Report 4 Predictability and grows of forecast
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1.Dynamics of forecast spread and errors in the barotropic instability experiment

For this exercise, the MAD shallow water model is being used. The error amplitude of the initial condition is set as 0.05 and the forecast length is set as 15 days, then generate 10 forecast ensembles and one reference solution with higher accuracy.

Figure 1 shows the evolution of the barotropic instability experiment of reference simulation. The initial condition consists of two balanced zonal jets located symmetrically with respect to the central latitude, applying periodic boundary conditions and a Gaussian height perturbation for each jet.

At the beginning when t=0h, the initial perturbations have evolved in MAD for 3 days, so the instability has already developed in the jet core, geostrophic adjustment triggers significant gravity waves that interact with the jet core; At t=72h, the instability is further developed, leading to fine structures and even larger vorticity gradients; At t=120h, the jets have been totally destabilized, vorticity is even larger but still sort of symmetrically distributed, the gravity waves are propagating in the whole domain. At t=360 h, the waves amplitudes are even greater, the system is fully chaotic and cannot be predicted anymore.

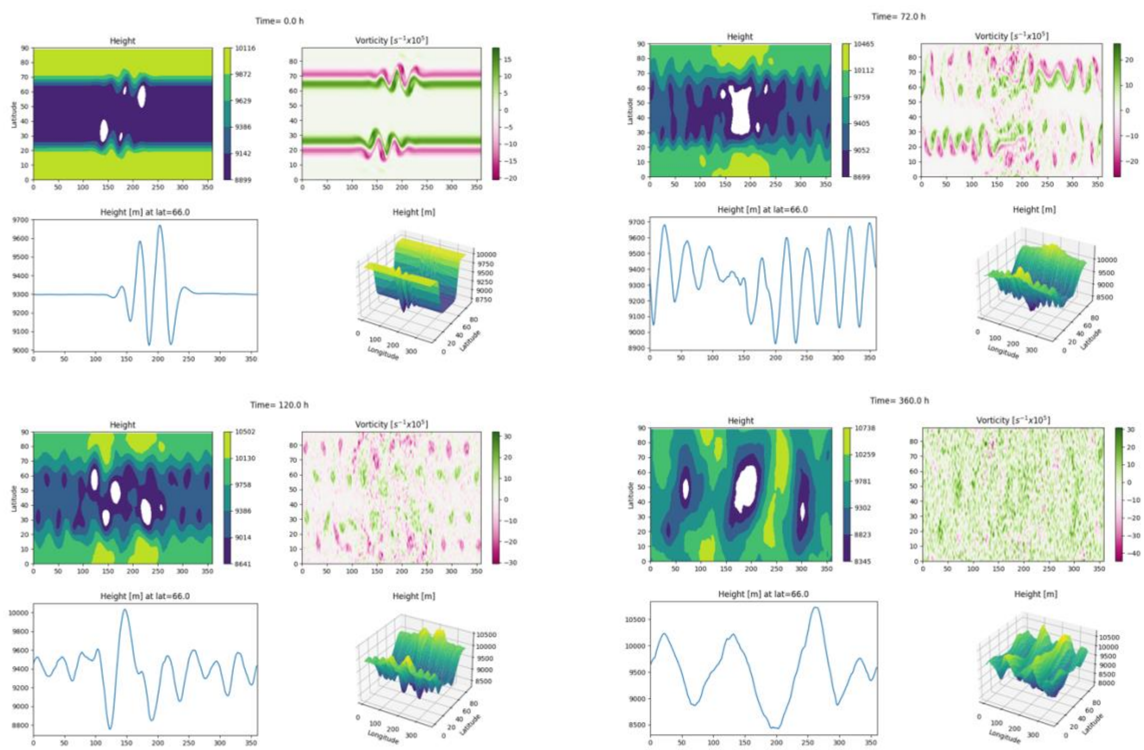

Figure 1 Snapshots of the reference solution showing t=0h, t=72h, t=120h and t=360h respectively

Figure 2 compares the trajectory spread dynamics with ensemble error dynamics. According to the spread of ensembles, their evolution can be divided into four phases.

