Variable Markups with Heterogeneous Demand and Productivity

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Markup dispersion

- Important for productivity, labor share, inequality, welfare, etc.

  Dixit, Stiglitz, 1976; Atkeson, Burstein, 2008; Dhingra, Morrow, 2019; Edmond, Midrigan, Xu 2015, 2023; Yeh, Macaluso, Hershbein 2022; Baqaee, Farhi, Sangani, 2024; Boar, Midrigan, 2024; Hasenzagl, Pérez, 2024.

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    $\longrightarrow$ Misallocation because high-markup (more-productive) firms are "too small"

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- Measured markups from production function estimation show:
  - Large markup dispersion concentrated in small firms
  - Both: Small firms with "high"-markups & large firms with "low"-markups
  - Indicative of relevant role of demand heterogeneity for markup dispersion

    De Loecker, Goldberg, Khandelwal, Pavcnik 2016; De Loecker, Eeckhout, Unger 2020; Raval 2023; Blum, Claro, Horstmann, Rivers, 2024.

What we want

1. Model of firm competition capable of matching distribution of markups and firm size
    - Generate small firms with high markups + large firms with low markups
    - Disentangle role of heterogeneity in productivity, demand, and market concentration

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$$
\begin{tikzcd}
 & & \text{Demand Elasticity Shifter} \arrow[d] & \\
\text{Productivity} \uparrow \arrow[r] \arrow[dr] & \text{Relative Size} \uparrow \arrow[r, "\text{Kimball}"] & \text{Elasticity} \downarrow \arrow[r] & \text{Markup} \uparrow \\
 & \text{Market Share} \uparrow \arrow[urr, "\text{Atkeson \& Burstein}"'] & &
\end{tikzcd}
$$

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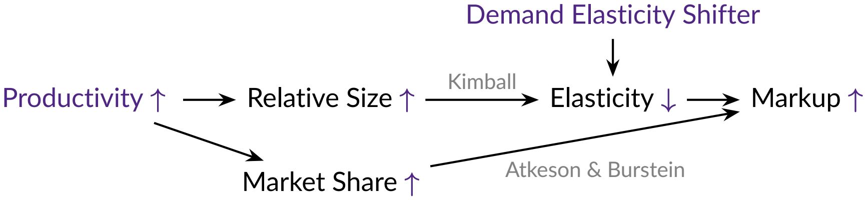

2. Measurement exercise to better understand markup distribution
   - Relative role of demand heterogeneity + productivity + concentration

What we do: Markups with productivity & demand heterogeneity

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3. Match *measured* distribution of markups & market shares (Colombia, India, US)
   - Markups from production function (Raval 2023, De Loecker, et al 2016, De Loecker, et al 2020)

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Soon: Misallocation and decomposition of firm heterogeneity (productivity & demand factors)

Model of Variable Markups

Firm problem(s)

1. Cost minimization: Choose *flexible* inputs ▸ details
   - Results in firm's cost function (productivity, input prices)
   - FOC used to estimate production function $\longrightarrow$ Measured markups ▸ opt. 1 ▸ opt. 2 ▸ opt. 3

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2. Profit maximization: Choose price given demand
   - Demand for goods within a market comes from *Kimball* market aggregator
   - Demand for market's goods from CES aggregator: $\frac{P_m}{P} = \alpha_m \left(\frac{Y_m}{Y}\right)^{-\frac{1}{\gamma}}$ ▸ details
   - Firms act strategically within but not across markets (take P and Y as given)

Demand within markets: Kimball

- Output within markets $\{Y_i^m\}$ aggregated into Y_m with *Kimball* aggregators

$$1 = \sum_{i=1}^{N_m} \Upsilon_i \left(\frac{y_i^m}{Y_m} \right) \qquad \left(\text{CES: } \Upsilon \left(\frac{y_i^m}{Y_m} \right) = \left(\frac{y_i^m}{Y_m} \right)^{\frac{\nu-1}{\nu}} \right)$$

Key: Firm-specific functions $\Upsilon_i \longrightarrow$ Idiosyncratic demand shifters

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- Firm (inverse) demand

Properties

$$\frac{p_i^m}{P_m} = \frac{\Upsilon_i' \left(\frac{y_i^m}{Y_m} \right)}{\sum_j \Upsilon_j' \left(\frac{y_j^m}{Y_m} \right) \frac{y_j^s}{Y_m}} \qquad \left(\text{CES: } \frac{p_i^m}{P_m} = \left(\frac{y_i^m}{Y_m} \right)^{\frac{-1}{\nu}} \right)$$

- P_m: Market m' ideal price index, i.e., $P_m Y_m = \sum_i p_i^m y_i^m$

The firm problem

$$\max \quad p_i^m y_i^m - C_i\left(y_i^m\right)$$

$$\text{s.t.} \qquad \underbrace{\frac{p_i^m}{P_m} = \frac{\Upsilon_i'\left(\frac{y_i^m}{Y_m}\right)}{\sum_j \Upsilon_j'\left(\frac{y_j^m}{Y_m}\right)\frac{y_j^s}{Y_m}}}_{\text{Own Demand}}; \qquad \underbrace{1 = \sum_{i=1}^{N_m} \Upsilon_i\left(\frac{y_i^m}{Y_m}\right)}_{\text{Market Aggregation}}; \qquad \underbrace{\frac{P_m}{P} = \alpha_m\left(\frac{Y_m}{Y}\right)^{-\frac{1}{\gamma}}}_{\text{Market Demand}}.$$

- Maximize over quantities (*Cournot*) or prices (*Bertrand*)

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Next: Use demand structure to characterize markups analytically

Markups and Demand Elasticities

Optimal pricing + Demand elasticity

$$p_i^m = \underbrace{\frac{1}{1 - \frac{1}{\overline{\varepsilon}_i^m}}}_{\mu_i^m\text{: Markup}} C_i'\left(y_i^m\right) \qquad \text{where} \qquad \underbrace{\overline{\varepsilon}_i^m \equiv -\left(\frac{\partial \log p_i^m}{\partial \log y_i^m}\right)^{-1}}_{\text{Firm's Demand Elasticity}}$$

- Demand Elasticity depends on more than Kimball aggregator Υ_i through competition!

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- **Key:** Demand elasticity depends only on own-elasticities and market shares $\{\varepsilon_i^m, \sigma_i^m\}$

Proposition: Equilibrium demand elasticity — Cournot

▸ Bertrand

$$\frac{1}{\bar{\varepsilon}_i^m} = \underbrace{\frac{1}{\gamma}}_{\text{Market Elasticity}} \sigma_i^m + \underbrace{\left(\frac{1}{\varepsilon_i^m}\left(1-\sigma_i^m\right) + \frac{1}{\bar{\varepsilon}_{-i}^m}\sigma_i^m\right)}_{\text{Variety Elasticity}}\left(1-\sigma_i^m\right)$$

where $\boldsymbol{\sigma_i^m}$ is firm i's market revenue share (Domar weight), $\boldsymbol{\varepsilon_i^m}$ its "own elasticity", and

$$\frac{1}{\bar{\varepsilon}_{-i}^m} \equiv E_\sigma\left[\frac{1}{\varepsilon_j^m}\middle| j \neq i\right] = \sum_{j\neq i} \frac{1}{\varepsilon_j^m}\frac{\sigma_j^m}{1-\sigma_i^m}$$

is the average elasticity of its competitors.

