Logistic Regression and Default of Credit Cards

A recent study conducted by the financial services and researcher company, The Ascent, was directed to answer the question, "What's the total credit card debt in the U.S.?" The outcome of this research exposed that the total credit card balances in the United States were around \$890 billion and that approximately 9.1% of all credit card balances across the nation were 90 days or more delinquent as of the first quarter of 2020 (Frankel & Rosen). From a macro-perspective, the mentioned case illustrates how critical predictability or probability of default could be for financial institutions and their risk management departments when approving or denying credit applications. The following assessment and report aims to explore a customer credit card payments dataset and shows how the data mining technique of logistic regression could benefit the financial sector foreseeing credit card default payments based on customers payment trends and/or activities interrelated to their respective credit balances.

The multivariate dataset comes from the UCI's Center for Machine Learning and Intelligent Systems repository and it consists of 30000 observations and 24 attributes involving customers credit card's payments information. The same was data collected as the result of a research in Taiwan, aiming to forecast default payments based on consumers' history of payments, bill amounts, and/or payment delays among other qualities. The following list contains a brief description of each of the variables in the set:

```
ID: auto-generated identified LIMIT_BAL: amount of given credit AGE: years

GENDER: 1 = male; 2 = female MARRIAGE: (1 = married; 2 = single; 3 = others)

EDUCATION: (1 = graduate school; 2 = university; 3 = high school; 4 = others)

PAY_1 to PAY_6: the measurement scale for the repayment status is: -1 = pay duly; 1 = payment delay for one month; 2 = payment delay for two months... and so forth.

BILL_AMT 1 to BILL_AMT 6: amount of bill statement

PAY_AMT 1 to BILL_AMT 6: amount of previous payment

default.payment.next.month: (response variable) default payment yes = 1, no = 0
```

Exploratory Data Analysis (EDA): During the EDA process, commands like dim(), head(), str(), summary () were employed to better appreciate the data at hand. As displayed in figures 1.0A, and 1.0B, the data includes a variety of attributes, all of these expressed as integers, and it

confirms the number of observations and variables.

11) LIM	IIT_BAL	SEX	EDUCATION N	MARRIAGE	AGE	PAY_0	PAY_Z	PAY_3	PAY_4	PAY_5	PAY_6	BILL_AMT1	BILL_AMT2	BILL_AMT3
<int< th=""><th>></th><th><int></int></th><th><int></int></th><th><int></int></th><th><int></int></th><th><int></int></th><th><int></int></th><th><int></int></th><th><int></int></th><th><int></int></th><th><int></int></th><th><int></int></th><th><int></int></th><th><int></int></th><th><int></int></th></int<>	>	<int></int>	<int></int>	<int></int>	<int></int>	<int></int>	<int></int>	<int></int>	<int></int>	<int></int>	<int></int>	<int></int>	<int></int>	<int></int>	<int></int>
1	1	<u>20</u> 000	2	2	1	24	2	2	-1	-1	-2	-2	<u>3</u> 913	<u>3</u> 102	689
2	2	120000	2	2	2	26	-1	2	0	0	0	2	<u>2</u> 682	<u>1</u> 725	<u>2</u> 682
3	3	90000	2	2	2	34	0	0	0	0	0	0	29239	14027	<u>13</u> 559
4	1	50000	2	2	1	37	0	0	0	0	0	0	46990	48233	49291
5	5	50000	1	2	1	57	-1	0	-1	0	0	0	8617	<u>5</u> 670	35835
6	5	50000	1	1	2	37	0	0	0	0	0	0	64400	57069	57608
wit	th 10	more v	ariabl	es: BILL_AN	MT4 <int></int>	. BILL	L AMT5	<int></int>	BILL	_AMT6 -	<int>,</int>	PAY_AN	$\Pi 1 < int>$,	PAY_AMT2	<int>,</int>

Figure 1.0A – Dataset initial 6 observations.

