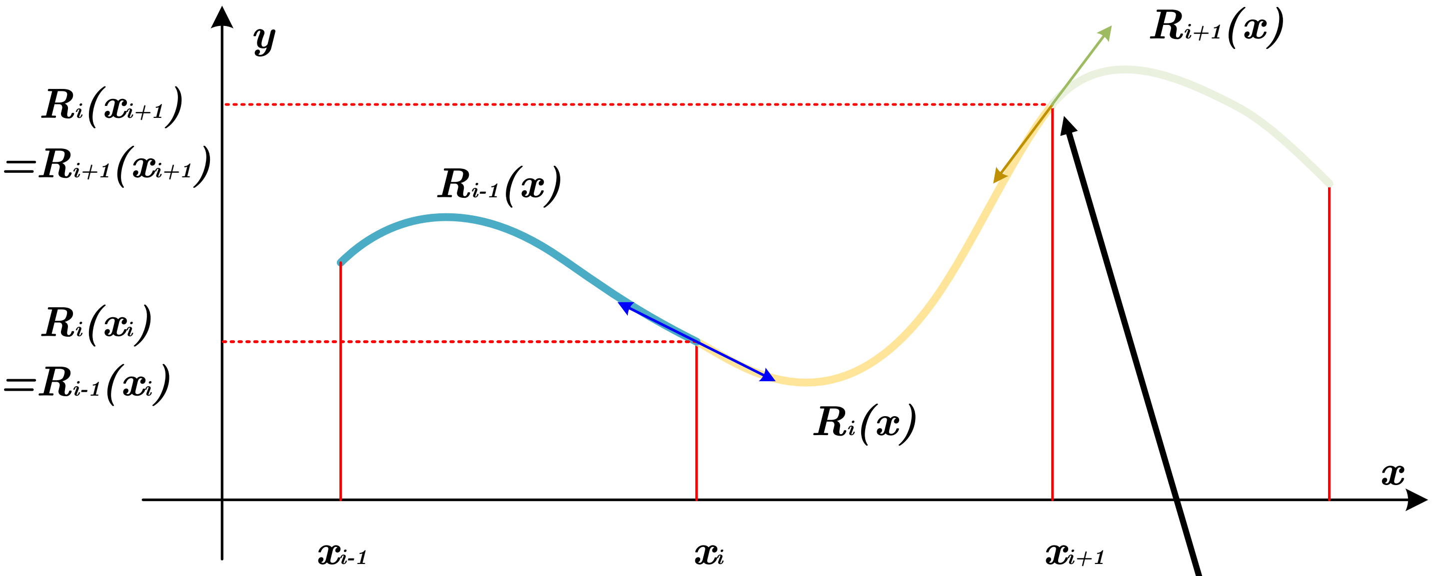
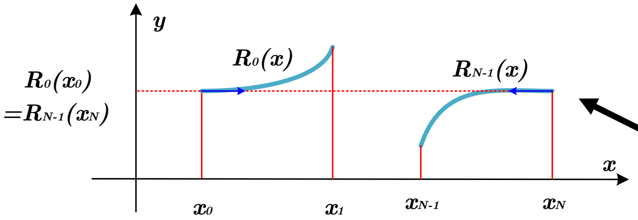
Cubic Spline Overview





The following 3 equations must be valid at x = x[i+1]:

1) R_i(x[i+1]) = R_{i+1} (x[i+1])
2) R'_i(x[i+1]) = R'_{i+1} (x[i+1])
2) R''_i(x[i+1]) = R''_{i+1} (x[i+1])

Except for the special cases x= x[0] and x=x[N]

- Matrix M: size 3*n by 3*n
  - Easier if matrix is viewed as n bands of 3 rows.
- Remember
  - The U column vector is the unknown (i.e: a[i], b[i], c[i] are the 3*n unknown).
  - You must fill all matrix coefficients as well as the V column vector.
  - Most of the coefficients are zero, so you must only initialize non zero coefficients

$$\begin{bmatrix} B_0 \\ B_1 \\ \\ B_i \\ \\ B_{N-2} \\ B_{N-1} \end{bmatrix} \begin{bmatrix} u_0 \\ u_1 \\ \\ u_i \\ \\ u_{N-2} \\ u_{N-1} \end{bmatrix} = \begin{bmatrix} v_0 \\ v_1 \\ \\ v_i \\ \\ v_{N-2} \\ v_{N-1} \end{bmatrix}$$

$$u_i = \begin{bmatrix} a_i \\ b_i \\ c_i \end{bmatrix}$$

- Fill the matrix band by band
- Use the last 2 rows of the last band to express the null tangents at x[0] and x[N] (R’_0(0) = 0 and R’_{N-1}(x[N]) = 0).

Notations:
Δi = x[i+1] - x[i]

R_i(x) is the ith polynomial
R_i(x) = a[i]*(x-xi)^3 + b[i]*(x-xi)^2 + c[i]*(x-xi) + d[i]

R_{i+1}(x) is the (i+1)th polynomial
R’_i(x) is the first derivative of the ith polynomial
R’’_i(x) is the second derivative of the ith polynomial

$$B_i = \begin{bmatrix} B_{i,0} & B_{i,1} & \cdots & B_{i,i-1} & B_{i,i} & B_{i,i+1} & \cdots & B_{i,N-2} & B_{i,N-1} \end{bmatrix}$$

First Row: we express the condition
R_i(x[i+1]) = y[i+1]

a[i]*Δi^3 + b[i]*Δi^2 + c[i]*Δi + y[i] = y[i+1]

Second Row: we express the first derivative condition at x[i+1]
R’_i(x[i+1]) = R’_{i+1}(x[i+1])

3* a[i]*Δi^2 + 2* b[i]*Δi + c[i] = c[i+1]

Third Row: we express the second derivative condition at x[i+1]
R’’_i(x[i+1]) = R’’_{i+1}(x[i+1])

6 * a[i]*Δi + 2* b[i] = 2 * b[i+1]