During the initial phase, which is model relaxation. The spread has a significant increase until reaches a maximum and then a sudden drop to the minimum, due to its peculiar generation of perturbation and model smoothing, and the spreads of 10 ensembles are almost overlapping in the phase. Following model relaxation, the spread experiences an exponential increase until a distinguishable trajectory can be seen. The ten ensembles have a very close starting point and each one has a slightly different increasing amplitude and ends up with different values. The third phase is the transition to chaos. The slope of the spread becomes gentler, and different forecast has the same trend but different trajectories. The last phase is chaotic, the spread growth becomes steady, it might still increase but the amplitude is very small in a long time. The spreads are almost saturated at the end.

The error growth pattern resembles the spread trajectory, following a very distinct tanh-like curve. The scale of error is comparable with the spread.

Clearly, the third phases in the dynamics are distinguishable on the resulting plot when different predictions show very different trajectories. According to the formula given below, the saturation spread E^* is around 50 m/s seen from the averaged trajectory spread, where the average error no longer increases, and error slope becomes zero.

$$\frac{dE}{dt} = s(E^* - E)(E - E_{min})$$

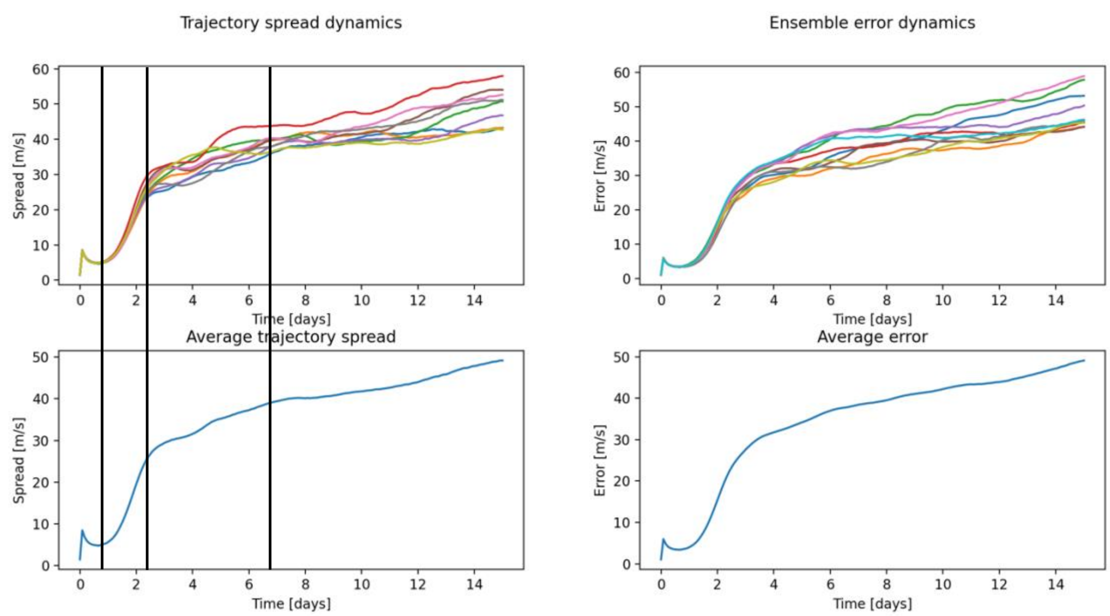

Figure 2 Dynamics of the spread (left) and the error (right), the upper plots are trajectories of ten ensembles and the lower plots are averaged value.

2.Reduction in the initial condition uncertainty

For this exercise, the error amplitude of the initial condition is set as 0.025 and then compare with the previous experiment.

Figure 3 shows the spread and error for the experiment with the initial perturbation uncertainty reduced by half. Comparing with the previous experiment, the magnitude of spread in the first phase is reduced. In the second phase, the spread is moved to the right, the time span of linear phase from around 1 day in previous experiment extends around 2 days. In the third phase, the pattern is similar, but time range lasts longer. In the last phase, one can notice that the saturation value is reduced from about 50m/s in previous experiment to around 44 m/s. Therefore, reduction by half in the initial condition uncertainty improves the predictability by increasing more predictable time (approximately one day), and the spread is more accurate with the smaller errors.

