employed even if she is ill-equipped for the task, lacking the entrepreneurial ability (productivity) or the resources (assets) needed to run a profitable business. Self-employment is then an occupation of last resort. [6]

To illustrate the relevance of this third case for government policy, and understand the mechanisms at work, consider the following simple setup: an agent that is unemployed at

[5]See for instance the review by Buera, Kaboski, and Shin (2015) and references therein.

[6]Paulson and Townsend (2005), elaborate on many of the same ideas developed in this paper on a short policy note. They show evidence from Thailand during the Asian crisis of 1997, after which "*entrepreneurial activity in Thailand increased [...] the number of business households more than doubled*", the authors further note that *"rising unemployment and falling real wages during the crisis led to changes in the types of people who started businesses—and in the types of businesses they started*".

the beginning of the period chooses whether to remain unemployed (U) or to become self-employed (S). The agent has a units of assets and a productivity of z, and derives utility only from consumption. Assume that the utility function is of the CRRA type $(u(c) = c^{1-\sigma}/1-\sigma)$.

If the agent chooses to remain unemployed, she will get a job with probability $p \in (0, 1)$, becoming employed (E). If she does get a job she will receive a wage $w > 0$, her consumption will then be $c^E = a + w$. If she does not get a job she will receive unemployment benefits $b \in [0, w)$, and her consumption will be $c^U = a + b$.

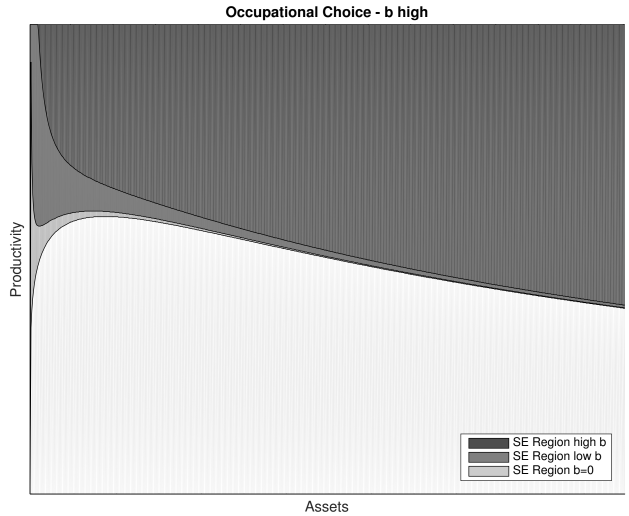
If the agent becomes self-employed she can produce consumption goods using her own assets. Her production depends on her assets and her productivity according to $f(a, z) = za^\alpha$, with $\alpha \in (0, 1)$. Consumption for the self-employed is: $c^S = a + za^\alpha$.

In the setup described above the agent will choose to become self-employed if z or a are high, so that $f(a, z) > pw + (1 - p) b$. Under these conditions selection to self-employment allows only for agents that are very productive or can produce enough given their assets. These conditions give the same selection as in Buera, Kaboski, and Shin (2011), where all agents can become workers $(p = 1)$ and there is no uncertainty over wages. The decision to become self-employed depends on the agent's own characteristics, productivity and wealth.

In contrast, the decision of poor agents to become self-employed is shaped by government policy, in this setup captured by the size of benefits b. If b is high it is not worthwhile for poor or unproductive agents to become self-employed, for instance if $f(a, z) < b$. This leaves only the selection mechanism discussed in the previous paragraph. But, if there is no safety net, i.e. $b = 0$, the situation changes. Unemployment risk makes the agent's consumption depend only on a if unemployment persists. As $a \to 0$ the agent values greatly any increase in consumption (i.e. $u'(c) \to \infty$). Becoming a self-employed increases the agent's consumption by $f(a, z)$. This is enough to make the agent choose self-employment, even when $f(a, z)$ is itself approaching 0.

Figure 1 shows the combinations of assets (a) and productivity (z) for which the agent will choose to become self-employed for three different levels of benefits. As made evident by the graph, benefits do not affect the decision of wealthy agents. Moreover, when benefits are high (the dark grey area) there is a clear negative relationship between assets and productivity (since they are substitutes in generating self-employment income). Importantly, as the assets approach zero the required productivity goes to infinity. As benefits increase the negative relation between assets and productivity breaks for poor agents. In fact, when there are no

FIGURE 1. Occupational Choice: Unemployment vs Self-Employment



Note: *Shaded areas show the combinations of assets and productivity for which the agent chooses to become self-employed. Light grey area has no unemployment benefits ($b = 0$). Grey area has low unemployment benefits ($b > 0$). Dark grey area has high unemployment benefits ($b \gg 0$).*

benefits (light grey area) and assets approach zero, agents become self-employed regardless of their productivity. Breaking the selection and allowing ill-equipped agents to start a business. In the words of Breza, Kaur, and Shamdasani (2017) these are "*disguised unemployed, or forced entrepreneurs*".

Government transfers, here in form of unemployment benefits, play a crucial role, since they affect the selection into self-employment. In our illustrative example a large enough transfer allows poor agents to remain unemployed longer, inducing a better allocation of resources, by allocating them to paid labor and preventing unproductive agents to engage in production.[7]

In Section 3 we extend the setup discussed above into a quantitative occupational choice model with search frictions. But before we discuss the model in detail we turn to empirical evidence of the mechanisms at play.

[7]The potential of productivity gains from unemployment insurance has been explored before, see for instance Acemoglu and Shimer (1999, 2000). As in our case the gains spam from allowing for longer search and better selection.

2. Data

We start by documenting three facts about the prevalence of self-employment in developing countries, taking Mexico as an example. According to the OECD (2012), "A high level of self-employment, combined with the predominance of micro-enterprises, is a distinctive feature of entrepreneurship in Mexico". Mexico has an enterprise birth-rate that doubles that of the USA, a self-employment rate of around 35%, and more than 90% of companies have less than 9 employees, compared to 60% in the U.S..

For our benchmark analysis we use the Mexican household survey (ENOE), a rotating panel in which we observe households for up to 5 quarters. We restrict attention to prime age males (23 to 65 years old) who are head of household and live in one of the ten largest municipalities of Mexico.We cover data from 1995.Q1 to 2015.Q4. In total we study 250 thousands individuals, and have around 1 million observations.

Table 1 shows some summary statistics on the labor force for our sample, for the whole Mexican labor force, and for the U.S. over the period 1995 - 2015[8]. It offers two main takeaways. First, our sample behaves in a similar way to the overall Mexican labor force. Second, Mexico, compared to the U.S., has a lower average unemployment rate, and a significantly higher self-employment rate. We now turn our attention to characterize the population of self-employed agents in the Mexican economy.

Labor Status	Our Sample	General Population	US
Worker	68.0%	57.9%	80.7%
Unemployed	2.6%	3.9%	6.3%
Self Employed	**29.5%**	38.1%	**12.9%**

Table 1. Sources: Mexico General Population: WDI - US: CPS

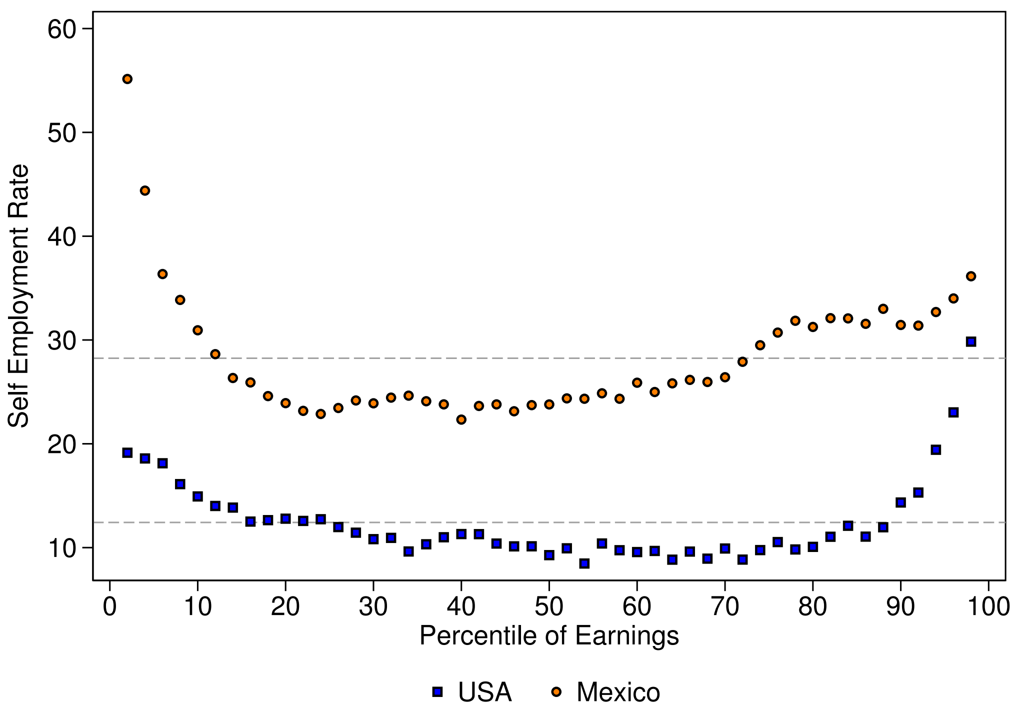
Figure 2 shows a U-shaped relation between the self-employment rate and earnings both for Mexico and for the U.S.[9]. Self-employment is more prevalent in the tails of the earnings

[8]Data for the U.S. comes fom the Current Population Survey, CPS.

[9]In particular, we first run a regression of the form $\log(w_{i,t}) = \alpha + \gamma_t + \beta X_{i,t} + \eta_{i,t}$, where X is a vector of individual level controls. We rank $\hat{\eta_{i,t}}$ and classify them in bins of 3% of the sample, and then compute the self-employment rate in each of these bins. The pattern we report is robust when we use raw earnings instead of controlling for observables. We use data from the Current Population Survey for the U.S.

distribution, but is higher in the left tail for Mexico, while it is more marked in the right tail of the distribution for the U.S.. Within the bottom 3% of individuals in Mexico the self-employment rate is around 55%, compared to 25% for the median earner. These numbers are 20% and 9% for the U.S.. This configures the first fact we document, namely that self-employment is more prevalent in the tails of the distribution, but the relevance of each tail varies between developed and developing countries. We will use the relationship between self-employment and earnings to inform the Model developed in Section 3.

FIGURE 2. Self-Employment rate by percentile of the earnings distribution - Mexico and the US



We now turn to our second fact, the transition rates across different labor status. Table 2 details the average transition rates in our sample period. We see that, at a quarterly frequency, the transition rates to unemployment are very low, while transitions out of unemployment are high, which translates into the low unemployment rate we already documented. More importantly, there is a large flow from unemployment to self-employment, with a transition rate higher than 25%.

	Worker	Unemployed	Self-Employment
Worker	90.2%	1.7%	**8.1%**
Unemployed	47.1%	26.7%	**26.9%**
Self-Employment	19.2%	2.0%	79.0%

Table 2. Quarterly transition rates

There are many competing hypothesis that are consistent with the unconditional moments we have showed so far. Other than the hypothesis we are proposing there are two main alternatives. The first one is that some individuals have strong preferences for independence at work, and they choose to be self-employed (despite lower income) in exchange for non-monetary rewards that self-employment offers to them. This mechanism has been proposed by Hurst and Pugsley (2016) to explain the patterns of self-employment in the U.S.. The second alternative is unobserved individual-level factors, like differences in job-finding rates that make individuals who are not easily employable to become self-employed. If this is the case, the econometrician would judge self-employment to be an inferior option, because she would not capture the right counterfactual.

If preferences for self-employment were the main driver of the variation in the data, we should observe that people who have external sources of income should be more prone to become self-employed, since they could enjoy the non-pecuniary benefits of work independence without dealing with low income levels. These individuals are in a way less constrainted. We use two variables as proxies for additional sources of income: the presence of a dual-earner (as in Chetty (2008)), and the reception of remittances income from relatives living abroad. Tables 3 and 5 show our results. All the standard errors are clustered at the city level and we include time fixed effects at the year-quarter level to control for aggregate changes in the transition rates, and dummies for educational level to control non-parametrically for education attainment. The tables show a consistent message. Individuals with additional sources of income have lower transition rates to self-employment.

In particular, Table 3 shows the relationship between the transitions out of unemployment and the presence of a dual earner. Individuals with a second earner have probabilities of transition to employment that are 3.2 percentage points higher and transitions to self-employment that are 3.9 percentage points lower (that is a 17% decrease, from 22.2 to

18.3) with respect to individuals without a second order. Table 4 shows that most of the variables related with job-search activities are not significantly different for individuals with and without a second earner. In particular, unemployed workers report being as likely to make plans to start their own business (column 5), to ask personally for a job (column 1), look for a temporary job (column 4) regardless of whether they have a second earner. They are different in that workers with a second earner are 1.5 years older and presumably as a consequence, they make less use of the internet to find a job (column 6).

Table 5 presents the results using remittances as a proxy for the resources of the individual. We see how individuals who receive remittances in times of unemployment transition at lower rates to self-employment. The coefficient, -8 percentage points, is economically significant, considering mean transition rates are 18.8 percentage points in our sub-sample, even after controlling for age and education. We would expect the coefficients in this regression to have an upward bias. It is reasonable to expect that people who need financial help from relatives and friends who live abroad are probably those who have lower probabilities of finding a good job, therefore creating an spurious positive coefficient in column (2) and a negative bias in column (1). The fact that we observe the opposite (people who receive remittances transition less to self-employment) is reassuring, although of course we cannot rule other sources of bias since the reception of remittances is not created by exogenous variation across individuals.

	(1)	(2)	(3)	(4)
	U→E	U→S	U→U	U→I
Second Earner$_{t-1}$	0.032***	-0.039**	0.007	-0.000
	[0.010]	[0.018]	[0.015]	[0.000]
Age	-0.008***	0.003***	0.005***	0.000
	[0.000]	[0.000]	[0.000]	[0.000]
Constant	0.835***	0.209	-0.044	-0.001
	[0.301]	[0.326]	[0.098]	[0.002]
Observations	8376	8376	8376	8376
Mean Dep. Variable	0.505	0.222	0.272	0.000104
Schooling Controls	Yes	Yes	Yes	Yes
Time Fixed Effect	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes

Table 3. Second Earner and Transitions from Unemployment

Note: *The LHS variable is an indicator variable that takes the value of one if individual i experienced the transition specified in each column. U denotes unemployment, E salaried work, S self-employment, I inactivity. The subsample consists of all the individuals in the original sample that were unemployed in period* $t-1$. *Second Earner*$_{t-1}$ *is an indicator variable that takes the value of one if the individual's couple was an income earner in the period in* $t-1$. *The standard errors were clustered at the city level. Schooling controls are a set of dummies for different schooling levels, Time fixed effects are dummies at the year-quarter level. The regressions are run by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Asked	Job Post	Public Ag	Temp	SE Plans	Internet	Newspaper	Need to Work	Age
Second Earner	-0.033	-0.009	-0.006	-0.002	0.002	-0.058***	-0.022	0.000	1.564***
	[0.023]	[0.010]	[0.008]	[0.003]	[0.001]	[0.019]	[0.017]	[0.000]	[0.528]
Constant	0.203***	0.021**	0.020**	0.008***	0.001	0.105***	0.069***	0.000	41.063***
	[0.022]	[0.009]	[0.008]	[0.002]	[0.001]	[0.018]	[0.016]	[.]	[0.497]
Observations	11214	11214	11214	11214	11214	11214	11214	11214	11214

Table 4. Second Earner and Job-Search Activities

Note: *The LHS variable is an indicator variable that takes the value of one if individual i perfomed the given activity to search for a job in the previous quarter. The last two columns correspond to weather or not the individual declares to have a need to work, and differences in age. The subsample consists of all the individuals in the original sample that were unemployed in period* $t-1$. *Second Earner*$_{t-1}$ *is an indicator variable that takes the value of one if the individual's couple was an income earner in the period in* $t-1$. *The standard errors were clustered at the city level. Schooling controls are a set of dummies for different schooling levels, Time fixed effects are dummies at the year-quarter level. The regressions are run by weighted OLS.* *, **, *and* ***, *denote significance at the 10%, 5%, and 1% level.*