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is the average elasticity of its competitors.

- Elasticity of large firms reflects market's elasticity (monopoly) over variety's elasticity.
- Elasticity of small firms reflects "own elasticity" (monopolistic competition)

Proposition: Equilibrium markups — Cournot

▸ Bertrand ▸ Aggregation

$$\frac{1}{\mu_i^m} = \underbrace{\frac{\gamma-1}{\gamma}}_{\text{Monopoly Markup}} + \underbrace{\left(\frac{1}{\gamma} - \frac{1}{\varepsilon_i^m}\right)\left(1-\sigma_i^m\right)}_{\text{“}i\text{” vs Market}} + \underbrace{\left(\frac{1}{\varepsilon_i^m} - E_\sigma\left[\frac{1}{\varepsilon_j^m}\right]\right)\sigma_i^m}_{\text{“}i\text{” vs Competitors “}j\text{”}}$$

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Higher markup μ_i^m if

- “Own elasticity” (ε_i^m) lower than market’s (γ)
- “Own variety” is elastic relative to market average (limiting substitution effects)

$$\begin{tikzcd}
 & \text{Demand Elasticity Shifter } \Upsilon_i \arrow[dr] & & \\
\text{Productivity} \uparrow \arrow[r] \arrow[dr] & \frac{y_i^m}{Y_m} \uparrow \arrow[r] & \varepsilon_i^m \downarrow \arrow[r] & \mu_i^m \uparrow \\
 & \sigma_i^m \uparrow \arrow[urr] & &
\end{tikzcd}$$

Estimation:

Matching the joint distribution of markups (μ) and market shares (σ)

Data: Manufacturing

Markups: Recover markups from cost minimization FOC

▸ details

$$\mu_i = \frac{\epsilon^x}{s^x} = \frac{\text{Output Elasticity wrt } x}{\text{Input } x \text{ Share}}$$

Data: Manufacturing

Markups: Recover markups from cost minimization FOC ▸ details

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Colombia (1985–1989)

- Firm level ▸ details
- Revenue + Expenditure
- $\epsilon^x_{G(i)}$: Cost-shares
  (Raval 2023; Foster, Haltiwanger, Syverson 2008)

India (2005–2008)

- Establishment level ▸ details
- Quantity + Prices
  by product & input
- ϵ^x_i: Trans-log technology
  (De Loecker, Goldberg, Khandelwal, Pavcnik 2016)
- Avoid "output-price bias" $\longrightarrow$
  Consistent markup estimates
  (Bond, Hashemi, Kaplan, Zoch 2021)

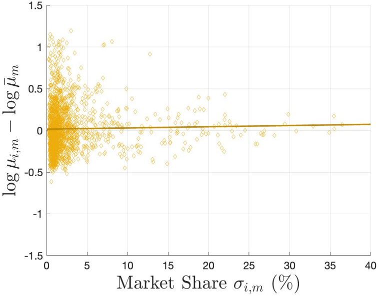
U.S. (1985–1989)

- Firm level ▸ details
- Publicly traded firms
- Revenue + Expenditure
- ϵ^x_i: Cobb-Douglas tech.
  (De Loecker, Eeckhout, Unger 2020)

Distribution of Markups and Market Shares

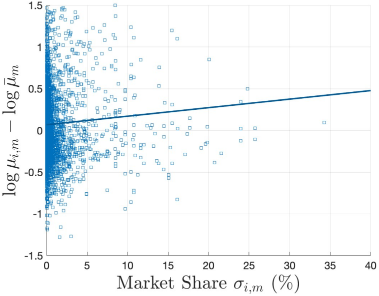
▸ Histograms

Colombia (1985–1989)



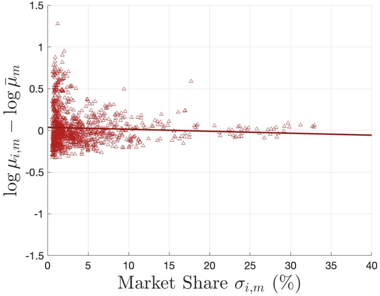
Avg. $V_\sigma(\log \mu_i^m) = 0.07$

India (2005–2008)



Avg. $V_\sigma(\log \mu_i^m) = 0.29$

U.S. (1985–1989)



Avg. $V_\sigma(\log \mu_i^m) = 0.03$

(***i***) Dispersion concentrated in small firms (***ii***) Both small-high-markup & large-low-markup firms

Distribution of Markups and Market Shares

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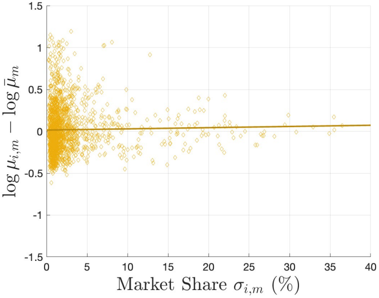

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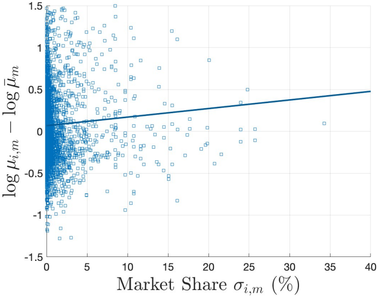

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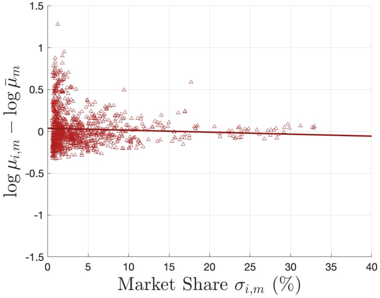


Avg. $V_\sigma(\log \mu_i^m) = 0.03$

Next: Recover $\{\varepsilon_i^m\}$ that match $\{\mu_i^m, \sigma_i^m\}$ distribution $\longrightarrow$ Role of elasticity dispersion

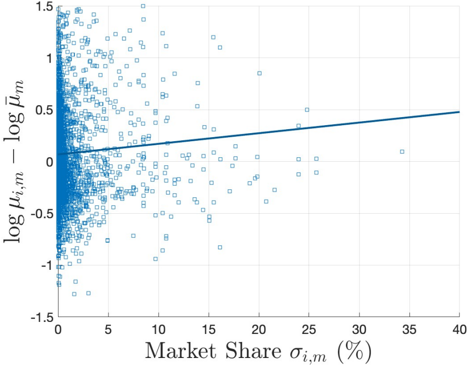
“Own” elasticities that match markups and market shares Col Ind US

Recover elasticities from equilibrium markups $\vec{\mu}^m = f\left(\vec{\sigma}^m, \vec{\varepsilon}^m\right)$

