NPAO Lab1 Report

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Topic 1: how the adjustment differs between the high latitudes and near the equator?

Giving a height perturbation based on a Gaussian function, two adjustment experiments are carried out with the same model setup, performing β-plane simulations with identical mean height 1000 m and perturbation amplitude 100 m, while one with perturbation at the equator and another at 45°N.

In the equator case, at first, gravity waves propagating westward, while eastward propagating equatorial Kelvin wave runs relatively faster than other slow modes. When the Kelvin wave is out of the domain, a pair of Rossby waves start to form and propagate westward (Fig.2(a)).

One major difference to the mid-latitude case can be seen from the adjustment of the perturbation height. 1 hour later, there is a rapid due to dispersion of gravity waves, much the same as that on the mid-latitude case, despite that no Coriolis effect in equator case. At 5 hours, the height at perturbation center (997m, Fig.1(b)) is much lower than that in mid-latitude case (1010.5m Fig.1(d)). The wave crest of the eastward Kalvin wave is comparatively higher, which means it is the most energetic wave (Fig.1(b)). While gravity waves in mid-latitude case (Fig.(d)) are almost symmetrically distributed, but the right side of perturbation is indeed slightly higher.

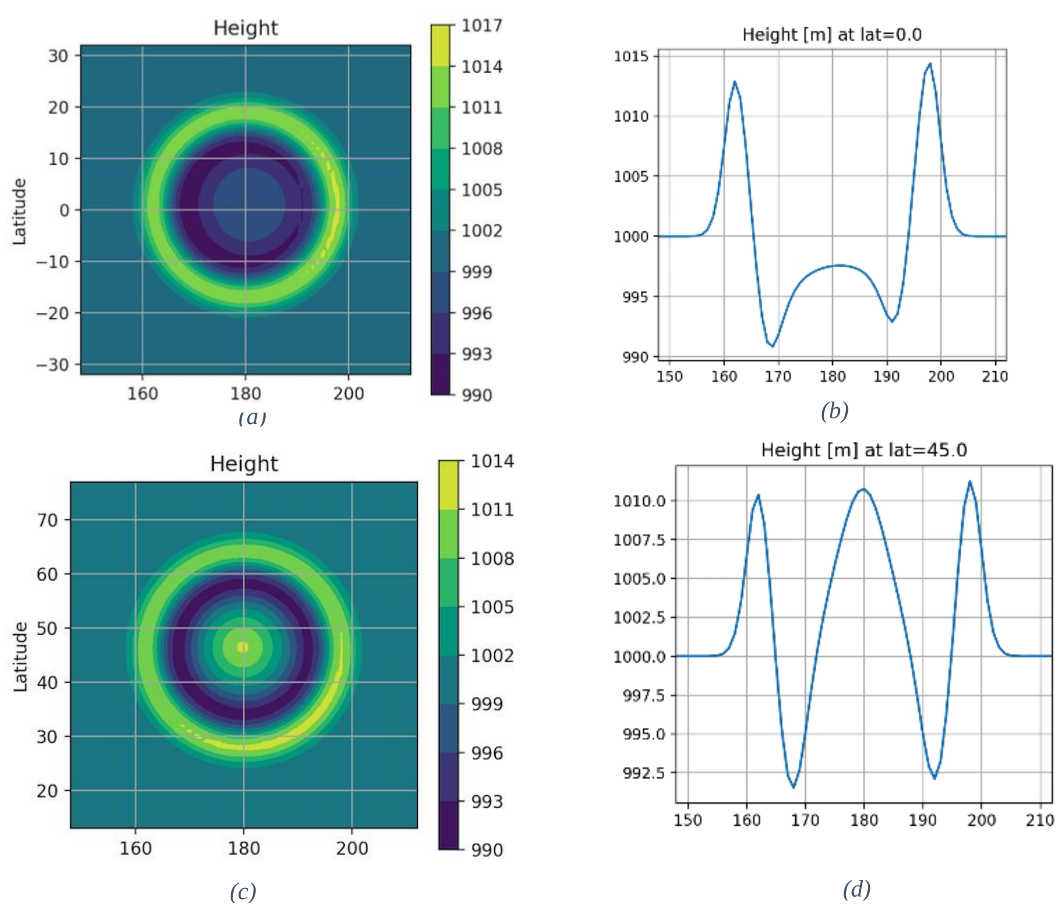
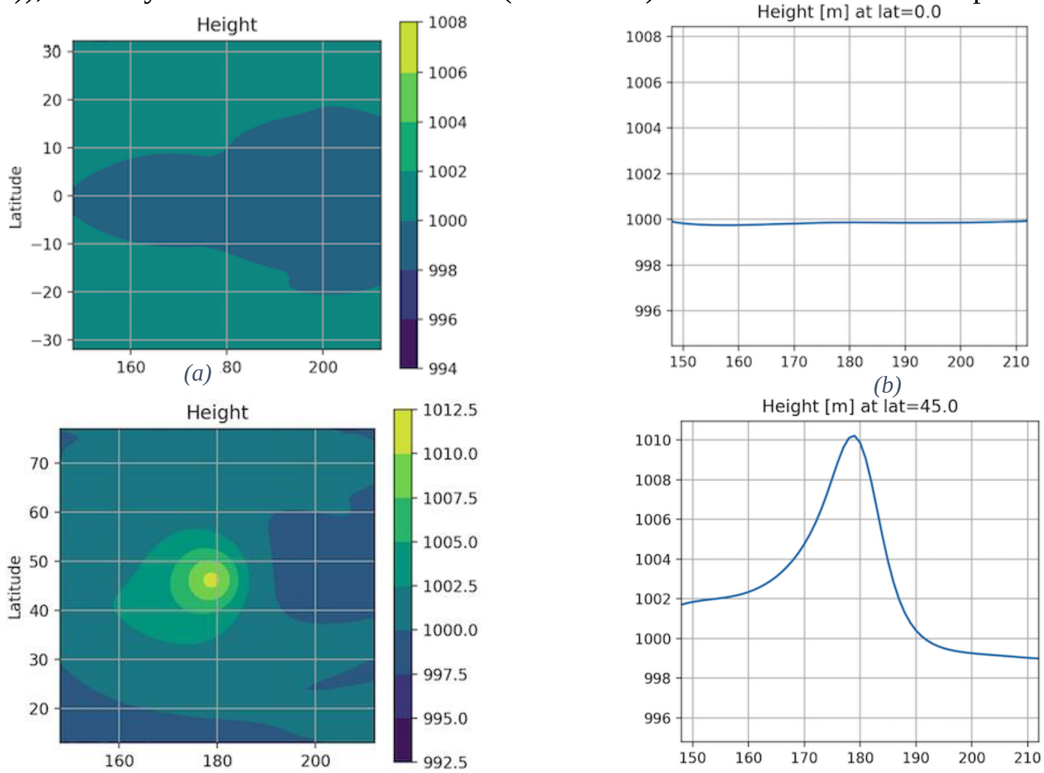


