Lesson 7a

DIRECTIONAL DERIVATIVES AND GRADIENTS

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OBJECTIVES:

At the end of the lesson, the student must be able to :

1. Compute the directional derivatives of a function in two and three variables using first-order partial derivatives.
2. Find the gradient of a function and apply its properties.

DIRECTIONAL DERIVATIVES OF A FUNCTION OF TWO VARIABLES

Directional derivatives allow us to compute the rates of change of a function with respect to distance in any direction.

DEFINITION 1: If $f(x, y)$ is a function of $x\ and\ y$, and if $u = u_1 i + u_2 j$ is a unit vector, then the directional derivative of f in the direction of $u\ at\ (x_o, y_o)$ is denoted by

$$\boldsymbol{D_u f(x_o, y_o) = \frac{d}{dx}[f(x_o + su_1, y_o + su_2)]_{s=o}}$$

provided this derivative exists

DIRECTIONAL DERIVATIVES OF A FUNCTION OF TWO VARIABLES

$\boldsymbol{D_u f(x_o, y_o)}$ can be interpreted as the $slope\ of\ the\ curve$ $z = f(x, y)$ in the direction of u $at\ the\ point$ $(x_o, y_o, f(x_o, y_o))$.

Analytically the directional derivative represents the $instantaneous\ rate\ of\ change\ of\ f(x, y)$ with respect to distance in the direction of $u\ at\ the\ point$.

Directional Derivative of Function of Three variables

DEFINITION 1: If $u = u_1 i + u_2 j + u_3 k$ is a unit vector, and if $f(x, y, z)$ is a function of x, y, z then the directional derivative of f in the direction of u at (x_o, y_o, z_o) is denoted by $\boldsymbol{D_u f(x_o, y_o, z_o)}$ and is defined by

$$\boldsymbol{D_u f(x_o, y_o, z_o) = \frac{d}{dx}[f(x_o + su_1, y_o + su_2, z_o + su_3),]_{s=o}}$$

provided this derivative exists.

Example 1

Example 1. Let $f(x,y) = xy$. Find and $D_u f(1,2)$ for the unit vector $u = \frac{\sqrt{3}}{2} i + \frac{1}{2} j$

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Solution:

using $\boxed{D_u f(x_0, y_0) = \frac{d}{ds}[f(x_0 + su_1, y_0 + su_2)]_{s=0}}$ thus,

$$D_u f(1,2) = \frac{d}{ds}\left[f(1 + \tfrac{\sqrt{3}}{2}s, 2 + \tfrac{1}{2}s)\right] \; at\ s = 0 \text{ and}$$

and $f(x,y) = xy$, then

$$f(1 + \tfrac{\sqrt{3}}{2}s, 2 + \tfrac{1}{2}s) = \left(1 + \tfrac{\sqrt{3}}{2}s\right)\left(2 + \tfrac{1}{2}s\right) \rightarrow \text{by multiplication}$$

$$f(1 + \tfrac{\sqrt{3}}{2}s, 2 + \tfrac{1}{2}s) = 2 + \left(\tfrac{1}{2} + \sqrt{3}\right)s + \tfrac{\sqrt{3}}{4}s^2$$

Since we have to find $D_u f(1,2)$ for the unit vector

$$u = \frac{\sqrt{3}}{2} i + \frac{1}{2} j$$

Then, $D_u f(1,2) = \frac{d}{ds}\left[2 + \left(\frac{1}{2} + \sqrt{3}\right)s + \frac{\sqrt{3}}{4}s^2\right]$ at s = 0

By taking derivative with respect to s,

$$D_u f(1,2) = \left[\left(\tfrac{1}{2} + \sqrt{3}\right) + \tfrac{\sqrt{3}}{4}(2s)\right] \text{ at s = 0}$$

$$D_u f(1,2) = \left[\left(\tfrac{1}{2} + \sqrt{3}\right)\right] \cong 2.23 \text{ (Final Answer)}$$

Directional Derivative of Function of Two or Three variables in terms of Partial Derivatives

For a function that is differentiable at a point, directional derivatives exist in every direction from the point and can be computed directly in terms of the first-order partial derivative of the function.