“Own” elasticities that match markups and market shares Col Ind US

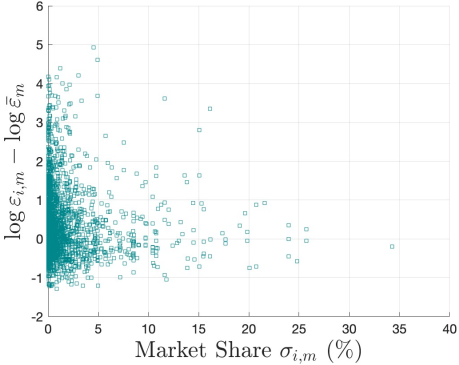
Recover elasticities from equilibrium markups $\vec{\mu}^m = f\left(\vec{\sigma}^m, \vec{\varepsilon}^m\right)$

India: Markups & Market Shares



Avg. $V_\sigma(\log \mu_i^m) = 0.29$

India: Recovered Elasticities & Market Shares



Avg. $V_\sigma(\log \varepsilon_{i,m}) = 0.96$

Turning off idiosyncratic demand shifters

Oligopolistic Competition with CES Demand: (Atkeson & Burstein 2008)

- Variation in market shares $\longrightarrow$ Variation in markups

Counterfactual: Match avg. market markup with $\tilde{\varepsilon}_m$: $\quad \frac{1}{\tilde{\mu}_i^m} = \frac{\gamma-1}{\gamma} + \left(\frac{1}{\gamma} - \frac{1}{\tilde{\varepsilon}_m}\right)\left(1 - \sigma_i^m\right)$

Turning off idiosyncratic demand shifters

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Oligopolistic Competition with VES Demand: (Atkeson & Burstein 2008 + Kimball 1995)

- Variation in market shares $+$ size $\longrightarrow$ Variation in markups

Counterfactual: Common Υ from Klenow & Willis (2016) $\longrightarrow$ $$\tilde{\varepsilon}_{i,m} = \nu_m \left(\frac{y_{i,m}}{Y_m}\right)^{-\frac{\theta_m}{\nu_m}}$$

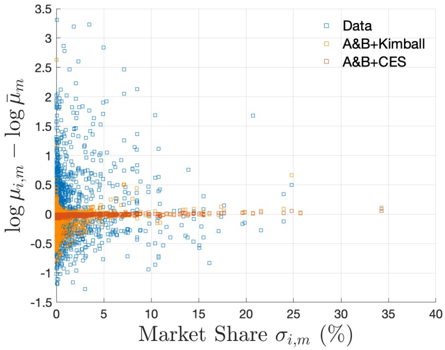
- Choose $\{\nu_m, \theta_m\}$ to match $\{\mu_i^m\}$ while being consistent with $\{\sigma_i^m\}$

▸ details

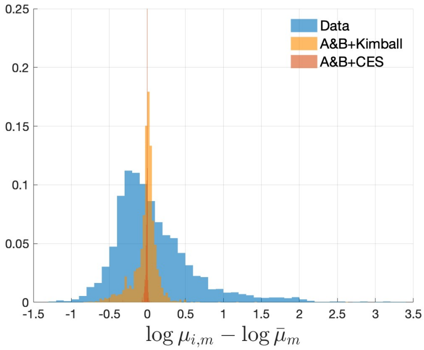
Elasticity dispersion is key for markup dispersion

Col US

India: Markups & Market Shares



India: Distribution of Markups



	Data/Full Model	A&B + Kimball		A&B + CES	
	$V_\sigma(\log \mu)$	$V_\sigma(\log \tilde{\mu})$	$\rho_\sigma(\log \mu, \log \tilde{\mu})$	$V_\sigma(\log \tilde{\mu})$	$\rho_\sigma(\log \mu, \log \tilde{\mu})$
India	0.29	0.011	0.18	0.0002	−0.03
Colombia	0.07	0.006	0.16	0.0005	0.02
US	0.03	0.002	0.25	0.0003	−0.004

Estimation:
Demand Parameters

Estimating demand parameters

- No conditions placed so far over demand aggregators Υ_i

▸ examples

- Standard functional forms give tractable elasticity: $\varepsilon_i^m = f\left(\frac{y_i^m}{Y}; \boldsymbol{\nu_i^m}, \theta_m\right)$

Estimating demand parameters

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- Standard functional forms give tractable elasticity: $\varepsilon_i^m = f\left(\frac{y_i^m}{Y}; \boldsymbol{\nu_i^m}, \theta_m\right)$

- Identify $\{\nu_i^m\} + \theta_m$ from changes in elasticities as size changes: ▸ θ estimates

$$\underbrace{d \log \varepsilon_i^m = -\left(\frac{\boldsymbol{\xi_i^m}}{\varepsilon_i^m}\right) \overbrace{\left(\frac{\varepsilon_i^m}{1+\varepsilon_i^m}\right)}^{\text{“Observed”}} d \log \sigma_i^m}_{\text{Regress change in elasticity on change in market share}} \qquad \text{where} \qquad \underbrace{\boldsymbol{\xi_i^m} \equiv -\frac{p_i^m}{P_m} \frac{\partial \log \varepsilon_i^m}{\partial \left(\frac{p_i^m}{P_m}\right)}}_{\text{Super-Elasticity}}$$

Estimating demand parameters

- No conditions placed so far over demand aggregators Υ_i ▸ examples

- Standard functional forms give tractable elasticity: $\varepsilon_i^m = f\left(\frac{y_i^m}{Y}; \boldsymbol{\nu_i^m}, \theta_m\right)$

- Identify $\{\nu_i^m\} + \theta_m$ from changes in elasticities as size changes: ▸ θ estimates

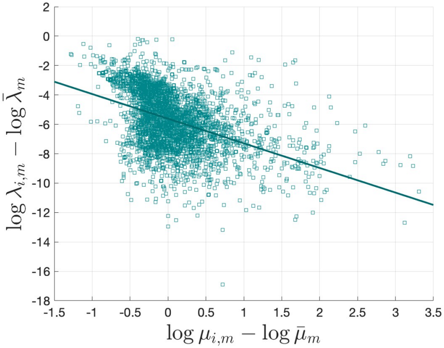
$$\underbrace{d\log\varepsilon_i^m = -\left(\frac{\boldsymbol{\xi_i^m}}{\varepsilon_i^m}\right)\overbrace{\left(\frac{\varepsilon_i^m}{1+\varepsilon_i^m}\right)}^{\text{“Observed”}} d\log\sigma_i^m}_{\text{Regress change in elasticity on change in market share}} \qquad \text{where} \qquad \underbrace{\boldsymbol{\xi_i^m} \equiv -\frac{p_i^m}{P_m}\frac{\partial\log\varepsilon_i^m}{\partial\left(\frac{p_i^m}{P_m}\right)}}_{\text{Super-Elasticity}}$$

- Choose θ_m to match regression coefficient + Given θ_m set $\{\nu_i^m\}$ to match $\left\{\sigma_i^m\left(\frac{y_i^m}{Y_m}\right)\right\}$
- Recover model objects like (relative) marginal costs $\left\{\frac{\lambda_i^m}{\lambda_m}\right\}$ ▸ details ▸ Corr.