	(1)	(2)	(3)	(4)	(5)
	U→E	U→S	U→U	U→I	U→S
Remitances$_{t-1}$	0.058	-0.080***	-0.033	0.055	
	[0.053]	[0.021]	[0.040]	[0.037]	
Age	-0.012***	0.002***	0.002**	0.008***	0.001***
	[0.000]	[0.000]	[0.001]	[0.001]	[0.000]
Latent Remittances					-0.045
					[0.036]
Constant	1.237***	0.147	-0.168***	-0.216	0.177
	[0.262]	[0.202]	[0.050]	[0.227]	[0.114]
Observations	8615	8615	8615	8615	25135
Mean Dep. Variable	0.463	0.188	0.256	0.0932	0.188
Schooling Controls	Yes	Yes	Yes	Yes	Yes
Time Fixed Effect	Yes	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes	Yes

Table 5. Remittances and Transitions from Unemployment

Note: *The LHS variable is an indicator variable that takes the value of one if individual i experienced the transition specified in each column. U denotes unemployment, E salaried work, S self-employment, I inactivity. The subsample consists of all the individuals in the original sample that were unemployed in period $t-1$. Remittances$_{t-1}$ is an indicator variable that takes the value of one if the individual reported to have received remittances in the period in $t-1$. The standard errors were clustered at the city level. Schooling controls are a set of dummies for different schooling levels, Time fixed effects are dummies at the year-quarter level. The regressions are run by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Asked	Job Posting	Public Ag.	Temp	SE Plans	Internet	Newspaper	Need to Work	Age
Receiver	0.183*	0.023	0.004	-0.007***	-0.004**	-0.035***	0.092	0.045	-0.250
	[0.103]	[0.023]	[0.012]	[0.002]	[0.002]	[0.004]	[0.089]	[0.051]	[2.385]
Constant	0.157***	0.007***	0.008***	0.007***	0.004**	0.035***	0.040***	0.043***	43.766***
	[0.007]	[0.002]	[0.002]	[0.002]	[0.002]	[0.004]	[0.004]	[0.004]	[0.212]
Observations	8200	8200	8200	8200	8200	8200	8200	8200	8200

Table 6. Remittances and Job Search Activities

Note: *The LHS variable is an indicator variable that takes the value of one if individual i perfomed the given activity to search for a job in the previous quarter. The last two columns correspond to weather or not the individual declares to have a need to work, and differences in age. The subsample consists of all the individuals in the original sample that were unemployed in period $t-1$. Second Earner$_{t-1}$ is an indicator variable that takes the value of one if the individual's couple was an income earner in the period in $t-1$. The standard errors were clustered at the city level. Schooling controls are a set of dummies for different schooling levels, Time fixed effects are dummies at the year-quarter level. The regressions are run by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

Finally we estimate regressions in a similar spirit of those in Katz and Krueger (2017). The idea is to take the universe of individuals who in period t have a job (either salary workers or self-employed), and check whether the transitions to self-employment are larger for those agents who were unemployed in the previous period. This exercise is different than the previous ones in which we conditioned the sample to those individuals that were unemployed in period $t-1$. Katz and Krueger (2017) using CPS data for the U.S. find that unemployed agents are more likely to transition to an alternative work arrangement job than agents who are employed. Table 7 reports the equivalent results for Mexico. The results are consistent with those of Katz and Krueger (2017). The transition rates of unemployed agents to self-employment are 20.9 percentage points higher than those exhibited by their counterparts who had a salaried job. This result hold after controlling for age, education, and after adding time and city fixed-effects.

	(1)	(2)	(3)	(4)
	SE	SE	SE	SE
U_{t-1}	0.209***	0.209***	0.208***	0.208***
	[0.003]	[0.003]	[0.003]	[0.003]
S_{t-1}	0.717***	0.717***	0.706***	0.706***
	[0.012]	[0.012]	[0.012]	[0.012]
Age			0.002***	0.002***
			[0.000]	[0.000]
Constant	0.080***	0.109***	-0.027	-0.038
	[0.005]	[0.005]	[395.990]	[167.520]
Observations	1033397	1033397	1033397	1033397
Mean Ent	0.285	0.285	0.285	0.285
Schooling Controls	No	No	Yes	Yes
City Fixed Effect	No	No	No	Yes
Time Fixed Effect	No	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes

TABLE 7. Transitions to Self-Employment. The LHS variable is an indicator variable that takes the value of one if the individual is self-employed. U_{t-1} and S_{t-1} are indicator variables that take the value of 1 if the individual was unemployed or self employed in the previous quarter respectively. Age is the age in years. All the standard errors are clustered at the city level. The sample is restricted to those individuals who have a job (either salary work or self-employment) in period t. Schooling controls are a set of dummies by education level to control non-parametrically for education. Time fixed effects are at the year-quarter level. We run the regressions by weighted OLS.

	(1)	(2)	(3)	(4)	(5)
	E	E	E	E	E
S_{t-1}	-0.268***	-0.268***	-0.255***	-0.254***	-0.340***
	[0.021]	[0.021]	[0.020]	[0.020]	[0.014]
Age			-0.006***	-0.006***	-0.004***
			[0.000]	[0.000]	[0.000]
Second Earner					0.022
					[0.018]
$S_{t-1}\times$ Second Earner					0.024**
					[0.011]
Constant	0.463***	0.417***	0.704***	0.684***	0.589
	[0.015]	[0.014]	[0.103]	[0.111]	[1203.540]
Observations	327250	327250	327250	327250	145945
Mean Emp	0.221	0.221	0.221	0.221	0.221
Schooling Controls	No	No	Yes	Yes	Yes
Time Fixed Effect	No	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes	Yes

TABLE 8. Transitions to Employment. The sample is restricted to those individuals who were not employed in period $t-1$ (either unemployed or self-employment). The LHS variable is an indicator variable that takes the value of one if the individual is employed. S_{t-1} is an indicator variable that take the value of 1 if the individual self employed in the previous quarter respectively. Age is the age in years. All the standard errors are clustered at the city level. Schooling controls are a set of dummies by education level to control non-parametrically for education. Time fixed effects are at the year-quarter level. We run the regressions by weighted OLS.

	(1)	(2)	(3)	(4)	(5)
	E	E	E	E	E
S_{t-1}	-0.066***	-0.069***	-0.084***	-0.086***	-0.144***
	[0.014]	[0.014]	[0.017]	[0.017]	[0.050]
Age			-0.009***	-0.009***	-0.007***
			[0.001]	[0.001]	[0.001]
Second Earner					0.035
					[0.057]
$S_{t-1}\times$ Second Earner					-0.024
					[0.058]
Constant	0.383***	0.456***	0.854***	0.863***	1.327***
	[0.017]	[0.064]	[0.226]	[0.228]	[0.075]
Observations	7320	7320	7320	7320	3205
Mean Emp	0.355	0.355	0.355	0.355	0.355
Schooling Controls	No	No	Yes	Yes	Yes
Time Fixed Effect	No	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes	Yes

TABLE 9. Transitions to Employment. The sample of the regression is universe of individuals who in period $t-2$ where unemployed, and in period $t-1$ where not employed. The LHS variable is an indicator variable that takes the value of one if the individual is employed in period t. S_{t-1} is an indicator variable that take the value of 1 if the individual self employed in the previous quarter respectively. Age is the age in years. All the standard errors are clustered at the city level. Schooling controls are a set of dummies by education level to control non-parametrically for education. Time fixed effects are at the year-quarter level. We run the regressions by weighted OLS.

The empirical evidence we have provided so far does not rule out potential explanations based on differences in unobservable characteristics. Problems in terms of people declaring themselves as unemployed when they are inactive would bias down the transitions from unemployment to employment and self-employment. Our balance tables show that the proportion of people that report making plans to start their own business is the same regardless of having a second earner, and the overall proportion of people making plans to start business while unemployed is minimal. On the other hand, assortative matching is an issue we do not have sufficient data to address, and the lack of exogenous variation prevent us from being confident to claim that we have a causal interpretation for the results we are finding.

Due to these concerns we move from variation at the individual level, to use variation at the regional level. We take advantage of the schedule in which a large workfare program was implemented in districts across India. The National Rural Employment Guarantee Act (NREGA) is a program that provides short-term work at market wages in rural India. The program was initially implemented in the poorer districts in India in 2006, and then extended in 2007 and 2008. We use information of the implementation of the program and microdata from The National Sample Survey Office (NSSO) to study the effect on self-employment of the implementation of this large workfare program.

If our mechanism is present in the data, then the creation of public salaried work positions should reduce self-employment, and also would let households spend more time looking for a job, therefore increasing unemployment. On the contrary, if self-employment was driven by preferences towards independence of a subset of the population, we should see an increase in wages in salaried work and no change in the composition of the workforce. The exercise we perform is very similar in spirit to that of Imbert and Papp (2015), who study the effect of this program on the composition of work between the public and private sector. Our exercise checks the effect of the program not by sector, but by occupational choice. Table 10 summarizes the results. The main finding is that in districts where the program was instituted first, the share of time dedicated to self-employment went down. This is true even when controlling for district fixed effects, time fixed effects, and individual-level controls.

The main concern with regressions of the type depicted in Table 10 is a matter of interpretation. If the public jobs that are created in the local markets are comparable to those that already existed, then the fact that self-employment went down is good enough evidence that some of the self-employed in India reveal their preferences for salaried work, and the mechanism we are suggesting has support in the data. However, and although this evidence is very suggestive, it is difficult to assure that the jobs posted under NREGA were not superior to jobs available in the economy prior to the implementation of the program. If NREGA jobs were superior, because they offer higher pay, higher flexibility, higher prestige, etc., then the fact that we see self-employment falling can mean just that individuals are switching to better jobs, not that they were self-employed due to slack in the labor market before the implementation of the program.

	(1) SE	(2) SE	(3) SE	(4) U
NREGA	-0.012	-0.038***	-0.015*	0.010**
	[0.008]	[0.005]	[0.008]	[0.004]
Observations	395662	395662	395662	395662
Avg LHS	0.719	0.719	0.719	0.0516
District Fixed Effect	No	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes
Year-Quarter Fixed Effect	No	No	Yes	Yes
Constant	Yes	Yes	Yes	Yes

Table 10. NREGA is the dif-dif coefficient that takes the value of one for all the district-year-quarter triplets in which the program is active. All the columns cluster the standard errors at the district level. Columns 1 to 3 analyze the effects on self-employment and differ on whether there are district and time fixed effects. Column four runs the same regression of column 3 but using unemployment as the dependent variable.

However, Breza et al. (2017) randomize market-level transitory positive labor demand shocks across Indian villages. They find that self-employment falls without an increase in the market wage. This rules out the preference for self-employment hypothesis, and, due to the randomization, rules out the relevance of unobserved individual characterstics to explain the data. Our findings with the NREGA are compatible with Breza et al. (2017) and show that the results hold beyond the experimental setting.

We have provided evidence from multiple sources which are consistent with our idea. Each of these pieces of evidence

In the next section we develop a model in which more restricted households are prone to self-employment, where the job-finding rates are contingent on current labor status, and where preferences towards self-employment is not the main driver to explain the facts we have established in this section. We will use this model to understand the relevance of this mechanism to explain low productivity in developing economies. We will also use it to study counterfactual worlds in which safety nets are more generous and credit access is less stringent.

3. Model

In this section we describe a quantitative occupational choice model with search frictions. The model extends the baseline macro-development model in Moll (2014) to allow for different occupations, namely employment (E), unemployment (U) and self-employment (S), and job search frictions that prevent agents from finding a job at will. We will estimate the model using the results presented in Section 2 and conduct counterfactuals to determine the effects of different policies, such as unemployment insurance and micro finance on the composition of the economy, aggregate productivity and welfare.

The model economy is populated by a continuum of agents. Time is continuous and goes forever. Agents are heterogenous not only with respect to their occupation $\{E, U, S\}$, but also with respect to their labor efficiency (ϵ), entrepreneurial ability (z), and asset holdings (a). If employed, an agent's labor income is given by we^{ϵ}, where w is the economy wide wage rate. If self-employed, an agent produces final goods using capital and labor. The agent's productivity is given by her entrepreneurial ability. Unemployed agents receive a constant income b.[10]

Any agent can become self-employed after paying an installation cost $\kappa(z)$ that depends on her productivity,[11] or become unemployed at no cost. In contrast an agent requires a job offer in order to become employed. Job offers are assumed to arrive following a Poisson process with arrival rates γ^U and γ^S that depend on whether the agent is unemployed or self-employed. Finally, employed agents are subject to job destruction shocks with arrival rate γ^E. Note that ϵ and z behave as latent states and are kept by the agent regardless of her occupation.

Labor search frictions will play a crucial role in the occupation choice of agents, and differentiate the model from the standard macro-development framework. As shown in Section 1, unemployment risk and a weak safety net (both prominent features of labor markets in

[10]It is possible to think about income while unemployed as coming from home production. We argue against that interpretation in our setup since home-production is already captured as part of the self-employment activities. For agents that have low entrepreneurial ability (z) engaging in self-employment is qualitatively different than engaging in home production.

[11]The relation between installation cost and productivity is similar to the indexing of fixed costs by technology type in Midrigan and Xu (2014) or Buera et al. (2011). It is also related to the findings by Li (2016), who shows evidence of a negative relation between borrowing constraints and productivity for young, unlisted firms in Japan.

developing economies) induce non-monotonicities in the occupational choice of agents. The potential for efficiency or welfare improving policies is rooted in the tight constraints faced by poor agents, who cannot afford to search for a job.

As is standard in the literature, there is limited access to credit markets. Employed and Unemployed agents face a borrowing constraint, with borrowing limit $-\infty < \underline{a} \leq 0$. Self-Employed agents can borrow to obtain productive capital, but they face a borrowing constraint that depends on her assets (which are used as collateral): $k \leq \lambda a$. These borrowing constraints capture information frictions or commitment problems, which we do not model explicitly. See, among others, Cagetti and De Nardi (2006) and Buera et al. (2011) for micro foundations of the borrowing constraint.

Finally, we allow for a "corporate sector", as in Kitao (2008), formed by competitive firms operating a constant returns to scale technology. The firms in this sector face no shocks or financial frictions. The presence of this sector induces an extra source of labor and capital demand.

We will solve for the stationary equilibrium of the model. Appendices A and B discuss the computational implementation of the model's solution.

3.1. *Stochastic Processes*

We assume that labor efficiency, ϵ, and productivity, z, follow independent Ornstein-Uhlenbeck processes:

$$d\epsilon = \mu^{\epsilon}(\epsilon)\, dt + \sigma^{\epsilon}(\epsilon)\, dW^{\epsilon} \qquad dz = \mu^{z}(z)\, dt + \sigma^{z}(z)\, dW^{z} \qquad \text{with: } \mu^{x}(x) = -\mu_x x \qquad \sigma^{x} = \sigma_x$$

For computational reasons we also impose two reflecting barriers on the processes, so that $\epsilon \in [\underline{\epsilon}, \overline{\epsilon}]$ and $z \in [\underline{z}, \overline{z}]$.

3.2. *Agent's Problem*

The problem of an agent depends on her occupation. We discuss the three occupations in turn.

