

Collatz-6

A new dynamical system in the 6-adic world

1. What is Collatz 6?

Collatz 6 is a variant of the Collatz map defined in a 6-adic structure. It combines **6-adic gravity** (division by 2, 3, and 6) with **$7n \pm 1$ jumps**, producing behaviors not seen in the classical problem—especially **gliders**, orbits that repeatedly escape downward collapse.

2. The Collatz 6 Map

For any integer n :

- if $n \equiv 0 \pmod{6} \rightarrow n / 6$
- if $n \equiv 2 \text{ or } 4 \pmod{6} \rightarrow n / 2$
- if $n \equiv 3 \pmod{6} \rightarrow n / 3$
- if $n \equiv 1 \pmod{6} \rightarrow (7n - 1) / 6^{v_6(7n - 1)}$
- if $n \equiv 5 \pmod{6} \rightarrow (7n + 1) / 6^{v_6(7n + 1)}$

Here $v_6(n)$ is the largest integer k such that 6^k divides n .

3. Pure Jumps ($v_6 = 0$)

A **pure jump** occurs when: $v_6(7n \pm 1) = 0$. Then: $T(n) = 7n \pm 1$ with **no division**. Pure jumps cause rapid growth and are the key mechanism behind gliders.

4. Gliders

A glider is an orbit that:

- repeatedly enters $n \equiv 1 \text{ or } 5 \pmod{6}$,
- triggers many $7n \pm 1$ jumps,
- exhibits infinitely many pure jumps,
- avoids long division sequences,
- and grows instead of converging.

Gliders are unique to the 6-adic system.

5. The Chaos Rim

The residue classes **1** and **5** modulo 6 form the **Chaos Rim**—the boundary where $7n \pm 1$ jumps occur. Orbits that frequently return to the Chaos Rim have the potential to become gliders.

6. Fibonacci Numbers

Fibonacci numbers often satisfy: $F_n \equiv 1 \text{ or } 5 \pmod{6}$, so they naturally fall on the Chaos Rim. They trigger many $7n \pm 1$ jumps, but for F_1 – F_{20} , **no pure jump has yet been observed**. Finding the first Fibonacci number with $v_6 = 0$ is an open challenge.

7. Escape Candidates

Some numbers show long upward drift with few divisions. The strongest known example is:

20737, which repeatedly returns to the Chaos Rim and grows persistently. It is the leading **escape candidate** for an unbounded orbit.

8. Purpose

This overview introduces the core ideas of Collatz 6 and invites exploration. Open problems include:

- First Fibonacci number with a pure jump
- Existence of true gliders
- Behavior of 20737
- Structure of the Chaos Rim
- Distribution of v_6 in $7n \pm 1$

Collatz 6 is a new mathematical world. Exploration begins here.

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