> str(df) # struc			Descriptive statis					
tibble [30,000 × 25]	(S3: tbl_df/tbl/data.frame)	ID	LIMIT_BAL	SEX	EDUCATION	MARRIAGE	AGE PAY	
\$ ID	: int [1:30000] 1 2 3 4 5 6 7 8 9 10					n. :0.000 Min.		:-2.0000
<pre>\$ LIMIT_BAL</pre>	: int [1:30000] 20000 120000 90000 50000 50000 50000							:-1.0000
\$ SEX	: int [1:30000] 2 2 2 2 1 1 1 2 2 1							: 0.0000
\$ EDUCATION	: int [1:30000] 2 2 2 2 2 1 1 2 3 3					an :1.552 Mean		:-0.0167
<pre>\$ MARRIAGE</pre>	: int [1:30000] 1 2 2 1 1 2 2 2 1 2							: 0.0000
\$ AGE	: int [1:30000] 24 26 34 37 57 37 29 23 28 35					ix. :3.000 Max.		: 8.0000
\$ PAY_0	: int [1:30000] 2 -1 0 0 -1 0 0 0 0 -2	PAY_2 Min. :-2.0000	PAY_3 Min. :-2.0000	PAY_4 Min. :-2.000	PAY_5 0 Min. :-2.000	PAY_6 00 Min. :-2.000	BILL_AMT1 0 Min. :-165580	
\$ PAY_2	: int [1:30000] 2 2 0 0 0 0 0 -1 0 -2	1st Ou.:-1.0000	1st Ou.:-1.0000					
\$ PAY_3	: int [1:30000] -1 0 0 0 -1 0 0 -1 2 -2	Median : 0.0000	Median : 0.0000	Median : 0.000				
S PAY 4	: int [1:30000] -1 0 0 0 0 0 0 0 0 -2	Mean :-0.1338	Mean :-0.1662					
\$ PAY 5	: int [1:30000] -2 0 0 0 0 0 0 0 0 -1	3rd Ou.: 0.0000	3rd Qu.: 0.0000					
\$ PAY 6	: int [1:30000] -2 2 0 0 0 0 0 -1 0 -1	Max. : 8,0000						
\$ BILL AMT1	: int [1:30000] 3913 2682 29239 46990 8617 64400 3679	BILL_AMT2	BILL AMT3	BILL_AMT4	BILL_AMT5	BILL_AMT6	PAY_AMT1	
\$ BILL_AMT2	: int [1:30000] 3102 1725 14027 48233 5670 57069 4120	Min. :-69777	Min. :-157264	Min. :-170000	Min. :-81334	Min. :-339603	Min. : 0	
\$ BILL_AMT3	: int [1:30000] 689 2682 13559 49291 35835 57608 4450	1st Qu.: 2985	1st Qu.: 2666	1st Qu.: 2327	1st Qu.: 1763	1st Qu.: 1256	1st Qu.: 1000	
\$ BILL_AMT4	: int [1:30000] 0 3272 14331 28314 20940 19394 542653	Median : 21200	Median : 20088	Median : 19052	Median : 18104	Median : 17071	Median : 2100	
\$ BILL_AMT5	: int [1:30000] 0 3455 14948 28959 19146 19619 483003	Mean : 49179	Mean : 47013	Mean : 43263		Mean : 38872	Mean : 5664	
\$ BILL_AMT6	: int [1:30000] 0 3261 15549 29547 19131 20024 473944	3rd Qu.: 64006	3rd Qu.: 60165	3rd Qu.: 54506	3rd Qu.: 50190	3rd Qu.: 49198	3rd Qu.: 5006	
\$ PAY_AMT1	: int [1:30000] 0 0 1518 2000 2000 2500 55000 380 332	Max. :983931	Max. :1664089	Max. : 891586			Max. :873552	
\$ PAY_AMT2	: int [1:30000] 689 1000 1500 2019 36681 1815 40000 6	PAY_AMT2	PAY_AMT3	PAY_AMT4	PAY_AMT5	PAY_AMT6	default.payment	.next.mon
\$ PAY_AMT3	: int [1:30000] 0 1000 1000 1200 10000 657 38000 0 43	Min. : 0		Min. : 0	Min. : 0.0			
\$ PAY_AMT4	: int [1:30000] 0 1000 1000 1100 9000 1000 20239 581	1st Qu.: 833	1st Qu.: 390	1st Qu.: 296	1st Qu.: 252.5			
\$ PAY_AMT5	: int [1:30000] 0 0 1000 1069 689 1000 13750 1687 100	Median : 2009 Mean : 5921	Median : 1800 Mean : 5226	Median : 1500 Mean : 4826	Median : 1500.6 Mean : 4799.4			
\$ PAY_AMT6	: int [1:30000] 0 2000 5000 1000 679 800 13770 1542 1	3rd Ou.: 5000	3rd Ou.: 4505	Mean : 4826 3rd Ou.: 4013	Mean : 4799.4 3rd Ou.: 4031.5			
	ext.month: int [1:30000] 1 1 0 0 0 0 0 0 0 0	Max. :1684259	Max. :896040	Max. :621000	Max. :426529.0			

