

# Discovery of an 8-length Prime Chain under a 3-adic Collatz-like Map

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## Abstract

This report presents the discovery of an 8-term prime chain generated by the 3-adic Collatz-like transition:

- If  $n \equiv 1 \pmod{3}$ :  $f(n) = (4n - 1) / 3$
- If  $n \equiv 2 \pmod{3}$ :  $f(n) = (4n + 1) / 3$
- If  $n \equiv 0 \pmod{3}$ : stop

A full search up to  $6 \times 10^8$  revealed exactly one prime chain of length 8, beginning at **46,000,363**. No other chain of equal or greater length was found within the tested range.

## 1. Definition

A *3-adic prime chain* is a finite sequence of primes  $(p_0, p_1, \dots, p_k)$  satisfying:

- $p_{i+1} = f(p_i)$
- all  $p_i$  are prime
- the chain stops when  $f(p_k)$  is composite or  $p_k \equiv 0 \pmod{3}$

This generalizes Cunningham chains by replacing the rule  $2p \pm 1$  with a modularly branched linear fractional transformation.

## 2. Discovered Length-8 Chain

**46000363, 61333817, 81778423, 109037897, 145383863, 193845151, 258460201, 344613601**

Each transition satisfies  $p_{i+1} = (4p_i \pm 1) / 3$ , with the sign determined by  $p_i \pmod{3}$ .

All eight values were verified prime using deterministic primality testing.

## 3. Search Method

A Python/Sympy implementation exhaustively tested all primes below  $6 \times 10^8$ . For each prime  $p$ , the map  $f$  was iteratively applied. Only sequences consisting entirely of primes were recorded. Prefix chains (e.g., the first 6 or 7 elements of the length-8 chain) were excluded when counting maximal chains.

### Results:

- Length 8: 1 chain
- Length 7: 0 maximal chains
- Length 6: several chains, many converging into the same orbit

Thus, the chain beginning at 46,000,363 is the only maximal chain of length  $\geq 8$  in the tested range.

## 4. Observations

### 4.1 Growth

The map grows approximately as  $f(n) \approx (4/3)n$ , so values increase rapidly. Combined with the decreasing density of primes, long chains become extremely rare.

### 4.2 3-adic branching

The rule depends entirely on  $n \pmod{3}$ , ensuring integrality while introducing a branching structure absent in classical Cunningham chains.

### 4.3 Uniqueness

No other chain of length  $\geq 8$  was found below  $6 \times 10^8$ . Whether longer chains exist beyond this range remains an open computational question.

## 5. Conclusion

This study reports the discovery of a unique 8-term 3-adic Collatz-like prime chain beginning at 46,000,363. It is the longest chain found in a full search up to  $6 \times 10^8$  and demonstrates the rarity of long prime chains under this map.

## 6. Code Availability

The Python scripts used in this study are included with the Zenodo submission.

## License

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