Collatz-5: Racing Gliders in Pentadica

Overview

Collatz-5 is the third entry in the Collatz-series mathematical game project, following *Collatz-3* (Boss-search in 3-adic space) and *Collatz-6* (chaotic gliders on the 6-adic edge). This work introduces a new discrete dynamical system based on a **5-adic Collatz**-type map, reimagined as a high-speed racing game set in the layered world of **Pentadica**.

Pentadica is a five-tiered mathematical universe governed by a simple rule:

- Numbers not divisible by 5 remain on the Outer Circuit, where they accelerate, oscillate, and generate long glider-like trajectories.
- Numbers that become divisible by 5 immediately fall into a deeper layer, losing energy and eventually converging to the stable core {1, 2, 3}.
- This creates a natural interpretation of the dynamics as a multi-layer racing circuit with hidden traps and gravitational sinks.

Each number acts as a "racer," and its behavior is characterized by three quantitative features:

1. **Steps** — total orbit length before reaching a fixed point.
2. **Laps** — number of iterations spent in the Outer Circuit (mod 5 ≠ 0).
3. **Maximum Value** — the peak magnitude reached during the orbit, representing how close the racer comes to "breaking out" of the system.

These three axes allow every number to be classified into a distinct racer-type:

- Instant Drop (non-glider)
- Short-Range Glider
- Endurance Glider
- High-Altitude Jumper
- Legendary Glider (breakout candidate)

A notable discovery is that numbers from the P(3,2) sequence naturally act as elite racers: they avoid divisibility by 2, 3, and 5, and frequently produce long-range gliders with dramatic oscillations. Among them, **774,840,977** emerges as a Legendary Glider, achieving exceptionally high step counts, long Outer-Circuit residence, and extreme peak values—appearing as if it might "escape" the system, though mathematically it never does.

5-adic Update Rule

The next value f(n) is determined by the residue of n modulo 5:

- mod 5 = 0 → f(n) = n / 5
- mod 5 = 1 → f(n) = (6n - 1) / 5
- mod 5 = 2 → f(n) = (6n - 2) / 5
- mod 5 = 3 → f(n) = (6n - 3) / 5
- mod 5 = 4 → f(n) = (6n + 1) / 5

Numbers with mod 5 ≠ 0 remain on the Outer Circuit;
numbers with mod 5 = 0 fall into a deeper layer.

Example of Step Calculation

Using the above rules, the first steps of the orbit of n = 161 are:

- 161 mod 5 = 1 → 193
- 193 mod 5 = 3 → 231
- 231 mod 5 = 1 → 277

Continuing this process yields the full trajectory: **161** → **193** → **231** → **…** → **15** → **3**

For this racer:

- Steps: 38
- Laps: 32
- Maximum Value: 1705
- Classification: *Endurance Glider*

Collatz-5 thus provides a playful yet rigorous exploration of 5-adic dynamics, orbit classification, and emergent behavior in discrete systems. It stands as a mathematical game, a visualization tool, and a narrative world built on number theory.

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