Figure 1.0B – Structure and descriptive statistics of the dataset.

The set contains a unique identifier named, ID, which could be excluded. The LIMIT_BAL covers a significant scope ranging from 10000 to 1000000. In regards the SEX variable could be converted to a factor as *dummy variables* "male", "female" and variables PAY_1 to PAY_6 could potentially be converted to factors as payment "on-time", or payment "delayed". Furthermore, the BILL_AMT and PAY_AMT attributes have significant figures which could indicates the need to trim outliers and/or scale the distribution—refer to boxplot

illustration on figure 1.1 for examples of original distribution.

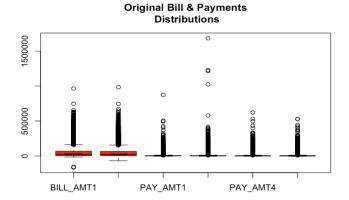


Figure 1.1 – Examples of dispersity across some variables distributions.

Preprocessing: As part of pre-processing, some variables were removed, renamed or

transformed. Initially, the ID variable was the only one removed; however, after running multiple iterations, additional variables were removed given their irrelevancy or low influence against the targeted variable.

Attributes like LIMIT_BAL, EDUCATION, PAY_AMT, BILL_AMT and EDUCATION were excluded using the subset() command while others were renamed and vectored as factors. As shown in the below code, the response variable had been named "default.payment.next.month" and for simplicity purposes for this assessment was renamed to "PROJECTED DEFAULT".

```
> # Irrelevant variables to remove given their low influence
> df <- subset(df, select = -c(ID, LIMIT_BAL, EDUCATION, PAY_AMT3, PAY_AMT5, BILL_AMT3, BILL_AMT4,
                               BILL_AMT5, BILL_AMT6))
> # Convert Marriage status to either married, or not married
> df$MARRIAGE<-factor(df$MARRIAGE == 1, levels = c(FALSE,TRUE), labels = c("NotMarried","Married"))</pre>
> # Convert all PAY variables to OnTime or Delayed
> df$PAY_0<-factor(df$PAY_0>= 1, levels = c(FALSE,TRUE), labels = c("OnTime", "Delayed"))
> df$PAY_2<-factor(df$PAY_2 >= 1, levels = c(FALSE,TRUE), labels = c("OnTime", "Delayed"))
> df$PAY_3<-factor(df$PAY_3 >= 1, levels = c(FALSE,TRUE), labels = c("OnTime", "Delayed"))
> df$PAY_4<-factor(df$PAY_4>= 1, levels = c(FALSE,TRUE), labels = c("OnTime","Delayed"))
> df$PAY_5<-factor(df$PAY_5>= 1, levels = c(FALSE,TRUE), labels = c("OnTime", "Delayed"))
> df$PAY_6<-factor(df$PAY_6>= 1, levels = c(FALSE,TRUE), labels = c("OnTime", "Delayed"))
> colnames(df)[colnames(df)=="default.payment.next.month"]<-"PROJECTED_DEFAULT" # Rename targeted variable
> df$PROJECTED_DEFAULT<-factor(df$PROJECTED_DEFAULT, levels=0:1, labels=c("No","Yes")) # Factor targeted variable
> df$SEX<-factor(df$SEX, levels=1:2, labels = c("Male", "Female"))
> # Preview Bill and Payment Amount distributions, without outliers, and scaled
> boxplot(df[10:15], col=3:6, outline=FALSE, notch=TRUE, main="Scaled Bill & Payments\n Distribution without Outliers")
> df[10:15] <- rm.outlier(df[10:15], fill = TRUE)
> df[10:15]<-scale(df[10:15], center=FALSE, scale=TRUE)
```

