Question

Q. Multiply (2+3i) (1-4i) and express the result in standard form.

$(2+3i)(1-4i)$

$2.1 - 2(4i) + 3i - (3i)(4i)$

$2 - 8i + 3i - 12i^2$

$2 - 8i + 3i + 12$

$\underline{\underline{14 - 5i}}$

Answer.

Q. Find the modulus and argument the complex number $z = 3+4i$.

$$|z| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

distance from origin (0,0) to point (3,4) on the Argand plane.

Argument of $z = 3+4i$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) \approx \underline{0.93 \text{ radians}}$$

angle represents the $dir^n$ of vector from origin the point in complex plane.

Q. given a resistor of $4\Omega$ and an inductor with a reactance of $3\Omega$, calculate the impedance of the circuit using complex numbers.

Impedance in an AC circuit

$$Z = R + jX$$
$$= 4 + (3i)\,\Omega$$

$R \to$ Resistance
$X \to$ reactance.

Q. In an AC circuit, the voltage is $V = 100\angle 30^\circ$ and the current is $I = 20\angle 0^\circ$. Calculate the phase difference the voltage and current using complex numbers.

phase difference.

$V = 100\angle 30^\circ$ $\quad$ $I = 20\angle 0^\circ$.

Convert to complex form.

$$V = 100\,(\cos(30^\circ) + j\sin(30^\circ))$$

$$= 100\left(\frac{\sqrt{3}}{2} + j\,\frac{1}{2}\right)$$

$$\boxed{I = 20\,(\cos 0 + j\sin 0) = 20}$$

phase difference

$$\Delta\theta = \angle V - \angle I = 30^\circ - 0^\circ = 30^\circ$$
$$= 0.52 \text{ Radians}$$

Q. Verify that matrix $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is unitary by showing $U^\dagger U = I$

where $U^\dagger$ is conjugate transpose of $U$.

If $U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then:

$$U^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

thus

$$U^\dagger U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.$$

hence

U is unitary.

Q. If the initial quantum state is $|\psi(0)\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

calculate the state $|\psi(t)\rangle$ after applying the unitary matrix.

$$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

If the initial quantum state is $|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

applying the unitary matrix U:

$$|\psi(t)\rangle = U|\psi(0)\rangle$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

indicating a state change in a quantum system, highlighting the impact of unitary transformation.

Q: matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Calculate its eigenvalues and determine if the system exhibits oscillatory behavior.

eigenvalues of $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

to find eigenvalues,

Characteristic polynomial:

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0$$

Eigenvalues:

$$\boxed{\lambda = i \text{ and } -i}$$

Q: A system has eigenvalues $-1+2i$ and $-1-2i$. Is the system stable?

Yes.

Since real parts of both eigenvalues are $(-1)$ negative. it is stable.

# Stability is essential in control systems to ensure that system responses don't diverge over time.