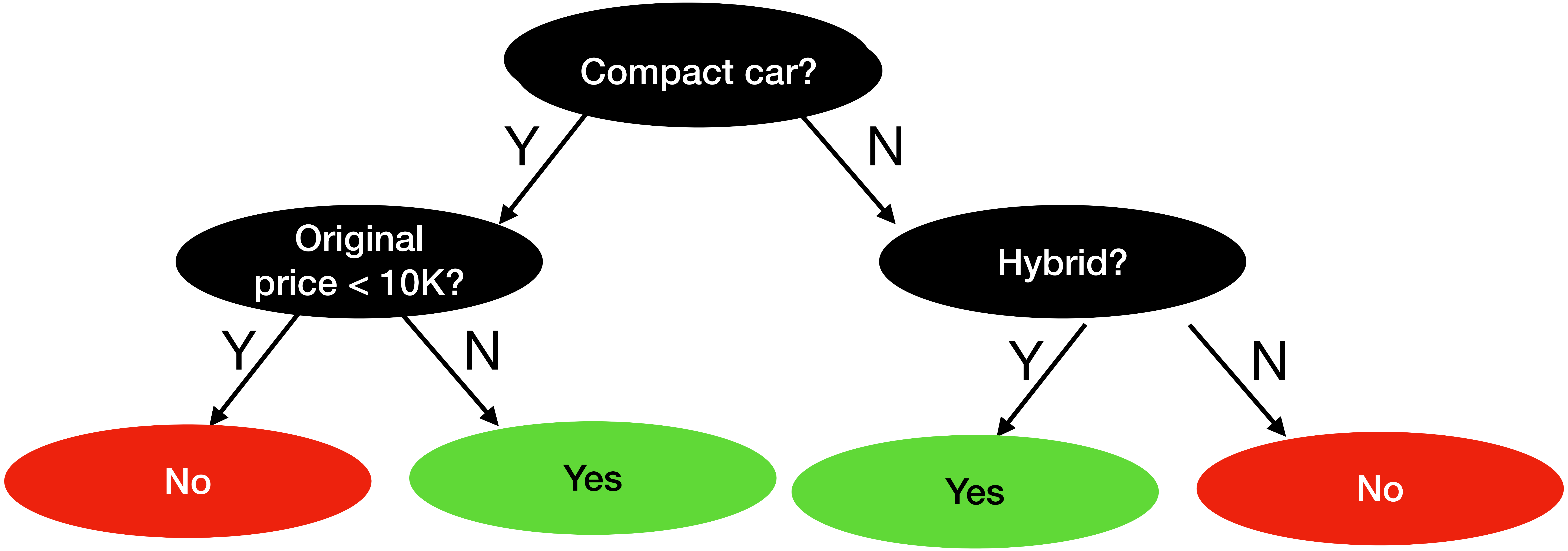
Decision Tree, finished product

To classify/predict: will the car be driven more than 200000 miles?



Training data: examples with feature values, correct class

Compact	Hybrid	Price	Class
No	No	9K	-
No	No	8K	-
No	No	7K	-
Yes	No	6K	-
No	No	10K	+
Yes	No	15K	+
No	Yes	17K	+
No	Yes	20K	+

+: Driven over 200,000 miles

-: Not driven over 200,000 miles

Transforming categorical & numerical features to Boolean "yes or no" questions

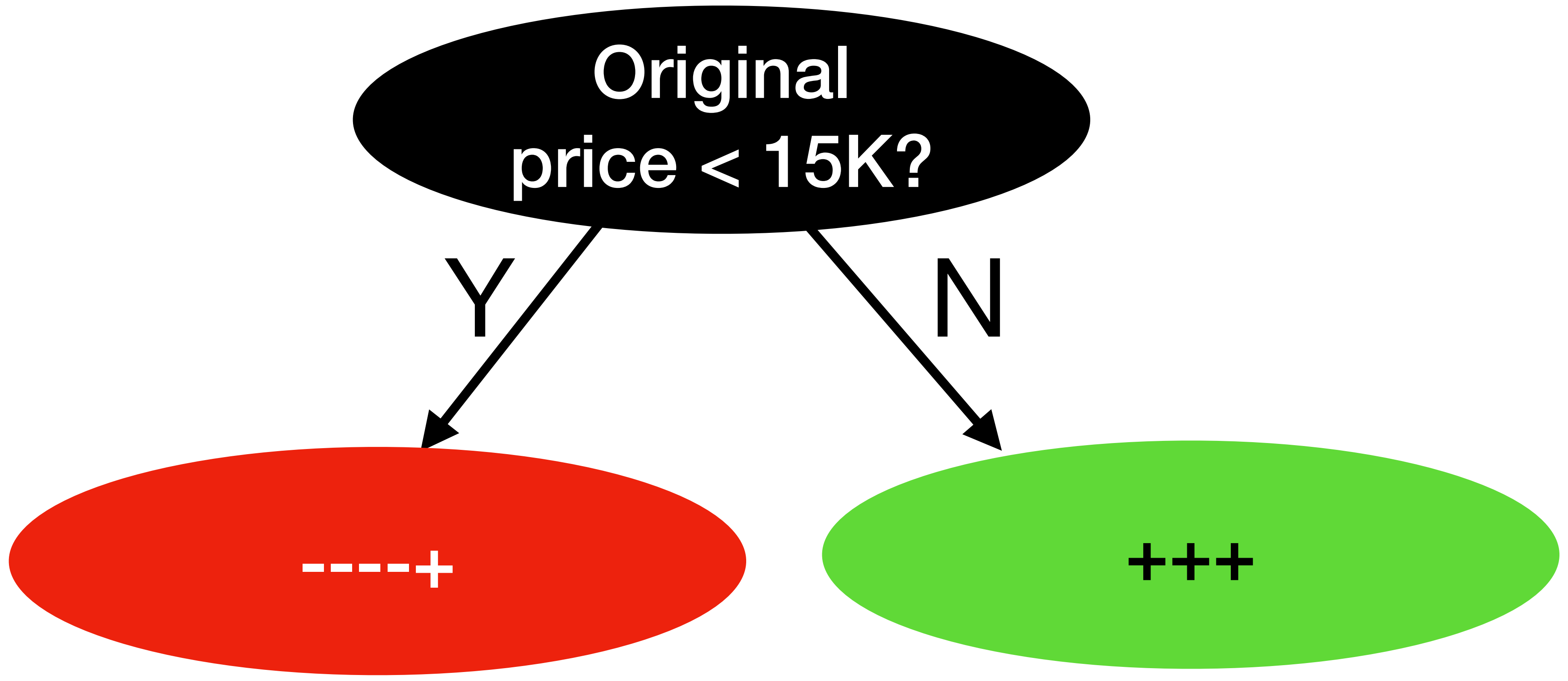
- **Categorical features (e.g., make of car):** Becomes one question/feature per value of the categorical feature: "Is it a Corolla? Is it a Fit?"
  - This strategy is generally known as "one-hot encoding."
- **Numerical features (e.g., price of car):** Becomes one Boolean "threshold" question per observed value: "Did it cost < 10K? Did it cost < 15K?"