THEOREM:

(a) If $f(x, y)$ is differentiable at (x_o, y_o) and if $u = u_1 i + u_2 j$ is a unit vector, then the directional derivative $\boldsymbol{D_u f(x_o, y_o)}$ exists and is given by

$$\mathbf{D_u\, f(x_o, y_o) = f_x(x_o, y_o)u_1 + f_y(x_o, y_o)u_2}$$

(b) If $f(x, y, z)$ is differentiable at (x_o, y_o, z_o) and if $u = u_1 i + u_2 j + u_3 k$ is a unit vector, then the directional derivative $\boldsymbol{D_u f(x_o, y_o, z_o)}$ exists and is given by

$$\mathbf{D_u\, f(x_o, y_o, z_o) = f_x(x_o, y_o, z_o)u_1 + f_y(x_o, y_o, z_o)u_2 + f_z(x_o, y_o, z_o)u_3}$$

EXAMPLE 1

Find $D_u f$ at P

1. $f(x,y) = ln(1 + x^2 + y^2)$; $\mathrm{P}(0.0)$; $u = \frac{1}{\sqrt{10}} i - \frac{3}{\sqrt{10}} j$

$$\boxed{D_{\mathbf{u}} f(x_0, y_0) = f_x(x_0, y_0) u_1 + f_y(x_0, y_0) u_2}$$

Solution :

EXAMPLE 1

Find $D_u f$ at P

1. $f(x,y) = ln(1+x^2+y^2)$; $\mathrm{P}(0.0)$; $u = \frac{1}{\sqrt{10}} i - \frac{3}{\sqrt{10}} j$

$$\boxed{D_{\mathbf{u}} f(x_0, y_0) = f_x(x_0, y_0) u_1 + f_y(x_0, y_0) u_2}$$

Solution :

$f_x(x,y) = \frac{2x}{1+x^2+y^2}$; $\quad f_x(0,0) = 0$

$f_y(x,y) = \frac{2y}{1+x^2+y^2}$; $\quad f_y(0,0) = 0$

$D_u f(0,0) = 0$

Example 2

Example 2: Find the directional derivative of $f(x, y, z) = x^2y - yz^3 + z$ at the point (1, -2, 0) in the direction of the vector $a = 2i + j - 2k$.

Required: $D_u f(1, -2, 0)$

$$\mathbf{D_u\,f(x_o, y_o, z_o)} \\ \mathbf{= f_x(x_o, y_o, z_o)u_1 + f_y(x_o, y_o, z_o)u_2 + f_z(x_o, y_o, z_o)u_3}$$

Solution:

Example 2

Example 2: Find the directional derivative of $f(x, y, z) = x^2y - yz^3 + z$ at the point (1, -2, 0) in the direction of the vector $a = 2i + j - 2k$.

Required: $D_u f(1, -2, 0)$

$$\mathbf{D_u\,f(x_o, y_o, z_o)} \\ = \mathbf{f_x(x_o, y_o, z_o)u_1 + f_y(x_o, y_o, z_o)u_2 + f_z(x_o, y_o, z_o)u_3}$$

Solution:

Since a is not a unit vector, we normalize it using the formula $u = \frac{a}{\|a\|}$

Thus, $a = 2i + j - 2k = \langle 2, 1, -2 \rangle$ and $\|a\| = \sqrt{(2)^2 + (1)^2 + (-2)^2} = 3$

$$u = \frac{\langle 2, 1, -2 \rangle}{3} = \left\langle \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} \right\rangle$$

con’t

Then get the partial derivatives of the function given
$f(x,y,z) = x^2y - yz^3 + z$
Thus, $f_x(x,y,z) = 2xy$, $f_y(x,y,z) = x^2 - z^3$, $f_z(x,y,z) = -3yz^2 + 1$
And $f_x(1,-2,0) = -4$, $f_y(1,-2,0) = 1$ $f_z(1,-2,0) = 1$
To solve $D_u f(1,-2,0)$, use the formula from previous slide

$$\mathbf{D_u\, f(x_o, y_o, z_o) = f_x(x_o, y_o, z_o)u_1 + f_y(x_o, y_o, z_o)u_2 + f_z(x_o, y_o, z_o)u_3}$$

Thus, substituting all the values from the unit vector u and partial derivatives of the given function, we get
$D_u f(1,-2,0) = (-4)\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)(1) + \left(-\frac{2}{3}\right)(1) = -3$ (Final Answer)