Figure 1 Response of shallow water model to a Gaussian perturbation at equatorial β-plane after 5 h (a) and its height profile at perturbation latitude (b). Same time for mid-latitude case (c) and (d). Mean height is 1000m, perturbation amplitude is 100 m.

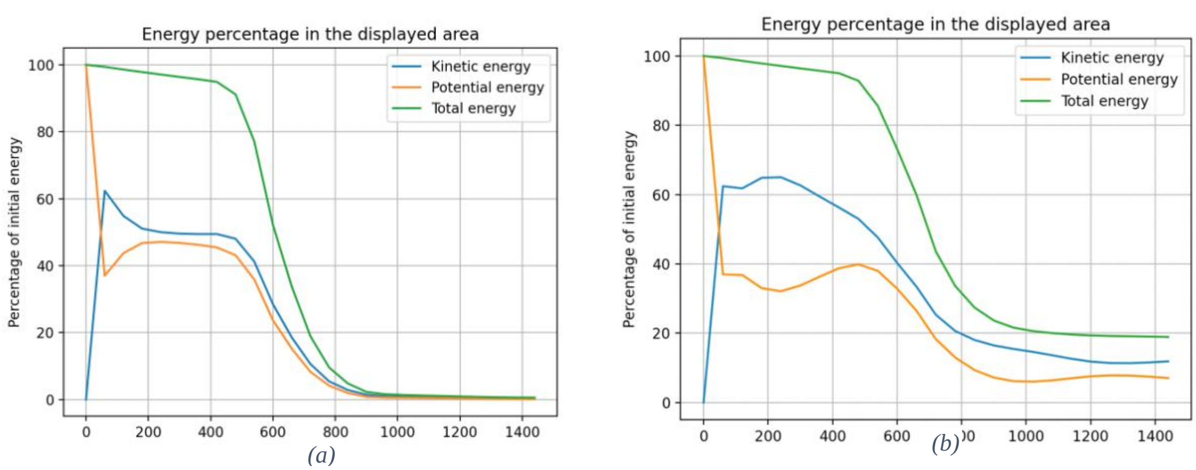
After 24 h, gravity waves have totally left the domain. There is no perturbation left at the location of the initial perturbation (Fig.2(a) and (b)), and characteristic windspeed is stabilized as 0.2m/s, we can see a pair of Rossby waves off-equator. However there remains an asymmetrically mountain-shaped adjusted state right at the location of disturbance in mid-latitude case (Fig.2(c) and (d)), vorticity in the center is conserved (not shown) and characteristic windspeed is 1.2 m/s.



(c) (d)

Figure 2 Response of shallow water model to a Gaussian perturbation at equatorial β-plane after 24 h (a) and its height profile at perturbation latitude (b). Same time for mid-latitude case (c) and (d).

Compare the energy distribution in the displayed area during the adjustment process, we can see soon after 24 hours, the adjustment of the same perturbation at the equator results in almost all energy being dispersed away, only a negligible amount of total energy is left. However, the adjusted state on the mid-latitude ß-plane has remained about 20 % of its initial total energy, about 10% of its initial potential energy, which is consistent with wave contour graph in Fig.2.



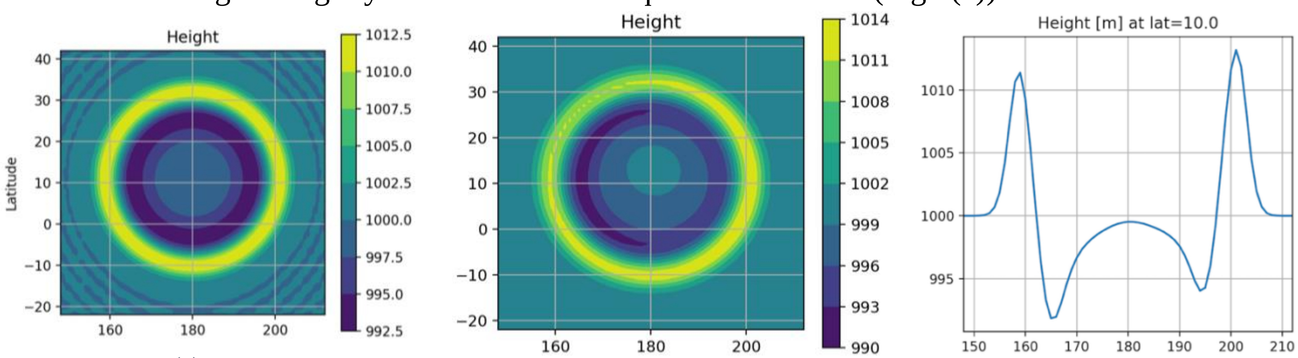
(a)

Figure 3 time evolution of energy within the displayed domain, equator case (a) and mid-latitude case (b)

Topic3: locate the initial perturbation slightly off the equator, compare geostrophic adjustment between the *ƒ* and ß planes.

Two experiments are carried out, same condition with topic 1, while keep perturbation located at 10º N, one with *ƒ*-plane while another using β-plane.