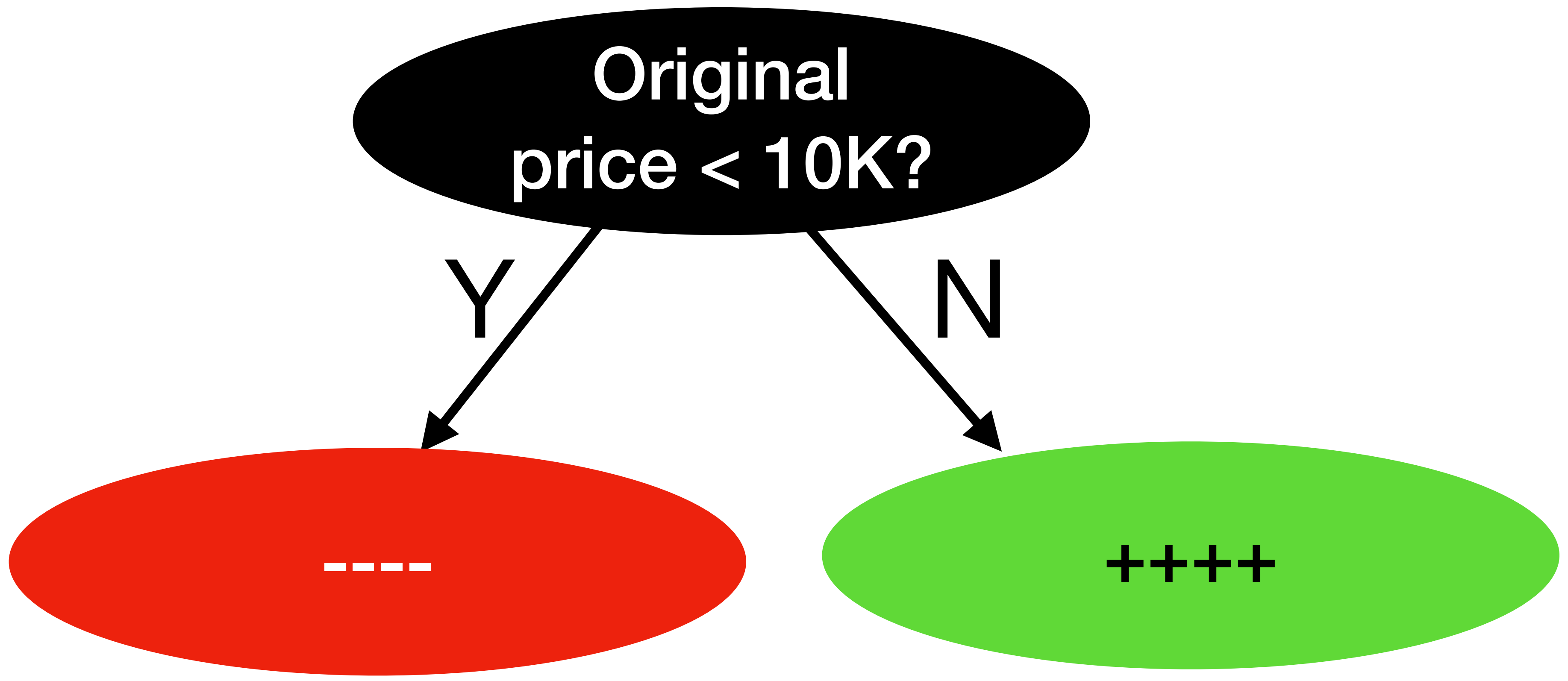
Strategy: find questions that make the labels agree within "pile"

