Prime Chain Dynamics in a 6-adic Collatz-like Map: Discovery of a Length-8 Chain

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Abstract

This report presents a search for prime number chains within a 6-adic growth system defined by the mapping $f(n) = (7n \pm k)/6$. Unlike the Collatz conjecture, which is conjectured to converge, this system exhibits geometric growth with a ratio of approximately $7/6 \approx 1.17$. Within the search range $n < 10^7$, we identified several stable chains of length 7 (L7) and a single instance of a chain of length 8 (L8).

1. Introduction

We investigate the behavior of prime numbers under a quasi-affine transformation in a 6-adic system:

$$\boldsymbol{f(n) = (7n + \delta)/6}$$

where $\delta \in \{-1, -5, +5, +1\}$ is chosen such that $7n + \delta$ is divisible by 6, ensuring $f(n)$ is an integer. The sequence continues only if each resulting value is prime. This mapping induces geometric growth, testing the persistence of primality as the sequence progresses.

2. Methodology

- Mapping Logic: $f(n) = (7n \pm k)/6$, where $k \in \{1, 5\}$
- Search Range: $3 \leq n < 10^7$
- Target: Prime chains of length $L \geq 7$ (i.e., sequences of 7 or more consecutive primes under the mapping)
- Termination: Chain ends immediately upon encountering a composite number.

3. Results

Within the defined range, the algorithm discovered:

- Total L7 Chains: 9 (e.g., $n_0 = 53$; 434437; ...)
- Total L8 Chains: 1 ($n_0 = 1099687$)

L8 Chain

1099687 → 1282969 → 1496797 → 1746263 → 2037307 → 2376859 → 2773003 → 3235171

4. Discussion

The rarity of L8 chains within the first 10^7 integers suggests that surviving the 7/6 acceleration while maintaining primality becomes increasingly difficult. Notably, the number 5 is a fixed point:

$$f(5) = (7 \cdot 5 - 5)/6 = 5$$

For all $n > 5$, the sequence diverges.

5. Future Work

The current scan was limited to 10^7. The existence of longer chains (e.g., L9 or beyond) remains an open question. Although the mapping permits unbounded growth, the decreasing density of primes may impose a practical limit. Further computational exploration is encouraged.

License

Code and data are available with the Zenodo submission.