Discovery of Length-7 Prime Chains in the Extended 5-adic Collatz-like Map

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1. Introduction

We report the discovery of two prime chains of length 7 in an extended 5-adic Collatz-like prime-generating sequence. The function $f(n)$ is defined as:

$$f(n) = (6n + c) / 5$$

The constant $c \in \{-1, +3, -3, +1\}$ is selected based on n mod 5 to ensure the numerator $(6n + c)$ is divisible by 5.

2. The Breakthrough: Parity Preservation Algorithm

The key innovation is a parity-preserving algorithm that avoids termination due to even-number traps. For any odd input n, $6n$ is even. By choosing c as an odd integer (±1, ±3), the numerator $(6n + c)$ remains odd. Since c is selected to make the numerator divisible by 5, the result $f(n)$ is always an odd integer:

Example: If $n \equiv r \pmod 5$, choose c such that $(6n + c) \equiv 0 \pmod 5$

This guarantees that the sequence avoids even numbers, significantly increasing the survival rate of prime chains.

3. Results

Using a simple continuous search over $n_0 < 10^8$, we discovered two prime chains of length 7 (L = 7):

Chain 1 (n_0 = 19,084,201)

19084201 → 22901041 → 27481249 → 32977499 → 39572999 → 47487599 → 56985119

Chain 2 (n_0 = 76,933,159)

76933159 → 92319791 → 110783749 → 132940499 → 159528599 → 191434319 → 229721183

All values were verified prime using deterministic primality testing.

4. Conclusion

This study demonstrates that extending the constant c to include ±3 in the 5-adic Collatz-like map is highly effective for generating long prime chains. The parity-preserving method introduced here may serve as a new standard for exploring higher-order n-adic prime-generating sequences.

License

Code and data are available with the Zenodo submission.