Price	Class
9K	-
8K	-
7K	-
6K	-
20K	+
17K	+
15K	+
10K	+

+: Driven over 200,000 miles

-: Not driven over 200,000 miles

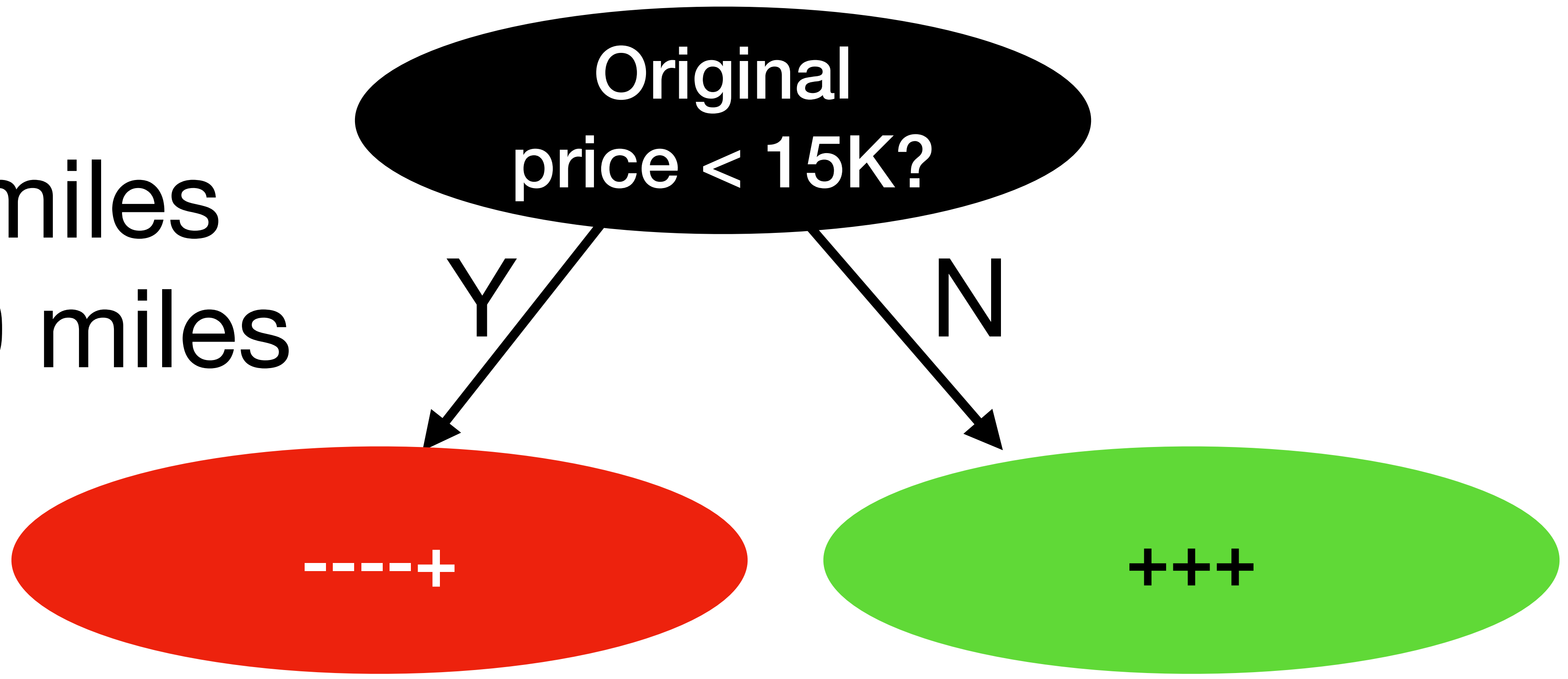




Reminder: the - and +'s are labeled data points

+: Driven over 200,000 miles

-: Not driven over 200,000 miles



Compact	Hybrid	Price	Class
No	No	9K	-
No	No	8K	-
No	No	7K	-
Yes	No	6K	-
Yes	No	10K	+