Employed agents receive an income of we^{ϵ} and are subject to job destruction shocks that arrive at a rate γ^{E}. If the shock arrives the agent becomes unemployed. The agent can also choose to become self-employed at any instant. The value for an employed agent that remains employed is given by $\tilde{V}^E$:

$$\begin{aligned}\rho\tilde{V}^E(a,\epsilon,z) &= \max_c u(c) + V_a^E(a,\epsilon,z)\dot{a} + \gamma^E\left(V^U(a,\epsilon,z) - V^E(a,\epsilon,z)\right) + \frac{E\left[dV^E\right]}{dt} \quad (1)\\ \text{s.t.} \quad & \dot{a} = we^{\epsilon} + ra - c\end{aligned}$$

Unemployed agents receive an income of b and are subject to job offers that arrive at a rate γ^U, they are free to reject an offer, depending on their current assets, labor efficiency and productivity. The agent can also choose to become self-employed. The value for an agent that remains unemployed is given by $\tilde{V}^U$:

$$\begin{aligned}\rho\tilde{V}^U(a,\epsilon,z) &= \max_c u(c) + V_a^U(a,\epsilon,z)\dot{a} + \gamma^U\left(\max\left\{V^E(a,\epsilon,z) - V^U(a,\epsilon,z), 0\right\}\right) + \frac{E\left[dV^U\right]}{dt} \quad (2)\\ \text{s.t.} \quad & \dot{a} = b + ra - c\end{aligned}$$

Finally we consider the problem of a self-employed agent who receives income from the profits, $\pi(a,z)$, generated by her productive activities. We allow self-employed agents to receive job offers at a rate γ^S. Upon arrival of an offer the agent is free to reject it. The agent can also choose to become unemployed and stop producing. The value for an agent that continues being self-employed is $\tilde{V}^S$:

$$\begin{aligned}\rho\tilde{V}^S(a,\epsilon,z) &= \max_c u(c) + V_a^S(a,\epsilon,z)\dot{a} + \gamma^S\left(\max\left\{V^E(a,\epsilon,z) - V^S(a,\epsilon,z), 0\right\}\right) + \frac{E\left[dV^S\right]}{dt} \quad (3)\\ \text{s.t.} \quad & \dot{a} = \pi(a,z) + ra - c\end{aligned}$$

In all cases the expected change of the value function, $E[dV^s]$ for $s \in \{E,U,S\}$), follows from Ito's lemma:

$$\frac{E[dV^s]}{dt} = \mu^{\epsilon}(\epsilon)V_{\epsilon}^s(a,\epsilon,z) + \frac{1}{2}(\sigma^{\epsilon}(\epsilon))^2 V_{\epsilon\epsilon}^s(a,\epsilon,z) + \mu^z(z)V_z^s(a,\epsilon,z) + \frac{1}{2}(\sigma^z(z))^2 V_{zz}^s(a,\epsilon,z)$$

When ϵ and z follow an Ornstein-Uhlenbeck process this reduces to:

$$\frac{E[dV^s]}{dt} = -\mu_{\epsilon}\epsilon V_{\epsilon}^s(a,\epsilon,z) + \frac{1}{2}\sigma_{\epsilon}^2 V_{\epsilon\epsilon}^s(a,\epsilon,z) - \mu_z z V_z^s(a,\epsilon,z) + \frac{1}{2}\sigma_z^2 V_{zz}^s(a,\epsilon,z)$$

The optimal consumption decision can be found in all cases from the first order condition of the agent's problem (see Achdou et al. (2017)):

$$\begin{aligned}u'(c) &= V_a^s(a,\epsilon,z)\\ c &= u'^{-1}\left(V_a^s(a,\epsilon,z)\right) \quad (4)\end{aligned}$$

It is only left to account for occupational choice of the agents. At every instant the value of an agent must reflect the upper envelope of the choices she has available. This works akin to a value matching condition in optimal stopping time problems (Stokey, 2009). The following conditions must hold:

$$V^E(a,\epsilon,z) = \max\left\{\tilde{V}^E(a,\epsilon,z), \tilde{V}^U(a,\epsilon,z), \tilde{V}^S(a-\kappa(z),\epsilon,z)\right\} \tag{5}$$

$$V^U(a,\epsilon,z) = \max\left\{\tilde{V}^U(a,\epsilon,z), \tilde{V}^S(a-\kappa(z),\epsilon,z)\right\} \tag{6}$$

$$V^S(a,\epsilon,z) = \max\left\{\tilde{V}^U(a,\epsilon,z), \tilde{V}^S(a,\epsilon,z)\right\} \tag{7}$$

For future reference let $\chi^o_{o'}(a,\epsilon,z)$ be an indicator function for the occupational choice of the agents:

$$\chi^{oU}(a,\epsilon,z) = \begin{cases} 1 & \text{if } V^o(a,\epsilon,z) = \tilde{V}^U(a,\epsilon,z) \\ 0 & \text{otw} \end{cases} \qquad \chi^{oS}(a,\epsilon,z) = \begin{cases} 1 & \text{if } V^o(a,\epsilon,z) = \tilde{V}^S(a-\kappa(z),\epsilon,z) \\ 0 & \text{otw} \end{cases}$$

$$\chi^{SS}(a,\epsilon,z) = \begin{cases} 1 & \text{if } V^S(a,\epsilon,z) = \tilde{V}^S(a,\epsilon,z) \\ 0 & \text{otw} \end{cases}$$

of course $\chi^{oo} = 1$ indicates no change in the agent's status.

3.3. *Self-Employed production technology*

The profits of a self-employed agent are given by:

$$\pi(a,z) = \max_{k\le\lambda a} e^z\left(k^\alpha n^{1-\alpha}\right)^\nu - wn - (r+\delta)k$$

where $\alpha \in (0,1)$ and $\nu \le 1$. The solution to the profit maximization problem when $\nu < 1$ is:

$$n(a,z) = \left(\frac{e^z\nu(1-\alpha)}{w}\right)^{\frac{1}{1-(1-\alpha)\nu}} (k(a,z))^{\frac{\alpha\nu}{1-(1-\alpha)\nu}} \tag{8}$$

$$\pi(a,z) = e^z(k(a,z))^{\nu\alpha}(n(a,z))^{(1-\alpha)\nu} - wn(a,z) - (r+\delta)k(a,z)$$

where capital demand is given by:

$$k(a,z) = \min\left\{\nu^{\frac{1}{1-\nu}} e^{\frac{z}{1-\nu}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{1-(1-\alpha)\nu}{1-\nu}} \left(\frac{1-\alpha}{w}\right)^{\frac{(1-\alpha)\nu}{1-\nu}}, \lambda a\right\} \tag{9}$$

If $\nu = 1$ the solution is:

$$n = \left(\frac{(1-\alpha)\,e^{z}}{w}\right)^{\frac{1}{\alpha}} k \qquad \pi\,(a,z) = \left(\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} e^{\frac{1}{\alpha}z} - (r+\delta)\right) k \qquad k\,(a,z) = \lambda a \mathbf{1}_{\{z \geq \underline{z}\}} \tag{10}$$

where $\underline{z} = \ln\left(\left(\frac{r+\delta}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\right)$ is the minimum value of z for which there is production.

3.4. *Firm's problem*

There is also a (representative) firm that produces using capital and labor using a CRS technology, as in Kitao (2008). This firm faces no financial constraints and rents capital and labor in the market. The firm produces the same (homogeneous) good as the self-employed. The problem of the firm is:

$$\max_{K,N} \quad AK^{\beta}N^{1-\beta} - (r+\delta)\,K - wN$$

The optimality conditions of the firm give:

$$\beta A K^{\beta-1} N^{1-\beta} = r + \delta \qquad (1-\beta)\,AK^{\beta}N^{-\beta} = w \tag{11}$$

Note that the factor demand of the firm are ill-defined, because of the CRS assumption the firm can demand any N as long as $\frac{N}{K} = \frac{1-\beta}{\beta}\frac{r+\delta}{w}$. The value of the firm's labor demand can be obtained as a residual of the labor market condition.

This firm is included to smooth the market clearing conditions. Without it the whole capital and labor demand depends on the agent's occupational choice. The importance of the firm can be changed by means of parameter A. Clearly when $A = 0$ the firm does not operate.

3.5. *Equilibrium*

An stationary equilibrium for this economy is a set of value functions $\left\{V^{s}, \tilde{V}^{s}\right\}_{s\in\{E,U,S\}}$, along with an optimal consumption function $\{c^{s}\}_{s\in\{E,U,S\}}$, labor and capital demand from self-employed $\{n, k\}$ and from the corporate sector $\{N, K\}$, prices $\{r, w\}$ and a distribution of agents for each occupation $\{G^{s}\}_{s\in\{E,U,S\}}$,[12] such that:

[12]Let g^{E}, g^{U} and g^{S} be the density functions of employed, unemployed and self-employed agents in the economy, with:

$$1 = \int\int\int g^{E}\,(a,\epsilon,z) + g^{U}\,(a,\epsilon,z) + g^{S}\,(a,\epsilon,z)\;da\,d\epsilon\,dz$$

So that $\int g^{i} d\,(a,\epsilon,z)$ gives the mass of agents with occupation i.

(1) Value functions are consistent with the agent's optimization. That is, they satisfy equations (1)-(3) and equations (5)-(7).

(2) Consumption (and thus asset accumulation) are consistent with the agent's optimization. That is, it is given by equation (4).

(3) Capital and labor demand solve the self-employed's profit maximization problem. That is, they are given by (8) and (9) if $\nu < 1$, or by (10) if $\nu = 1$.

(4) If $A > 0$ the corporate sector's first order conditions have to be satisfied (equation 11).

(5) Labor market clears $\left(N^D = N^S\right)$, where total labor demand is given by:

$$N^D = \int \left(\frac{(1-\alpha)\, z}{w}\right)^{\frac{1}{\alpha}} \left(\lambda a \mathbf{1}_{\{z \geq \underline{z}\}}\right) dG^S + N \ ,$$

and total labor supply is given by:

$$N^S = \int e^{\epsilon} dG^E \ .$$

(6) Capital market clears $\left(K^D = K^S\right)$, where total demand for capital is given by:

$$K^D = \int k\,(a, z)\, dG^S + K \ ,$$

and total (net) supply of capital is given by:

$$K^S = \int a dG^E + \int a dG^U + \int a dG^S \ .$$

(7) The distribution of agents is stationary. This is obtained if the distributions satisfy the system of Kolmogorov Forward Equations (KFE) defined by:

$$\begin{aligned} 0 = & -\frac{\partial}{\partial a}\left[\dot{a} g^o\,(a, \epsilon, z)\right] + \gamma^o 1_{V^{o'} > V^o}\left(g^{o'}\,(a, \epsilon, z) - g^o\,(a, \epsilon, z)\right) \\ & - \left(1 - \chi^{oo}\right) g^o\,(a, \epsilon, z) + \sum_{o' \neq o} \chi^{o' o} g^o\,(a, \epsilon, z) \\ & - \frac{\partial}{\partial \epsilon}\left[\mu^{\epsilon}\,(\epsilon)\, g^o\,(a, \epsilon, z)\right] + \frac{1}{2}\frac{\partial^2}{\partial \epsilon^2}\left[\left(\sigma^{\epsilon}\,(\epsilon)\right)^2 g^o\,(a, \epsilon, z)\right] \\ & - \frac{\partial}{\partial z}\left[\mu^{z}\,(z)\, g^o\,(a, \epsilon, z)\right] + \frac{1}{2}\frac{\partial^2}{\partial z^2}\left[\left(\sigma^{z}\,(z)\right)^2 g^o\,(a, \epsilon, z)\right] \end{aligned}$$

where o is an occupation, o' in the first line is the occupation to which the agent is sent by the labor market shock (the indicator function $1_{V^{o'} > V^o}$ is only relevant for job offers. The second line captures endogenous changes in occupation, the first term is the mass of agents in occupation o that leave to other occupations, and second term the mass from other occupations that choose to transition to occupation o. The final two lines capture smooth movement due to changes in the stochastic states ϵ and z.

4. Results

We use a preliminary calibration of the model described in Section 3 to test if the mechanism described in Section 1 is present. A complete calibration of the model that will allow us to evaluate policy is in progress.

For this exercise we choose to maintain the model simple. With this in mind we set the fixed cost of becoming a self-employed equal to zero ($\kappa(z) = 0 \, \forall_z$), set depreciation to zero ($\delta = 0$), and we eliminate the corporate sector by setting $A = 0$. More importantly we concentrate on a partial equilibrium or open economy version of our model in which the interest rate is taken as given. We set $r = 0.05$. This assumption is not entirely made for convenience, since we do not think that our economy is able to influence interest rates, which are likely given by the international market. Other parameters are calibrated to usual targets. Appendix C presents the complete list of parameters.

Its worthwhile to mention some relationships between parameters that play an important role in our results. Motivated by the evidence presented in Section 2 we set $\gamma^S < \gamma^U$, so that becoming an entrepreneur is costly for agents in terms of their likelihood of getting a job. We also set $0 < \sigma_\epsilon^2 < \sigma_z^2$ so that income risk is lower for workers than for the self-employed, this relaxes the usual assumption of the macro-development literature where there is no dispersion in labor earnings, and the only source of income risk is engaging in self-employment activities (e.g. Buera et al. (2011)).

Although not directly targeted by our calibration, the model is successful in replicating the U-shaped relationship between self-employment and earnings (Figure 2) as shown in Figure 3. To understand the intuition of this result let's start with the right tail of the distribution. The distribution of labor efficiency units is capped at a finite value, but very productive self-employed agents can scale their businesses as time goes by since productivity is persistent. Therefore, the prevalence of self-employed agents is larger in the right tail of the distribution where they will have high profits. The key implication of this paper compared with the existing literature is the left tail of the distribution. Why would an agent choose to become self-employed if she could be earning a higher wage by being employed? Due to low asset holdings and the lack of insurance, agents who become unemployed move rapidly towards

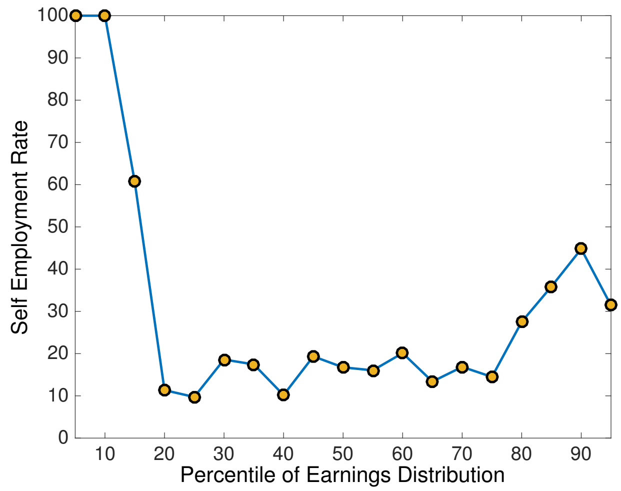


Figure 3. Self-Employment rate by percentile of earnings distribution

their borrowing constraint. It is then when they become self-employed. But because they have low entrepreneurial abilities they generate low profits.

The existing literature is unable to match the shape of the curve. The literature explains the prevalence of self-employment across the income distribution (not just at the top) through agents who, because of preferences or because they are in the process of scaling their businesses, engage in self-employment but do not generate high income. But this explanation does not capture the sharp increase of self-employment rates at the lowest levels of income.

Occupation	Becnhmark
Self Employed	28.49%
Worker	68.47%
Unemployment	3.04%

Table 11. Model Agent Composition

We first solve the model for two different government policies. The first one provides a safety net for unemployed agents through unemployment income $b > 0$. In particular we set the replacement ratio of unemployment benefits to 10 percent of the wage. The second one does away with the safety net and sets $b = 0$. The occupational choice of unemployed agents is presented in Figure 4. The results confirm the basic intuition given in Section 1 and Figure 1.

When $b > 0$, agent's consumption is bounded from below and they opt into self-employment only if they are productive enough, given a level of assets. Moreover, there is a negative relation between the minimum productivity needed to become self-employed and the agent's wealth. Naturally agents with a higher labor efficiency (higher ϵ) are less inclined to become self-employed, requiring a higher productivity given a level of assets.

When $b = 0$, the monotone relation between productivity and assets is broken when assets are sufficiently low. Since agent's have no income while unemployed other than the return on their assets they are unable to tolerate states of unemployment with low assets. They then opt into self-employment as a last resort to obtain income. Importantly, they become more likely to do this the lower their asset level is, increasing the range of productivities for which they become self-employed.

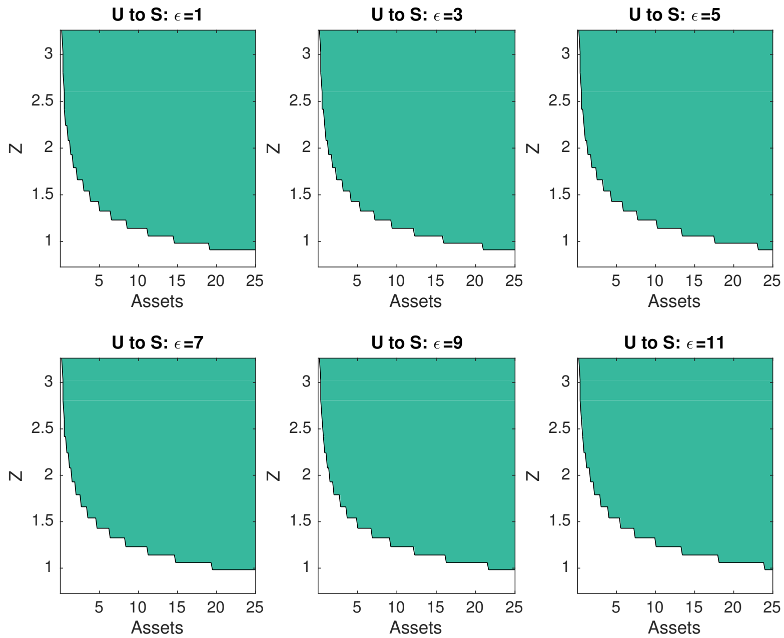
When comparing two economies with different levels of benefits the share of low-asset/low-productivity self-employed is higher for the economy with less generous benefits. This will impact in turnt the pool of workers, and the productivity of the economy. Quantifying these impacts requires a more complete calibration of our model which is currently in progress. In the remaining of this section we provide a preview of the results.