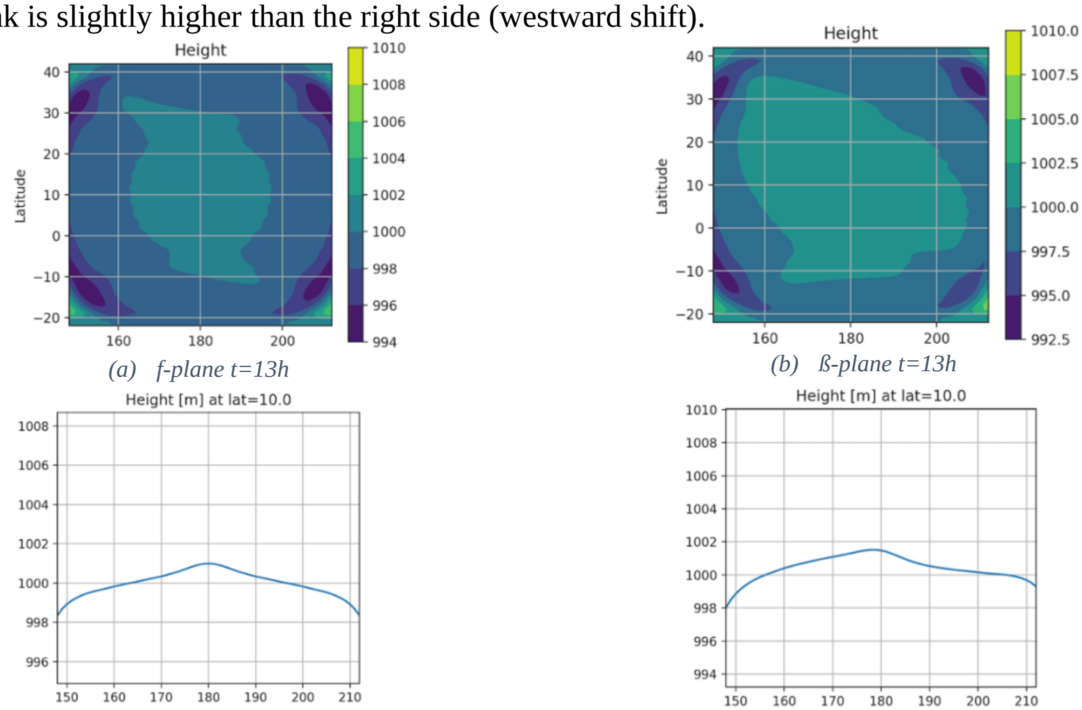
During geostrophic adjustment process: in *ƒ*-plane case, the waves propagate symmetrically in all directions (Fig.3 (a)). While in ß-plane case, the waves propagate asymmetrically (Fig.3 (b)), due to beta effect. To the right of disturbance, wave crest is slighter higher than its equivalent left side, and wave trough is slightly shallower than its equivalent left side (Fig3 (c)).



(a) *(b)* *(c)*

Figure 3 Response of shallow water model to a Gaussian perturbation at 10ºN f-plane after 6 h (a) β-plane (b) and β-plane height profile at perturbation latitude (c).

At 13 hours: at *ƒ*-plane case, height profile at the perturbation latitude is symmetrically distributed. Small peak at the location of initial disturbance. In ß-plane case, the wave height in the left side of peak is slightly higher than the right side (westward shift).