Compact	Hybrid	Price	Class
Yes	No	15K	+
No	Yes	17K	+
No	Yes	20K	+

Entropy: $\sum_i -p_i \log_2 p_i$ where p_i is the probability of symbol

++++

Entropy 0

0 lg 0 - 1 lg 1 = 0 - 0 = 0

++--

Entropy 1

-1/2 lg 1/2 - 1/2 lg 1/2 = - 1/2(-1) - 1/2(-1) = 1

+---

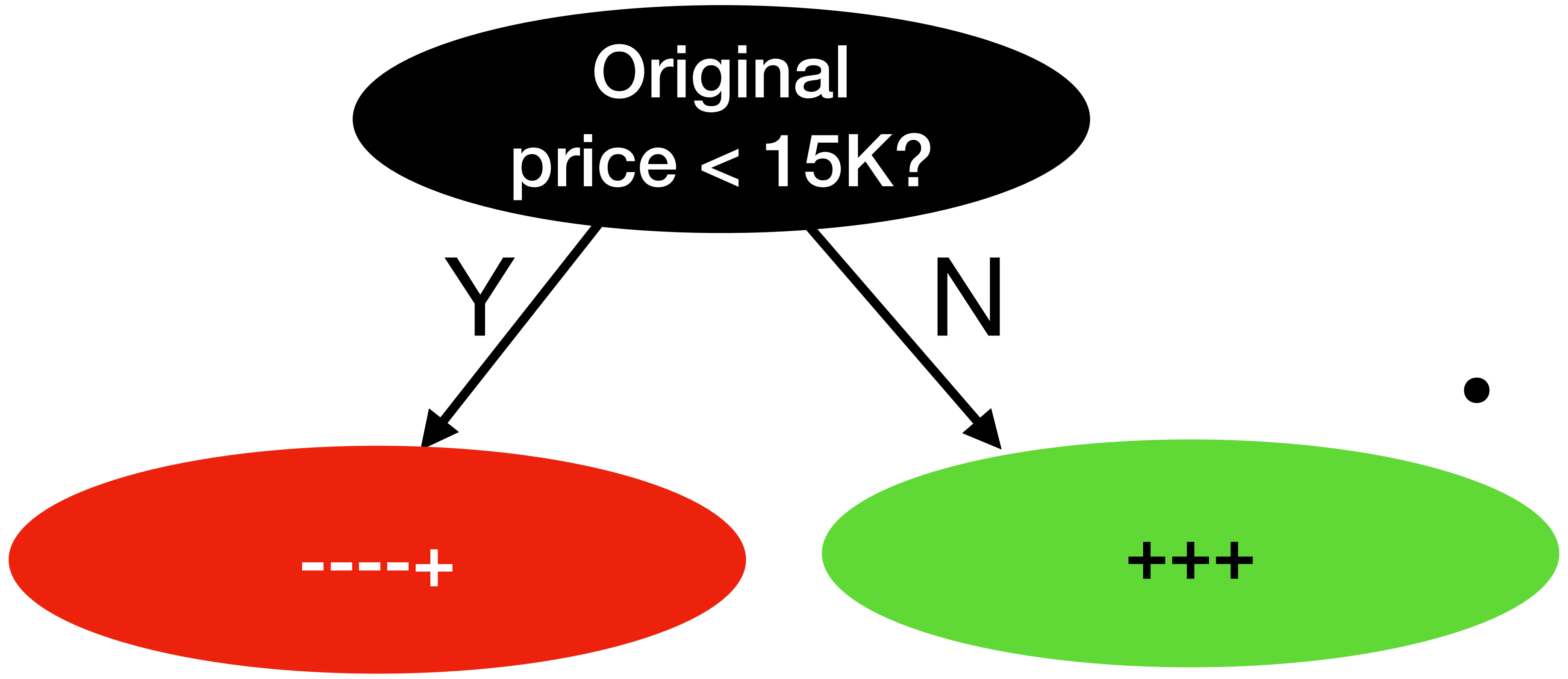
Entropy 0.81

-3/4 lg 3/4 - 1/4 lg 1/4

Expected Entropy

- Each question makes two piles of examples - yes to question, no to question - and calculates entropy for each pile's labels
- Expected entropy: entropies weighted by the branch's fraction of examples
- We pick the question that yields the best expected entropy

- Yes entropy:
  -1/5 lg 1/5 - 4/5 lg 4/5 = 0.72

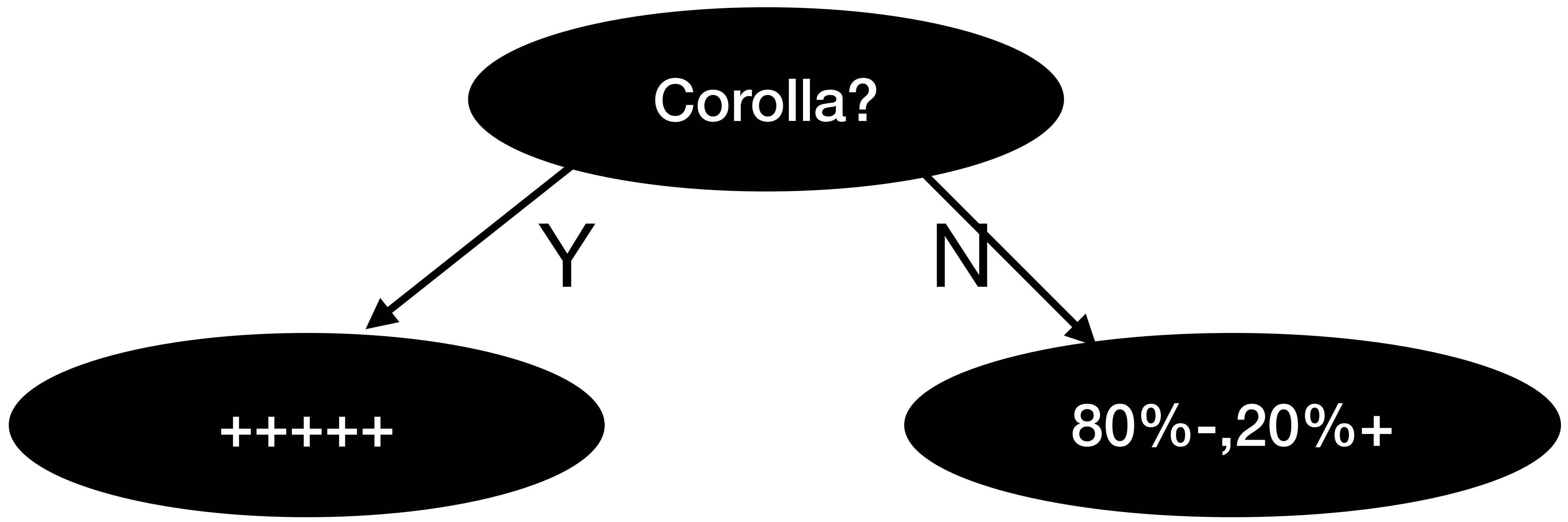


- "No pile" entropy: 0
  (all labels agree)