We use our model to study the effects of an increase in unemployment benefits, from a benchmark value of 1% of the wage to 20% of the wage. Before discussing the results we lay out the mechanism by which this increase affects our model. Inherit in the model is a tradeoff of unemployed agents: higher job finding rate vs higher current earnings.

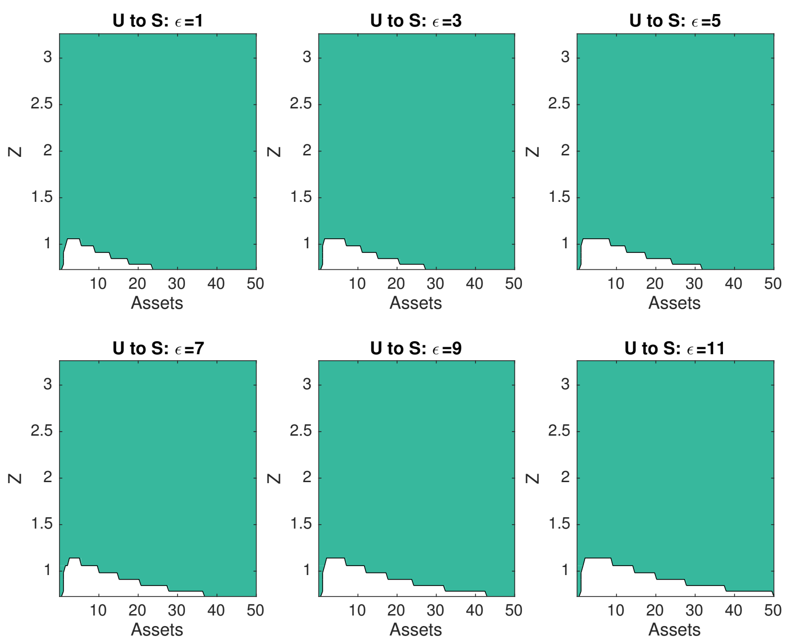
This tradeoff comes from the option all agents have to become self-employed. If they do so they can obtain higher earnings, but the likelihood of finding a job goes down. The decision of the agent will depend on her current assets, current entrepreneurial ability and additional sources of income. If an agent has enough assets or borrowing ability she can tolerate longer unemployment spells without decreasing her consumption. If, on the contrary the agent has not assets, or cannot borrow, she can only consume her current income, in which case waiting for a job can result painful. Agents in the latter situation are more likely to become self-employed. The experiment we conduct is meant to increase the "tolerance to unemployment" of this type of agents by providing them a higher income.

We find that increasing unemployment benefits increases unemployment by 4.2% and reduces SE by 4.1%, with small effects on the employment rate. Yet output and total assets increase (0.20% and 1.97% respectively). This is due to a reallocation of capital to

Figure 4. Occupational Choice: Unemployment to Self-Employment



(A) Positive Unemployment Benefits



(B) No Unemployment Benefits

more productive agents. Even though the fraction of self-employed is reduced by 4.1%, the production of intermediate goods is only reduced in 0.11%, and this is largely driven by our aggregation function which exhibits a love-for-variety effect. Overall welfare increases after the intervention. The results are summarized in tables 12 and 13. Our result shows that low unemployment rates can be a misleading indicator of the functioning of markets in developing economies once one accounts for the occupational choice. Welfare is not monotonic on the unemployment rate, nor is production or productivity. Unemployment in this counterfactual world is increased because agents can wait longer for a match before having to restore to self-employment.

Occupation	Becnhmark	U Insurance
Self Employed	28.49%	24.39%
Worker	68.47%	68.33%
Unemployment	3.04%	7.27%

Table 12. Model Agent Composition - Experiments

Aggregate	U Insurance
Output	0.20%
Intermediate Goods	-0.11%
Labor	0.36%
Total Assets	1.97%
Welfare	2.26%

Table 13. Experiment Effects (Percentage change with respect to the benchmark)

5. Concluding Remarks

We show how the decisions of agents to become self-employed can have aggregate effects on productivity and output, emphasizing the role of low-ability agents in the allocation of resources in an economy. Contrary to previous research which studies the role of financial development to improve the allocation of resources towards high-ability agents that face barriers to accumulate assets and engage in entrepreneurial activities, we highlight the role of constrained agents with low ability in determining who becomes self-employed. We calibrated a heterogeneous agent model with micro data on labor transitions from the main municipalities from Mexico, a representative developing economy. We show how our mechanism is crucial to understand the prevalence of self-employment across the income-distribution of urban Mexico, and how policies that expand safety nets increase output and welfare. We also show this policies increase unemployment in our model economy, a result of the possibility of agents to wait in unemployment for a job-offer instead of moving quickly towards self-employment after a job-loss.

References

Acemoglu, D. and R. Shimer (1999). Efficient unemployment insurance. *Journal of Political Economy 107*(5), 893–928.

Acemoglu, D. and R. Shimer (2000). Productivity gains from unemployment insurance. *European Economic Review 44*(7), 1195 – 1224.

Achdou, Y., J. Han, J.-M. Lasry, P.-L. Lions, and B. Moll (2017, August). Income and wealth distribution in macroeconomics: A continuous-time approach. Working Paper 23732, National Bureau of Economic Research.

Breza, E., S. Kaur, and Y. Shamdasani (2017). Testing for labor rationing: Revealed preference estimates from demand shocks. Unpublished Manuscript.

Buera, F. J., J. P. Kaboski, and Y. Shin (2011, August). Finance and development: A tale of two sectors. *American Economic Review 101*(5), 1964–2002.

Buera, F. J., J. P. Kaboski, and Y. Shin (2012, March). The macroeconomics of microfinance. Working Paper 17905, National Bureau of Economic Research.

Buera, F. J., J. P. Kaboski, and Y. Shin (2015). Entrepreneurship and financial frictions: A macrodevelopment perspective. *Annual Review of Economics 7*(1), 409–436.

Cagetti, M. and M. De Nardi (2006). Entrepreneurship, frictions, and wealth. *Journal of Political Economy 114*(5), 835–870.

Chetty, R. (2008). Moral hazard versus liquidity and optimal unemployment insurance. *Journal of Political Economy 116*(2), 173–234.

Hurst, E. G. and B. W. Pugsley (2016, October). Wealth, Tastes, and Entrepreneurial Choice. In J. Haltiwanger, E. Hurst, J. Miranda, and A. Schoar (Eds.), *Measuring Entrepreneurial Businesses: Current Knowledge and Challenges*, NBER Chapters, Chapter 3, pp. 111–151. National Bureau of Economic Research, Inc.

Imbert, C. and J. Papp (2015, April). Labor market effects of social programs: Evidence from india's employment guarantee. *American Economic Journal: Applied Economics 7*(2), 233–63.

Katz, L. F. and A. B. Krueger (2017, May). The role of unemployment in the rise in alternative work arrangements. *American Economic Review 107*(5), 388–92.

Kitao, S. (2008). Entrepreneurship, taxation and capital investment. *Review of Economic Dynamics 11*(1), 44 – 69.

Li, H. (2016). Leverage and productivity. Unpublished Manuscript.

Midrigan, V. and D. Y. Xu (2014, February). Finance and misallocation: Evidence from plant-level data. *American Economic Review 104*(2), 422–58.

Moll, B. (2014, October). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review 104*(10), 3186–3221.

OECD (2012). *Entrepreneurship at a Glance 2012*. OECD Publishing.

Paulson, A. and R. Townsend (2005). Financial constraints and entrepreneurship: Evidence from the thai financial crisis. *Economic Perspectives, Federal Reserve Bank of Chicago 29*(3), 34–48.

Poschke, M. (2013a). Entrepreneurs out of necessity: a snapshot. *Applied Economics Letters 20*(7), 658–663.

Poschke, M. (2013b). Who becomes an entrepreneur? labor market prospects and occupational choice. *Journal of Economic Dynamics and Control 37*(3), 693 – 710.

Stokey, N. L. (2009). *The Economics of Inaction: Stochastic Control Models with Fixed Costs*. Princeton University Press.

Appendix A. Solution Method - Uniform Grid

A.1. *Solution to HJB equations*

The model is solved using an implicit finite difference as the one shown in Achdou et al. (2017). Consider grids over assets, labor efficiency and entrepreneurial ability:

$$\vec{a} = [a_1, \ldots, a_{n_a}] \qquad \vec{\epsilon} = [\epsilon_1, \ldots, \epsilon_{n_\epsilon}] \qquad \vec{z} = [z_1, \ldots, z_{n_z}]$$

with n_a, n_ϵ and n_z elements respectively, and constant distance between grid points of Δa, $\Delta \epsilon$ and Δz. Let i denote the index of the asset dimension, j of the labor efficiency, and k of the entrepreneurial ability.

We assume that the cost of becoming a self-employed $(\kappa(z))$ can be represented exactly as: $\kappa(z_k) = \kappa_k = l_k \Delta a$, where $l_k \in \mathbb{N}$.

Denote $V^o_{ijk} = V^o(a_i, \epsilon_j, z_k)$ and let the backward and forward difference of the value function approximate the derivative:

$$V^o_a(a_i, \epsilon_j, z_k) \approx \frac{V^o_{i+1,jk} - V^o_{ijk}}{\Delta a} = \partial_a V^o_{ijk,F} \qquad V^o_a(a_i, \epsilon_j, z_k) \approx \frac{V^o_{ijk} - V^o_{i-1,jk}}{\Delta a} = \partial_a V^o_{ijk,B}$$

A similar definition is used for $\partial_\epsilon V^o_{ijk,F}$ and $\partial_z V^o_{ijk,F}$, and the backward differences. The second difference with respect to ϵ and z are defined as:

$$V^o_{\epsilon\epsilon}(a_i, \epsilon_j, z_k) \approx \frac{V^o_{i,j+1,k} - 2V^o_{ijk} + V^o_{i,j-1,k}}{(\Delta\epsilon)^2} = \partial_{\epsilon\epsilon} V^o_{ijk}$$

$$V^o_{zz}(a_i, \epsilon_j, z_k) \approx \frac{V^o_{i,j,k+1} - 2V^o_{ijk} + V^o_{i,j,k-1}}{(\Delta z)^2} = \partial_{zz} V^o_{ijk}$$

The problem to solve is (in general):

$$\begin{aligned}\rho V^o_{ijk} = u(c_{ijk}) + \partial_a V^o_{ij}\left(y^o_{ijk} + ra_i - c_{ijk}\right) + \tilde{\gamma}^o_{ijk}\left(V^{o'}_{ijk} - V^o_{ijk}\right) \\ - \mu_\epsilon \epsilon_j \partial_\epsilon V^o_{ijk} + \frac{1}{2}\sigma^2_\epsilon \partial_{\epsilon\epsilon} V^o_{ijk} - \mu_z z_k \partial_z V^o_{ijk} + \frac{1}{2}\sigma^2_z \partial_{zz} V^o_{ijk}\end{aligned}$$

The implicit method works as VFI, solving the following equation on $V^{o,n+1}_{ijk}$ given a value for $V^{o,n}_{ijk}$:

$$\begin{aligned}\frac{V^{o,n+1}_{ijk} - V^{o,n}_{ijk}}{\Delta} + \rho V^{o,n+1}_{ijk} = u\left(c^{o,n}_{ijk}\right) + \partial_a V^{o,n+1}_{ijk}\left(y^o_{ijk} + ra_i - c^{o,n}_{ijk}\right) + \tilde{\gamma}^{o,n}_{ijk}\left(V^{o',n+1}_{ijk} - V^{o,n+1}_{ijk}\right) \\ - \mu_\epsilon \epsilon_j \partial_\epsilon V^{o,n+1}_{ijk} + \frac{1}{2}\sigma^2_\epsilon \partial_{\epsilon\epsilon} V^{o,n+1}_{ijk} - \mu_z z_k \partial_z V^{o,n+1}_{ijk} + \frac{1}{2}\sigma^2_z \partial_{zz} V^{o,n+1}_{ijk}\end{aligned}$$

Note that the (known) value at iteration n is used to compute consumption, and the drift of the assets, which we will call savings for convenience:

$$s^{o,n}_{ijk} = y^o_{ijk} + ra_i - c^{o,n}_{ijk} \qquad \text{where} \qquad c^{o,n}_{ijk} = u'^{-1}\left(\partial_a V^{o,n}_{ijk}\right)$$

It is also used to define if the agent is willing to change after a job offer. We have:

$$\tilde{\gamma}^{U,n}_{ijk} = \gamma^U 1_{V^{E,n}_{ijk} > V^{U,n}_{ijk}} \qquad \tilde{\gamma}^{S,n}_{ijk} = \gamma^S 1_{V^{E,n}_{ijk} > V^{U,n}_{ijk}} \qquad \tilde{\gamma}^{E,n}_{ijk} = \gamma^E$$

It is possible to solve for $V^{o,n+1}$, but it is first necessary to determine whether to use the forward or backward approximation to the first derivatives of the value function. The "upwind scheme" presented in Achdou et al. (2017) is used for this.

Since consumption can be defined with the backward or forward difference approximation we get:

$$s^{o,n}_{ijk,B} = y^o_{ijk} + ra_i - u'^{-1}\left(\partial_a V^{o,n}_{ijk,B}\right) \qquad s^{o,n}_{ijk,F} = y^o_{ijk} + ra_i - u'^{-1}\left(\partial_a V^{o,n}_{ijk,F}\right)$$

The idea is to use the backward difference when the implied drift is negative, and the forward difference when the drift is positive. Yet there are cases for which $s^{o,n}_{ijk,F} < 0 < s^{o,n}_{ijk,B}$, in these cases we set savings equal to zero, so the derivative is not used, in any case the FOC of the problem gives the exact derivate of the value function as: $\partial_a \overline{V}^{o,n}_{ijk} = u'\left(y_{jk} + ra_i\right)$.