Distribution of marginal costs, markups, and market shares

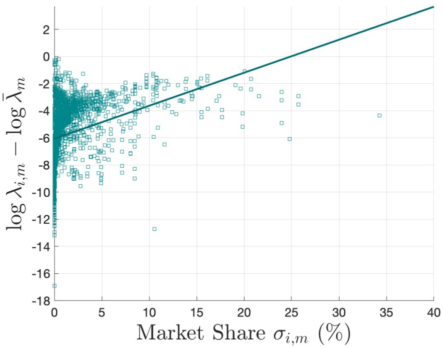
Col US

India: Mrg. Costs & Markups



Avg. $V_\sigma(\log \lambda_{i,m}) = 1.49$

India: Mrg. Costs & Market Shares



Avg. $\rho_\sigma(\log \lambda_{i,m}, \mu_{i,j}) = -0.61$

- Firms with *lower* marginal costs tend to have *higher* markups …but large variation
- Firms with *higher* market share have *higher* marginal costs!

Prod. vs. Demand.

Conclusions

Conclusions

- Analytical model of variable markups with idiosyncratic demand elasticity shifters
  - Merge variable elasticity of demand + oligopolistic competition
- Match observed distribution of markups and firm size
  - Account for high-markup small firms and low-markup large firms
- Variation in elasticities of demand is **key** to account for markup dispersion

Soon:

- US Annual Survey of Manufactures + US Economic Census + Chilean Data
- Role of different heterogeneity dimensions for misallocation

Extra

Cost minimization (and markup estimation)

Firm Problems Data

$$C\left(y \middle| \left\{p_n\right\}_{n=1}^{N}, \left\{K_m\right\}_{m=1}^{M}\right) = \min_{\left\{x_n\right\}_{n=1}^{N}} \sum_{n=1}^{N} p_n \cdot x_n \quad \text{s.t. } \overline{y} \leq zF\left(x_1, \ldots, x_N, K_1, \ldots, K_M\right)$$

Variable inputs: $\left\{x_n\right\}_{n=1}^{N}$ Fixed inputs: $\left\{K_m\right\}_{m=1}^{M}$ Scale: y

Cost minimization (and markup estimation)

◂ Firm Problems ◂ Data

$$C\left(y|\left\{p_n\right\}_{n=1}^N,\left\{K_m\right\}_{m=1}^M\right)=\min_{\left\{x_n\right\}_{n=1}^N}\sum_{n=1}^N p_n\cdot x_n \quad \text{s.t. } \overline{y}\leq zF\left(x_1,\ldots,x_N,K_1,\ldots,K_M\right)$$

Variable inputs: $\left\{x_n\right\}_{n=1}^N$ Fixed inputs: $\left\{K_m\right\}_{m=1}^M$ Scale: y

- Optimality links markup with input elasticities ϵ_{x_n} and input shares s_{x_n} (observed)

$$\underbrace{\mu}_{\text{Markup}}=\frac{p}{\lambda}=\frac{py}{\underbrace{p_n x_n}_{\text{Input Share}}}\epsilon_{x_n}=\frac{\epsilon_{x_n}}{s_{x_n}}$$

  - Marginal cost $\lambda=C'(y)$ is the relevant multiplier

- Use IO production function estimation to recover elasticity ϵ_{x_n} and markups

Demand across markets: CES

back

- Final good producers aggregate across markets m:

$$\min_{\{Y_m\}} \sum_{m=1}^{M} P_m Y_m \qquad \text{s.t. } Y \leq \left(\sum_{m=1}^{M} \alpha_m Y_m^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

- Markets face a constant elasticity of demand γ

$$\frac{P_m}{P} = \alpha_m \left(\frac{Y_m}{Y} \right)^{-\frac{1}{\gamma}}$$

- We assume there are *many* markets so firms do not act strategically across markets
    - Take Y and P as given

Market shares and demand

back

Lemma: Firm demand satisfies

$$\frac{\partial Y_m}{\partial y_i^m} = \frac{p_i^m}{P_m} \qquad \text{and} \qquad \frac{\partial P_m}{\partial p_i^m} = \frac{y_i^m}{Y_m}$$

So that market share σ_i^m satisfy

$$\sigma_i^m \equiv \frac{p_i^m y_i^m}{P_m Y_m} = \frac{y_i^m}{Y_m}\frac{\partial Y_m}{\partial y_i^m} = \frac{p_i^m}{P_m}\frac{\partial P_m}{\partial p_i^m}.$$

Market shares and demand

back

Lemma: Firm demand satisfies

$$\frac{\partial Y_m}{\partial y_i^m} = \frac{p_i^m}{P_m} \qquad \text{and} \qquad \frac{\partial P_m}{\partial p_i^m} = \frac{y_i^m}{Y_m}$$

So that market share σ_i^m satisfy

$$\sigma_i^m \equiv \frac{p_i^m y_i^m}{P_m Y_m} = \frac{y_i^m}{Y_m}\frac{\partial Y_m}{\partial y_i^m} = \frac{p_i^m}{P_m}\frac{\partial P_m}{\partial p_i^m}.$$

- Demand system restricts responses to changes in firms' output and prices
- This links firms' choices of output and prices to changes their market shares $\{\sigma_i^m\}$

Proposition: Equilibrium elasticities — Bertrand

◂ back

$$\bar{\varepsilon}_i^m = \underbrace{\gamma}_{\text{Market Elasticity}} \sigma_i^m + \underbrace{\varepsilon_i^m \frac{E_\sigma\left[\varepsilon_j^m | j \neq i\right]}{E_\sigma\left[\varepsilon_j^m\right]}}_{\text{Variety Elasticity}} \left(1 - \sigma_i^m\right)$$

where σ_i^m is firm i's market share, ε_i^m its "own elasticity", and $E_\sigma\left[x_j\right] = \sum_j x_j \sigma_j^m$ is the average with respect to expenditure in market m.

Proposition: Equilibrium elasticities — Bertrand

back

$$\bar{\varepsilon}_i^m = \underbrace{\gamma}_{\text{Market Elasticity}} \sigma_i^m + \underbrace{\varepsilon_i^m \frac{E_\sigma\left[\varepsilon_j^m | j \neq i\right]}{E_\sigma\left[\varepsilon_j^m\right]}}_{\text{Variety Elasticity}} \left(1 - \sigma_i^m\right)$$

where σ_i^m is firm i's market share, ε_i^m its "own elasticity", and $E_\sigma\left[x_j\right] = \sum_j x_j \sigma_j^m$ is the average with respect to expenditure in market m.