Scaled Bill & Payments Distribution without Outliers

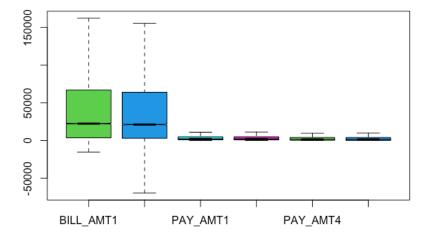


Figure 1.2 – Illustration of variable distributions after scaled and excluding outliers

Moreover, like alluded before, the PAY 0 to PAY 6 variables, which relate to timeliness

of credit card payments, were consolidated to indicate if payments were either submitted "OnTime", or "Delayed. Finally, the continuous variables of BILL_AMT and PAY_AMT were scaled down and centered. Once completed these mentioned preprocessing steps the dataframe ended up with a better structured and organized shape, ready for further analysis and modeling. Figure 1.3 shows a summary of the final frame to model using the logistic regression algorithm.

> summary(df) #	Stats check				
SEX	MARRIAGE	AGE	PAY_0	PAY_2	PAY_3
Male :11888	NotMarried:16341	Min. :21.00	OnTime :23182	OnTime :25562	OnTime :25787
Female:18112	Married :13659	1st Qu.:28.00	Delayed: 6818	Delayed: 4438	Delayed: 4213
		Median :34.00			
		Mean :35.49			
		3rd Qu.:41.00			
		Max. :79.00			
PAY_4	PAY_5	PAY_6	BILL_AMT1	BILL_AMT2	
OnTime :26490	OnTime :27032	OnTime :26921	Min. :-1.84948	Min. :-0.80	830
Delayed: 3510	Delayed: 2968	Delayed: 3079	1st Qu.: 0.03975	1st Qu.: 0.03	458
			Median : 0.24999	Median : 0.24	558
			Mean : 0.57181	Mean : 0.56	933
			3rd Qu.: 0.74936	3rd Qu.: 0.74	140
			Max. : 8.34170	Max. : 8.61	815
PAY_AMT1	PAY_AMT2	PAY_AM	_		JECTED_DEFAULT
Min. : 0.0000					:23364
1st Qu.: 0.0596		•	•	: 0.006444 Yes	: 6636
Median : 0.1252	28 Median : 0.09)253 Median : (0.09377 Median :	0.082086	
Mean : 0.336				0.284457	
3rd Qu.: 0.2986		•	0.25089 3rd Qu.:	0.218895	
Max. :30.1266	67 Max. :56.51	.795 Max. :3	3.06472 Max. :	:28.847221	

Figure 1.3 – Post-EDA and preprocessing dataset of 15 independent variables.

Algorithm Intuition: In simple terms, the logistic regression model forecasts the likelihood of an event to either be one way or the other. In analysis of data, the capacity of foreseen the possibilities of an outcome can help consumers of such data extrapolating assumptions or potential consequences. In supervised learning, logistic regression is one of the fundamental techniques when "finding the best-fitting set of parameters and modeling the relationship between a set of variables". The coefficients generated through this method are key interpreting degrees of probability or conversely levels of uncertainty. In this case, the dataset is comprised of independent categorical and continuous variables (as seen it above) and the response variable dichotomous, the model is handled as a binary logistic regression (Hosmer, Lemeshow, & Sturdivant, 2013).