The equation is then:

$$\begin{aligned}
\frac{V^{o,n+1}_{ijk} - V^{o,n}_{ijk}}{\Delta} + \rho V^{o,n+1}_{ijk} = u\left(c^{o,n}_{ijk}\right) + \tilde{\gamma}^{o,n}_{ijk}\left(V^{o',n+1}_{ijk} - V^{o,n+1}_{ijk}\right) + \\
+ \partial_a V^{o,n+1}_{ijk,B}\left[s^{o,n}_{ijk,B}\right]^- + \partial_a V^{o,n+1}_{ijk,F}\left[s^{o,n}_{ijk,F}\right]^+ \\
- \mu_\epsilon \epsilon_j \left(\partial_\epsilon V^{o,n+1}_{ijk,B} 1_{\epsilon_j>0} + \partial_\epsilon V^{o,n+1}_{ijk,F} 1_{\epsilon_j<0}\right) + \frac{1}{2}\sigma^2_\epsilon \partial_{\epsilon\epsilon} V^{o,n+1}_{ijk} \\
- \mu_z z_k \left(\partial_z V^{o,n+1}_{ijk,B} 1_{z_k>0} + \partial_z V^{o,n+1}_{ijk,F} 1_{z_k<0}\right) + \frac{1}{2}\sigma^2_z \partial_{zz} V^{o,n+1}_{ijk}
\end{aligned}$$

Replacing by the definitions of backward and forward derivatives:

$$\begin{aligned}
\frac{V^{o,n+1}_{ijk} - V^{o,n}_{ijk}}{\Delta} + \rho V^{o,n+1}_{ijk} = u\left(c^{o,n}_{ijk}\right) + \tilde{\gamma}^{o,n}_{ijk}\left(V^{o',n+1}_{ijk} - V^{o,n+1}_{ijk}\right) + \\
+ \frac{V^o_{ijk} - V^o_{i-1,jk}}{\Delta a}\left[s^{o,n}_{ijk,B}\right]^- + \frac{V^o_{i+1,jk} - V^o_{ijk}}{\Delta a}\left[s^{o,n}_{ijk,F}\right]^+ \\
- \mu_\epsilon \epsilon_j \left(\frac{V^o_{ijk} - V^o_{i,j-1,k}}{\Delta\epsilon} 1_{\epsilon_j>0} + \frac{V^o_{i,j+1,k} - V^o_{ijk}}{\Delta\epsilon} 1_{\epsilon_j<0}\right) + \frac{1}{2}\sigma^2_\epsilon \frac{V^o_{i,j+1,k} - 2V^o_{ijk} + V^o_{i,j-1,k}}{(\Delta\epsilon)^2} \\
- \mu_z z_k \left(\frac{V^o_{ijk} - V^o_{ij,k-1}}{\Delta z} 1_{z_k>0} + \frac{V^o_{ij,k+1} - V^o_{ijk}}{\Delta z} 1_{z_k<0}\right) + \frac{1}{2}\sigma^2_z \frac{V^o_{i,j,k+1} - 2V^o_{ijk} + V^o_{i,j,k-1}}{(\Delta z)^2}
\end{aligned}$$

Grouping terms we get:

$$\frac{V_{ijk}^{o,n+1}-V_{ijk}^{o,n}}{\Delta}+\rho V_{ijk}^{o,n+1}=u\left(c_{ijk}^{o,n}\right)+\tilde{\gamma}_{ijk}^{o,n}V_{ijk}^{o',n+1}+\left(\begin{array}{c}\frac{\left[s_{ijk,B}^{o,n}\right]^{-}}{\Delta a}-\frac{\left[s_{ijk,F}^{o,n}\right]^{+}}{\Delta a}-\tilde{\gamma}_{ijk}^{o,n}\\ -\frac{\mu_\epsilon\epsilon_j}{\Delta\epsilon}\left(1_{\epsilon_j>0}-1_{\epsilon_j<0}\right)-\frac{\sigma_\epsilon^2}{(\Delta\epsilon)^2}\\ -\frac{\mu_z z_k}{\Delta z}\left(1_{z_k>0}-1_{z_k<0}\right)-\frac{\sigma_z^2}{(\Delta z)^2}\end{array}\right)V_{ijk}^{o}$$

$$-\frac{\left[s_{ijk,B}^{o,n}\right]^{-}}{\Delta a}V_{i-1,jk}^{o}+\frac{\left[s_{ijk,F}^{o,n}\right]^{+}}{\Delta a}V_{i+1,jk}^{o}$$

$$+\left(\frac{\mu_\epsilon\epsilon_j}{\Delta\epsilon}1_{\epsilon_j>0}+\frac{\sigma_\epsilon^2}{2\left(\Delta\epsilon\right)^2}\right)V_{i,j-1,k}^{o}+\left(-\frac{\mu_\epsilon\epsilon_j}{\Delta\epsilon}1_{\epsilon_j<0}+\frac{\sigma_\epsilon^2}{2\left(\Delta\epsilon\right)^2}\right)V_{i,j+1,k}^{o}$$

$$+\left(\frac{\mu_z z_k}{\Delta z}1_{z_k>0}+\frac{\sigma_z^2}{2\left(\Delta z\right)^2}\right)V_{ij,k-1}^{o}+\left(-\frac{\mu_z z_k}{\Delta z}1_{z_k<0}+\frac{\sigma_z^2}{2\left(\Delta z\right)^2}\right)V_{i,j,k+1}^{o}$$

We now name the coefficients for ease of use:

$$\frac{V_{ijk}^{o,n+1}-V_{ijk}^{o,n}}{\Delta}+\rho V_{ijk}^{o,n+1}=u\left(c_{ijk}^{o,n}\right)+\tilde{\gamma}_{ijk}^{o,n}V_{ijk}^{o',n+1}+\left(\tilde{x}_{ijk}+\tilde{y}_j+\tilde{w}_k\right)V_{ijk}^{o,n+1}$$

$$+\tilde{x}_{ijk}^{-}V_{i-1,jk}^{o,n+1}+\tilde{x}_{ijk}^{+}V_{i+1,jk}^{o,n+1}+\tilde{y}_j^{-}V_{i,j-1,k}^{o,n+1}+\tilde{y}_j^{+}V_{i,j+1,k}^{o,n+1}+\tilde{w}_k^{-}V_{ij,k-1}^{o,n+1}+\tilde{w}_k^{+}V_{i,j,k+1}^{o,n+1}$$

where:

$$\tilde{x}_{ijk}=\frac{\left[s_{ijk,B}^{o,n}\right]^{-}}{\Delta a}-\frac{\left[s_{ijk,F}^{o,n}\right]^{+}}{\Delta a}-\tilde{\gamma}_{ijk}^{o,n}$$

$$\tilde{x}_{ijk}^{-}=-\frac{\left[s_{ijk,B}^{o,n}\right]^{-}}{\Delta a}$$

$$\tilde{x}_{ijk}^{+}=\frac{\left[s_{ijk,F}^{o,n}\right]^{+}}{\Delta a}$$

$$\begin{aligned}\tilde{y}_j &= -\tfrac{\mu_\epsilon\epsilon_j}{\Delta\epsilon}\left(1_{\epsilon_j>0}-1_{\epsilon_j<0}\right)-\tfrac{\sigma_\epsilon^2}{(\Delta\epsilon)^2} & \tilde{w}_k &= -\tfrac{\mu_z z_k}{\Delta z}\left(1_{z_k>0}-1_{z_k<0}\right)-\tfrac{\sigma_z^2}{(\Delta z)^2}\\ \tilde{y}_j^{-} &= \tfrac{\mu_\epsilon\epsilon_j}{\Delta\epsilon}1_{\epsilon_j>0}+\tfrac{\sigma_\epsilon^2}{2(\Delta\epsilon)^2} & \tilde{w}_k^{-} &= \tfrac{\mu_z z_k}{\Delta z}1_{z_k>0}+\tfrac{\sigma_z^2}{2(\Delta z)^2}\\ \tilde{y}_j^{+} &= -\tfrac{\mu_\epsilon\epsilon_j}{\Delta\epsilon}1_{\epsilon_j<0}+\tfrac{\sigma_\epsilon^2}{2(\Delta\epsilon)^2} & \tilde{w}_k^{+} &= -\tfrac{\mu_z z_k}{\Delta z}1_{z_k<0}+\tfrac{\sigma_z^2}{2(\Delta z)^2}\end{aligned}$$

A.1.1. *Boundary Conditions*

A final loose end before writing up the linear system in matrix form is what to do with the boundaries of the different grids. At the lower boundary of the asset grid the agent is subject to a no-borrowing constraint. Hence it has to be the case that the agent does not try to borrow. The drift has to be non-negative at that point, which implies that $\tilde{x}_{1jk}^{-}=0$ for all (j,k). At the upper boundary a similar constraint can be imposed, so that $\tilde{x}_{n_a jk}^{+}=0$. This should arise naturally if the upper boundary is high enough. Notice that imposing these restrictions implies that V_{0j}^{n+1} and $V_{n_a+1,j}^{n+1}$ are not part of the system.

The barriers on ϵ and z imply that:

$$V^o_{\epsilon^-}\left(a,\underline{\epsilon},z\right)=V^o_{\epsilon^+}\left(a,\bar{\epsilon},z\right)=V^o_{z^-}\left(a,\epsilon,\underline{z}\right)=V^o_{z^+}\left(a,\epsilon,\bar{z}\right)=0$$

that is, the negative (and positive) drift of the value function at the lower (and upper) boundary must be zero. In terms of our approximation that is:

$$\partial_\epsilon V^o_{i1k,B}=\frac{V^o_{i1k}-V^o_{i0k}}{\Delta\epsilon}=0 \qquad \partial_\epsilon V^o_{in_\epsilon k,F}=\frac{V^o_{i,n_\epsilon+1,k}-V^o_{in_\epsilon k}}{\Delta\epsilon}=0$$

$$\partial_z V^o_{ij1,B}=\frac{V^o_{ij1}-V^o_{ij0}}{\Delta z}=0 \qquad \partial_z V^o_{ijn_z,F}=\frac{V^o_{ij,n_z+1}-V^o_{ijn_z}}{\Delta z}=0$$

This in turn implies that:

$$V^o_{i0k}=V^o_{i1k} \qquad V^o_{i,n_\epsilon+1,k}=V^o_{in_\epsilon k}$$

$$V^o_{ij0}=V^o_{ij1} \qquad V^o_{ij,n_z+1}=V^o_{ijn_z}$$

Using these results we can write the HJB equation at the boundaries:

$$\begin{aligned}\frac{V^{o,n+1}_{i1k}-V^{o,n}_{i1k}}{\Delta}+\rho V^{o,n+1}_{i1k}&=u\left(c^{o,n}_{i1k}\right)+\tilde{\gamma}^{o,n}_{ijk}V^{o',n+1}_{i1k}+\left(\tilde{x}_{i1k}+\tilde{y}_1+\tilde{\mathbf{y}}^-_{\mathbf{1}}+\tilde{w}_k\right)V^o_{i1k}\\&+\tilde{x}^-_{i1k}V^o_{i-1,1k}+\tilde{x}^+_{i1k}V^o_{i+1,1k}+\tilde{y}^+_1V^o_{i,2,k}+\tilde{w}^-_kV^o_{i1,k-1}+\tilde{w}^+_kV^o_{i,1,k+1}\end{aligned}$$

$$\begin{aligned}\frac{V^{o,n+1}_{in_\epsilon k}-V^{o,n}_{in_\epsilon k}}{\Delta}+\rho V^{o,n+1}_{in_\epsilon k}&=u\left(c^{o,n}_{in_\epsilon k}\right)+\tilde{\gamma}^{o,n}_{ijk}V^{o',n+1}_{in_\epsilon k}+\left(\tilde{x}_{in_\epsilon k}+\tilde{y}_{n_\epsilon}+\tilde{\mathbf{y}}^+_{\mathbf{n}_\epsilon}+\tilde{w}_k\right)V^o_{in_\epsilon k}\\&+\tilde{x}^-_{in_\epsilon k}V^o_{i-1,n_\epsilon k}+\tilde{x}^+_{in_\epsilon k}V^o_{i+1,n_\epsilon k}+\tilde{y}^-_{n_\epsilon}V^o_{i,n_\epsilon-1,k}+\tilde{w}^-_kV^o_{in_\epsilon,k-1}+\tilde{w}^+_kV^o_{i,n_\epsilon,k+1}\end{aligned}$$

$$\begin{aligned}\frac{V^{o,n+1}_{ij1}-V^{o,n}_{ij1}}{\Delta}+\rho V^{o,n+1}_{ij1}&=u\left(c^{o,n}_{ij1}\right)+\tilde{\gamma}^{o,n}_{ijk}V^{o',n+1}_{ij1}+\left(\tilde{x}_{ij1}+\tilde{y}_j+\tilde{w}_1+\tilde{\mathbf{w}}^-_{\mathbf{1}}\right)V^o_{ij1}\\&+\tilde{x}^-_{ij1}V^o_{i-1,j1}+\tilde{x}^+_{ij1}V^o_{i+1,j1}+\tilde{y}^-_jV^o_{i,j-1,1}+\tilde{y}^+_jV^o_{i,j+1,1}+\tilde{w}^+_1V^o_{i,j,1+1}\end{aligned}$$

$$\begin{aligned}\frac{V^{o,n+1}_{ijn_z}-V^{o,n}_{ijn_z}}{\Delta}+\rho V^{o,n+1}_{ijn_z}&=u\left(c^{o,n}_{ijn_z}\right)+\tilde{\gamma}^{o,n}_{ijk}V^{o',n+1}_{ijn_z}+\left(\tilde{x}_{ijn_z}+\tilde{y}_j+\tilde{w}_{n_z}+\tilde{\mathbf{w}}^+_{\mathbf{n_z}}\right)V^o_{ijn_z}\\&+\tilde{x}^-_{ijn_z}V^o_{i-1,jn_z}+\tilde{x}^+_{ijn_z}V^o_{i+1,jn_z}+\tilde{y}^-_jV^o_{i,j-1,n_z}+\tilde{y}^+_jV^o_{i,j+1,n_z}+\tilde{w}^-_{n_z}V^o_{ij,n_z-1}\end{aligned}$$

A.1.2. *System Solution*

The equations above describe a system of $n_a \times n_\epsilon \times n_z \times 3$ equations, its best to define the value function a stack of three value functions, one for each occupation:

$$V = \left[V^U; V^S; V^E\right]^T$$
$$V^o = \text{vec}\left[V^o_{ijk}\right]$$

The system is:

$$\frac{1}{\Delta}\left(V^{n+1} - V^n\right) + \rho V^{n+1} = u^n + A^n V^{n+1}$$

where n^n is the stacked version of: $u^n = \left[u^{U,n}; u^{S,n}; u^{E,n}\right]$ and $u^{o,n} = \text{vec}\left[u\left(c^{o,n}_{ijk}\right)\right]$ and consumption is computed as:

$$c^{o,n}_{ijk} = u^{'-1}\left(\partial_a V^{o,n}_{ijk,B}\right) 1_{s^{o,n}_{ijk,B}<0} + u^{'-1}\left(\partial_a V^{o,n}_{ijk,F}\right) 1_{s^{o,n}_{ijk,F}>0} + \left(y^o_{jk} + ra_i\right) 1_{s^{o,n}_{ijk,F}<0<s^{o,n}_{ijk,B}}$$

(note that at the boundaries there is no backward or forward drift correspondingly).

Matrix A^n is given by:

$$A^n = B^n + C + D$$

$$B^n = \begin{bmatrix} B^n_{UU} & \mathbf{0} & B^n_{UE} \\ \mathbf{0} & B^n_{SS} & B^n_{SE} \\ B^n_{EU} & \mathbf{0} & B^n_{EE} \end{bmatrix} \qquad C = \begin{bmatrix} \tilde{C} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tilde{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \tilde{C} \end{bmatrix} \qquad D = \begin{bmatrix} \tilde{D} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tilde{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \tilde{D} \end{bmatrix}$$

The matrices B^n_{oo} are sparse and they only contain elements in the diagonal, upper diagonal and lower diagonal. Consider $X_o = \left[\tilde{x}^o_{ijk}\right]$, $X^-_o = \left[\tilde{x}^{o,-}_{ijk}\right]$ and $X^+_o = \left[\tilde{x}^{o,+}_{ijk}\right]$, all three dimensional matrix that contain the coefficients $\tilde{x}$ (note that $\tilde{x}$ is already adjusted for the boundaries). Then we have: $\text{diag}\left(B^n_{oo}\right) = \text{vec}\left(X_o\right)$, $\text{diag}^+\left(B^n_{oo}\right) = \text{vec}\left(X^+_o\right)$ and $\text{diag}^-\left(B^n_{oo}\right) = \text{vec}\left(X^-_o\right)$, where the upper diagonal and lower diagonal are adjusted not to include the zero terms of the boundaries. The matrices $B^n_{oo'} = \gamma^o I$.

Matrices $\tilde{C}$ and $\tilde{D}$ are also sparse. Their construction takes advantage of the fact that the elements of $\tilde{C}$ only vary with j and the elements of $\tilde{D}$ only vary with k. We first construct

$\tilde{C}$. Consider a sparse matrix Ω of order $n_\epsilon \times n_\epsilon$, such that:

$$\Omega_{1,1} = \tilde{y}_1 + \tilde{y}_1^-$$
$$\Omega_{j,j} = \tilde{y}_j$$
$$\Omega_{n_\epsilon,n_\epsilon} = \tilde{y}_{n_\epsilon} + \tilde{y}_{n_\epsilon}^+$$
$$\Omega_{j,j+1} = \tilde{y}_1^+$$
$$\Omega_{j-1,j} = \tilde{y}_1^-$$

$$\Omega = \begin{bmatrix} \tilde{y}_1 + \tilde{y}_1^- & \tilde{y}_1^+ & & & & \\ \tilde{y}_2^- & \tilde{y}_2 & \tilde{y}_2^+ & & & \\ & \ddots & \ddots & \ddots & & \\ & & \tilde{y}_{n_\epsilon-1}^- & \tilde{y}_{n_\epsilon-1} & \tilde{y}_{n_\epsilon-1}^+ & \\ & & & & \tilde{y}_{n_\epsilon-1}^+ & \tilde{y}_{n_\epsilon} + \tilde{y}_{n_\epsilon}^+ \end{bmatrix}$$

Then we can get $\tilde{C} = I_{n_z} \otimes (\Omega \otimes I_{n_a})$.

For $\tilde{D}$ the first part of the construction is equivalent, with Ω being built with coefficients $\tilde{w}$ instead of $\tilde{y}$. Then $\tilde{D} = (\Omega \otimes I_{n_a n_\epsilon})$.