- Expected entropy: 5/8(0.72) + 3/8(0) = 0.45

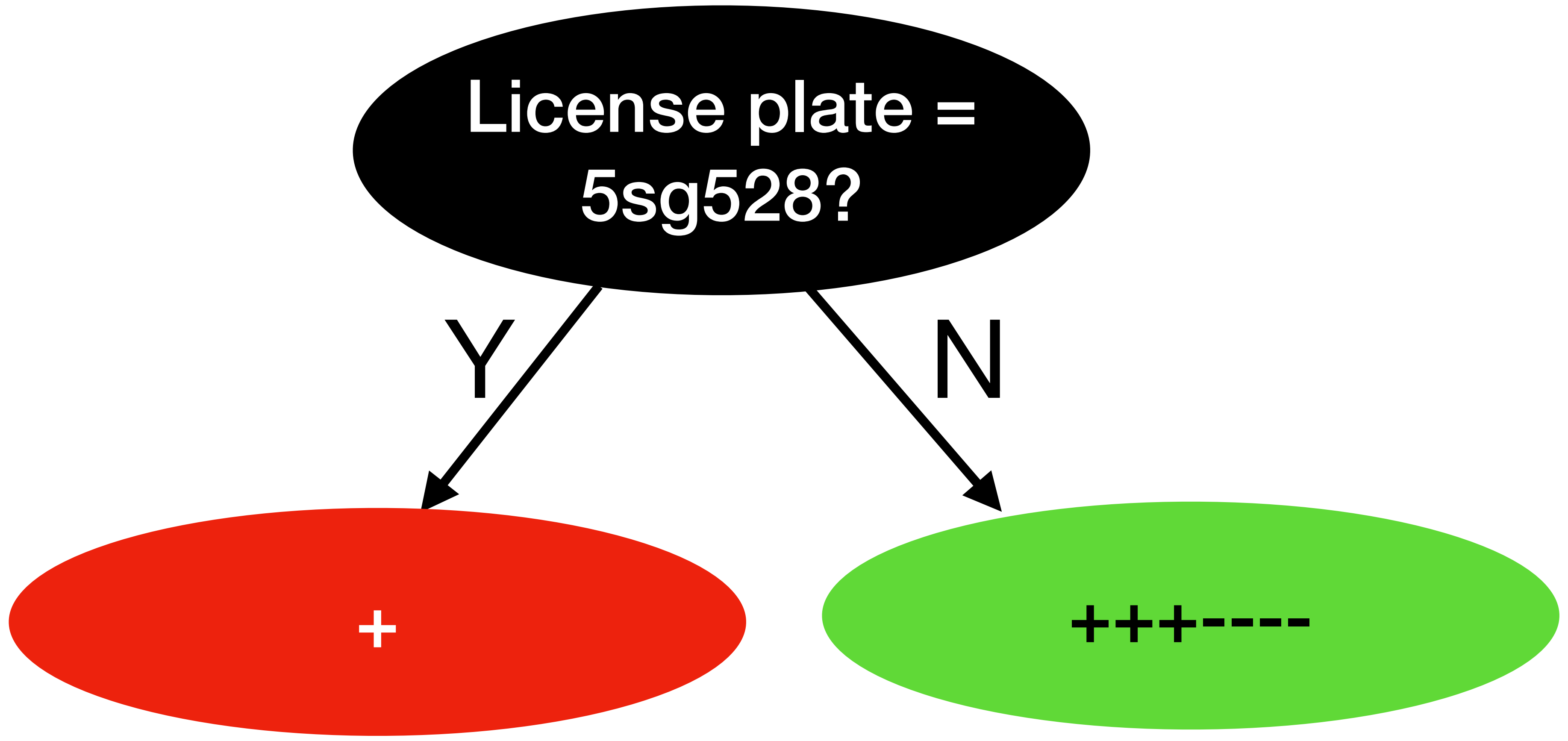
Expected Entropy, example 2

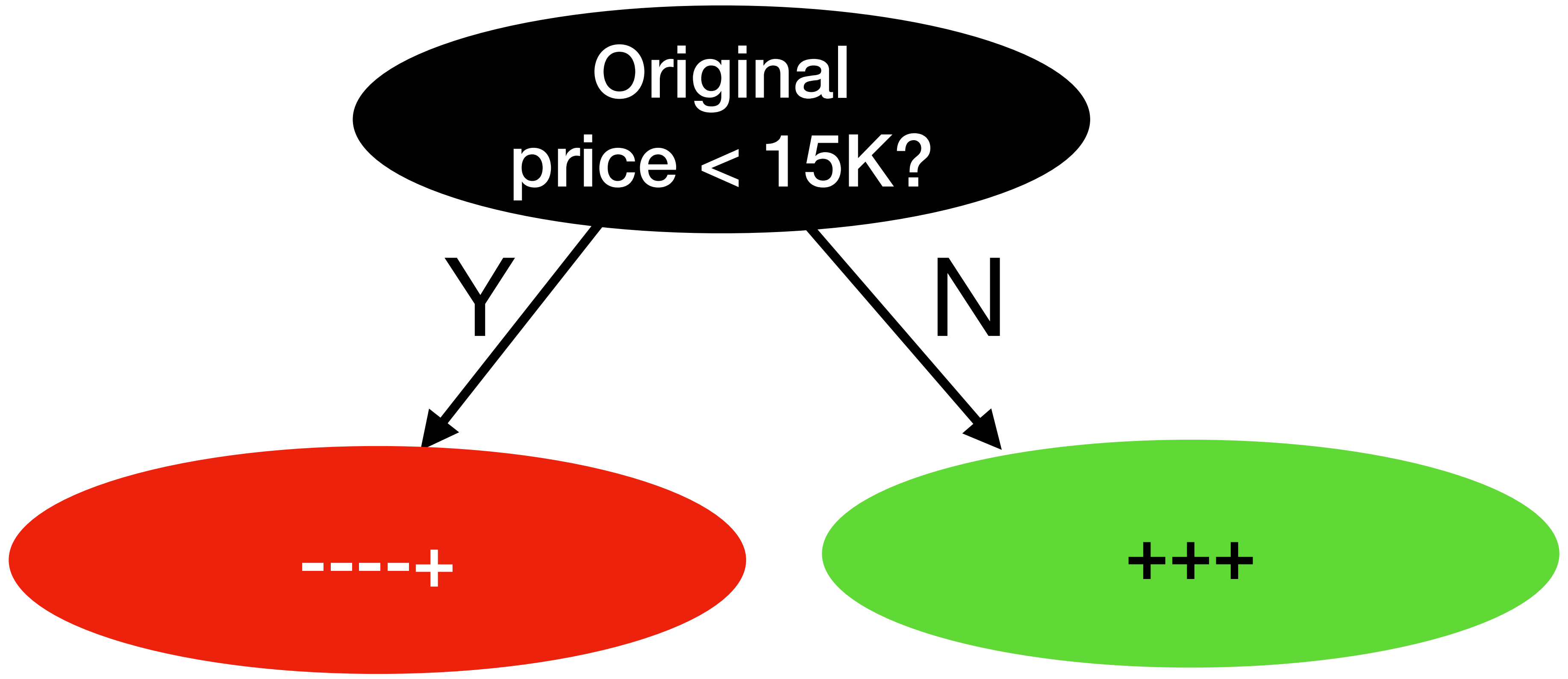
- Question "Is it a Corolla?" affects 5 cars in a 2005 point training set
- All 5 Corollas driven over 200000 miles, so that branch is entropy 0
- 20% of the remaining cars were driven over 200,000 miles, so entropy of that branch is $-0.2 \log_2 0.2 - 0.8 \log_2 0.8 = 0.72$
- Expected entropy is then 5/2005*0 + 2000/2005*0.72 = 0.72