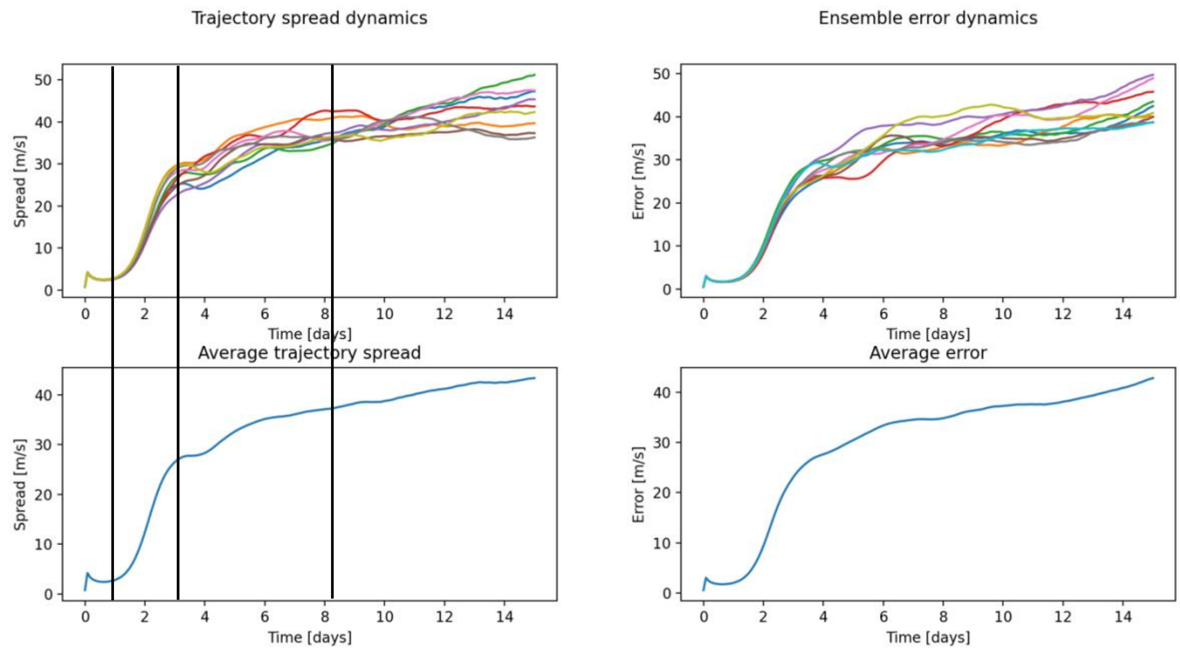

Figure 3 Dynamics of the spread (left) and the error (right), the upper plots are trajectories of ten ensembles and the lower plots are averaged value, with the initial error amplitude 0.025.

If the initial perturbation uncertainty is further reduced to 0.01, the result is just improved slightly, as figure 4 shown blow. It indicates the nature of this experiment, reducing the initial uncertainty improves the predictability and decreases errors to a certain degree, but can hardly make huge improvement by further reduction of the initial condition uncertainty.

