Angle of Attack and Slocum Vehicle Model

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1 Intro

While underwater, the glider dead reckons its position relative to the last GPS fix. For a glider without a DVL, the inputs are vehicle depth and pitch. Without the use of a DVL, any error between the final dead reckoned position and GPS surface fix is attributed to water currents (see /doco/how-it-works/water-velocity-calculations.txt). The horizontal position is the horizontal velocity integrated over time. It is clear that it is important to get the horizontal velocity as accurate as possible. We don't directly measure the horizontal velocity. It is a function of the measured vertical speed vs [m/s] (time derivative of pressure sensor output), measured pitch θ [rad], and estimated angle of attack α [rad]:

$$\tan(\theta + \alpha) = \frac{-vs}{hs} \tag{1}$$

for vehicle coordinate system shown in Fig. 1.

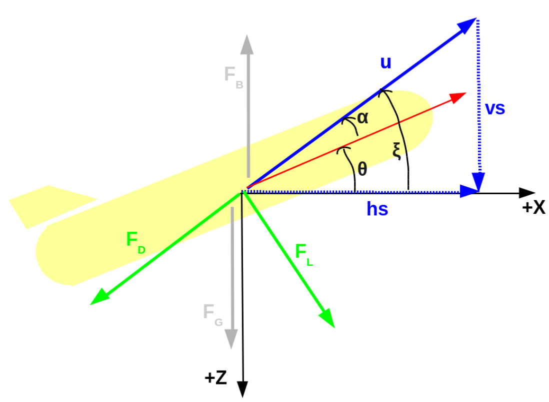

Figure 1: Slocum glider global reference frame.

The purpose of this document is to describe the method for calculating the estimated vehicle angle of attack.

Please note that this is a working document.

2 Model Coefficients

2.1 Geometry

length	L	1.5	m
hull diameter	D	0.22	m
sweep angle	Λ	0.785	rad
wing area	S	0.0927	m^2
wing span	b	0.99	m
weight in air (shallow)	m_{Gs}	52	kg
weight in air (deep)	m_{Gd}	56	kg

Slocum Electric Geometry. Note that wing span b and area S includes both fins

2.2 Equations of Motion in Global Refence Frame

We create a simplified model of a glider in quasi-steady flight, following the methods of [4]:

$$\sum F_x = 0 = -F_D \cos\zeta - F_L \sin\zeta \tag{2}$$

$$\sum F_z = 0 = F_G - F_B + F_D \sin\zeta - F_L \cos\zeta \tag{3}$$

with drag force F_D, lift force F_L, buoyancy force F_B, force due to gravity F_G, and glide angle ζ, where $\zeta = \theta + \alpha$. Drag force and lift force are functions of surrounding fluid density ρ, reference area A_f, relative velocity v, and coefficients of drag and lift C_D and C_L respectively:

$$F_D = \frac{1}{2}\rho C_D A_f v|v| \tag{4}$$

$$F_L = \frac{1}{2}\rho C_L A_f v|v| \tag{5}$$

We substitute eqns. 4 and 5 into eqn. 2 to find:

$$C_D \cos\zeta = -C_L \sin\zeta \tag{6}$$

2.3 Lift

The total coefficient of lift C_L is the sum of lift due to the hull C_{Lh} and lift due to the wings C_{Lw}

$$C_L = C_{Lh} + C_{Lw} \tag{7}$$

2.3.1 Body Lift

For small angles of attack, Hoerner [2, pg. 13-2 and 19-13] defines non-dimensional lateral force coefficient due to the hull C_{Lh} as a function of angle of attackα.

$$\begin{aligned} C_{Lh} &= \frac{dC_{Lh}}{d\alpha}\alpha \\ &= \frac{L}{D}\frac{dC_{Lh}}{d\alpha}\alpha \end{aligned} \tag{8}$$

for which $\frac{dC_{Lh}}{d\alpha} = 0.003\ \text{deg}^{-1} = 0.172\ \text{rad}^{-1}$, and D and L are the diameter and length of the hull respectively. We find the coefficient of lift due to the hull C_{Lh}:

$$C_{Lh} = 1.172\beta \tag{9}$$

2.3.2 Wing Lift

The coefficient of lift due to the wings C_{Lw} is determined using lift theory of swept wings.

