Covering Groups

1 Schur Multiplier

A_n as quotient group.

preimage of $\pi \in A_n$

[] instead of ()

Definition 1.1. *$\tilde{G}$ is a covering group of G if $Z(\tilde{G}) \leq \tilde{G}'$ and $\tilde{G}/Z(\tilde{G}) \cong G$.*

double, triple cover

Theorem 1.2 (Schur Theorem, *universal cover*)**.** *Every finite perfect group G has a unique maximal covering group $\tilde{G}$, with the property that every other covering group is a quotient of $\tilde{G}$.*

Example 1.3 (non-perfect)**.** *$\mathbb{Z}_2 \times \mathbb{Z}_2 = \langle \alpha \rangle \times \langle \beta \rangle$ has four maximal covering groups: one Q_8 and three D_8.*

$Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$, $Z(Q_8) = \{\pm 1\}$, $Q_8/Z(Q_8) \cong \mathbb{Z}_2 \times \mathbb{Z}_2$.

$D_8 = \langle a \rangle : \langle b \rangle$,

2 Double Covers of A_n and S_n

Now we define $2.S_n$.

Firstly, let G be a set of order $2n!$, with a map φ onto S_n such that each $\pi \in S_n$ has exactly two preimages denoted as $+\pi$ and $-\pi$.

Intuitively, we should define the multiplication of G as $+\pi + \sigma = +(\pi\sigma)$ and $+\pi - \sigma = -(\pi\sigma)$.

WLOG, we denote $+(1\ 2)$ as $[1\ 2]$ and $-(1\ 2)$ as $-[1\ 2]$. Then for each 2-cycle $\pi \in S_n$, taking $(+\pi)^{-1}$ to be a preimage of π, define the products (of 3 elements in G) $[i\ j]^{+\pi}$ and $[i\ j]^{-\pi}$ to be a same preimage of $(i^\pi\ j^\pi)$, we denote it as $-[i^\pi\ j^\pi]$. That is $[i\ j]^{\pm\pi} = -[i^\pi\ j^\pi]$.

Then define the preimage of $(a_i, a_{i+1}, \cdots, a_j) by [a_i\ a_{i+1}][a_{i+1}\ a_{i+2}] \cdots [a_{j-1}\ a_j] = [a_i\ a_{i+1}][a_i\ a_{i+2}] \cdots [a_i\ a_j]$.

3 Triple Covers of A_6 and A_7