Learning Evidence

Some questions

Basic Arithmetic of Complex Numbers.

Q. Simplify the complex number $(2+5i) - (3-4i)$

$$(2+5i) - (3-4i) = 2-3+5i+4i$$
$$= 9i - 1$$

Q. multiply $(1+2i)(4-3i)$ and express the result in standard form.

$$(1+2i)(4-3i) = 1.4 + 1.(-3i) + 2i.4 + 2i.(-3i)$$
$$= 4 - 3i + 8i - 6i^2$$

Since $i^2 = -1$

$$= 4 - 3i + 8i + 6$$
$$= 10 + 5i.$$

Q. Plot the complex number $z = 3+2i$ on the Argand plane. Calculate its modulus & argument:

Modulus:

$$|z| = \sqrt{a^2+b^2} = \sqrt{3^2+2^2} = \sqrt{9+4}$$
$$= \sqrt{13} \approx 3.61.$$

Argument:

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{2}{3}\right) \approx 33.69^\circ$$

Q. Convert the complex number $z = -2+2i$ to polar form:

- Modulus: $|z| = \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = 2\sqrt{2}$
- Argument: $\theta = \tan^{-1}\left(\frac{2}{-2}\right) = \tan^{-1}(-1) = 135^\circ.$

polar form $z = -2+2i$ is $2\sqrt{2}\,\mathrm{cis}\,135$

Q. Apply a unitary matrix to a quantum state $|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ using the matrix $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$:

$$U \cdot |\psi(0)\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

After applying unitary matrix the new state is $|\psi(t)\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Q. Calculate the eigenvalues of the matrix $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$:

Characteristic equation is

$$\det(A - \lambda I) = \det \begin{pmatrix} -\lambda & -1 \\ 1 & -\lambda \end{pmatrix} = \lambda^2 + 1 = 0$$

Solving for λ:

$$\lambda^2 = -1 \Rightarrow \lambda = \pm i$$

eigenvalues are i and $-i$

Q. Analyse the stability of a system with eigenvalues $-2+i$ and $-2-i$:

→ Real parts of eigenvalues are both -2. negative it is stable.

→ Imaginary parts $i, -i$ indicate oscillatory behavior.

∴ Real parts are negative, oscillations will decay over time.

Q. Calculate the impedance of an AC circuit with a resistor of 5Ω and an inductor with a reactance of 3Ω.

→ Impedance $Z = R + jX$, where $R = 5$ and $X = 3$

$Z = 5 + 3i$

→ magnitude of impedance is

$|Z| = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34} \approx 5.83\,\Omega$.

Q. Calculate the phase difference between voltage $V = 120\angle 30°$ & current $I = 10\angle 0°$:

phase difference ϕ between voltage and current.

difference b/w angles.

$\phi = 30° - 0° = 30°$.

Voltage leads the current by 30°.

Q. Simplify $(6+2i) - (3-4i)$:

$(6+2i) - (3-4i) = 6-3 + (2i - (-4i)) = 3+6i$.

Q. divide $(5+6i)$ by $(1-3i)$

$$\frac{5+6i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{(5+6i)(1+3i)}{1^2+3^2} = \frac{-13+21i}{10} = -1.3 + 2.1i.$$

Ans.

Q. find Complex conjugate of $z = 2-7i$ and verify that $z.\bar{z}$ is a real number.

Complex conjugate of $z = 2-7i$ is $\bar{z} = 2+7i$.

$$z.\bar{z} = (2-7i)(2+7i) = 4+14i-14i-49i^2$$
$$= 4+49$$
$$= 53$$

Is a real number.

Q. Convert the complex number $z = -3+3i$ into polar and exponential forms. What are the modulus & argument?

modulus:

$$|z| = \sqrt{(-3)^2+3^2} = \sqrt{9+9} = 3\sqrt{2}$$

argument:

$$\theta = \tan^{-1}\left(\frac{3}{-3}\right) = 135^\circ$$

polar form is $z = 3\sqrt{2} \text{ cis } 135^\circ$

exponential form is $z = 3\sqrt{2}e^{i135^\circ}$

Q. Given the unitary matrix $U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, calculate new quantum state after applying U to the initial state $|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ for $\theta = \frac{\pi}{4}$.

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Given $U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ for $\theta = \frac{\pi}{4}$

Unitary matrix

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

applying U to $|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$U \cdot |\psi(0)\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

new state is $|\psi(t)\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

Q. Show Pauli Matrix $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is unitary & explain its significance in quantum computing.

$$\sigma_x^{\dagger} = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_x^{\dagger}\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= I$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In quantum computing, σ_x represents a bit-flip operation, flipping a qubit from $|0\rangle$ to $|1\rangle$ or vice versa.