EXAMPLE 3

3. $f(x,y,z) = 4x^5y^2z^2$; $P(2,-1,1)$; $u = \frac{1}{3}i + \frac{2}{3}j - \frac{2}{3}k$

Solution: $$\mathbf{D_u\, f(x_o, y_o, z_o) = f_x(x_o, y_o, z_o)u_1 + f_y(x_o, y_o, z_o)u_2 + f_z(x_o, y_o, z_o)u_3}$$

EXAMPLE 3

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$$\mathbf{D_u\, f(x_o, y_o, z_o) = f_x(x_o, y_o, z_o)u_1 + f_y(x_o, y_o, z_o)u_2 + f_z(x_o, y_o, z_o)u_3}$$

Solution:

$f_x(x,y,z) = 20x^4y^2z^2$; $\quad f_x(2,-1,1) = 320$

$f_y(x,y,z) = 8x^5yz^2$; $\quad f_y(2,-1,1) = -256$

$f_z(x,y,z) = 8x^5y^2z$; $\quad f_z(2,-1,1) = 256$

$$D_uf(2,-1,1) = 320\left(\frac{1}{3}\right) - 256\left(\frac{2}{3}\right) + 256\left(-\frac{2}{3}\right)$$

$$= \frac{320}{3} - \frac{512}{3} - \frac{512}{3}$$

$$D_uf(2,-1,1) = -\frac{704}{3}$$

EXAMPLE 4

4. $f(x,y,z) = ln(x^2 + 2y^2 + 4z^2)$; $P(-1,2,4)$; $u = -\frac{3}{13}i - \frac{4}{13}j - \frac{12}{13}k$

Solution:

EXAMPLE 4

4. $f(x,y,z) = ln(x^2 + 2y^2 + 4z^2)$; $P(-1,2,4)$; $u = -\frac{3}{13}i - \frac{4}{13}j - \frac{12}{13}k$

Solution:

$f_x(x,y,z) = \frac{2x}{x^2+2y^2+4z^2}$; $f_x(-1,2,4) = \frac{-2}{73}$

$f_y(x,y,z) = \frac{4y}{x^2+2y^2+4z^2}$; $f_y(-1,2,4) = \frac{8}{73}$

$f_z(x,y,z) = \frac{8z}{x^2+2y^2+4z^2}$; $f_z(-1,2,4) = \frac{32}{73}$

$$D_u f(-1,2,4) = \frac{-2}{73}\left(\frac{-3}{13}\right) + \frac{8}{73}\left(\frac{-4}{13}\right) + \frac{32}{73}\left(\frac{-12}{13}\right)$$

$$= \frac{6}{949} - \frac{32}{949} - \frac{384}{949}$$

$$D_u f(-1,2,4) = \frac{-410}{949}$$

THE GRADIENT

The gradient vector can be interpreted as the "direction and rate of fastest increase". If the gradient of a function is non-zero at a point p, the direction of the gradient is the direction in which the function increases most quickly from p, and the magnitude of the gradient is the rate of increase in that direction. Further, the gradient is the zero vector at a point if and only if it is a stationary point (where the derivative vanishes). The gradient thus plays a fundamental role in optimization theory, where it is used to maximize a function by gradient ascent.

Gradient of a Function of Two and Three Variables

DEFINITION:

(a) If f is a function of x *and* y, then the *gradient of f is defined by*

$$\nabla f(x,y) = f_x(x,y)i + f_y(x,y)j \text{ ....(1)}$$

(b) If f is a function of x, y, *and* z, then the *gradient of f is defined by*

$$\nabla f(x,y,z) = f_x(x,y,z)i + f_y(x,y,z)j + f_z(x,y,z)k \text{................(2)}$$

The symbol ∇ (read "del") is an inverted delta.