(a) f-plane t=13h *(b) ß-plane t=13h*

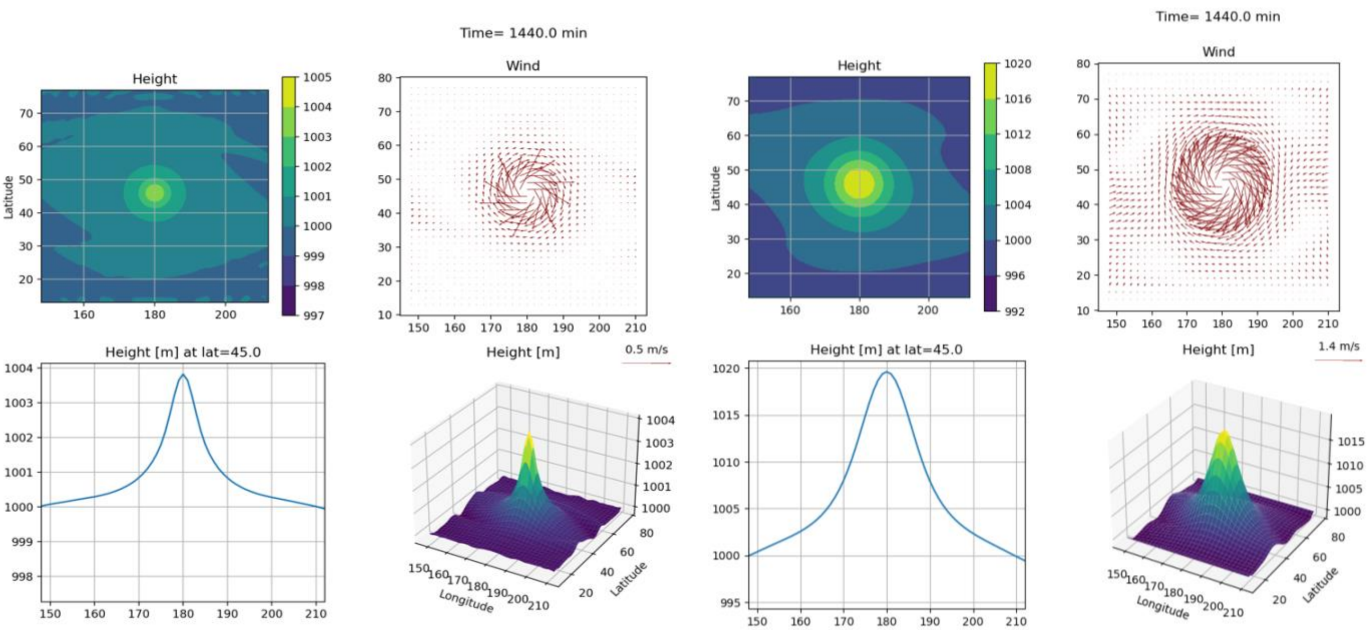
(c) Height profile f-plane t=13h *(d) Height profile ß-plane t=13h*

Figure 4 Response of shallow water model to a Gaussian perturbation at 10ºN f-plane (a) & (c) and ß-plane (b) & (d) after 13 hours

Topic4: How the spatial extent of the initial perturbation influences the adjusted state?

For this experiment, we run the model on mid-latitude ʄ-plane. In theory, the horizontal extent of the adjusted state L_{adj} is comparable in scale to the Rossby deformation radius, $\lambda_{R,}$ which is $\sqrt{gH}/f$. At 45° N ʄ-plane, the calculated Rossby deformation radius is $\frac{\sqrt{9.81*1000}}{1.028e-4} = 964km$, approximately equals to 8.5 degrees in this model.

For the first run, we choose horizonal extent in 5 degrees, which is less than Rossby deformation radius. At adjusted state, wave crest at perturbation is 1003.5m, adjusted characteristic windspeed is 0.5 m/s, adjusted radius is gradually spreading to Rossby deformation radius.



(a) Horizonal extent = 5 degrees *(b) Horizonal extent = 15 degrees*

Figure 5 Response of shallow water model to a Gaussian perturbation at mid-latitude f-plane at 24h, the left with 5 degrees horizonal extent of initial perturbation, the right side with 15 degrees extent.

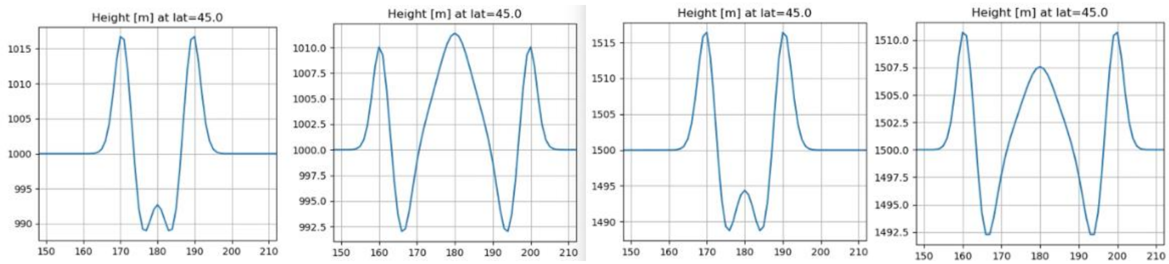
For the second run, we choose horizonal extent in 15 degrees, which is larger than Rossby deformation radius. At adjusted state, wave crest at perturbation is 1018 m, adjusted characteristic windspeed is 1.4 m/s, adjusted radius is quite close to its perturbation radius.