- Elasticity of larger firms reflects market's elasticity (monopoly) more than variety's elasticity. Elasticity of smaller firms reflects "own elasticity" (monopolistic competition)

Proposition: Equilibrium markups — Bertrand

back

$$\frac{1}{\mu_i^m} = 1 - \frac{1}{\gamma\sigma_i^m + \varepsilon_i^m \left[1 - \frac{\varepsilon_i^m}{E_\sigma\left[\varepsilon_j^s\right]}\sigma_i^m\right]}$$

Aggregating Markups

Proposition: Market markup

▸ details ◂ back

$$\frac{1}{\mu_m} = \underbrace{\left(1 - \frac{1}{\gamma}\right)}_{\text{Monopoly Markup}} + \underbrace{\left(\frac{1}{\gamma} - E_\sigma\left[\frac{1}{\varepsilon_i^m}\right]\right)(1 - \text{HHI})}_{\text{Concentration}} + \underbrace{2\text{Cov}_\sigma\left(\sigma_i^m, \frac{1}{\varepsilon_i^m}\right)}_{\text{Distribution}}$$

- $\text{HHI} = \sum_i (\sigma_i^m)^2$: market's Herfindahl-Hirschman index
- $\text{Cov}_\sigma(x_j, y_j) = \sum_{j=1}^{N_s} (x_j)(y_j - E_\sigma[y_j])\,\sigma_j^m$: sales-weighted covariance

Proposition: Market markup

details back

$$\frac{1}{\mu_m} = \underbrace{\left(1 - \frac{1}{\gamma}\right)}_{\text{Monopoly Markup}} + \underbrace{\left(\frac{1}{\gamma} - E_\sigma\left[\frac{1}{\varepsilon_i^m}\right]\right)(1 - \text{HHI})}_{\text{Concentration}} + \underbrace{2\text{Cov}_\sigma\left(\sigma_i^m, \frac{1}{\varepsilon_i^m}\right)}_{\text{Distribution}}$$

- $\text{HHI} = \sum_i \left(\sigma_i^m\right)^2$: market's Herfindahl-Hirschman index
- $\text{Cov}_\sigma\left(x_j, y_j\right) = \sum_{j=1}^{N_s}\left(x_j\right)\left(y_j - E_\sigma\left[y_j\right]\right)\sigma_j^m$: sales-weighted covariance

Two key forces

1. Concentration: $\uparrow \mu_m$ if varieties are less elastic than the market (Edmond, Midrigan, Xu 2015)
2. Distribution of elasticities: $\downarrow \mu_m$ if sales are concentrated in firms with a low ε_i^m
   - Large firms care more about market elasticity $\gamma < \bar{\varepsilon}_m$.
     It is small (niche) firms who increase avg. markups when their varieties are less elastic.

How to aggregate within markets

back

$$\underbrace{\mu_m = \frac{P_m}{\lambda_m}}_{\text{Market's Markup}} \qquad \text{where} \qquad \underbrace{\lambda_m = \sum_{i=1}^{N_m} \lambda_i^m \frac{y_i^m}{Y_m}}_{\text{Market's Mrg Cost}}$$

How to aggregate within markets

back

$$\underbrace{\mu_m = \frac{P_m}{\lambda_m}}_{\text{Market's Markup}} \qquad \text{where} \qquad \underbrace{\lambda_m = \sum_{i=1}^{N_m} \lambda_i^m \frac{y_i^m}{Y_m}}_{\text{Market's Mrg Cost}}$$

Correct measure of markups is weighted harmonic mean of markups:

$$\mu_m = \left[\sum_{i=1}^{N_m} \lambda_i^m \frac{y_i^m}{P_m Y_m}\right]^{-1} = \left[\sum_{i=1}^{N_m} \frac{1}{\mu_i^m} \sigma_i^m\right]^{-1}$$

Equilibrium markups depend on weighted harmonic mean of elasticity

$$\frac{1}{\mu_m} = \sum_{i=1}^{N_m} \frac{1}{\mu_i^m} \sigma_i^m = \sum_{i=1}^{N_m} \left(1 - \frac{1}{\bar{\varepsilon}_i^m}\right) \sigma_i^m = 1 - \frac{1}{\bar{\varepsilon}_m}$$

Kimball Aggregators

back

Firm-specific parameters $\{\nu_i^m\}$ control "own elasticities" $\{\varepsilon_i^m\}$

1. Klenow & Willis (2016): $$\varepsilon_i^m = \nu_i^m \left(\frac{y_i^m}{Y_m}\right)^{-\frac{\theta_m}{\nu_i^m}}$$

2. Dotsey & King (2005): $$\varepsilon_i^m = \nu_i^m \left(1 - \frac{\theta_m}{1+\theta_m}\frac{y_i^m}{Y_m}\right)^{-1}$$

3. CES: $$\varepsilon_i^m = \nu_i^m$$

Super-elasticity is key for estimation:

- Klenow & Willis (2016): $$\xi_i^m = \theta_m \cdot \left(\frac{y_i^m}{Y_m}\right)^{-\frac{\theta_m}{\nu_i^m}} \quad \rightarrow \quad \frac{\xi_i^m}{\varepsilon_i^m} = \frac{\theta_m}{\nu_i^m}; \qquad \frac{y_i^m}{Y_m} = \left(\frac{\varepsilon_i^m}{\nu_i^m}\right)^{-\frac{\nu_i^m}{\theta_m}}$$

- Choose θ_m to match regression coefficient + Given θ_m set $\{\nu_i^m\}$ to match $\left\{\sigma_i^m\left(\frac{y_i^m}{Y_m}\right)\right\}$

Relative output, prices, and marginal costs

back

- **Relative Output:** Inverting the "own-elasticity" for the Klenow & Willis Υ we get

$$\frac{y_i^m}{Y_m} = \left(\frac{\varepsilon_i^m}{\nu_i^m}\right)^{-\frac{\nu_i^m}{\theta_m}}$$

- **Relative Prices:** Obtained to be consistent with market shares

$$\frac{p_i^m}{P_m} = \sigma_i^m \frac{Y_m}{y_i^m}$$

- **Marginal Costs:** Using markups definition we get

$$\frac{\lambda_i}{\lambda} = \frac{\frac{p_i}{\mu_i}}{\sum \frac{p_i}{\mu_i}\frac{y_i}{Y}} = \frac{\frac{1}{\mu_i}\frac{p_i}{P}}{\sum \frac{1}{\mu_j}\frac{p_j y_j}{PY}} = \frac{\frac{1}{\mu_i}\frac{p_i}{P}}{\sum \frac{\sigma_j}{\mu_j}} = \frac{\bar{\mu}}{\mu_i}\frac{p_i}{P}$$

where $\lambda \equiv \sum \frac{p_i}{\mu_i}\frac{y_j}{Y}$ is the market's marginal cost

Estimate Kimball Parameters $\{\nu_m, \theta_m\}$

back

1. **Measure:** $\{\sigma_i^m, \mu_i^m\}$

2. **Recover:** Elasticity $\bar{\varepsilon}_i^m = \frac{\mu_i^m}{\mu_i^m - 1}$ and "own-elasticity" $\{\varepsilon_i^m\}$ from eqm. markups

3. **Match observed market shares:** Under Klenow & Willis (2016)

$$\sigma_i^m = \frac{\Upsilon'\left(\frac{y_i^m}{Y_m}\right)\frac{y_i^m}{Y_m}}{\sum_j \Upsilon'\left(\frac{y_j^m}{Y_m}\right)\frac{y_j^m}{Y_m}} = \frac{exp\left(\frac{1}{\theta}\left(1-\left(\frac{y_i^m}{Y_m}\right)^{\frac{\theta}{\nu}}\right)\right)\frac{y_i^m}{Y_m}}{\sum_j exp\left(\frac{1}{\theta}\left(1-\left(\frac{y_j^m}{Y_m}\right)^{\frac{\theta}{\nu}}\right)\right)\frac{y_j^m}{Y_m}}$$