Q. Calculate eigenvalues of Hermitian matrix $H = \begin{pmatrix} 2 & -i \\ i & 2 \end{pmatrix}$.

Are eigenvalues real?

Characteristic equation is

$$\det(H - \lambda I) = \det\begin{pmatrix} 2-\lambda & i \\ i & 2-\lambda \end{pmatrix} = (2-\lambda)^2 - (-i)(i)$$

$$= (2-\lambda)^2 - 1$$

$$= 0.$$

Solving for λ $\quad (2-\lambda)^2 = 1$

$2-\lambda = \pm 1 \qquad \lambda = 3 \text{ or } 1$

Both are real.

Q. For a control system represented by matrix $A = \begin{pmatrix} 0 & 1 \\ -4 & -4 \end{pmatrix}$,

Calculate eigenvalues and determine if system is stable.

characteristic eqn is $\det(A - \lambda I) = \det\begin{pmatrix} -\lambda & 1 \\ -4 & -4-\lambda \end{pmatrix}$

$$= \lambda^2 + 4\lambda + 4 = 0$$

solving for λ

$$\lambda = \frac{-4 \pm \sqrt{16-16}}{2} = -2$$

$(\lambda = -2)$ negative

stable $\because$ Both Real parts are negative.

Q. Given the transfer function $H(s) = \frac{s+2}{s^2+2s+5}$, determine the poles & zeros of system. Are there any complex poles, and what do they signify for the system's behavior?

Zeros: numerator gives $s+2=0$

$s=-2$

Poles: denominator is $s^2+2s+5=0$

$$s = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm \sqrt{-16}}{2} = -1 \pm 2i$$

system is stable ∵ real parts of poles is negative

Q. Impedance.

In a series circuit with $R = 10\,\Omega$, $L = 0.1\,H$, and $C = 10\,\mu F$, calculate the impedance at a frequency of 100 Hz.

$R = 10\,\Omega$, $L = 0.1\,H$, $C = 10\,\mu F$, & $f = 100\,Hz$

Inductive Reactance $X_L = 2\pi f L = 2\pi \times 100 \times 0.1 = 62.83\,\Omega$

Capacitive Reactance $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 100 \times 10 \times 10^{-6}} = 159.15\,\Omega$

total impedance $Z = R + j(X_L - X_C) = 10 + j(62.83 - 159.15)$

$= 10 - j\,96.32$

Magnitude $|Z| = \sqrt{10^2 + (-96.32)^2} = \sqrt{100 + 9277.18}$

$= \sqrt{9377.18} \approx 96.85\,\Omega$

Q given to load with complex power S = 50 + 30i V A, calculate the real power P, reactive power Q, & power factor.

$S = 50 + 30i$ VA

$P = 50$ W $Q = 30$ VAR.

Power factor is

$$\text{Power factor} = \frac{P}{|S|} = \frac{50}{\sqrt{50^2+30^2}} = \frac{50}{\sqrt{2500+900}}$$

$$= \frac{50}{\sqrt{3400}}$$

$$\approx \frac{50}{58.31}$$

$$\approx 0.86.$$

Question

Q. Multiply (2+3i) (1-4i) and express the result in standard form.

$(2+3i)(1-4i)$

$2.1 - 2(4i) + 3i - (3i)(4i)$

$2 - 8i + 3i - 12i^2$

$2 - 8i + 3i + 12$

$14 - 5i$

Answer.

Q. Find the modulus and argument the complex number $z = 3+4i$.

$|z| = \sqrt{3^2+4^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$ units

distance from origin (0,0) to point (3,4) on the Argand plane.

Argument of $z = 3+4i$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) \approx 0.93$ radians

angle represents the dir^n of vector from origin the point in complex plane.

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Q. given a resistor of 4Ω and an inductor with a reactance of 3Ω, calculate the impedance of the circuit using complex numbers.

Impedance in an AC circuit

$$Z = R + jX$$
$$= 4 + (3i)\,\Omega$$

$R \rightarrow$ Resistance
$X \rightarrow$ reactance.

Q. In an AC circuit, the voltage is $V = 100\angle 30^\circ$ and the current is $I = 20\angle 0^\circ$. Calculate the phase difference the voltage and current using complex numbers.

phase difference.

$V = 100\angle 30^\circ$ $\quad$ $I = 20\angle 0^\circ$.