$$C_{Lw} = \frac{dC_{Lw}}{d\alpha}\alpha \tag{10}$$

The aspect ratio for the wings AR_w is

$$\begin{align} AR_w &= \frac{b^2}{S} \tag{11} \\ &= 10.573 \tag{12} \end{align}$$

For wings with aspect ratio >4, [2, pg. 15-7] gives

$$\begin{align} \frac{dC_{Lw}}{d\alpha^\circ} &= \frac{\cos\Lambda}{10 + 20/AR_w} \tag{13} \\ &= \frac{\cos 0.785}{10 + 20/10.573} \tag{14} \\ &= 0.0595 \text{ deg}^{-1} \tag{15} \end{align}$$

for which we find the coefficient of lift to be

$$\begin{align} C_{Lw} &= \frac{dC_{Lw}}{d\alpha}\alpha \tag{16} \\ &= (0.0595 * \frac{180}{\pi})\alpha \tag{17} \\ &= 3.40987\alpha \tag{18} \end{align}$$

2.4 Drag

The total coefficient of drag C_D is the sum of drag due to pressure C_{Do} and induced drag C_{Di}

$$C_L = C_{Lh} + CLw \tag{19}$$

Reynolds number R_e is the ratio of inertial to viscous forces, given in [1, Table A.2] as

$$R_e = \frac{uL}{\nu} \tag{20}$$

for speed u, characteristic length L, and fluid kinematic viscosity ν, where ν is taken to be $1.35\text{x}10^{-6} m^2/s$ at 10°deg C. Fig. 2 shows Reynolds Number as a function of characteristic glide speed u for a Slocum glider.

Hoerner [3, pg. 6-16] defines subcritical conditions to be $R_e < 10^5$, and the transitional range around R_e 10^6. Slocum gliders generally operate between these subcritical and transitional ranges, however it should be noted that turbulent flow may be tripped due to inconsistencies in the hull smoothness.

2.4.1 Pressure Drag

Hoerner [3, pg. 3-12] finds the coefficient of drag due to pressure C_{Do} to be:

$$C_{Do} = 0.44\frac{D}{L} + 4C_f(R_e)\frac{L}{D} + 4C_f(R_e)\frac{D}{L}^2 \tag{21}$$

where friction-drag coefficient C_f is a function of Reynolds Number (fig. 3).

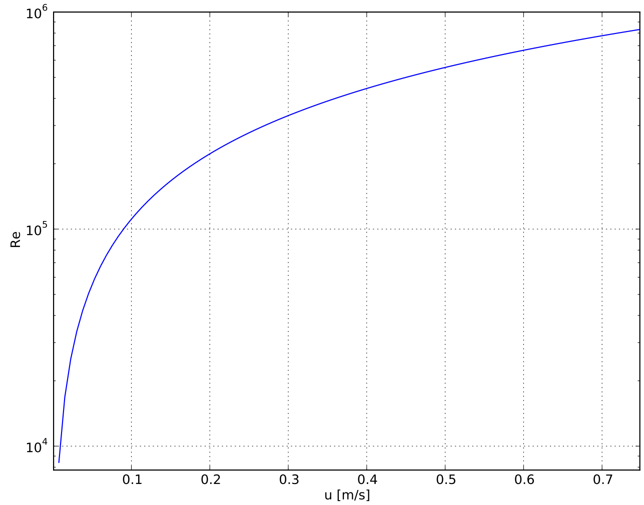

Figure 2: Reynolds Number as a function of glide speed u

2.4.2 Induced Drag

From [3, pg. 7-2] we find the induced drag coefficient, or drag due to lift C_{Di}