   Given $\{\nu_m, \theta_m\}$, we choose $\left\{\frac{y_i^m}{Y_m}\right\}$ to match market shares $\{\sigma_i^m\}$

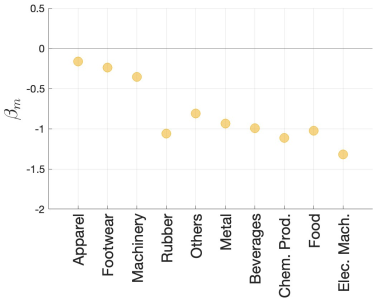
4. We choose $\{\nu_m, \theta_m\}$ to match $\{\mu\left(\varepsilon_i^m\right)\}$

Estimated β

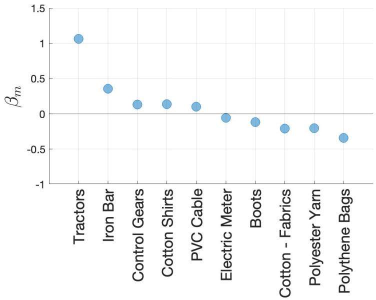
back

$$\Delta \log \varepsilon_i^m = \beta \, \Delta \log \sigma_i^m \qquad \text{where} \quad \beta = -\left(\frac{\xi_i^m}{\varepsilon_i^m}\right)\left(\frac{\varepsilon_i^m}{1+\varepsilon_i^m}\right) = -\left(\frac{\theta_m}{\nu_i^m}\right)\left(\frac{\varepsilon_i^m}{1+\varepsilon_i^m}\right)$$

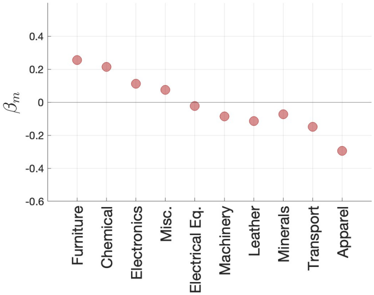
β Colombia (1985–1989)



β India (2005–2008)



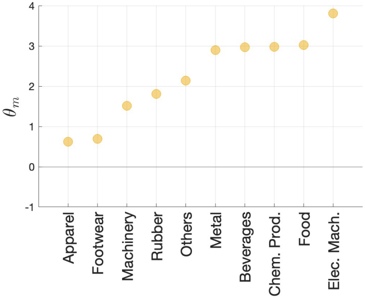
β U.S. (1985–1989)



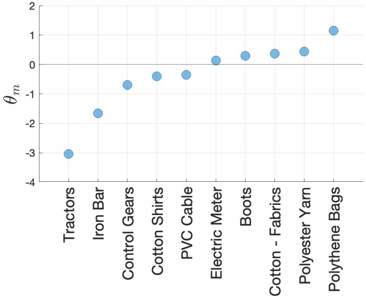
Matched θ

◂ back

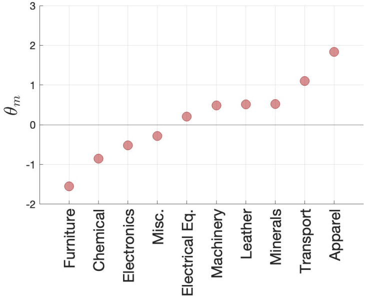
θ Colombia (1985–1989)



θ India (2005–2008)



θ U.S. (1985–1989)



Markups Estimation: Colombia

◂ Firm Problems ◂ Data

1. Data: 21 Manufacturing Industries 1980—1989 (Encuesta Anual Manufacturera)
   - Firm level: Total Revenues + Input Expenditures
2. Revenue-Based Production Function Estimation: (Raval 2023)
   - Cost share method to recover output elasticities ϵ_x (Foster, Haltiwanger, Syverson 2008)

$$\epsilon^x_{m,g} = \frac{E(x_i P^x_i | G(i) = g)}{E(x_i p^x_i + w_i p^w_i + k_i p^k_i | G(i) = g)} = \frac{\text{Avg. Input Expenditure in Group}}{\text{Avg. Cost in Group}}$$

   - Allows elasticities + labor-to-materials cost ratio to vary within markets
   - Assume (i) Constant Returns to Scale (ii) FOC holds for all inputs (on average)
3. Markups: $\mu = \epsilon^x_{m,g} \cdot \frac{p_i q_i}{p_x x_i}$; Each market has G elasticity groups

Markups Estimation: India

◂ Firm Problems ◂ Data

1. Data: 23 Manufacturing Industries 2001–2008
   - Product level: Prices + Quantities
   - Establishment level: Input prices + quantities
2. Quantity-Based Production Function Estimation: (De Loecker, Goldberg, Khandelwal, Pavcnik 2016)
   - Control function approach to recover output elasticities ϵ^X
     (Olley, Pakes 1996; Levinhson, Petrin 2003; Ackerberg, Caves, Frazer 2015)
   - Trans-log production function at industry level (same across products, estimated w/ single-product)
   - Returns-to-scale by industry (Close to CRS: 0.96–1.04)
   - Robust to output price and input allocation biases
3. Markups: $\mu = \epsilon_i^X \cdot \frac{p_i q_i}{p_X X_i}$; Establishment specific output elasticity (depends on input level)

Markups Estimation: US

◂ Firm Problems ◂ Data

1. Data: 19 Manufacturing Industries 1980—1989
   - Firm level: Total Revenues + Input Expenditures
   - Publicly-traded firms
2. Revenue-Based Production Function Estimation: (De Loecker, Eeckhout, Unger 2020)
   - Control function approach to recover output elasticities ϵ^x
     (Olley, Pakes 1996; Levinhson, Petrin 2003; Ackerberg, Caves, Frazer 2015)
   - Cobb-Douglas production $\longrightarrow$ Constant output elasticities within industry
   - Returns-to-scale by industry (Increasing Returns: 1.05–1.2)
   - Time-varying output elasticities
3. Markups: $\mu = \epsilon^x_{mt} \cdot \frac{p_i q_i}{p_x x_i}$; Each market-year pair mt has an output elasticity

Blum, Claro, Horstmann and Rivers (2024)

◂ back

1. Data: Production function estimation over Chilean multiproduct firms

   - Product Level: Quantities + Prices

   - Firm Level: Input expenditures

2. Production Function Estimation:

   - Gandhi, Navarro and Rivers (2020) on single product firms to estimate output elasticities

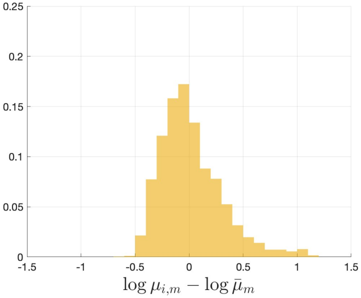
   - Profit maximization $\longrightarrow$ Markups are a general function of prices, quantities and a demand shifter.

   - Recover markups after estimating output elasticities.