EXAMPLE

1. Find ∇w if $w = ln\sqrt{x^2 + y^2 + z^2}$

Solution: Recall:

$$\nabla f(x, y, z) = f_x\,(x, y, z)i + f_y\,(x, y, z)j + f_z\,(x, y, z)k$$

EXAMPLE

1. Find ∇w if $w = ln\sqrt{x^2 + y^2 + z^2}$

Solution: Recall:

$$\nabla f(x,y,z) = f_x(x,y,z)i + f_y(x,y,z)j + f_z(x,y,z)k$$

$$w_x = \frac{1}{2}\left(\frac{2x}{x^2 + y^2 + z^2}\right) = \frac{x}{x^2 + y^2 + z^2}$$

$$w_y = \frac{1}{2}\left(\frac{2y}{x^2 + y^2 + z^2}\right) = \frac{y}{x^2 + y^2 + z^2}$$

$$w_z = \frac{1}{2}\left(\frac{2z}{x^2 + y^2 + z^2}\right) = \frac{z}{x^2 + y^2 + z^2}$$

$$\nabla w = \frac{x}{x^2 + y^2 + z^2}i + \frac{y}{x^2 + y^2 + z^2}j + \frac{z}{x^2 + y^2 + z^2}k$$

EXAMPLE

2. Find the gradient of f at the indicated point.

$$f(x, y, z) = yln(x + y + z); \qquad (-4, 5, 0)$$

Solution: Recall: $\nabla f(x, y, z) = f_x(x, y, z)i + f_y(x, y, z)j + f_z(x, y, z)k$

EXAMPLE

2. Find the gradient of f at the indicated point.

$$f(x, y, z) = yln(x + y + z); \qquad (-4, 5, 0)$$

Solution: Recall: $\nabla f(x, y, z) = f_x\,(x, y, z)i + f_y\,(x, y, z)j + f_z\,(x, y, z)k$

$f_x(x, y, z) = \frac{y}{x+y+z};$ $\qquad f_x(-4, 5, 0) = 5$

$f_y(x, y, z) = \frac{y}{x+y+z} + ln(x, y, z);$ $\qquad f_y(-4, 5, 0) = 5$

$f_z(x, y, z) = \frac{y}{x+y+z};$ $\qquad f_z(-4, 5, 0) = 5$

$$\nabla f(x, y, z) = 5i + 5j + 5k$$

Directional Derivatives of a Function Using Gradient

Recall: Formula of Directional Derivatives of a Function

1) $\mathbf{D_u\, f(x_o, y_o) = f_x(x_o, y_o)u_1 + f_y(x_o, y_o)u_2}$

2) $\mathbf{D_u\, f(x_o, y_o, z_o) = f_x(x_o, y_o, z_o)u_1 + f_y(x_o, y_o, z_o)u_2 + f_z(x_o, y_o, z_o)u_3}$

Recall: Formula of Gradient of a Function

3) $\boldsymbol{\nabla f(x, y) = f_x\,(x, y)i + f_y\,(x, y)j}$

4) $\boldsymbol{\nabla f(x, y, z) = f_x\,(x, y, z)i + f_y\,(x, y, z)j + f_z\,(x, y, z)k}$

Formulas 1 and 2 can be expressed as a dot product where

1) $\mathbf{D_u\, f(x_o, y_o) = (f_x(x_o, y_o)}\boldsymbol{i} + \mathbf{f_y(x_o, y_o)}\boldsymbol{j}) \cdot (\boldsymbol{u_1 i + u_2 j})$

2) $\mathbf{D_u\, f(x_o, y_o, z_o) = (f_x(x_o, y_o, z_o)i + f_y(x_o, y_o, z_o)j + f_z(x_o, y_o, z_o)}\boldsymbol{j}) \cdot (\boldsymbol{u_1 i + u_2 j + u_3 k})$

The 2 formulas can be written as

$$\mathbf{D_u\, f(x_o, y_o)} = \boldsymbol{\nabla f(x_o, y_o) \cdot u}$$

and

$$\mathbf{D_u\, f(x_o, y_o, z_o)} = \boldsymbol{\nabla f(x_o, y_o, z_o) \cdot u}$$

The formula

$$D_{\mathbf{u}}f(x_0, y_0) = \nabla f(x_0, y_0) \cdot \mathbf{u}$$

Can be interpreted to mean that the slope of the surface $z = f(x, y)$ at the point $(x_{0,}\, y_0)$ in the direction of u is the dot product of the gradient with u.