$$
\begin{aligned}
C_{Di} &= \frac{C_L^2}{AR\pi} && (22)\\
&= \frac{C_{Lh}^2}{AR_h\pi} + \frac{C_{Lw}^2}{AR_w\pi} && (23)\\
&= \frac{(1.172\alpha)^2}{0.147\pi} + \frac{(3.41\alpha)^2}{10.573\pi} && (24)\\
&= \frac{dC_{Di}}{\alpha^2}\alpha^2 && (25)\\
&= 3.331\alpha^2 && (26)
\end{aligned}
$$

where the aspect ratio of the hull $AR_h = D/L$

2.5 Putting it all together

$$
\begin{aligned}
C_L &= C_{Lh} + C_{Lw} && (27)\\
&= (\frac{dC_{Lh}}{d\alpha} + \frac{dC_{Lw}}{d\alpha})\alpha && (28)\\
&= (1.172 + 3.408)\alpha && (29)\\
&= 4.580\alpha && (30)
\end{aligned}
$$

$$
\begin{aligned}
C_D &= C_{Do} + C_{Di} && (31)\\
&= C_{Do} + \frac{dC_{Di}}{d\alpha^2}\alpha^2 && (32)\\
&= C_{Do}(R_e) + 3.33\alpha^2 && (33)
\end{aligned}
$$

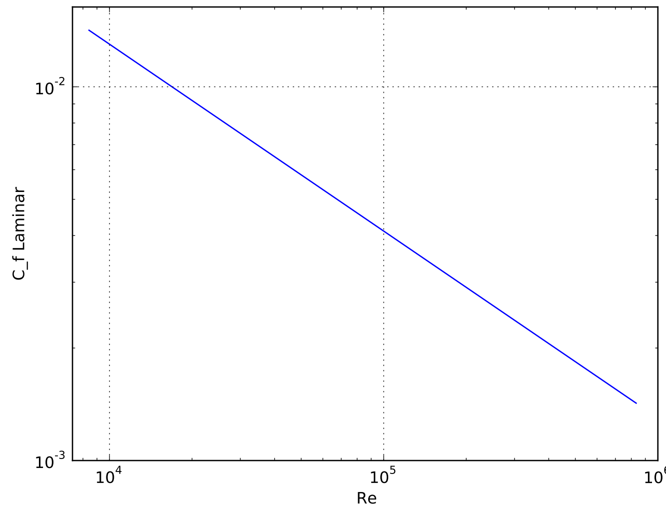

Figure 3: Laminar Skin-Friction coefficient C_F as a function of Reynolds Number. From [3, pg. 6-16]

Referring back to eq. 6:

$$C_D \cos\zeta = -C_L \sin\zeta \tag{34}$$

$$(C_{Do} + \frac{dC_{Di}}{d\alpha^2}\alpha^2)\cos\zeta = -(\frac{dC_{Lh}}{d\alpha} + \frac{dC_{Lw}}{d\alpha})\alpha)\sin\zeta \tag{35}$$

$$(C_{Do}(R_e) + 3.33\alpha^2)\cos(\theta+\alpha) = -(4.580)\alpha)\sin(\theta+\alpha) \tag{36}$$

For given pitch angle θ and glide speed u (which allows us to solve for R_e and therefore C_{Do}, we can solve for angle of attack α. It gets a bit messy, so as suggested by [5], we solve using an iterative approach. Figure 4 shows calculated angle of attack for a range of pitch and vehicle depth rates, for which vehicle vertical depth rate $vs = u\sin(\theta+\alpha)$.

