STD – 10 MATHS

CHAPTER - 1

REAL NUMBER

EXERCISE - 1.1 Q-4 Q-5

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

- Let x be any positive integer and $y = 3$.

By Euclid's division algorithm, then,

$x = 3q + r$ for some integer $q \geq 0$ and $r = 0, 1, 2$, as $r \geq 0$ and $r < 3$.

Therefore, $x = 3q$, $3q + 1$ and $3q + 2$

Now as per the question given, by squaring both the sides, we get,

$$x^2 = (3q)^2$$

$$= 9q^2$$

$$= 3 \times 3q^2$$

Let $3q^2 = m$

Therefore, $x^2 = 3m$**...........1**

$x^2 = (3q + 1)^2$

$= (3q)^2 + 1^2 + 2 \times 3q \times 1$

$= 9q^2 + 1 + 6q$

$= 3(3q^2 + 2q) + 1$

Substitute, $3q^2 + 2q = m$, to get,

$x^2 = 3m + 1$................2

$x^2 = (3q + 2)^2$

$= (3q)^2 + 2^2 + 2 \times 3q \times 2$

$= 9q^2 + 4 + 12q$

$= 3\,(3q^2 + 4q + 1) + 1$

Again, substitute, $3q^2 + 4q + 1 = m$, to get,

$x^2 = 3m + 1$..............3

Hence, from equation 1, 2 and 3, we can say that, the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

- **Let x be any positive integer and $y = 3$.**

By Euclid's division algorithm, then,

$x = 3q + r$, where $q \geq 0$ and $r = 0, 1, 2$, as $r \geq 0$ and $r < 3$.

Therefore, putting the value of r, we get,

$x = 3q$

Or

$x = 3q + 1$

Or

$x = 3q + 2$

Now, by taking the cube of all the three above expressions, we get,

Case (i): When $r = 0$, then,

$x^2 = (3q^3)$

$= 27q^3$

$= 9\ (3q^3)$

$= 9m$; where $m = 3q^3$

Cast (ii): When r = 1, then,

$x^3 = (3q + 1)^3$

$= (3q)^3 + 1^3 + 3 \times 3q \times 1\ (3q + 1)$

$= 27q^3 + 1 + 27q^2 + 9q$

Taking 9 as common factor, we get,

$x^3 = 9\ (3q^3 + 3q^2 + q) + 1$

Putting $(3q^3 + 3q^2 + q) = m$, we get,

$x^3 = 9m + 1$

Case (iii): When $r = 2$, then,

$x^3 = (3q + 2)^3$

$= (3q)^3 + 2^3 + 3 \times 3q \times 2(3q + 2)$

$= 27q^3 + 54q^2 + 36q + 8$

Taking 9 as common factor, we get,

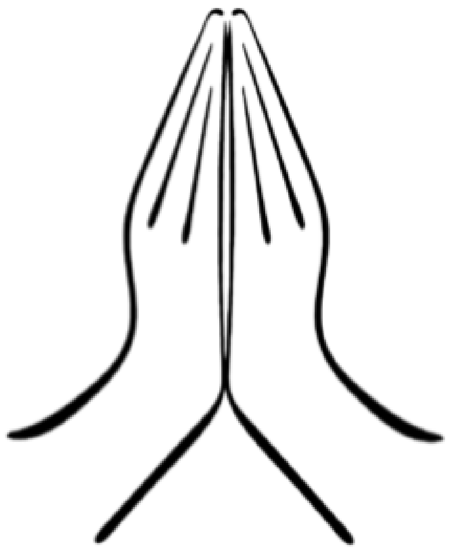
$x^3 = 9(3q^3 + 6q^2 + 4q) + 8$

Putting $(3q^3 + 6q^2 + 4q) = m$, we get,

$x^3 = 9m + 8$

Therefore, from all the three cases explained above, it is proved that the cube of any positive integer s of the form 9m, 9m + 1 or 9m + 8.

Thanks



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