That said, this logistic regression approach employs the named independent variables of the dataset, which were divided to create two subsets (training and test), to identify how the generated coefficients relate with the dependable variable and what kind of influence these apply to the response variable. As mentioned, the dataframe was divided into two subsets, training and test data. Showing in figure 2.0, 70% of the total dataset was set apart to train the model, while the rest 30%, for testing purposes.

```
# Set the seed, create data subsets
# Set the seed value to ensure that result is reproducible
set.seed(1234)
# Divide the data into train/test subsets
ind<-sample(2, nrow(df), replace=TRUE, prob=c(0.7, 0.3))
train.data<-df [ind == 1,]
test.data<-df [ind == 2,]</pre>
```

Figure 2.0 – Seed value set and dataset divided into training and test data.

Followed the separating of the data, the model was created with the glm() command, using the train.data as the source and specifying the family as binomial(). The below code output shows the immediate result including the coefficients and the model's intercept.

```
> # Build and interpret the model. Store the method output in a variable model
> model <- glm(PROJECTED_DEFAULT ~., data = train.data, family = binomial())</pre>
                     # Output the coefficients and intercept
Call: glm(formula = PROJECTED_DEFAULT ~ ., family = binomial(), data = train.data)
Coefficients:
                      SEXFemale MARRIAGEMarried
    (Intercept)
                                                                      PAY ODelayed
                                                                                      PAY 2Delayed
                                                             AGE
      -1.846898
                      -0.151393
                                        0.149031
                                                         0.002088
                                                                         1.321684
                                                                                          0.168766
   PAY_3Delayed
                   PAY_4Delayed
                                    PAY_5Delayed
                                                  PAY_6Delayed
                                                                        BILL_AMT1
                                                                                         BILL_AMT2
       0.411422
                       0.333658
                                        0.256419
                                                         0.451449
                                                                         -0.253182
                                                                                          0.319032
       PAY_AMT1
                       PAY_AMT2
                                        PAY_AMT4
                                                         PAY_AMT6
                      -0.244426
      -0.258284
                                       -0.116164
                                                        -0.072972
Degrees of Freedom: 20971 Total (i.e. Null); 20956 Residual
Null Deviance:
Residual Deviance: 18740
                               AIC: 18780
```

Figure 2.1 – Initial model with a residual deviance score of 18740.

Next, the summary() command was used to assess the statistics of the model. Illustrated on figure 2.2, the residuals deviance ranged from -1.7754 to 3.4498, with a median of -0.5160. At first sight, the included estimates show positive and negative numbers, underlining the kind of influence these will exert on the targeted variable. Also, most of the coefficients values are under the 0.05 probability rate (see asterisks interpretation), signaling that these values are outside of the standard distribution ranges and should be considered statistically significant. The model produced an Akaike Information Criterion (AIC) rate of 18775 after only 5 iterations.

Finally, based on the illustration of Appendix A., interpretation of the initial model is as follow: Starting from an intercept estimated value of -1.846898, the variable of SEX shows that

a female would be -0.151393 or 15% less-probable to default in their credit card payment. Similarly, few attributes below, the PAY_0 variable indicates that when a costumer fails to submit the payment on time, it has a probability greater than 132% of credit cards default.

The model's odds ratio is captured in below figure 2.2. By definition, the odds ratio is interpreted as a measure of association as it approximates how much more likely or unlikely (in terms of odds) it is for the outcome to be present among those subjects (Hosmer, Lemeshow, & Sturdivant, 2013). In simple words, the odds ratio variable measure the changes of odds of one variable to change when another increases one unit. Case in point, the odds of a customer to default on credit cards payments increases over 370% when history shows that the first payment was delayed.

> exp(coef(model))	# Odds rati	os				
(Intercept)	SEXFemale M	ARRIAGEMarried	AGE	PAY_0Delayed	PAY_2Delayed	PAY_3Delayed
0.1577257	0.8595102	1.1607093	1.0020905	3.7497304	1.1838435	1.5089620
PAY_4Delayed	PAY_5Delayed	PAY_6Delayed	BILL_AMT1	BILL_AMT2	PAY_AMT1	PAY_AMT2
1.3960656	1.2922935	1.5705869	0.7763267	1.3757956	0.7723757	0.7831538
PAY_AMT4	PAY_AMT6					
0.8903291	0.9296270					

Figure 2.2 – Model's odds ratios.

