Length-7 Prime Chains under the Collatz Map

Hiroshi Harada

February 19, 2026

1. Abstract

In the context of the Collatz conjecture, numbers typically converge to 1. However, local "divergent" behavior can be observed where sequences of prime numbers are generated through the iterative map $f(x) = (3x + 1)/2$. This report documents a rare find of a length-7 prime chain starting from $x = 89599$ and discusses the modular arithmetic properties that sustain such chains.

2. Definition

A **Generalized Cunningham Chain under the Collatz Map** is defined as a sequence $(x_0, x_1, \ldots, x_k)$ where:

1. $x_{i+1} = (3x_i + 1)/2$
2. Each x_i is a prime number.

The transformation $f(x)$ represents the increasing step in the standard $3n + 1$ problem, applied to odd integers.

3. Findings

Through a systematic search utilizing primorial-based sieving, the following length-7 chain was identified:

- $x_0 = 89599$
- $x_1 = 134399$
- $x_2 = 201599$
- $x_3 = 302399$
- $x_4 = 453599$
- $x_5 = 680399$
- $x_6 = 1020599$

Termination: The subsequent term $x_7 = 1530899$ is a composite number (divisible by 41), terminating the chain at length 7.

4. Observations: The "99" Invariance

A notable feature of this chain is that all terms end in the digits "99". This is not coincidental.
For $x_{n+1} \equiv x_n \pmod{100}$ to hold under $f(x)$, we require:

$$(3x + 1)/2 \equiv x \pmod{100} \Rightarrow x \equiv -1 \pmod{200}$$

The starting value $89599 \equiv 199 \pmod{200}$ satisfies this condition. This congruence ensures that the chain remains within a specific residue class, potentially avoiding small prime factors such as 2 and 5 for several iterations.

5. Future Work

While a length-7 chain was found within the tested range, theoretical density suggests that longer chains (length 8 or more) may exist at higher magnitudes, particularly for $x > 10^{12}$. Further exploration using optimized sieving and parallel computation is encouraged.

6. Code Availability

The Python script used for this discovery is included in the Zenodo record associated with this report. It enables reproducibility and invites further exploration by the community.

License