This problem can now be expressed as:

$$T^n V^{n+1} = t^n$$

where:

$$T^n = \left(\frac{1}{\Delta} + \rho\right) I_{3n_a n_\epsilon n_z} - A^n \qquad t^n = u^n + \frac{1}{\Delta} V^n$$

A.1.3. *Algorithm*

(1) Compute matrices C and D. These matrices do not change with equilibrium prices or iterations.
(2) Take as given w, r.
(3) Solve for earnings in each state: y^o_{ijk} for each combination of (a, ϵ, z) and occupation. These values don't change with iterations.
(4) Guess a value for V^n, a $3n_a n_\epsilon n_z$ vector. It is easier to store it as three separate matrices of dimension $n_a \times n_\epsilon \times n_z$.

(5) Compute the backward and forward drift: $s^{o,n}_{ijk,B}$ and $s^{o,n}_{ijk,F}$ for $i = \{2, \ldots, n_a\}$ and $i = \{1, \ldots, n_a - 1\}$ respectively, and all (j, k, o).

$$s^{o,n}_{ijk,B} = y^o_{jk} + ra_i - u^{'-1}\left(\partial_a V^{o,n}_{ijk,B}\right) \qquad s^{o,n}_{ijk,F} = y^o_{jk} + ra_i - u^{'-1}\left(\partial_a V^{o,n}_{ijk,F}\right)$$

These values are stored in six matrices of dimensions $n_a \times n_\epsilon \times n_z$ (two per occupation, one with backward drift and the other one with forward drift).

(6) For all (i, j, k, o) compute consumption as:

$$c^{o,n}_{ijk} = u^{'-1}\left(\partial_a V^{o,n}_{ijk,B}\right) 1_{s^{o,n}_{ijk,B}<0} + u^{'-1}\left(\partial_a V^{o,n}_{ijk,F}\right) 1_{s^{o,n}_{ijk,F}>0} + \left(y^o_{jk} + ra_i\right) 1_{s^{o,n}_{ijk,F}<0<s^{o,n}_{ijk,B}}$$

These values are stored in three matrices of dimensions $n_a \times n_\epsilon \times n_z$.

(7) Compute the utility vector as: $u^n = \left[u^{U,n}; u^{S,n}; u^{E,n}\right]$ and $u^{o,n} = \text{vec}\left[u\left(c^{o,n}_{ijk}\right)\right]$.

(8) Compute the adjusted shock arrival rates:

$$\tilde{\gamma}^{U,n}_{ijk} = \gamma^U 1_{V^{E,n}_{ijk} > V^{U,n}_{ijk}} \qquad \tilde{\gamma}^{S,n}_{ijk} = \gamma^S 1_{V^{E,n}_{ijk} > V^{U,n}_{ijk}} \qquad \tilde{\gamma}^{E,n}_{ijk} = \gamma^E$$

(9) Compute the three $n_a \times n_\epsilon \times n_z$ matrices $X_o = \left[\tilde{x}^o_{ijk}\right]$, $X^-_o = \left[\tilde{x}^{o,-}_{ijk}\right]$ and $X^+_o = \left[\tilde{x}^{o,+}_{ijk}\right]$.

(10) Compute matrix $B^n = \begin{bmatrix} B^n_{UU} & \mathbf{0} & B^n_{UE} \\ \mathbf{0} & B^n_{SS} & B^n_{SE} \\ B^n_{EU} & \mathbf{0} & B^n_{EE} \end{bmatrix}$, where $\text{diag}\left(B^n_{oo}\right) = \text{vec}\left(X_o\right)$, $\text{diag}^+\left(B^n_{oo}\right) = \text{vec}\left(X^+_o\right)$ and $\text{diag}^-\left(B^n_{oo}\right) = \text{vec}\left(X^-_o\right)$, where the upper diagonal and lower diagonal are adjusted not to include the zero terms of the boundaries. The matrices $B^n_{oo'} = \gamma^o I$.

(11) Compute the matrix $A^n = B^n + C + D$.

(12) Compute the matrix T and vector t:

$$T^n = \left(\frac{1}{\Delta} + \rho\right) I_{3n_a n_\epsilon n_z} - A^n \qquad t^n = u^n + \frac{1}{\Delta} V^n$$

(13) Compute $V^{n+1/2}$ as:

$$V^{n+1/2} = \left(T^n\right)^{-1} t^n$$

(14) Divide the vector $V^{n+1/2}$ into three matrices of $n_a \times n_\epsilon \times n_z$: $V^{U,n+1/2}$, $V^{S,n+1/2}$, and $V^{E,n+1/2}$.

(15) Compute the adjusted self-employed value, taking into account the cost of becoming a self-employed:

$$\tilde{V}^S_{ijk} = \begin{cases} -\infty & i \le l_k \\ V^{S,n+1/2}_{i-l_k,j,k} & \text{otw} \end{cases}$$

(16) Compute $V^{U,n+1}$, $V^{S,n+1}$, and $V^{E,n+1}$ as follows:

$$
\begin{aligned}
V_{ijk}^{U,n+1} &= \max\left\{V_{ijk}^{U,n+1/2}, \tilde{V}_{ijk}^{S}\right\} \\
V_{ijk}^{S,n+1} &= \max\left\{V_{ijk}^{U,n+1/2}, V_{ijk}^{S,n+1/2}\right\} \\
V_{ijk}^{E,n+1} &= \max\left\{V_{ijk}^{U,n+1/2}, \tilde{V}_{ijk}^{S}, V_{ijk}^{E,n+1/2}\right\}
\end{aligned}
$$

(a) Define the following matrices as indicators of the occupation choice: $\left[\tilde{\chi}_{ijk}^{oo'}\right]$

$$
\tilde{\chi}_{ijk}^{US} = \begin{cases} 1 & \text{if } V_{ijk}^{U,n+1} = \tilde{V}_{ijk}^{S} \\ 0 & \text{otw} \end{cases} \qquad \tilde{\chi}_{ijk}^{SU} = \begin{cases} 1 & \text{if } V_{ijk}^{S,n+1} = V_{ijk}^{U,n+1/2} \\ 0 & \text{otw} \end{cases}
$$

$$
\tilde{\chi}_{ijk}^{EU} = \begin{cases} 1 & \text{if } V_{ijk}^{E,n+1} = V_{ijk}^{U,n+1/2} \\ 0 & \text{otw} \end{cases} \qquad \tilde{\chi}_{ijk}^{ES} = \begin{cases} 1 & \text{if } V_{ijk}^{E,n+1} = \tilde{V}_{ijk}^{S} \\ 0 & \text{otw} \end{cases}
$$

These functions are 1 if the agent changes occupations at (i, j, k).

(b) Define now the vectors $\chi^{oo'} = \text{vec}\left(\tilde{\chi}^{oo'}\right)$ to be used later. χ is a vector of length $n_a n_\epsilon n_z$.

A.2. *Solution to KFE equations*

Before solving the KFE the transition matrix A has to be modified to include the endogenous transitions between unemployment and self-employment. For this we use the indicators χ constructed as part of the value function iteration.

Now, consider a transition matrix P:

$$
P = \begin{bmatrix} P^{UU} & P^{US} & A^{UE} \\ P^{SU} & P^{SS} & A^{SE} \\ P^{EU} & P^{ES} & A^{EE} \end{bmatrix}
$$

note that since there are not endogenous transitions to employment the last column of matrices are just as in matrix A. The other columns are modified only if there are endogenous transitions. Note that each matrix $P^{oo'}$ is of size $n_a n_\epsilon n_z \times n_a n_\epsilon n_z$.

(1) Make all matrices $P^{oo'} = A^{oo'}$ and $P^{oo} = A^{oo}$.

(2) For matrix P make zero any (column) entry related to an endogenous transition, since these states are not reached. For all m and q in $\{1, \ldots, n_a n_\epsilon n_z\}$:

$$
P_{mq}^{*U} = 0 \qquad \text{if } \chi^{US}(q) = 1
$$

$$P^{*S}_{mq} = 0 \qquad \text{if } \chi^{SU}(q) = 1$$

$$P^{*E}_{mq} = 0 \qquad \text{if } \chi^{EU}(q) = 1 \quad \text{or} \quad \chi^{ES}(q) = 1$$

where $* \in \{U, S, E\}$.

(3) For matrix P adjust entries to take into account endogenous transitions coming from other occupation o into occupation $o^{'}$. This implies moving the columns of P^{o*} that were set to 0 because of transitions into $P^{*o^{'}}$. For all m and q in $\{1, \ldots, n_a n_\epsilon n_z\}$:

$$P^{*S}_{m,q-l_q} = P^{*S}_{m,q-l_q} + A^{*U}_{mq} \qquad \text{if } \chi^{US}(q) = 1$$

$$P^{*U}_{mq} = P^{*U}_{mq} + A^{*S}_{mq} \qquad \text{if } \chi^{SU}(q) = 1$$

$$P^{*U}_{mq} = P^{*U}_{mq} + A^{*E}_{mq} \qquad \text{if } \chi^{EU}(q) = 1$$

$$P^{*S}_{m,q-l_q} = P^{*S}_{m,q-l_q} + A^{*E}_{mq} \qquad \text{if } \chi^{ES}(q) = 1$$

where l_q maps the index of the agent after paying the l_k units of adjustment cost.

(4) Finally as explained in Moll's example for stopping time (multiple assets with adjustment costs) the diagonal elements with transitions have to be adjusted:

$$P^{UU}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{US}(m) = 1$$

$$P^{SS}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{SU}(m) = 1$$

$$P^{EE}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{EU}(m) = 1 \quad \text{or} \quad \chi^{ES}(m) = 1$$

Moll says: "To see why the $^{-1}/_{\Delta}$ term shows up, consider the time-discretized process for g:

$$\dot{g}_t = P^T g_t \longrightarrow g_{t+\Delta t} = (\Delta P + I)^T g_t$$

where I is the identity matrix. The transition matrix $\tilde{P} = \Delta P + I$ needs to have all entries in the adjustment region $\tilde{C}_{mm} = 0$ and hence $\Delta P + I = 0$. Without the adjustment, the matrix P is singular.

The system to solve is:

$$P^T g = 0$$

A simple way to solve the system is to make one of the elements of g to be equal to an arbitrary number, and replace such row of P^T by a row of zeros with a one in the diagonal. Call this matrix $\tilde{P}^T$ and let $\tilde{\imath} = [0, \ldots, 0, 0.1, 0, \ldots, 0]^T$ then solve for:

$$\tilde{g} = \left[\tilde{P}^T\right]^{-1} \tilde{\imath}$$

finally define g as:

$$g = \frac{1}{\sum_{ijk} \left(\tilde{g}^U_{ijk} + \tilde{g}^S_{ijk} + \tilde{g}^E_{ijk}\right) \Delta a} \tilde{g}$$

A.3. *Solution to stationary equilibrium*

(1) Guess a value for prices (w, r).
 (a) Note that only the value of r is needed. Given it we have:

$$\frac{N}{K} = \left(\frac{r+\delta}{\beta A}\right)^{\frac{1}{1-\beta}} \qquad w = (1-\beta) A \left(\frac{N}{K}\right)^{-\beta} = (1-\beta) A \left(\frac{r+\delta}{\beta A}\right)^{\frac{-\beta}{1-\beta}}$$

(2) Solve HJB equations.
(3) Solve KFE equation.
(4) Clear the labor market by choosing N (of aggregate firm).
(5) Obtain K form firm's problem.
(6) Obtain residual from capital market clearing condition.

Appendix B. Solution Method - Non-Uniform Grid

B.1. *Solution to HJB equations*

The model is solved using an implicit finite difference as the one shown in Achdou et al. (2017). Consider grids over assets, labor efficiency and entrepreneurial ability:

$$\vec{a} = [a_1, \ldots, a_{n_a}] \qquad \vec{\epsilon} = [\epsilon_1, \ldots, \epsilon_{n_\epsilon}] \qquad \vec{z} = [z_1, \ldots, z_{n_z}]$$

with n_a, n_ϵ and n_z elements respectively. The grids $\vec{\epsilon}$ and $\vec{z}$ have constant distance between grid points of $\Delta\epsilon$ and Δz. The grid on assets $\vec{a}$ does not, let $\Delta a_{i,+} = a_{i+1} - a_i$ and $\Delta a_{i,-} = a_i - a_{i-1}$. Let i denote the index of the asset dimension, j of the labor efficiency, and k of the entrepreneurial ability.

Denote $V^o_{ijk} = V^o(a_i, \epsilon_j, z_k)$ and let the backward and forward difference of the value function approximate the derivative:

$$V^o_a(a_i, \epsilon_j, z_k) \approx \frac{V^o_{i+1,jk} - V^o_{ijk}}{\Delta a_{i,+}} = \partial_a V^o_{ijk,F} \qquad V^o_a(a_i, \epsilon_j, z_k) \approx \frac{V^o_{ijk} - V^o_{i-1,jk}}{\Delta a_{i,-}} = \partial_a V^o_{ijk,B}$$

A similar definition is used for $\partial_\epsilon V^o_{ijk,F}$ and $\partial_z V^o_{ijk,F}$, and the backward differences. The second difference with respect to ϵ and z are defined as:

$$V^o_{\epsilon\epsilon}(a_i, \epsilon_j, z_k) \approx \frac{V^o_{i,j+1,k} - 2V^o_{ijk} + V^o_{i,j-1,k}}{(\Delta\epsilon)^2} = \partial_{\epsilon\epsilon} V^o_{ijk}$$

$$V_{zz}^{o}\left(a_i,\epsilon_j,z_k\right)\approx\frac{V_{i,j,k+1}^{o}-2V_{ijk}^{o}+V_{i,j,k-1}^{o}}{\left(\Delta z\right)^2}=\partial_{zz}V_{ijk}^{o}$$

The problem to solve is (in general):

$$\begin{aligned}\rho V_{ijk}^{o}=u\left(c_{ijk}\right)+\partial_a V_{ij}^{o}\left(y_{ijk}^{o}+ra_i-c_{ijk}\right)+\tilde{\gamma}_{ijk}^{o}\left(V_{ijk}^{o'}-V_{ijk}^{o}\right)\\ -\mu_\epsilon\epsilon_j\partial_\epsilon V_{ijk}^{o}+\frac{1}{2}\sigma_\epsilon^2\partial_{\epsilon\epsilon}V_{ijk}^{o}-\mu_z z_k\partial_z V_{ijk}^{o}+\frac{1}{2}\sigma_z^2\partial_{zz}V_{ijk}^{o}\end{aligned}$$

The implicit method works as VFI, solving the following equation on $V_{ijk}^{o,n+1}$ given a value for $V_{ijk}^{o,n}$:

$$\begin{aligned}\frac{V_{ijk}^{o,n+1}-V_{ijk}^{o,n}}{\Delta}+\rho V_{ijk}^{o,n+1}=u\left(c_{ijk}^{o,n}\right)+\partial_a V_{ijk}^{o,n+1}\left(y_{ijk}^{o}+ra_i-c_{ijk}^{o,n}\right)+\tilde{\gamma}_{ijk}^{o,n}\left(V_{ijk}^{o',n+1}-V_{ijk}^{o,n+1}\right)\\ -\mu_\epsilon\epsilon_j\partial_\epsilon V_{ijk}^{o,n+1}+\frac{1}{2}\sigma_\epsilon^2\partial_{\epsilon\epsilon}V_{ijk}^{o,n+1}-\mu_z z_k\partial_z V_{ijk}^{o,n+1}+\frac{1}{2}\sigma_z^2\partial_{zz}V_{ijk}^{o,n+1}\end{aligned}$$

Note that the (known) value at iteration n is used to compute consumption, and the drift of the assets, which we will call savings for convenience:

$$s_{ijk}^{o,n}=y_{ijk}^{o}+ra_i-c_{ijk}^{o,n}\qquad\text{where}\qquad c_{ijk}^{o,n}=u'^{-1}\left(\partial_a V_{ijk}^{o,n}\right)$$

It is also used to define if the agent is willing to change after a job offer. We have:

$$\tilde{\gamma}_{ijk}^{U,n}=\gamma^U 1_{V_{ijk}^{E,n}>V_{ijk}^{U,n}}\qquad\tilde{\gamma}_{ijk}^{S,n}=\gamma^S 1_{V_{ijk}^{E,n}>V_{ijk}^{U,n}}\qquad\tilde{\gamma}_{ijk}^{E,n}=\gamma^E$$

It is possible to solve for $V^{o,n+1}$, but it is first necessary to determine whether to use the forward or backward approximation to the first derivatives of the value function. The "upwind scheme" presented in Achdou et al. (2017) is used for this.