Besides, we can see before reaching adjusted state, the waves are propagating symmetrically. But the adjusted state is not fully symmetric, this is due to numerical reason.

In general, for a fixed perturbation amplitude, the larger initial extent will lead to a larger adjusted state, higher wave height, farther horizontal extent and stronger characteristic windspeed.

Topic5: How mean height influences the adjusted state?

Choose mid-latitude f-plane, keep other variables unchanged. In the first run, mean height is set to 1000 m and determine the phase speed of the emitted gravity wave. When time = 150 min, the location of wave crest is at 190 degrees. When time is 330 min, the same wave crest arrives at 200 degrees. So, the phase speed of this emitted gravity wave is 10degree/180 mins.



(a) Mean height=1000m Time = 150 min *(b) Mean height=1000m Time = 330 min* *(c) Mean height=1500m Time = 135 min* *(d) Mean height=1500m Time = 270 min*

Figure 6 height profile at the latitude of perturbation with 1000 m mean height at time = 150 (a), and at time =330, (b), with 1500 m mean height at time = 135 (c), and at time =270 (d)

In the second run, change mean height to 1500 m and then re-run the same model. When time = 135 min, the location of wave crest is at 190 degrees. When time is 270 min, the same wave crest arrives at 200 degrees. So, the phase speed of this emitted gravity wave is 10 degree/135 mins.

In conclusion, the higher the mean height, the faster the phase speed. This is in accord with dispersion relation, the phase speed of emitted inertial gravity wave in is given by:

$$v_{ph} = \frac{\nu}{n_x} = \pm af\sqrt{\frac{1}{n_x^2} + \frac{gD}{f^2a^2}}$$

It also states the larger the mean height D, the larger the phase speed.
By simple calculation, 1 degree approximately equals to 111 km, the observed phase speed of the first run is:

$$V = \frac{10\text{degrees} * 111\text{km}}{180 * 60\text{s}} = 102\ m/s$$

Theoretical phase speed roughly equals to $\sqrt{gD} = \sqrt{9.81 * 1000} = 99$ m/s, almost same with observed value.

Similarly, for the second run, phase speed is:

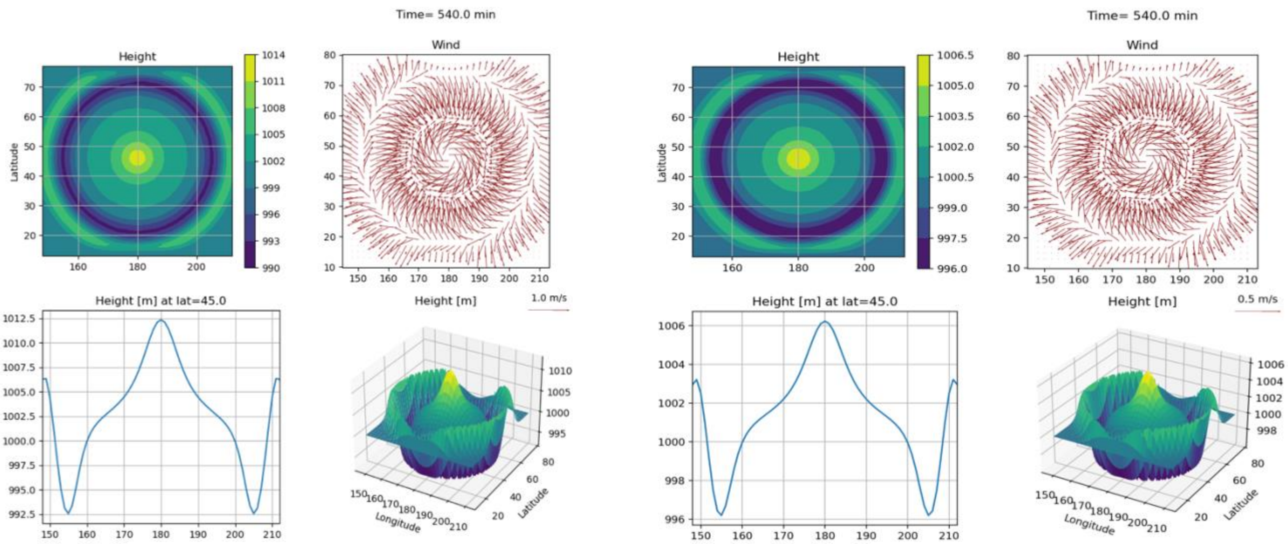
$$V = \frac{10\text{degrees} * 111\text{km}}{135 * 60\text{s}} = 137\ m/s$$

Theoretical phase speed roughly equals to $\sqrt{gD} = \sqrt{9.81 * 1500} = 121$ m/s, quite close with observed value. The theoretical value is always smaller than observed value, the discrepancy between observed value and theoretical value may explained by integer wave number term n_x.