While assessing the accuracy of the model, the *caret* and *e1071* libraries were utilized. Per the graphic bellow, the confusionMatrix() command illustrates that within 95% confidence intervals the accuracy of the model is rated at $\sim 65\%$.

```
> confusionMatrix(df$PROJECTED_DEFAULT[1:length(train.data$PROJECTED_DEFAULT)],
                 train.data$PROJECTED_DEFAULT, dnn=c("Predicted","Actual"))
Confusion Matrix and Statistics
        Actual
Predicted
     cted No Yes
No 12583 3603
      Yes 3765 1021
               Accuracy : 0.6487
                 95% CI : (0.6422, 0.6551)
    No Information Rate: 0.7795
    P-Value [Acc > NIR] : 1.0000
Mcnemar's Test P-Value: 0.0607
            Sensitivity: 0.7697
            Specificity: 0.2208
         Pos Pred Value : 0.7774
         Neg Pred Value : 0.2133
            Prevalence: 0.7795
         Detection Rate: 0.6000
  Detection Prevalence : 0.7718
      Balanced Accuracy : 0.4953
       'Positive' Class : No
```

Another area evaluated within the model was the precision, recall, and specificity. These

metrics are normally used to overall quality of a model. Precision highlights the accuracy of a predicted positive outcome, recall measures the strength of the model, and specificity the model's ability to predict a negative outcome (Bruce & Bruce, 2017). Figure 2.4 shows the receiver operating characteristics (ROC) curve. The closer the generated line gets to the top 1.0 parameter the more accurate the model will be considered.

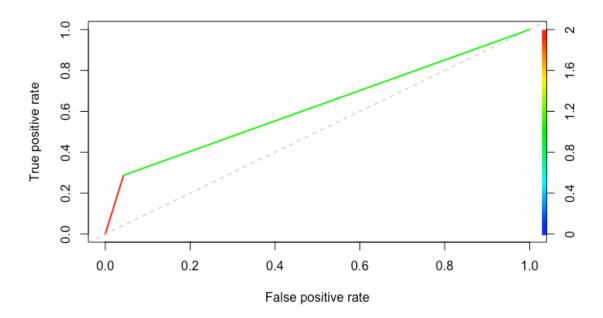


Figure 2.4 – ROC Curve for original model.

Finally, the model was assessed in terms diagnostic analysis of residuals, the delta

between what the model predicted versus what was actually observed. Illustrated on figure 2.5, the graphic shows the level of usefulness the model based on the partial residuals out of the model (ranges between As previously mentioned, the angle of the residuals is influence by the dispersed variation of the data values. Of notice, in logistic regression the prime objective is to find that the model that gives you that maximum likelihood estimation (Bruce & Bruce, 2017).

```
Deviance Residuals:

Min 1Q Median 3Q Max

-1.7754 -0.5665 -0.5160 -0.3587 3.4498
```

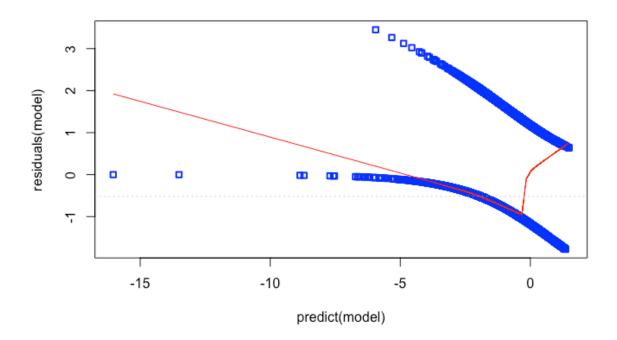


Figure 2.5 – Model residuals graphic.