Convert to complex form

$$V = 100\left(\cos(30^\circ) + j\sin(30^\circ)\right)$$

$$= 100\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)$$

$$\boxed{I = 20\left(\cos 0 + j\sin 0\right) = 20}$$

phase difference

$$\Delta\theta = \angle V - \angle I = 30^\circ - 0^\circ = 30^\circ$$
$$= 0.52 \text{ Radians}$$

Q. Verify that matrix $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is unitary by showing $U^{\dagger} U = I$

where $U^{\dagger}$ is conjugate transpose of U.

If $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then:

$$U^{\dagger} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

<u>thus</u>

$$U^{\dagger} U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.$$

<u>hence</u>

U is unitary.

Q. If the initial quantum state is $|\psi(0)\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

calculate the state $|\psi(t)\rangle$ after applying the unitary matrix.

$$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

If the initial quantum state is $|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

applying the unitary matrix U:

$$|\psi(t)\rangle = U|\psi(0)\rangle$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

indicating a state change in a quantum system, highlighting the impact of unitary transformation.

Q: matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Calculate its eigenvalues and determine if the system exhibits oscillatory behavior.

eigenvalues of $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

to find eigenvalues,

Characteristic polynomial:

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0$$

Eigenvalues:

$$\boxed{\lambda = i \text{ and } -i}$$

Q: A system has eigenvalues $-1+2i$ and $-1-2i$. Is the system stable?

Yes.

Since real parts of both eigenvalues are (-1) negative. it is stable.

Stability is essential in control systems to ensure that system responses don't diverge over time.

Q.

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

Find eigenvalues by solving $\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 1 & 2 \\ 1 & 3-\lambda & 0 \\ 2 & 0 & 2-\lambda \end{bmatrix}$$

determinant.

$$\det(A - \lambda I) = [(4-\lambda)((3-\lambda)(2-\lambda))] - (1)(0) - 2(1)(2-\lambda)$$

expanding the terms

$$(4-\lambda)((3-\lambda)(2-\lambda)) = (4-\lambda)(6 - 5\lambda + \lambda^2)$$
$$= 24 - 23\lambda + 6\lambda^2 - \lambda^3$$

$\lambda^3 - 6\lambda^2 + 5\lambda = 0$ giving eigenvalues $\rightarrow$ $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = 5$

Q. Power Method for Dominant Eigenvalue.

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$x_1 = A x_0$

$$x_1 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

normalize x_1 & Compute the Rayleigh quotient for an eigenvalue.

$$\lambda_1 = \frac{x_1^T A x_1}{x_1^T x_1} = \frac{(1)(2)(1) + (1)(-1)(1)}{(1^2 + 1^2)} = 1.5$$

$\lambda = 3$

Q. matrix: $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Perform QR Factorization of A. $A = QR$, where.

$$Q = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}, \quad R = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$$

multiply RQ to form the new matrix A_1:

$$A_1 = RQ = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

eigenvalues are $\lambda_1 = 3$, $\lambda_2 = -1$

8.7.5.

Q.

— Pauli(e) $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

hermitian check

$A = A^H$ (matrix equals its conjugate transpose)

conjugate transpose A^H

$$A^H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$A \neq A^H$
matrix is not hermitian

Unitary check

$A^H A = I$
(Conjugate transpose is inverse of matrix)

$A^H A$

$$A^H A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A^H A = I$, matrix is unitary.

normal.

$[A A^H = A^H A]$

$$A A^H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow A A^H = A^H A$$

normal

Q. $A = \begin{bmatrix} 1 & 1+i \\ 1+i & i \end{bmatrix}$

hermitian check

$A = A^H$

conjugate transpose A^H is

$A^H = \begin{bmatrix} 1 & 1-i \\ 1-i & -i \end{bmatrix}$

$A \neq A^H$, matrix not hermitian.

unitary check

$A^H A = I$

$A^H A = \begin{bmatrix} 1 & 1-i \\ 1-i & -i \end{bmatrix} \cdot \begin{bmatrix} 1 & 1+i \\ 1+i & i \end{bmatrix} = \begin{bmatrix} 2+i(0) & 2+i(0) \\ 2+i(0) & 2+i(0) \end{bmatrix}$

$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$A^H A \neq I$ matrix is not unitary

normal check

$\xrightarrow[\text{cond}^n]{}$ $A^H A = A A^H$

here.

$A^H A \neq A A^H \rightarrow$ not normal.

as well as.

Q.

$$A = \begin{bmatrix} 1 & 0 & 1+i \\ 0 & 2 & 0 \\ 1-i & 0 & 1 \end{bmatrix}$$

(1) Find the eigenvalues of A.

$\det(A - \lambda I) = 0$.