Since consumption can be defined with the backward or forward difference approximation we get:

$$s_{ijk,B}^{o,n}=y_{ijk}^{o}+ra_i-u'^{-1}\left(\partial_a V_{ijk,B}^{o,n}\right)\qquad s_{ijk,F}^{o,n}=y_{ijk}^{o}+ra_i-u'^{-1}\left(\partial_a V_{ijk,F}^{o,n}\right)$$

The idea is to use the backward difference when the implied drift is negative, and the forward difference when the drift is positive. Yet there are cases for which $s_{ijk,F}^{o,n}<0<s_{ijk,B}^{o,n}$, in these cases we set savings equal to zero, so the derivative is not used, in any case the FOC of the problem gives the exact derivate of the value function as: $\partial_a\overline{V}_{ijk}^{o,n}=u'\left(y_{jk}+ra_i\right)$.

The equation is then:

$$
\begin{aligned}
\frac{V_{ijk}^{o,n+1}-V_{ijk}^{o,n}}{\Delta}+\rho V_{ijk}^{o,n+1}=u\left(c_{ijk}^{o,n}\right)+\tilde{\gamma}_{ijk}^{o,n}\left(V_{ijk}^{o',n+1}-V_{ijk}^{o,n+1}\right)+& \\
+\partial_a V_{ijk,B}^{o,n+1}\left[s_{ijk,B}^{o,n}\right]^{-}+\partial_a V_{ijk,F}^{o,n+1}\left[s_{ijk,F}^{o,n}\right]^{+}& \\
-\mu_\epsilon \epsilon_j\left(\partial_\epsilon V_{ijk,B}^{o,n+1}1_{\epsilon_j>0}+\partial_\epsilon V_{ijk,F}^{o,n+1}1_{\epsilon_j<0}\right)+\frac{1}{2}\sigma_\epsilon^2\partial_{\epsilon\epsilon}V_{ijk}^{o,n+1}& \\
-\mu_z z_k\left(\partial_z V_{ijk,B}^{o,n+1}1_{z_k>0}+\partial_z V_{ijk,F}^{o,n+1}1_{z_k<0}\right)+\frac{1}{2}\sigma_z^2\partial_{zz}V_{ijk}^{o,n+1}&
\end{aligned}
$$

Replacing by the definitions of backward and forward derivatives:

$$
\begin{aligned}
\frac{V_{ijk}^{o,n+1}-V_{ijk}^{o,n}}{\Delta}+\rho V_{ijk}^{o,n+1}=&\ u\left(c_{ijk}^{o,n}\right)+\tilde{\gamma}_{ijk}^{o,n}\left(V_{ijk}^{o',n+1}-V_{ijk}^{o,n+1}\right)+ \\
&+\frac{V_{ijk}^{o}-V_{i-1,jk}^{o}}{\Delta a_{i,-}}\left[s_{ijk,B}^{o,n}\right]^{-}+\frac{V_{i+1,jk}^{o}-V_{ijk}^{o}}{\Delta a_{i,+}}\left[s_{ijk,F}^{o,n}\right]^{+} \\
&-\mu_\epsilon\epsilon_j\left(\frac{V_{ijk}^{o}-V_{i,j-1,k}^{o}}{\Delta\epsilon}1_{\epsilon_j>0}+\frac{V_{i,j+1,k}^{o}-V_{ijk}^{o}}{\Delta\epsilon}1_{\epsilon_j<0}\right)+\frac{1}{2}\sigma_\epsilon^2\frac{V_{i,j+1,k}^{o}-2V_{ijk}^{o}+V_{i,j-1,k}^{o}}{\left(\Delta\epsilon\right)^2} \\
&-\mu_z z_k\left(\frac{V_{ijk}^{o}-V_{ij,k-1}^{o}}{\Delta z}1_{z_k>0}+\frac{V_{ij,k+1}^{o}-V_{ijk}^{o}}{\Delta z}1_{z_k<0}\right)+\frac{1}{2}\sigma_z^2\frac{V_{i,j,k+1}^{o}-2V_{ijk}^{o}+V_{i,j,k-1}^{o}}{\left(\Delta z\right)^2}
\end{aligned}
$$

Grouping terms we get:

$$
\begin{aligned}
\frac{V_{ijk}^{o,n+1}-V_{ijk}^{o,n}}{\Delta}+\rho V_{ijk}^{o,n+1}=&\ u\left(c_{ijk}^{o,n}\right)+\tilde{\gamma}_{ijk}^{o,n}V_{ijk}^{o',n+1}+\left(\begin{array}{c}\frac{\left[s_{ijk,B}^{o,n}\right]^{-}}{\Delta a_{i,-}}-\frac{\left[s_{ijk,F}^{o,n}\right]^{+}}{\Delta a_{i,+}}-\tilde{\gamma}_{ijk}^{o,n} \\ -\frac{\mu_\epsilon\epsilon_j}{\Delta\epsilon}\left(1_{\epsilon_j>0}-1_{\epsilon_j<0}\right)-\frac{\sigma_\epsilon^2}{\left(\Delta\epsilon\right)^2} \\ -\frac{\mu_z z_k}{\Delta z}\left(1_{z_k>0}-1_{z_k<0}\right)-\frac{\sigma_z^2}{\left(\Delta z\right)^2}\end{array}\right)V_{ijk}^{o} \\
&-\frac{\left[s_{ijk,B}^{o,n}\right]^{-}}{\Delta a_{i,-}}V_{i-1,jk}^{o}+\frac{\left[s_{ijk,F}^{o,n}\right]^{+}}{\Delta a_{i,+}}V_{i+1,jk}^{o} \\
&+\left(\frac{\mu_\epsilon\epsilon_j}{\Delta\epsilon}1_{\epsilon_j>0}+\frac{\sigma_\epsilon^2}{2\left(\Delta\epsilon\right)^2}\right)V_{i,j-1,k}^{o}+\left(-\frac{\mu_\epsilon\epsilon_j}{\Delta\epsilon}1_{\epsilon_j<0}+\frac{\sigma_\epsilon^2}{2\left(\Delta\epsilon\right)^2}\right)V_{i,j+1,k}^{o} \\
&+\left(\frac{\mu_z z_k}{\Delta z}1_{z_k>0}+\frac{\sigma_z^2}{2\left(\Delta z\right)^2}\right)V_{ij,k-1}^{o}+\left(-\frac{\mu_z z_k}{\Delta z}1_{z_k<0}+\frac{\sigma_z^2}{2\left(\Delta z\right)^2}\right)V_{i,j,k+1}^{o}
\end{aligned}
$$

We now name the coefficients for ease of use:

$$
\begin{aligned}
\frac{V_{ijk}^{o,n+1}-V_{ijk}^{o,n}}{\Delta}+\rho V_{ijk}^{o,n+1}=&\ u\left(c_{ijk}^{o,n}\right)+\tilde{\gamma}_{ijk}^{o,n}V_{ijk}^{o',n+1}+\left(\tilde{x}_{ijk}+\tilde{y}_j+\tilde{w}_k\right)V_{ijk}^{o,n+1} \\
&+\tilde{x}_{ijk}^{-}V_{i-1,jk}^{o,n+1}+\tilde{x}_{ijk}^{+}V_{i+1,jk}^{o,n+1}+\tilde{y}_j^{-}V_{i,j-1,k}^{o,n+1}+\tilde{y}_j^{+}V_{i,j+1,k}^{o,n+1}+\tilde{w}_k^{-}V_{ij,k-1}^{o,n+1}+\tilde{w}_k^{+}V_{i,j,k+1}^{o,n+1}
\end{aligned}
$$

where:

$$\tilde{x}_{ijk} = \frac{\left[s^{o,n}_{ijk,B}\right]^-}{\Delta a_{i,-}} - \frac{\left[s^{o,n}_{ijk,F}\right]^+}{\Delta a_{i,+}} - \tilde{\gamma}^{o,n}_{ijk}$$

$$\tilde{x}^-_{ijk} = -\frac{\left[s^{o,n}_{ijk,B}\right]^-}{\Delta a_{i,-}}$$

$$\tilde{x}^+_{ijk} = \frac{\left[s^{o,n}_{ijk,F}\right]^+}{\Delta a_{i,+}}$$

$$\begin{aligned}
\tilde{y}_j &= -\tfrac{\mu_\epsilon \epsilon_j}{\Delta\epsilon}\left(1_{\epsilon_j>0} - 1_{\epsilon_j<0}\right) - \tfrac{\sigma^2_\epsilon}{(\Delta\epsilon)^2} & \tilde{w}_k &= -\tfrac{\mu_z z_k}{\Delta z}\left(1_{z_k>0} - 1_{z_k<0}\right) - \tfrac{\sigma^2_z}{(\Delta z)^2} \\
\tilde{y}^-_j &= \tfrac{\mu_\epsilon \epsilon_j}{\Delta\epsilon} 1_{\epsilon_j>0} + \tfrac{\sigma^2_\epsilon}{2(\Delta\epsilon)^2} & \tilde{w}^-_k &= \tfrac{\mu_z z_k}{\Delta z} 1_{z_k>0} + \tfrac{\sigma^2_z}{2(\Delta z)^2} \\
\tilde{y}^+_j &= -\tfrac{\mu_\epsilon \epsilon_j}{\Delta\epsilon} 1_{\epsilon_j<0} + \tfrac{\sigma^2_\epsilon}{2(\Delta\epsilon)^2} & \tilde{w}^+_k &= -\tfrac{\mu_z z_k}{\Delta z} 1_{z_k<0} + \tfrac{\sigma^2_z}{2(\Delta z)^2}
\end{aligned}$$

B.1.1. *Boundary Conditions*

A final loose end before writing up the linear system in matrix form is what to do with the boundaries of the different grids. At the lower boundary of the asset grid the agent is subject to a no-borrowing constraint. Hence it has to be the case that the agent does not try to borrow. The drift has to be non-negative at that point, which implies that $\tilde{x}^-_{1jk} = 0$ for all (j,k). At the upper boundary a similar constraint can be imposed, so that $\tilde{x}^+_{n_a jk} = 0$. This should arise naturally if the upper boundary is high enough. Notice that imposing these restrictions implies that V^{n+1}_{0j} and $V^{n+1}_{n_a+1,j}$ are not part of the system.

The barriers on ϵ and z imply that:

$$V^o_{\epsilon^-}\left(a, \underline{\epsilon}, z\right) = V^o_{\epsilon^+}\left(a, \bar{\epsilon}, z\right) = V^o_{z^-}\left(a, \epsilon, \underline{z}\right) = V^o_{z^+}\left(a, \epsilon, \bar{z}\right) = 0$$

that is, the negative (and positive) drift of the value function at the lower (and upper) boundary must be zero. In terms of our approximation that is:

$$\partial_\epsilon V^o_{i1k,B} = \frac{V^o_{i1k} - V^o_{i0k}}{\Delta\epsilon} = 0 \qquad \partial_\epsilon V^o_{in_\epsilon k,F} = \frac{V^o_{i,n_\epsilon+1,k} - V^o_{in_\epsilon k}}{\Delta\epsilon} = 0$$

$$\partial_z V^o_{ij1,B} = \frac{V^o_{ij1} - V^o_{ij0}}{\Delta z} = 0 \qquad \partial_z V^o_{ijn_z,F} = \frac{V^o_{ij,n_z+1} - V^o_{ijn_z}}{\Delta z} = 0$$

This in turn implies that:

$$V^o_{i0k} = V^o_{i1k} \qquad V^o_{i,n_\epsilon+1,k} = V^o_{in_\epsilon k}$$

$$V^o_{ij0} = V^o_{ij1} \qquad V^o_{ij,n_z+1} = V^o_{ijn_z}$$

Using these results we can write the HJB equation at the boundaries:

$$\frac{V_{i1k}^{o,n+1}-V_{i1k}^{o,n}}{\Delta}+\rho V_{i1k}^{o,n+1}=u\left(c_{i1k}^{o,n}\right)+\tilde{\gamma}_{ijk}^{o,n}V_{i1k}^{o',n+1}+\left(\tilde{x}_{i1k}+\tilde{y}_{1}+\tilde{\mathbf{y}}_{\mathbf{1}}^{-}+\tilde{w}_{k}\right)V_{i1k}^{o}$$
$$+\tilde{x}_{i1k}^{-}V_{i-1,1k}^{o}+\tilde{x}_{i1k}^{+}V_{i+1,1k}^{o}+\tilde{y}_{1}^{+}V_{i,2,k}^{o}+\tilde{w}_{k}^{-}V_{i1,k-1}^{o}+\tilde{w}_{k}^{+}V_{i,1,k+1}^{o}$$

$$\frac{V_{in_\epsilon k}^{o,n+1}-V_{in_\epsilon k}^{o,n}}{\Delta}+\rho V_{in_\epsilon k}^{o,n+1}=u\left(c_{in_\epsilon k}^{o,n}\right)+\tilde{\gamma}_{ijk}^{o,n}V_{in_\epsilon k}^{o',n+1}+\left(\tilde{x}_{in_\epsilon k}+\tilde{y}_{n_\epsilon}+\tilde{\mathbf{y}}_{\mathbf{n}_\epsilon}^{+}+\tilde{w}_{k}\right)V_{in_\epsilon k}^{o}$$
$$+\tilde{x}_{in_\epsilon k}^{-}V_{i-1,n_\epsilon k}^{o}+\tilde{x}_{in_\epsilon k}^{+}V_{i+1,n_\epsilon k}^{o}+\tilde{y}_{n_\epsilon}^{-}V_{i,n_\epsilon-1,k}^{o}+\tilde{w}_{k}^{-}V_{in_\epsilon,k-1}^{o}+\tilde{w}_{k}^{+}V_{i,n_\epsilon,k+1}^{o}$$

$$\frac{V_{ij1}^{o,n+1}-V_{ij1}^{o,n}}{\Delta}+\rho V_{ij1}^{o,n+1}=u\left(c_{ij1}^{o,n}\right)+\tilde{\gamma}_{ijk}^{o,n}V_{ij1}^{o',n+1}+\left(\tilde{x}_{ij1}+\tilde{y}_{j}+\tilde{w}_{1}+\tilde{\mathbf{w}}_{\mathbf{1}}^{-}\right)V_{ij1}^{o}$$
$$+\tilde{x}_{ij1}^{-}V_{i-1,j1}^{o}+\tilde{x}_{ij1}^{+}V_{i+1,j1}^{o}+\tilde{y}_{j}^{-}V_{i,j-1,1}^{o}+\tilde{y}_{j}^{+}V_{i,j+1,1}^{o}+\tilde{w}_{1}^{+}V_{i,j,1+1}^{o}$$

$$\frac{V_{ijn_z}^{o,n+1}-V_{ijn_z}^{o,n}}{\Delta}+\rho V_{ijn_z}^{o,n+1}=u\left(c_{ijn_z}^{o,n}\right)+\tilde{\gamma}_{ijk}^{o,n}V_{ijn_z}^{o',n+1}+\left(\tilde{x}_{ijn_z}+\tilde{y}_{j}+\tilde{w}_{n_z}+\tilde{\mathbf{w}}_{\mathbf{n_z}}^{+}\right)V_{ijn_z}^{o}$$
$$+\tilde{x}_{ijn_z}^{-}V_{i-1,jn_z}^{o}+\tilde{x}_{ijn_z}^{+}V_{i+1,jn_z}^{o}+\tilde{y}_{j}^{-}V_{i,j-1,n_z}^{o}+\tilde{y}_{j}^{+}V_{i,j+1,n_z}^{o}+\tilde{w}_{n_z}^{-}V_{ij,n_z-1}^{o}$$

B.1.2. *System Solution*

The equations above describe a system of $n_a \times n_\epsilon \times n_z \times 3$ equations, its best to define the value function a stack of three value functions, one for each occupation:

$$V=\left[V^U;V^S;V^E\right]^T$$
$$V^o=\mathrm{vec}\left[V_{ijk}^o\right]$$

The system is:

$$\frac{1}{\Delta}\left(V^{n+1}-V^n\right)+\rho V^{n+1}=u^n+A^nV^{n+1}$$

where n^n is the stacked version of: $u^n = \left[u^{U,n};u^{S,n};u^{E,n}\right]$ and $u^{o,n} = \mathrm{vec}\left[u\left(c_{ijk}^{o,n}\right)\right]$ and consumption is computed as:

$$c_{ijk}^{o,n}=u^{'-1}\left(\partial_a V_{ijk,B}^{o,n}\right)1_{s_{ijk,B}^{o,n}<0}+u^{'-1}\left(\partial_a V_{ijk,F}^{o,n}\right)1_{s_{ijk,F}^{o,n}>0}+\left(y_{jk}^o+ra_i\right)1_{s_{ijk,F}^{o,n}<0<s_{ijk,B}^{o,n}}$$

(note that at the boundaries there is no backward or forward drift correspondingly).