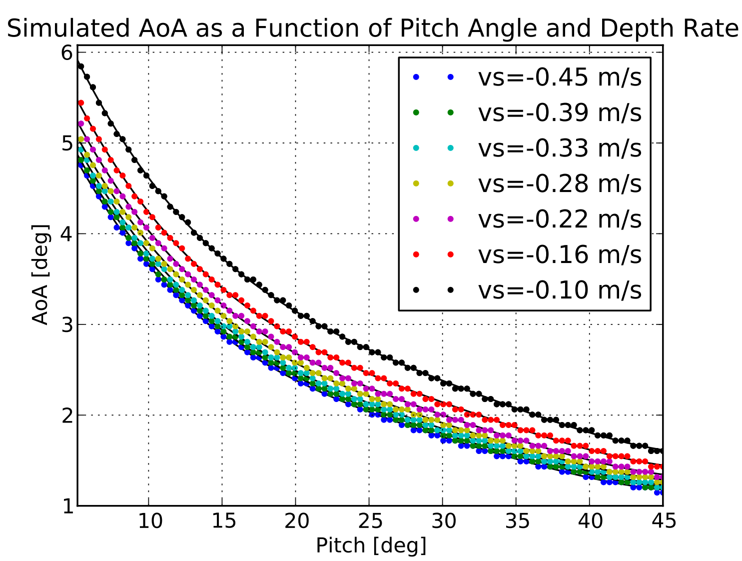

Figure 4: Angle of attack α as a function of pitch angle θ for several vehicle depth rates vs.

2.6 Universal Equation for Angle of Attack as a Function of Pitch and Depth Rate

In order to minimize computational cost of the angle of attack, we can characterize the angle of attack as a function of a given pitch angle and depth rate. The following equations were found to minimize the error between angle of attack computed using eq. 34 and angle of attack computed using the universal equation. For climbing ($\theta > 0$ and vertical speed $vs < 0$):

$$\alpha = 0.4830\theta^4 - 1.1073\theta^3 + 0.9834\theta^2 - 0.4521\theta + 0.1246 \tag{37}$$

$$- 0.00873 + 0.045 * 10^{-0.750} \frac{-0.220}{vs} \tag{38}$$

and for diving ($\theta < 0$ and vertical speed $vs > 0$): :

$$\alpha = - 0.4830\theta^4 - 1.1073\theta^3 - 0.9834\theta^2 - 0.4521\theta - 0.1246 \tag{39}$$

$$- 0.00873 + 0.045 * 10^{-0.750} \frac{-0.220}{vs} \tag{40}$$

Figure 5 shows how angle of attack solved using these two equations match up to simulated angle of attack.

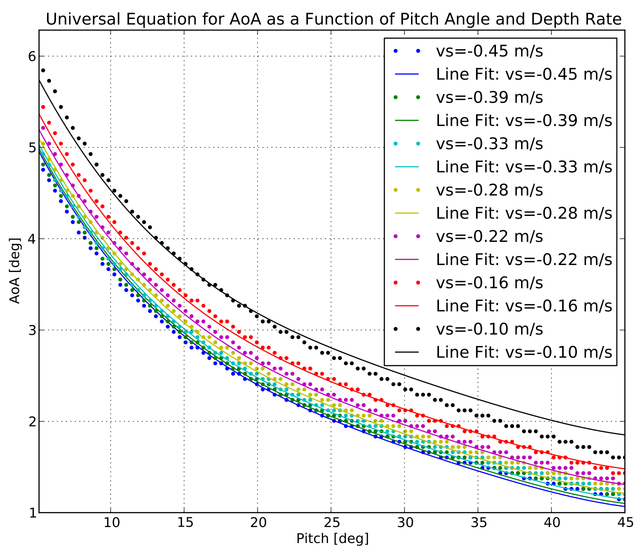

Figure 5: Fit line to AoA

3 Experimental Performance

TBW

References

[1] J. N. Newman, *Marine Hydrodynamics* MIT Press, August 1977.

[2] Sighard F. Hoerner, *Fluid Dynamic Lift*. Bakersfield, CA, 2nd Edition, 1985.

[3] Sighard F. Hoerner, *Fluid Dynamic Drag*. Bakersfield, CA, 2nd Edition, 1965.

[4] L. Merckelbach, D. Smeed, and G. Griffiths, *Vertical Water Velocities from Underwater Gliders*. Journal of Atmospheric and Oceanic Technology, 2009.

[5] L. Merckelbach, *Improved algorithm for deadreckoning for Slocum gliders*. January 29, 2010.