

# Finding Extreme Points: Takeaways

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## Concepts

- A derivative is the slope of the tangent line at any point along a curve.
- Let  $x$  be a point on the curve and  $h$  be the distance between two points, then the mathematical formula for the slope as  $h$  approaches zero is given as:
$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$
- Differentiation is the process of finding a function's derivative.
- Finding the derivative of:  $f(x) = x^2$  :
  - $f'(x) = 2x$
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- Three ways of notating a curve's derivative:
  - $f'(x)$
  - $\frac{df}{dx}$  \*Only use if derivative is a function
  - $\frac{d}{dx} f(x)$
- A critical point is a point where the slope changes direction from negative slope to positive slope or vice-versa. Critical points represent extreme values, which can be classified as a minimum or extreme value.
- Critical points are found by setting the derivative function to 0 and solving for  $x$ .
- Critical point classification:
  - When the slope changes direction from positive to negative it can be a maximum value.
  - When the slope changes direction from negative to positive, it can be a minimum value.
  - If the slope doesn't change direction, like at  $x=0$  for  $f(x) = x^3$ , then it can't be a minimum or maximum value.

- Each maximum or minimum value points are known as local extrema.
- Classifying local extrema:
  - A point is a relative minimum if a critical point is the lowest point in a given interval.
  - A point is a relative maximum if a critical point is the highest point in a given interval.
- Instead of using the definition of the derivative, we can apply derivative rules to easily calculate the derivative functions.
- Derivative rules:
  - Power rule: Let  $x^n$  be some power, then
    - Example: Let  $f(x) = x^3$ . In our function,  $n$  would be 3. Using the power rule, it's derivative would be  $3x^2$  or  $f'(x) = 3x^2$ .
  - Sum rule:
    - Example:  $f(x) = x^2 + x$ .  $f'(x) = 2x + 1$ .
  - Constant factor rule:
    - Example:  $f(x) = 5x^2$ .  $f'(x) = 10x$ .
- Derivative of  $\sin(x)$  is always  $\cos(x)$  and derivative of  $\cos(x)$  is always  $-\sin(x)$ .
- Once you found the critical points of a function, you can analyze the direction of the slope around the points using a sign chart to classify the point as a minimum or maximum. We can test points around our points of interest to see if there is a sign change as well as what the change is.

## Resources

- [Derivative rules](#)
- [Sign chart](#)