Data: Distribution of Markups

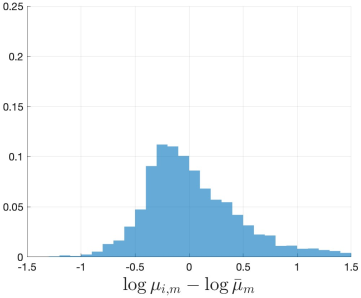
back

Colombia (1985–1989)



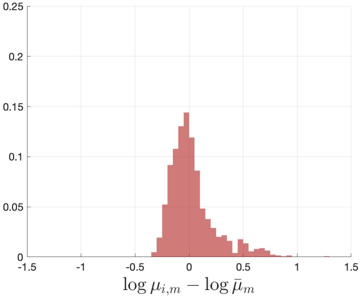
Avg. $V_\sigma(\log \mu_i^m) = 0.07$

India (2005–2008)



Avg. $V_\sigma(\log \mu_i^m) = 0.29$

U.S. (1985–1989)

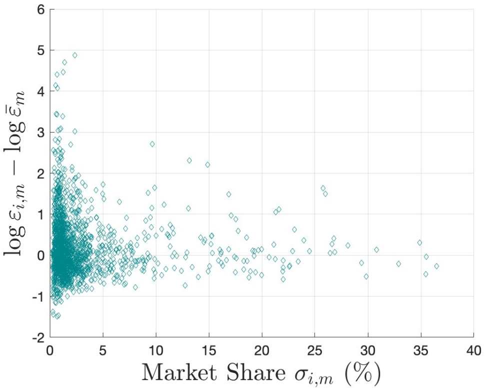


Avg. $V_\sigma(\log \mu_i^m) = 0.03$

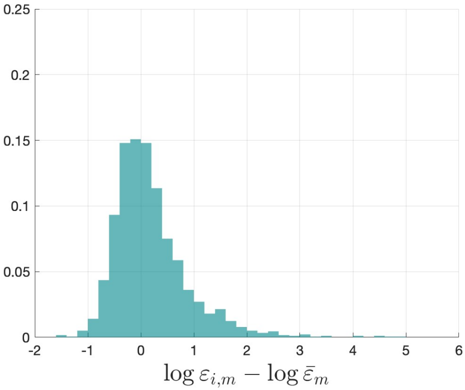
“Own” elasticities for Colombia

back

Recovered Elasticities & Market Shares



Distribution of Own Elasticities

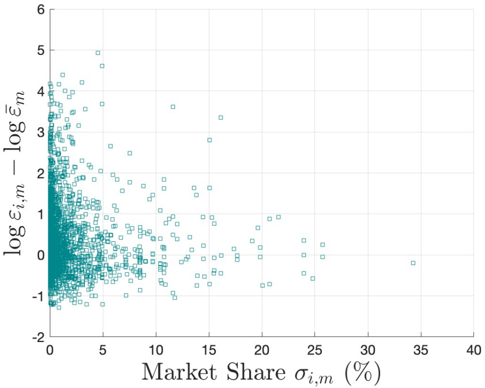


Avg. $V_\sigma(\log \varepsilon_{i,m}) = 1.02$ Avg. $V_\sigma(\log \mu_{i,m}) = 0.07$

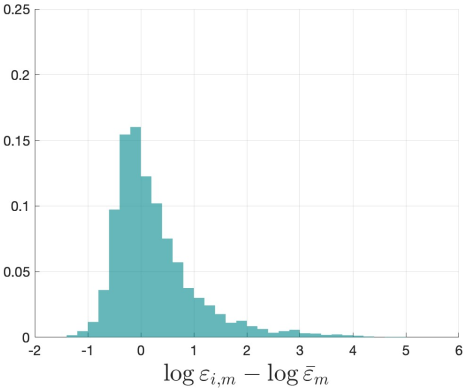
“Own” elasticities for India

back

Recovered Elasticities & Market Shares



Distribution of Own Elasticities

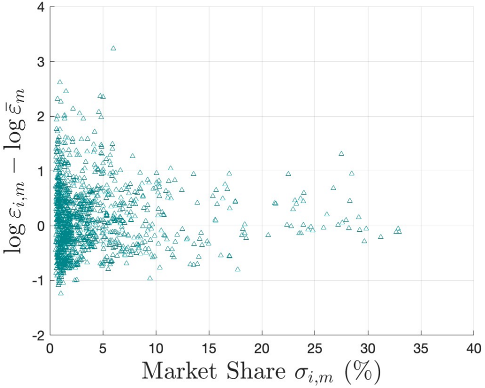


Avg. $V_\sigma(\log \varepsilon_{i,m}) = 0.96$ Avg. $V_\sigma(\log \mu_{i,m}) = 0.29$

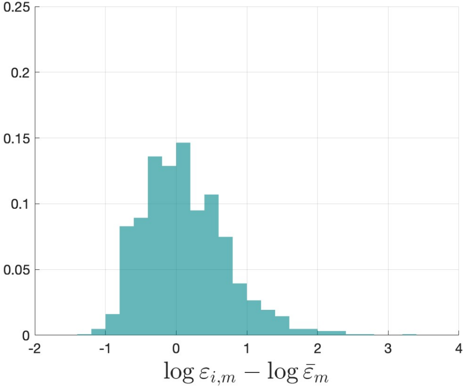
“Own” elasticities for the U.S.

back

Recovered Elasticities & Market Shares



Distribution of Own Elasticities

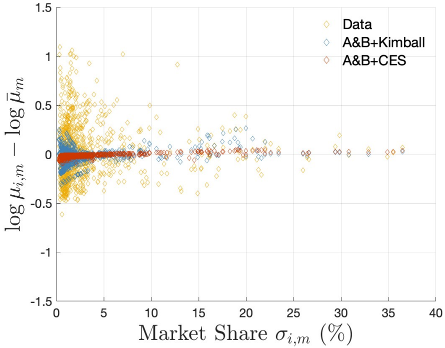


Avg. $V_\sigma(\log \varepsilon_{i,m}) = 0.30$ Avg. $V_\sigma(\log \mu_{i,m}) = 0.03$

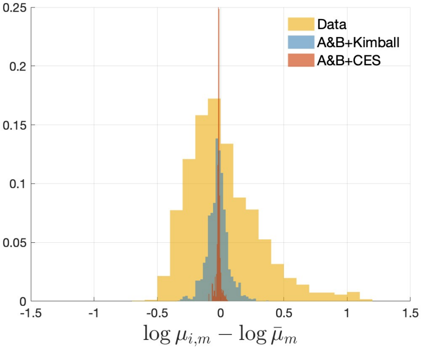
Markup Counterfactual Colombia

back

Distribution of Markups & Market Shares



Distribution of Markups

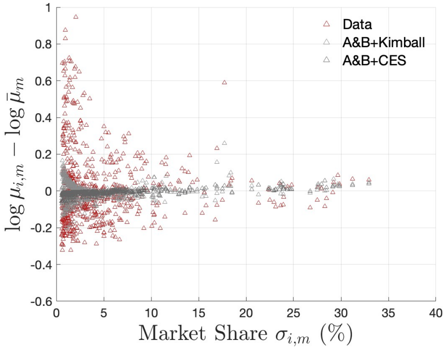


	Data	A&B + Kimball		A&B + CES	
	$V_\sigma(\log \mu)$	$V_\sigma(\log \tilde{\mu})$	$\rho_\sigma(\log \mu, \log \tilde{\mu})$	$V_\sigma(\log \tilde{\mu})$	$\rho_\sigma(\log \mu, \log \tilde{\mu})$
Colombia	0.07	0.006	0.16	0.0005	0.02