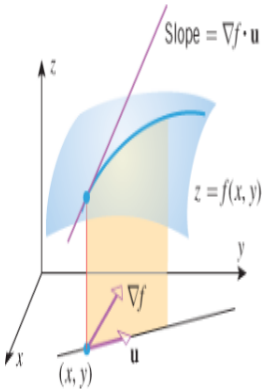

Figure 13.6.4

Example

1. Find the directional derivative of $f(x,y,z) = x^2y - yz^3 + z$ at the point (1, -2, 0) in the direction of the vector $a = 2i + j - 2k$ using the above formulas.

Required: $D_u f(1,-2,0)$
Recall: $\mathbf{D_u\, f(x_o, y_o, z_o) = (f_x(x_o, y_o, z_o)i + f_y(x_o, y_o, z_o)j + f_z(x_o, y_o, z_o)}\boldsymbol{j}) \cdot (\boldsymbol{u_1 i + u_2 j + u_3 k})$

or $$\mathbf{D_u\, f(x_o, y_o, z_o)} = \nabla \boldsymbol{f(x_o, y_o, z_o) \cdot u}$$

Example

1. Find the directional derivative of $f(x, y, z) = x^2y - yz^3 + z$ at the point (1, -2, 0) in the direction of the vector $a = 2i + j - 2k$ using the above formulas.

Required: $D_u f(1, -2, 0)$

Solution:

from the formula, $D_u f(1, -2, 0) = \nabla f(1, -2, 0) \cdot u$

where $u = \frac{a}{\|a\|} = \frac{2i+j-2k}{3} = \left\langle \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} \right\rangle$ and $\nabla f(1, -2, 0) = f_x(1, -2, 0)i + f_y(1, -2, 0) + f_z(1, -2, 0)$

By taking partial derivatives of the function f: $f_x(x, y, z) = 2xy$, $f_y(x, y, z) = x^2 - z^3$, $f_z(x, y, z) = -3yz^2 + 1$

And substituting the point (1, -2, 0): $f_x(1, -2,0) = -4$, $f_y(1, -2, 0) = 1$, $f_z(1, -2,0) = 1$

Thus, $\nabla f(1, -2, 0) = -4i + j + k$

Therefore, $D_u f(1, -2, 0) = \nabla f(1, -2, 0) \cdot u = \langle -4, 1, 1 \rangle \cdot \left\langle \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} \right\rangle = -4\left(\frac{2}{3}\right) + 1\left(\frac{1}{3}\right) + 1\left(\frac{-2}{3}\right) = -3$

PROPERTIES OF THE GRADIENT

THEOREM:

Let f be a function of either two variables or three variables, and let P denote the point $\mathrm{P}(x_o, y_o)$ or $\mathrm{P}(x_o, y_o, z_o)$ respectively. Assume that f is differentiable at P.

(a) If $\nabla f = 0 \ at \ P$, then all directional derivatives of $f \ at \ P$ are zero.

(b) If $\nabla f \neq 0 \ at \ P$, then among all possible directional derivatives of $f \ at \ P$, the derivative in the direction of ∇f at P has the largest value. The value of this largest directional derivative is $\|\nabla f\|$ at P.

(c) If $\nabla f \neq 0 \ at \ P$, then among all possible directional derivatives of $f \ at \ P$, the derivative in the direction opposite to that of ∇f at P has the smallest value. The value of this smallest directional derivative is $-\|\nabla f\|$ at P.

EXAMPLE

1. Find a unit vector in the direction in which f increases most rapidly at P, and find the rate of change of $f\ at\ P$ in that direction (also as the maximum value of a directional derivative at P).

$f(x, y) = \sqrt{x^2 + y^2}$; $P(4,3)$

Solution: Recall $u = \frac{\nabla f(x,y)}{\|\nabla f(x,y)\|}$ where: $\nabla f(x, y) = f_x(x, y)i + f_y(x, y)j$

EXAMPLE

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Solution:

$f(x, y) = \sqrt{x^2 + y^2}\,; \quad P(4,3)$

$f_x(x, y) = \frac{x}{\sqrt{x^2+y^2}}\,; \quad f_x(4,3) = \frac{4}{5} \qquad f_y(x, y) = \frac{y}{\sqrt{x^2+y^2}}\,; \quad f_y(4,3) = \frac{3}{5}$

$$\nabla f(x, y) = f_x\,(x, y)i + f_y\,(x, y)j$$
$$= \frac{x}{\sqrt{x^2+y^2}}\ i + \frac{y}{\sqrt{x^2+y^2}}\ j$$
$$\nabla f(4,3) = \frac{4}{5} i + \frac{3}{5}\ j$$