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 0 & 1+i \\ 0 & 2-\lambda & 0 \\ 1-i & 0 & 1-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (2-\lambda) \det \begin{bmatrix} 1-\lambda & 1+i \\ 1-i & 1-\lambda \end{bmatrix}$$

$$\det \begin{bmatrix} 1-\lambda & 1+i \\ 1-i & 1-\lambda \end{bmatrix} = (1-\lambda)^2 - (1+i)(1-i)$$

$$= (1-\lambda)^2 - (1^2 - i^2)$$

$$= (1-\lambda^2) - (2)$$

characteristic equation

$$(2-\lambda)\left((1-\lambda)^2 - 2\right) = 0$$

Solve for λ:

$2 - \lambda = 0$ gives $\lambda_1 = 2$

Solve $(1-\lambda)^2 - 2 = 0$ or $(1-\lambda)^2 = 2$

$1 - \lambda = \pm\sqrt{2} \Rightarrow \lambda_2 = 1 + \sqrt{2}, \ \lambda_3 = 1 - \sqrt{2}$

eigenvalues are →

$\lambda_1 = 2, \ \lambda_2 = 1+\sqrt{2}, \ \lambda_3 = 1-\sqrt{2}$

(2) eigenvectors

Solving $(A-\lambda I)x=0$

for $\lambda_1=2$

$$(A-2I)=\begin{bmatrix}-1 & 0 & 1+i\\ 0 & 0 & 0\\ 1-i & 0 & -1\end{bmatrix}$$

Solving the system of equations will give the eigenvector for $\lambda_1=2$

$$v_1=\begin{bmatrix}1\\0\\1\end{bmatrix}$$

eigenvectors

Solving $(A-\lambda I)x=0$

$(A-(1+\sqrt{2})I)x=0$

$(1+\sqrt{2})I \Rightarrow A$.

for $\lambda_2=1+\sqrt{2}$

results $\Rightarrow$

$$V_2=\begin{bmatrix}1\\0\\a_2\end{bmatrix}$$

for $\lambda_3=1-\sqrt{2}$

results $\Rightarrow$

$$V_3=\begin{bmatrix}1\\0\\a_3\end{bmatrix}$$

(3)

Normalize each eigenvector

Norm of $v_1 = \sqrt{1^2+0^2+1^2}$
$= \sqrt{2}$

$$\boxed{\begin{aligned} v_2 &= \sqrt{1+a_2^2} \\ v_3 &= \sqrt{1+a_3^2} \end{aligned}}$$

Normalized eigenvector

$$v_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Q. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1+i \\ 0 & 1-i & 2 \end{bmatrix}$

(1)

eigenvalue of A. $\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1+i \\ 0 & 1-i & 2-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (1-\lambda) \det \begin{bmatrix} 1-\lambda & 2+i \\ 1-i & 2-\lambda \end{bmatrix}$$

$= (1-\lambda)(2-\lambda) - (1+i)(1-i)$
$= (1-\lambda)(2-\lambda)$
$= -2$

characteristic eqn.
$[(1-\lambda)(1-\lambda)(2-\lambda)-2]=0$

Solve for λ.

$\rightarrow$ $1-\lambda = 0$ gives $\lambda_1 = 1$
$\rightarrow$ $(1-\lambda)(2-\lambda) = 2$ giving $\lambda_2 = 2$

② eigenvectors:

for $\lambda_1 = 1$:

Solve $(A - I)x = 0$

$$A - I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1+i \\ 0 & 1-i & 1 \end{bmatrix}$$

this system gives us:

$$0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 = 0$$

$$(1-i)\, x_2 + x_3 = 0$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ -1+i \end{bmatrix}$$

for $\lambda_2 = 2$:

Solve $(A - 2I)x = 0$

$$A - 2I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 1+i \\ 0 & 1-i & 0 \end{bmatrix}$$

gives

$$-x_1 = 0 \quad \& \quad -x_2 + (1+i)\, x_3 = 0$$

eigenvector.

$$v_2 = \begin{bmatrix} 0 \\ 2 \\ i-1 \end{bmatrix}$$

(3) Normalize each eigenvector $\longrightarrow$

$$\text{Norm of } v_1 = \sqrt{1^2+1^2+(-1+i)(-1+i)}$$
$$= \sqrt{4}$$
$$= 2.$$

normalized eigenvector is.

$$v_1 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ -1+i \end{bmatrix}$$

$$\text{Norm of } v_2 = \sqrt{0^2+1^2+(i-1)\overline{(i-1)}}$$
$$= \sqrt{1+i^2+0+2(1)}$$
$$= \sqrt{3}$$

$$v_2 = \frac{1}{\sqrt{3}}\begin{bmatrix} 0 \\ 1 \\ i-1 \end{bmatrix}$$