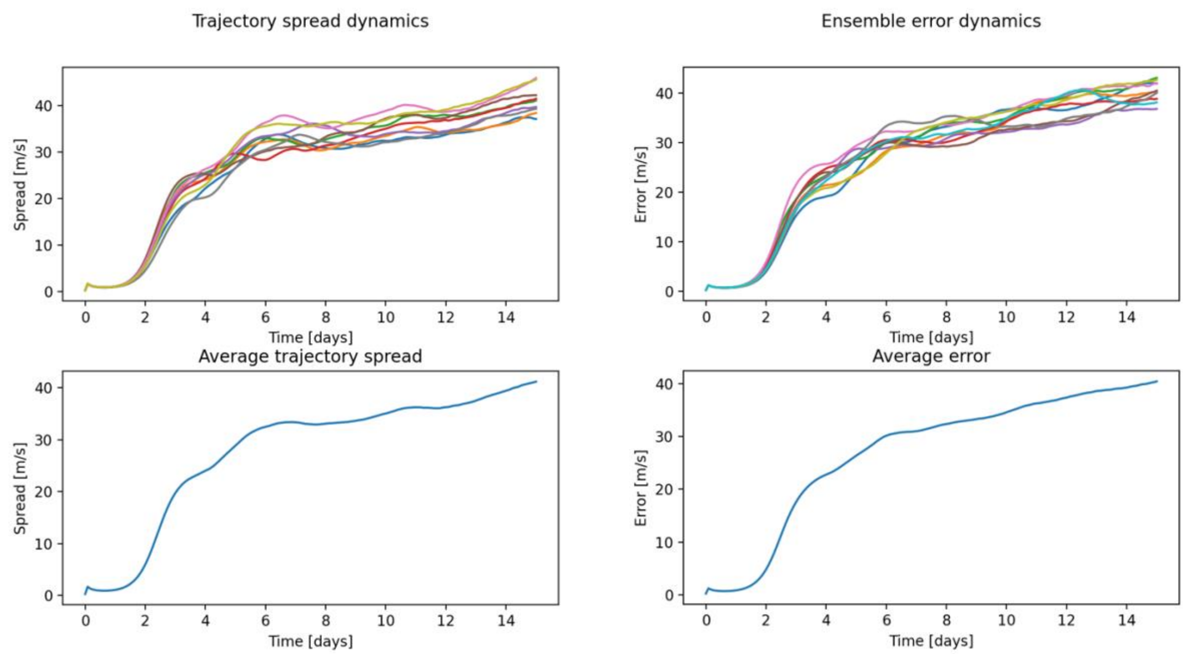

Figure 4 Dynamics of the spread (left) and the error (right), the upper plots are trajectories of ten ensembles and the lower plots are averaged value, with the initial error amplitude 0.01

3.Estimate the largest local Lyapunov exponent

For this exercise, one can find the best linear fit $f(t) = \lambda_1(t - t_0)$ for the ensemble spread data $\log\ E_i(t)$ over the interval (t_0, t_1), where the spread curve exhibit exponential grows.

The initial perturbation uncertainty for this experiment is 0.01 from previous experiment, and the average spread is analyzed. The largest local Lyapunov exponent should be slightly larger than this value, as I chose the average spread. The time range is from 1 day to 2.5 day, which is from 86400 seconds to 216000 seconds, the spread range is from 1.13069 to 11.7969, so according to the formula, the exponent λ_1 calculated as:

$$\lambda_1 = \frac{\log(12.8233379) - \log(1.1272662)}{216000 - 86400} = \frac{2.431471375}{129600} = 1.87614 \times\ 10\text{^} - 5$$

It is positive, meaning the system is chaotic. Note, the largest local Lyapunov exponent is larger than above value. Using data in spread.txt file, the Lyapunov exponents for 10 ensembles can also be estimated with the same method, the results are listed in the table below. And the largest local Lyapunov exponent is estimated as 1.93519E-05.

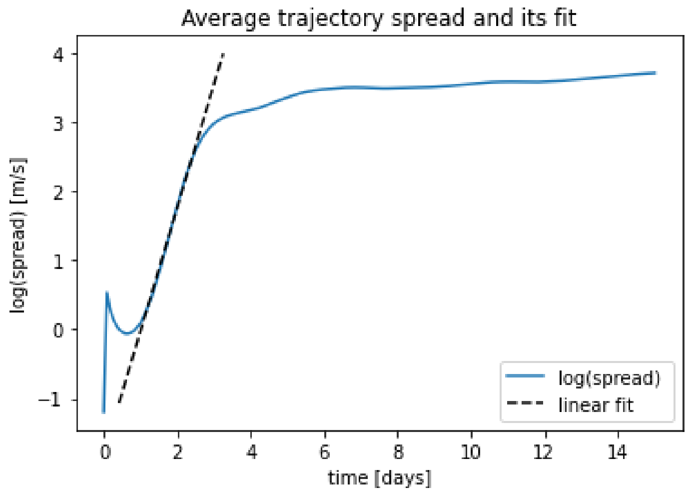

Figure 5 Average trajectory spread and its fit, with the initial error amplitude 0.01

Phase Start	Phase End	Lyapunov
1.218099	14.697388	1.92159E-05
1.10458	12.906012	1.89678E-05
1.131721	14.242272	1.95407E-05
1.083011	11.251467	1.80614E-05
1.0922	13.412728	1.93519E-05
1.153746	13.764643	1.91288E-05
1.092773	12.624929	1.88808E-05
1.105956	10.859464	1.7626E-05
1.16331	11.651138	1.77788E-05
1.127266	12.823338	1.87614E-05

Table 1 Lyapunov exponent estimates for the ensemble, with the initial error amplitude 0.01

Conclusion

The lab experiment demonstrates that two randomly chosen trajectories initially close enough but can be far apart at the end, and their average difference is even comparable with their amplitude. By reducing initial condition uncertainty, one can improve predictability to a certain degree, but cannot improve too much. Similarly, predictability of weather is naturally limited even with a perfect model and perfect initial conditions. In addition, the sign of Lyapunov exponent can indicate whether the system is chaotic. In our experiments, Lyapunov exponent in the linear phase is always positive and thus indicates the system is chaotic.