The maximum value of the directional derivative $\|\nabla f(4,3)\| = \sqrt{\frac{16}{25} + \frac{9}{25}} = 1$

This maximum occurs in the direction of $\nabla f(4,3)$. The unit vector in this direction is

$$u = \frac{\nabla f(4,3)}{\|\nabla f(4,3)\|} = \frac{4}{5}\ i + \frac{3}{5}\ j$$

EXAMPLE

2. Find a unit vector in the direction in which f increases most rapidly at P, and find the rate of change of f at P in that direction (also as the maximum value of a directional derivative at P).

$$f(x, y, z) = x^3z^2 + y^3z + z - 1; \qquad P(1, 1, -1)$$

Solution: Recall $u = \frac{\nabla f(x,y,z)}{\|\nabla f(x,y,z)\|}$

where: $\nabla f(x, y, z) = f_x(x, y, z)i + f_y(x, y, z)j + f_z(x, y, z)k$

EXAMPLE

2. Find a unit vector in the direction in which f increases most rapidly at P, and find the rate of change of f at P in that direction (also as the maximum value of a directional derivative at P).

$$f(x,y,z) = x^3z^2 + y^3z + z - 1; \qquad P(1,1,-1)$$

Solution:

$$f_x(x,y,z) = 3x^3z^2; \qquad f_x(1,1,-1) = 3$$
$$f_y(x,y,z) = 3y^2z; \qquad f_y(1,1,-1) = -3$$
$$f_z(x,y,z) = y^3 + 1 + 2x^3z; \qquad f_z(1,1,-1) = 0$$

$$\nabla f(x,y,z) = f_x\,(x,y,z)i + f_y\,(x,y,z)j + f_z\,(x,y,z)k$$
$$= 3x^3z^2\,i + 3y^2z\,j + (\,y^3+1 + 2x^3z\,)k$$
$$\nabla f(1,1,-1) = 3i - 3j$$

The maximum value of the directional derivative $\|\nabla f(1,1,-1)\| = \sqrt{9+9} = 3\sqrt{2}$ This maximum occurs in the direction of $\nabla f(1,1,-1)$. The unit vector in this direction is

$$u = \frac{\nabla f(1,1,-1)}{\|\nabla f(1,1,-1)\|} = \frac{3i-3j}{3\sqrt{2}} = \frac{i}{\sqrt{2}} - \frac{j}{\sqrt{2}}$$

EXAMPLE

3. Let $f(x, y) = x^2e^y$. Find the maximum value of a directional derivative at (-2, 0) and find the unit vector in the direction in which the maximum value occurs.

Required: $\|\nabla f(-2,0)\|$ and u

Solution: Recall $u = \frac{\nabla f(x,y)}{\|\nabla f(x,y)\|}$

where: $\nabla f(x, y) = f_x(x, y, z)i + f_y(x, y, z)j$

EXAMPLE

3. Let $f(x,y) = x^2e^y$. Find the maximum value of a directional derivative at (-2, 0) and find the unit vector in the direction in which the maximum value occurs.

Required: $\|\nabla f(-2,0)\|$ and u

Solution:

Taking the partial derivatives, $f_x(x,y) = 2xe^y$ and $f_y(x,y) = x^2e^y$

And evaluating it at the given point, $f_x(-2,0) = -4$ and $f_y(-2,0) = 4$

Using the formula $\nabla f(x,y) = f_x(x,y)i + f_y(x,y)j$

Thus, $\nabla f(-2,0) = -4i + 4j$

The maximum value of the directional derivative is

$\|\nabla f(-2,0)\| = \sqrt{(-4)^2 + (4)^2} = \sqrt{32} = 4\sqrt{2}$ (Final Answer)

Finding the unit vector u by using the formula $u = \frac{\nabla f(x,y)}{\|\nabla f(x,y\|}$

$u = \frac{\nabla f(-2,0)}{\|\nabla f(-2,0)\|} = \frac{-4i+4j}{4\sqrt{2}} = \frac{-1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$ (Final Answer)