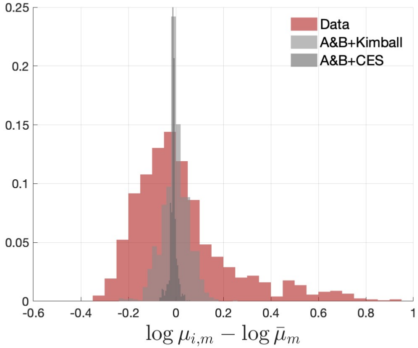
Markup Counterfactual U.S.

back

Distribution of Markups & Market Shares



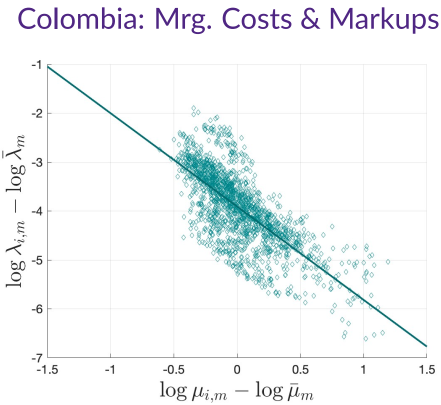
Distribution of Markups

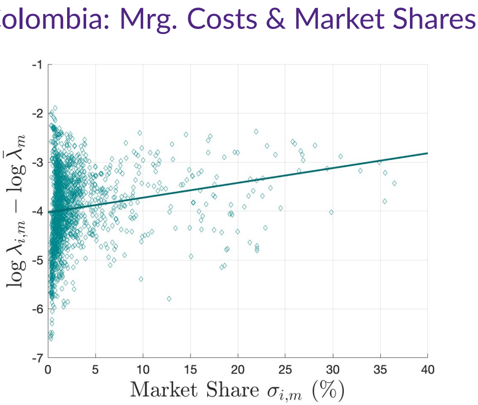


	Data	A&B + Kimball		A&B + CES	
	$V_\sigma(\log \mu)$	$V_\sigma(\log \tilde{\mu})$	$\rho_\sigma(\log \mu, \log \tilde{\mu})$	$V_\sigma(\log \tilde{\mu})$	$\rho_\sigma(\log \mu, \log \tilde{\mu})$
US	0.03	0.002	0.25	0.0003	−0.004

Distribution of marginal costs, markups, and market shares

back



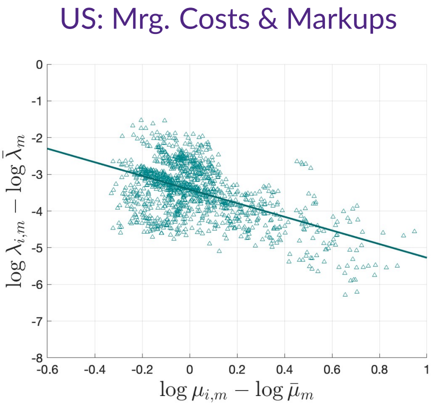


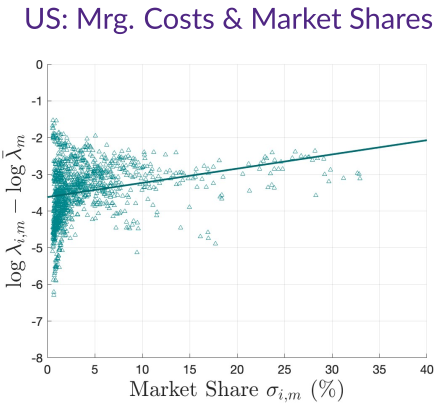
Avg. $V_\sigma(\log \lambda_{i,m}) = 0.33$

Avg. $\rho_\sigma(\log \lambda_{i,m}, \mu_{i,j}) = -0.89$

Distribution of marginal costs, markups, and market shares

back





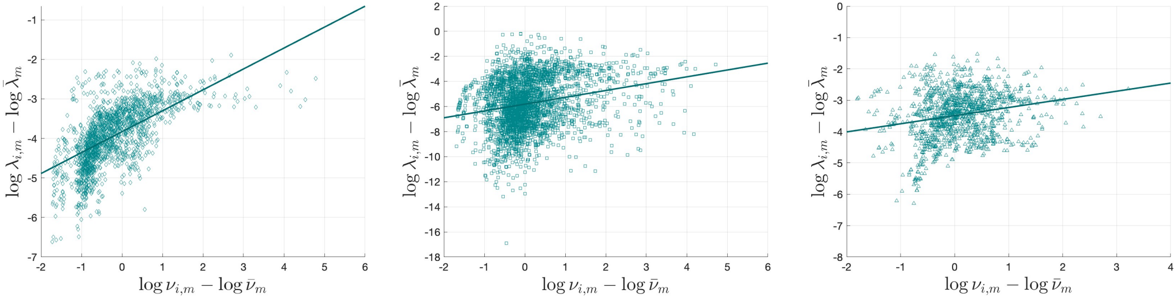
$$\text{Avg. } V_\sigma(\log \lambda_{i,m}) = 0.24$$

$$\text{Avg. } \rho_\sigma(\log \lambda_{i,m}, \mu_{i,j}) = -0.76$$

Demand elasticity shifters $\{\nu_i^m\}$ and marginal costs $\{\lambda_i^m\}$

back

Colombia (1985–1989) **India** (2005–2008) **U.S.** (1985–1989)



Avg. $\rho_\sigma(\log\nu_i^m, \log\lambda_i^m) = 0.41$

Avg. $\rho_\sigma(\log\nu_i^m, \log\lambda_i^m) = 0.47$

Avg. $\rho_\sigma(\log\nu_i^m, \log\lambda_i^m) = 0.31$

Variances and correlations: Colombia

back

	μ	ε	ν	λ	$\frac{y}{Y}$	$\frac{p}{P}$
μ	0.07					
ε	-0.87	1.02				
ν	-0.47	0.43	1.05			
λ	-0.89	0.70	0.41	0.33		
$\frac{y}{Y}$	0.25	-0.26	0.34	0.10	1.21	
$\frac{p}{P}$	-0.69	0.68	0.42	0.73	-0.18	0.14

Variances and correlations: US

back

	μ	ε	ν	λ	$\frac{y}{Y}$	$\frac{p}{P}$
μ	0.03					
ε	-0.93	0.30				
ν	-0.85	0.48	0.30			
λ	-0.76	0.46	0.31	0.24		
$\frac{y}{Y}$	0.13	-0.09	-0.08	0.69	0.81	
$\frac{p}{P}$	-0.55	0.49	0.20	0.45	-0.21	0.14

Variances and correlations: India

back

	μ	ε	ν	λ	$\frac{y}{Y}$	$\frac{p}{P}$
μ	0.29					
ε	-0.79	0.96				
ν	-0.73	0.86	0.87			
λ	-0.61	0.51	0.40	1.49		
$\frac{y}{Y}$	0.01	-0.04	-0.03	0.01	0.78	
$\frac{p}{P}$	-0.29	0.26	0.20	0.85	-0.22	0.85