Why Expected Entropy?

- A question that just helps 1 example be classified correctly is not that useful
- Expected entropy rewards questions that help classify many examples





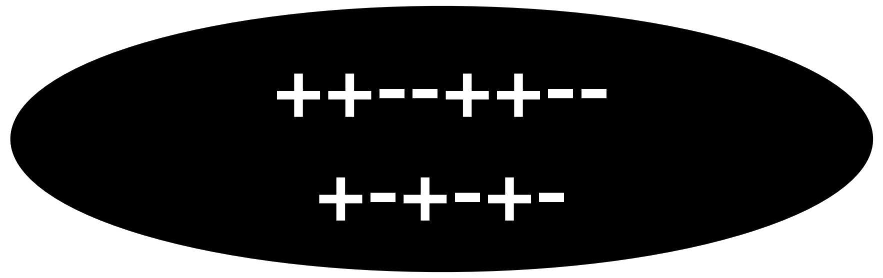
- Exp. entropy: 1/8(0) + 7/8(0.98) = 0.86
- Exp. entropy: 5/8(0.72) + 3/8(0) = 0.45

Decision stumps

Compact	Hybrid	Price	Class
No	No	9K	-
No	No	8K	-
No	No	7K	-
Yes	No	6K	-
No	No	10K	+
Yes	No	15K	+
No	Yes	17K	+
No	Yes	20K	+

- If we don't recur, we can just pick the single best question according to its expected entropy
- The classifications at the leaves are the "majority vote" of the training examples that land there

Decision Trees recursively split the training data to minimize entropy



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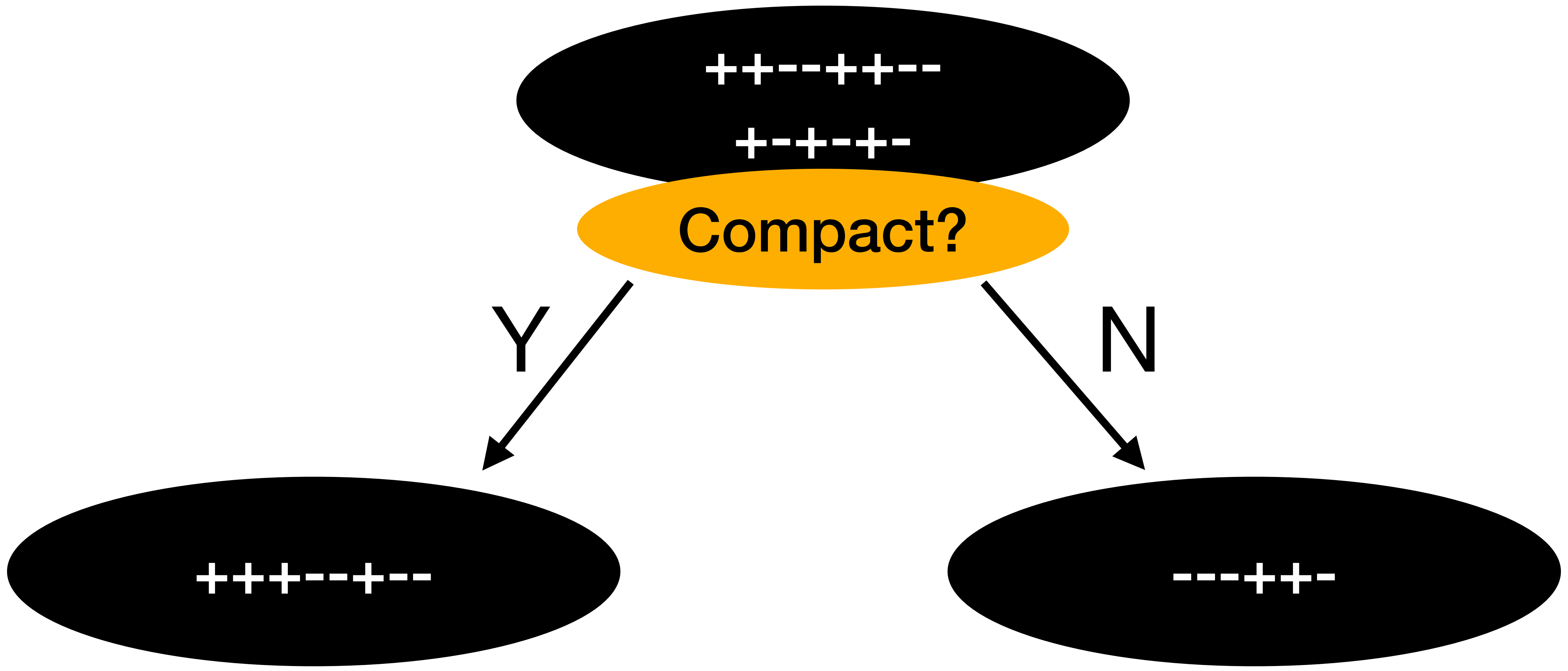