Summary: Based on the aforesaid evaluation, it is accurate to say that the model yields a level of practicality while identifying potential default customers by considering indicators like

previously missed payments, payments amount trend, or account balance (reflected on billed amount). One of the reasons the residual model shows a significant amount of negative values is due to the nature of the PAY_AMT variable, which in essence goes against letting the customer default the credit card account. Nevertheless, the model satisfies the initial intend of this study validating the feasibility for financial institutions to monitor their costumers' payment patters or characteristics.

As intended, prior of creating the model the entire dataset was preprocessed and structured for max efficiency. The training data was allotted with 70% of the original set and the rest 30% was preserved for the test data. In regard to the model, the generated coefficients depicted consistent high z-score values and significantly low probability values. In terms of residual deviance the model produced a residual deviance of 18743 on 20956 degrees of freedom, representing an 18% decrease of residual deviance.

In close, this results accentuate an ample business niche in where data analytics can contributed in a unique way the financial sector among many other branches. This assessment intended to weigh the logistic regression model capacity and efficacy while predicting potential credit card defaulting customer accounts. The elaborated R code, process and graphics displays how the inquiry was conducted. In order to generate a more robust model additional data inputs like income, expenses, household characteristics, job details, additional assets, etc. may be required. Additional recommendations include improving the quality of data and/or consistency of data collected, dynamic updates to a customer's profile based on registered expenses, and synchronizing near-real-time transactions to predict consumers inclinations or future expenses.

From a macro-perspective, the mentioned case illustrates how critical predictability or probability of default could be for financial institutions and their risk management departments when approving or denying credit applications. The following assessment and report aims to explore a customer credit card payments dataset and shows how the data mining technique of logistic regression could benefit the financial sector foreseeing credit card default payments based on customers payment trends and/or activities interrelated to their respective credit balances.

References

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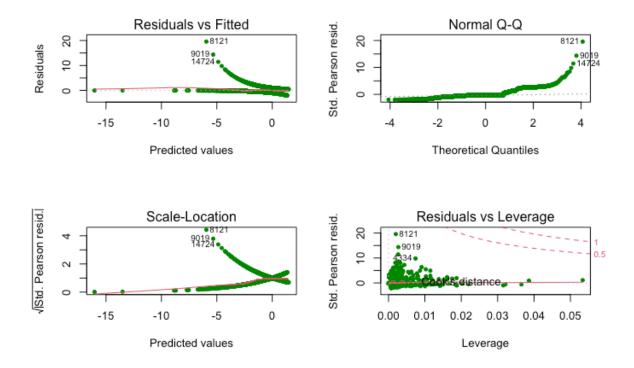
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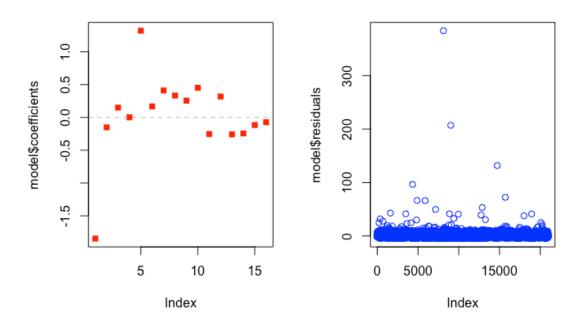
Appendix

```
> summary(model)
                     # Output the p value for each coefficient
Call:
glm(formula = PROJECTED_DEFAULT ~ ., family = binomial(), data = train.data)
Deviance Residuals:
                  Median
             1Q
   Min
                               30
                                      Max
-1.7754 -0.5665 -0.5160 -0.3587
                                   3.4498
Coefficients:
                Estimate Std. Error z value Pr(>|z|)
                           0.083275 -22.178 < 2e-16 ***
(Intercept)
               -1.846898
                           0.037697 -4.016 5.92e-05 ***
SEXFemale
               -0.151393
MARRIAGEMarried 0.149031
                           0.041621
                                     3.581 0.000343 ***
                0.002088
                           0.002227 0.938 0.348408
AGE
PAY_0Delayed
                1.321684
                           0.049752 26.565 < 2e-16 ***
PAY_2Delayed
                0.168766
                           0.067588 2.497 0.012525 *
                           0.067690 6.078 1.22e-09 ***
PAY_3Delayed
                0.411422
                           0.074731 4.465 8.01e-06 ***
PAY_4Delayed
                0.333658
PAY_5Delayed
                0.256419
                           0.082360 3.113 0.001849 **
PAY_6Delayed
                0.451449
                           0.070667 6.388 1.68e-10 ***
BILL_AMT1
               -0.253182
                           0.109119 -2.320 0.020328 *
BILL_AMT2
                0.319032
                           0.110110 2.897 0.003763 **
PAY_AMT1
                           0.046768 -5.523 3.34e-08 ***
               -0.258284
PAY_AMT2
                           0.053516 -4.567 4.94e-06 ***
               -0.244426
                           0.033550 -3.462 0.000535 ***
PAY_AMT4
               -0.116164
PAY_AMT6
               -0.072972
                           0.028968 -2.519 0.011768 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 22126 on 20971 degrees of freedom
Residual deviance: 18743 on 20956 degrees of freedom
AIC: 18775
Number of Fisher Scoring iterations: 5
```