Topic6: How perturbation amplitude influences the adjusted state?

Fix the mean height 1000 m and the horizonal extent of perturbation, f-plane 45ºN, one run is set up 100 m perturbation amplitude, another is 50 m.

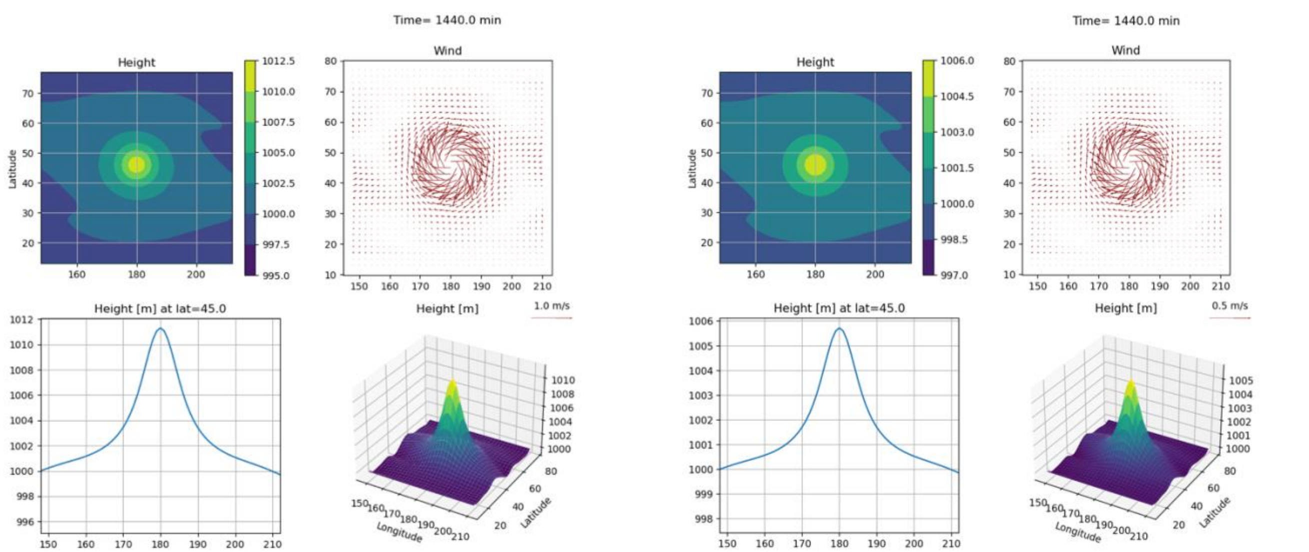
Compare to the second run, we can see more energetic gravity waves. Gravity waves heights, windspeed and divergence are as twice as the second run. But amplitude does not affect their phase speed.



(a) With 100m amplitude at 9 h *(b) with 50 m amplitude at 9 h*

Figure 7 Response of shallow water model to a Gaussian perturbation at mid-latitude f-plane at 9 h, the left with 100 m perturbation amplitude, the right side with 50 m amplitude.

At the end, we can see scaled up adjusted state for larger perturbation amplitude, stronger wind, stronger height. We can see a linear dependency of adjusted state one amplitude (the wave crest and characteristic windspeed of the first run are twice as the second run).



(a) With 100m amplitude at 24 h *(b) With 50m amplitude at 24 h*

Figure 8 Response of shallow water model to a Gaussian perturbation at mid-latitude f-plane at 24 h, the left (a) with 100 m perturbation amplitude, the right side (b) with 50 m amplitude.

Topic2: dispersion relation for equatorial inertia-gravity and Rossby waves