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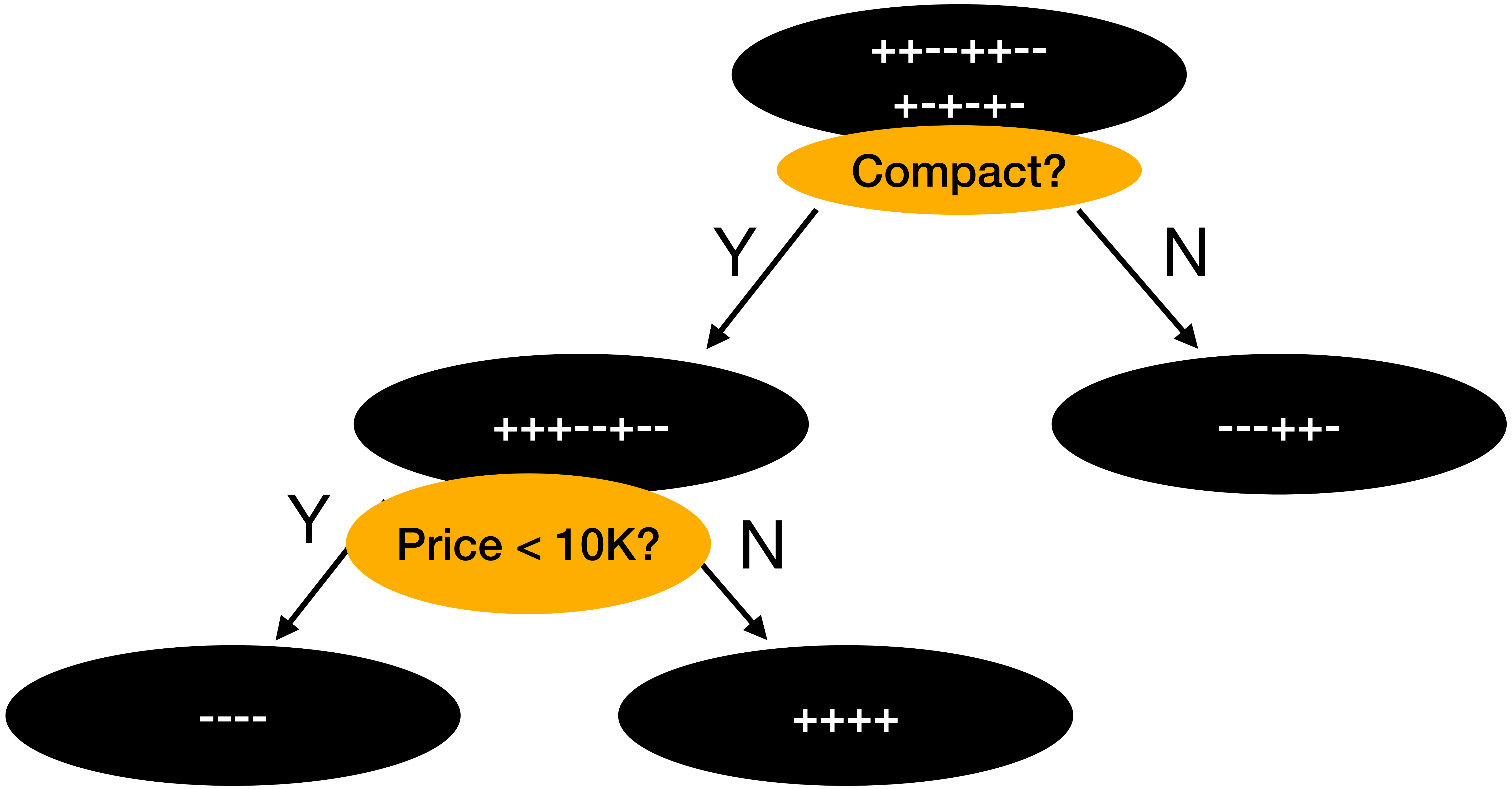

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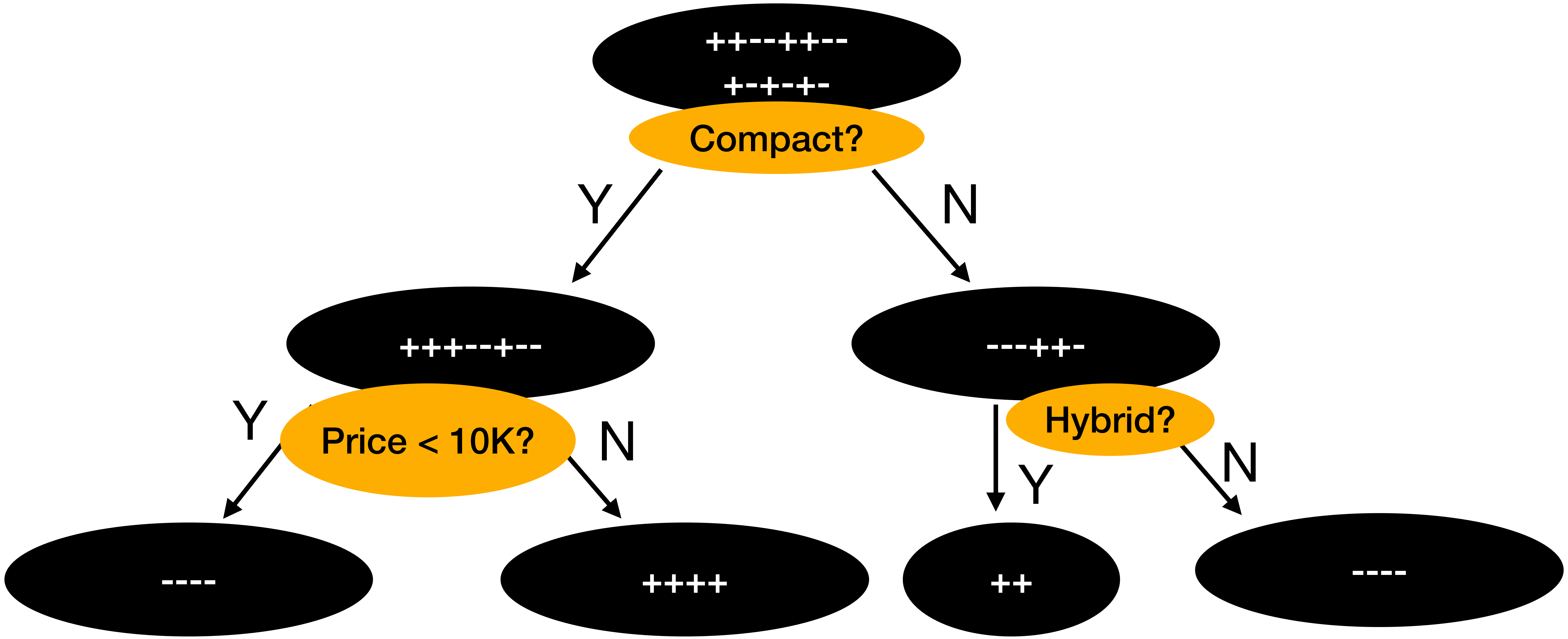