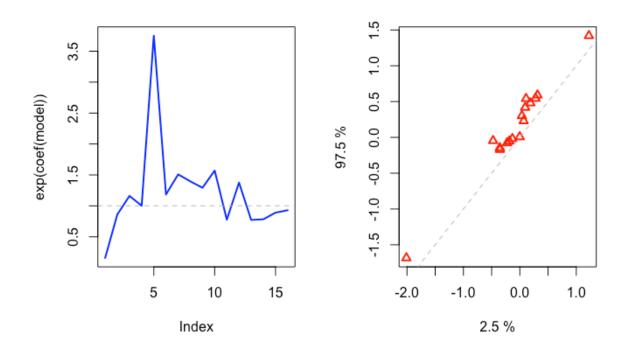
Appendix A. Initial model descriptive statistics.



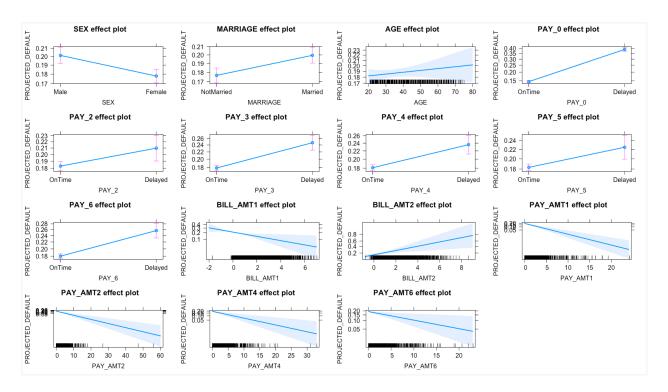
Appendix A1. Training data initial model.



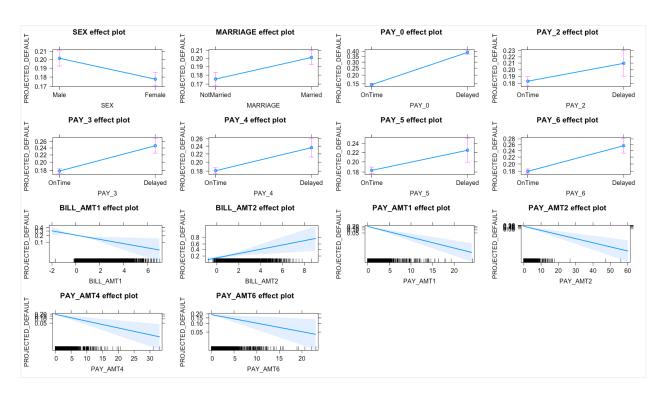
Appendix A2. Training data model coefficients and residuals.



Appendix A2. Training data model Odds ratios and confidence intervals.



Appendix A3 – Model 1, all-Effects plot.



Appendix A4 – Reduced model all-Effects plot.