Matrix A^n is given by:

$$A^n=B^n+C+D$$

$$B^n=\begin{bmatrix}B_{UU}^n & \mathbf{0} & B_{UE}^n\\ \mathbf{0} & B_{SS}^n & B_{SE}^n\\ B_{EU}^n & \mathbf{0} & B_{EE}^n\end{bmatrix}\qquad C=\begin{bmatrix}\tilde{C} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \tilde{C} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \tilde{C}\end{bmatrix}\qquad D=\begin{bmatrix}\tilde{D} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \tilde{D} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \tilde{D}\end{bmatrix}$$

The matrices B^n_{oo} are sparse and they only contain elements in the diagonal, upper diagonal and lower diagonal. Consider $X_o = \left[\tilde{x}^o_{ijk}\right]$, $X^-_o = \left[\tilde{x}^{o,-}_{ijk}\right]$ and $X^+_o = \left[\tilde{x}^{o,+}_{ijk}\right]$, all three dimensional matrix that contain the coefficients $\tilde{x}$ (note that $\tilde{x}$ is already adjusted for the boundaries). Then we have: $\text{diag}\left(B^n_{oo}\right) = \text{vec}\left(X_o\right)$, $\text{diag}^+\left(B^n_{oo}\right) = \text{vec}\left(X^+_o\right)$ and $\text{diag}^-\left(B^n_{oo}\right) = \text{vec}\left(X^-_o\right)$, where the upper diagonal and lower diagonal are adjusted not to include the zero terms of the boundaries. The matrices $B^n_{oo'} = \gamma^o I$.

Matrices $\tilde{C}$ and $\tilde{D}$ are also sparse. Their construction takes advantage of the fact that the elements of $\tilde{C}$ only vary with j and the elements of $\tilde{D}$ only vary with k. We first construct $\tilde{C}$. Consider a sparse matrix Ω of order $n_\epsilon \times n_\epsilon$, such that:

$$\Omega_{1,1} = \tilde{y}_1 + \tilde{y}^-_1$$

$$\Omega_{j,j} = \tilde{y}_j$$

$$\Omega_{n_\epsilon,n_\epsilon} = \tilde{y}_{n_\epsilon} + \tilde{y}^+_{n_\epsilon}$$

$$\Omega_{j,j+1} = \tilde{y}^+_1$$

$$\Omega_{j-1,j} = \tilde{y}^-_1$$

$$\Omega = \begin{bmatrix} \tilde{y}_1 + \tilde{y}^-_1 & \tilde{y}^+_1 & & & & \\ \tilde{y}^-_2 & \tilde{y}_2 & \tilde{y}^+_2 & & & \\ & \ddots & \ddots & \ddots & & \\ & & \tilde{y}^-_{n_\epsilon-1} & \tilde{y}_{n_\epsilon-1} & \tilde{y}^+_{n_\epsilon-1} & \\ & & & & \tilde{y}^+_{n_\epsilon-1} & \tilde{y}_{n_\epsilon} + \tilde{y}^+_{n_\epsilon} \end{bmatrix}$$

Then we can get $\tilde{C} = I_{n_z} \otimes \left(\Omega \otimes I_{n_a}\right)$.

For $\tilde{D}$ the first part of the construction is equivalent, with Ω being built with coefficients $\tilde{w}$ instead of $\tilde{y}$. Then $\tilde{D} = \left(\Omega \otimes I_{n_a n_\epsilon}\right)$.

This problem can now be expressed as:

$$T^n V^{n+1} = t^n$$

where:

$$T^n = \left(\frac{1}{\Delta} + \rho\right) I_{3 n_a n_\epsilon n_z} - A^n \qquad t^n = u^n + \frac{1}{\Delta} V^n$$

B.1.3. *Algorithm*

(1) Compute matrices C and D. These matrices do not change with equilibrium prices or iterations.

(2) Take as given w, r.

(3) Solve for earnings in each state: y^o_{ijk} for each combination of (a, ϵ, z) and occupation. These values don't change with iterations.

(4) Guess a value for V^n, a $3n_a n_\epsilon n_z$ vector. It is easier to store it as three separate matrices of dimension $n_a \times n_\epsilon \times n_z$.

(5) Compute the backward and forward drift: $s^{o,n}_{ijk,B}$ and $s^{o,n}_{ijk,F}$ for $i = \{2, \ldots, n_a\}$ and $i = \{1, \ldots, n_a - 1\}$ respectively, and all (j, k, o).

$$s^{o,n}_{ijk,B} = y^o_{jk} + ra_i - u^{'-1}\left(\partial_a V^{o,n}_{ijk,B}\right) \qquad s^{o,n}_{ijk,F} = y^o_{jk} + ra_i - u^{'-1}\left(\partial_a V^{o,n}_{ijk,F}\right)$$

These values are stored in six matrices of dimensions $n_a \times n_\epsilon \times n_z$ (two per occupation, one with backward drift and the other one with forward drift).

(6) For all (i, j, k, o) compute consumption as:

$$c^{o,n}_{ijk} = u^{'-1}\left(\partial_a V^{o,n}_{ijk,B}\right) 1_{s^{o,n}_{ijk,B} < 0} + u^{'-1}\left(\partial_a V^{o,n}_{ijk,F}\right) 1_{s^{o,n}_{ijk,F} > 0} + \left(y^o_{jk} + ra_i\right) 1_{s^{o,n}_{ijk,F} < 0 < s^{o,n}_{ijk,B}}$$

These values are stored in three matrices of dimensions $n_a \times n_\epsilon \times n_z$.

(7) Compute the utility vector as: $u^n = \left[u^{U,n}; u^{S,n}; u^{E,n}\right]$ and $u^{o,n} = \text{vec}\left[u\left(c^{o,n}_{ijk}\right)\right]$.

(8) Compute the adjusted shock arrival rates:

$$\tilde{\gamma}^{U,n}_{ijk} = \gamma^U 1_{V^{E,n}_{ijk} > V^{U,n}_{ijk}} \qquad \tilde{\gamma}^{S,n}_{ijk} = \gamma^S 1_{V^{E,n}_{ijk} > V^{U,n}_{ijk}} \qquad \tilde{\gamma}^{E,n}_{ijk} = \gamma^E$$

(9) Compute the three $n_a \times n_\epsilon \times n_z$ matrices $X_o = \left[\tilde{x}^o_{ijk}\right]$, $X^-_o = \left[\tilde{x}^{o,-}_{ijk}\right]$ and $X^+_o = \left[\tilde{x}^{o,+}_{ijk}\right]$.

(10) Compute matrix $B^n = \begin{bmatrix} B^n_{UU} & \mathbf{0} & B^n_{UE} \\ \mathbf{0} & B^n_{SS} & B^n_{SE} \\ B^n_{EU} & \mathbf{0} & B^n_{EE} \end{bmatrix}$, where $\text{diag}\left(B^n_{oo}\right) = \text{vec}\left(X_o\right)$, $\text{diag}^+\left(B^n_{oo}\right) = \text{vec}\left(X^+_o\right)$ and $\text{diag}^-\left(B^n_{oo}\right) = \text{vec}\left(X^-_o\right)$, where the upper diagonal and lower diagonal are adjusted not to include the zero terms of the boundaries. The matrices $B^n_{oo'} = \gamma^o I$.

(11) Compute the matrix $A^n = B^n + C + D$.

(12) Compute the matrix T and vector t:

$$T^n = \left(\frac{1}{\Delta} + \rho\right) I_{3n_a n_\epsilon n_z} - A^n \qquad t^n = u^n + \frac{1}{\Delta} V^n$$

(13) Compute $V^{n+1/2}$ as:

$$V^{n+1/2} = \left(T^n\right)^{-1} t^n$$

(14) Divide the vector $V^{n+1/2}$ into three matrices of $n_a \times n_\epsilon \times n_z$: $V^{U,n+1/2}$, $V^{S,n+1/2}$, and $V^{E,n+1/2}$.

(15) Compute the adjusted self-employed value, taking into account the cost of becoming a self-employed:

$$\tilde{V}^{S}_{ijk} = \begin{cases} -\infty & i \leq l_k \\ V^{S,n+1/2}_{i-l_k,j,k} & \text{otw} \end{cases}$$

(16) Compute $V^{U,n+1}$, $V^{S,n+1}$, and $V^{E,n+1}$ as follows:

$$\begin{aligned} V^{U,n+1}_{ijk} &= \max\left\{V^{U,n+1/2}_{ijk}, \tilde{V}^{S}_{ijk}\right\} \\ V^{S,n+1}_{ijk} &= \max\left\{V^{U,n+1/2}_{ijk}, V^{S,n+1/2}_{ijk}\right\} \\ V^{E,n+1}_{ijk} &= \max\left\{V^{U,n+1/2}_{ijk}, \tilde{V}^{S}_{ijk}, V^{E,n+1/2}_{ijk}\right\} \end{aligned}$$

(a) Define the following matrices as indicators of the occupation choice: $\left[\tilde{\chi}^{oo'}_{ijk}\right]$

$$\tilde{\chi}^{US}_{ijk} = \begin{cases} 1 & \text{if } V^{U,n+1}_{ijk} = \tilde{V}^{S}_{ijk} \\ 0 & \text{otw} \end{cases} \qquad \tilde{\chi}^{SU}_{ijk} = \begin{cases} 1 & \text{if } V^{S,n+1}_{ijk} = V^{U,n+1/2}_{ijk} \\ 0 & \text{otw} \end{cases}$$

$$\tilde{\chi}^{EU}_{ijk} = \begin{cases} 1 & \text{if } V^{E,n+1}_{ijk} = V^{U,n+1/2}_{ijk} \\ 0 & \text{otw} \end{cases} \qquad \tilde{\chi}^{ES}_{ijk} = \begin{cases} 1 & \text{if } V^{E,n+1}_{ijk} = \tilde{V}^{S}_{ijk} \\ 0 & \text{otw} \end{cases}$$

These functions are 1 if the agent changes occupations at (i, j, k).

(b) Define now the vectors $\chi^{oo'} = \text{vec}\left(\tilde{\chi}^{oo'}\right)$ to be used later. χ is a vector of length $n_a n_\epsilon n_z$.

B.2. *Solution to KFE equations*

Before solving the KFE the transition matrix A has to be modified to include the endogenous transitions between unemployment and self-employment. For this we use the indicators χ constructed as part of the value function iteration.

Now, consider a transition matrix P:

$$P = \begin{bmatrix} P^{UU} & P^{US} & A^{UE} \\ P^{SU} & P^{SS} & A^{SE} \\ P^{EU} & P^{ES} & A^{EE} \end{bmatrix}$$

note that since there are not endogenous transitions to employment the last column of matrices are just as in matrix A. The other columns are modified only if there are endogenous transitions. Note that each matrix $P^{oo'}$ is of size $n_a n_\epsilon n_z \times n_a n_\epsilon n_z$.

(1) Make all matrices $P^{oo'} = A^{oo'}$ and $P^{oo} = A^{oo}$.

(2) For matrix P make zero any (column) entry related to an endogenous transition, since these states are not reached. For all m and q in $\{1, \ldots, n_a n_\epsilon n_z\}$:

$$P^{*U}_{mq} = 0 \qquad \text{if } \chi^{US}(q) = 1$$

$$P^{*S}_{mq} = 0 \qquad \text{if } \chi^{SU}(q) = 1$$

$$P^{*E}_{mq} = 0 \qquad \text{if } \chi^{EU}(q) = 1 \quad \text{or} \quad \chi^{ES}(q) = 1$$

where $* \in \{U, S, E\}$.

(3) For matrix P adjust entries to take into account endogenous transitions coming from other occupation o into occupation o'. This implies moving the columns of P^{o*} that were set to 0 because of transitions into $P^{*o'}$. For all m and q in $\{1, \ldots, n_a n_\epsilon n_z\}$:

$$P^{*S}_{m,q-l_q} = P^{*S}_{m,q-l_q} + A^{*U}_{mq} \qquad \text{if } \chi^{US}(q) = 1$$

$$P^{*U}_{mq} = P^{*U}_{mq} + A^{*S}_{mq} \qquad \text{if } \chi^{SU}(q) = 1$$

$$P^{*U}_{mq} = P^{*U}_{mq} + A^{*E}_{mq} \qquad \text{if } \chi^{EU}(q) = 1$$

$$P^{*S}_{m,q-l_q} = P^{*S}_{m,q-l_q} + A^{*E}_{mq} \qquad \text{if } \chi^{ES}(q) = 1$$

where l_q maps the index of the agent after paying the l_k units of adjustment cost.

(4) Finally as explained in Moll's example for stopping time (multiple assets with adjustment costs) the diagonal elements with transitions have to be adjusted:

$$P^{UU}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{US}(m) = 1$$

$$P^{SS}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{SU}(m) = 1$$

$$P^{EE}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{EU}(m) = 1 \quad \text{or} \quad \chi^{ES}(m) = 1$$

Moll says: "To see why the $^{-1}/_{\Delta}$ term shows up, consider the time-discretized process for g:

$$\dot{g}_t = P^T g_t \longrightarrow g_{t+\Delta t} = (\Delta P + I)^T g_t$$

where I is the identity matrix. The transition matrix $\tilde{P} = \Delta P + I$ needs to have all entries in the adjustment region $\tilde{C}_{mm} = 0$ and hence $\Delta P + I = 0$. Without the adjustment, the matrix P is singular.

The system to solve is:

$$P^T g = 0$$

A simple way to solve the system is to make one of the elements of g to be equal to an arbitrary number, and replace such row of P^T by a row of zeros with a one in the diagonal. Call this matrix $\tilde{P}^T$ and let $\tilde{\imath} = [0, \ldots, 0, 0.1, 0, \ldots, 0]^T$ then solve for:

$$\tilde{g} = \left[\tilde{P}^T\right]^{-1} \tilde{\imath}$$

finally define g as:

$$g = \frac{1}{\sum_{ijk} \left(\tilde{g}^U_{ijk} + \tilde{g}^S_{ijk} + \tilde{g}^E_{ijk}\right) \Delta a} \tilde{g}$$

B.3. *Solution to stationary equilibrium*

(1) Guess a value for prices (w, r).
   (a) Note that only the value of r is needed. Given it we have:

$$\frac{N}{K} = \left(\frac{r+\delta}{\beta A}\right)^{\frac{1}{1-\beta}} \qquad w = (1-\beta) A \left(\frac{N}{K}\right)^{-\beta} = (1-\beta) A \left(\frac{r+\delta}{\beta A}\right)^{\frac{-\beta}{1-\beta}}$$

(2) Solve HJB equations.
(3) Solve KFE equation.
(4) Clear the labor market by choosing N (of aggregate firm).
(5) Obtain K form firm's problem.
(6) Obtain residual from capital market clearing condition.

Appendix C. List of parameters

The following are the parameters of the model:

Shocks		Preferences	
μ_ϵ	Labor Efficiency - Drift	ρ	Discount Factor
σ_ϵ	Labor Efficiency - Variance	σ	CRRA Parameter
μ_z	Entrepreneurial Ability - Drift	Technology	
σ_z	Entrepreneurial Ability- Variance	α	Self-Employed Technology
$\underline{\epsilon}$	Labor Efficiency - Lower Bound	β	Firm's Technology
$\overline{\epsilon}$	Labor Efficiency - Upper Bound	A	Firm's Productivity
$\underline{z}$	Entrepreneurial Ability - Lower Bound	δ	Capital Depreciation
$\overline{z}$	Entrepreneurial Ability- Upper Bound	λ	Equity Constraint
γ^U	Job Offer Arrival Rate - Unemployed	$\kappa(z)$	Installation Costs
γ^S	Job Offer Arrival Rate - Self-employed	b	Unemployed Income
γ^E	Job Destruction Arrival Rate	$\underline{a}$	Borrowing Constraint
		$\overline{a}$	Asset Barrier