Pseudocode

- DecisionTreeNode(examples):
  - If the examples all agree on a label, return a leaf with that label
  - Iterate through all questions about features to get question Q with best expected entropy
  - If the expected entropy for Q isn't an improvement over the current entropy, stop and make this a leaf with classification according to majority rule
  - Recursively create a "yes" branch with examples that answer "yes" to Q
  - Recursively create a "no" branch with examples that answer "no" to Q

Small sample run: Data and features

Underwater	Legs	Octopus?
No	8	-
No	8	-
No	4	-
Yes	0	-
Yes	8	+
Yes	8	+

Possible features to create questions about:

- Underwater?
- Legs < 0?
- Legs < 4?
- Legs < 8?

Small sample run - Choosing the first feature

Underwater	Legs	Octopus?
No	8	-
No	8	-
No	4	-
Yes	0	-
Yes	8	+
Yes	8	+

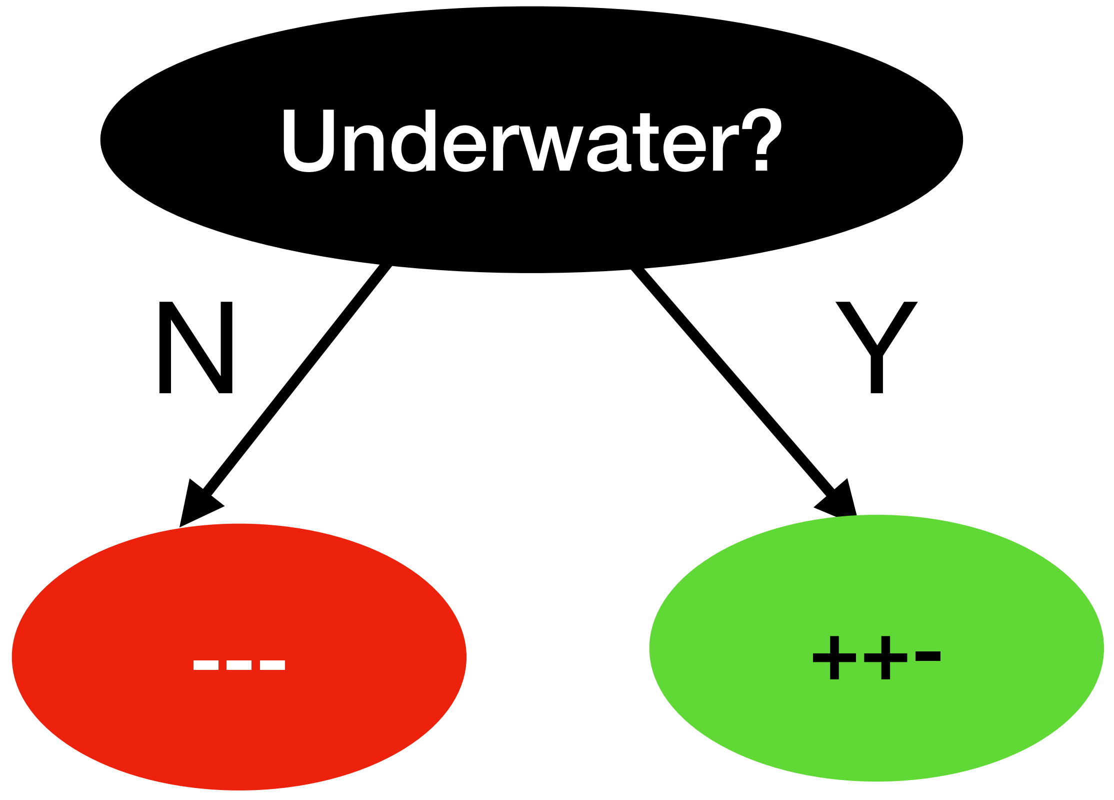
Underwater split:
Answer no: --- (entropy 0)
Answer yes: -++
(entropy $-1/3 \log_2 1/3 - 2/3 \log_2 2/3 = 0.918$)
Expected entropy:
$3/6 * 0 + 3/6 * 0.918 = 0.459$

Best legs split, legs < 8
Answer no: --++ (entropy 1)
Answer yes: -- (entropy 0)
Expected entropy:
$4/6*1 + 2/6*0 = 2/3 = 0.667$

We choose "Underwater?" as the first question.

Small sample run - Choosing the 2nd feature

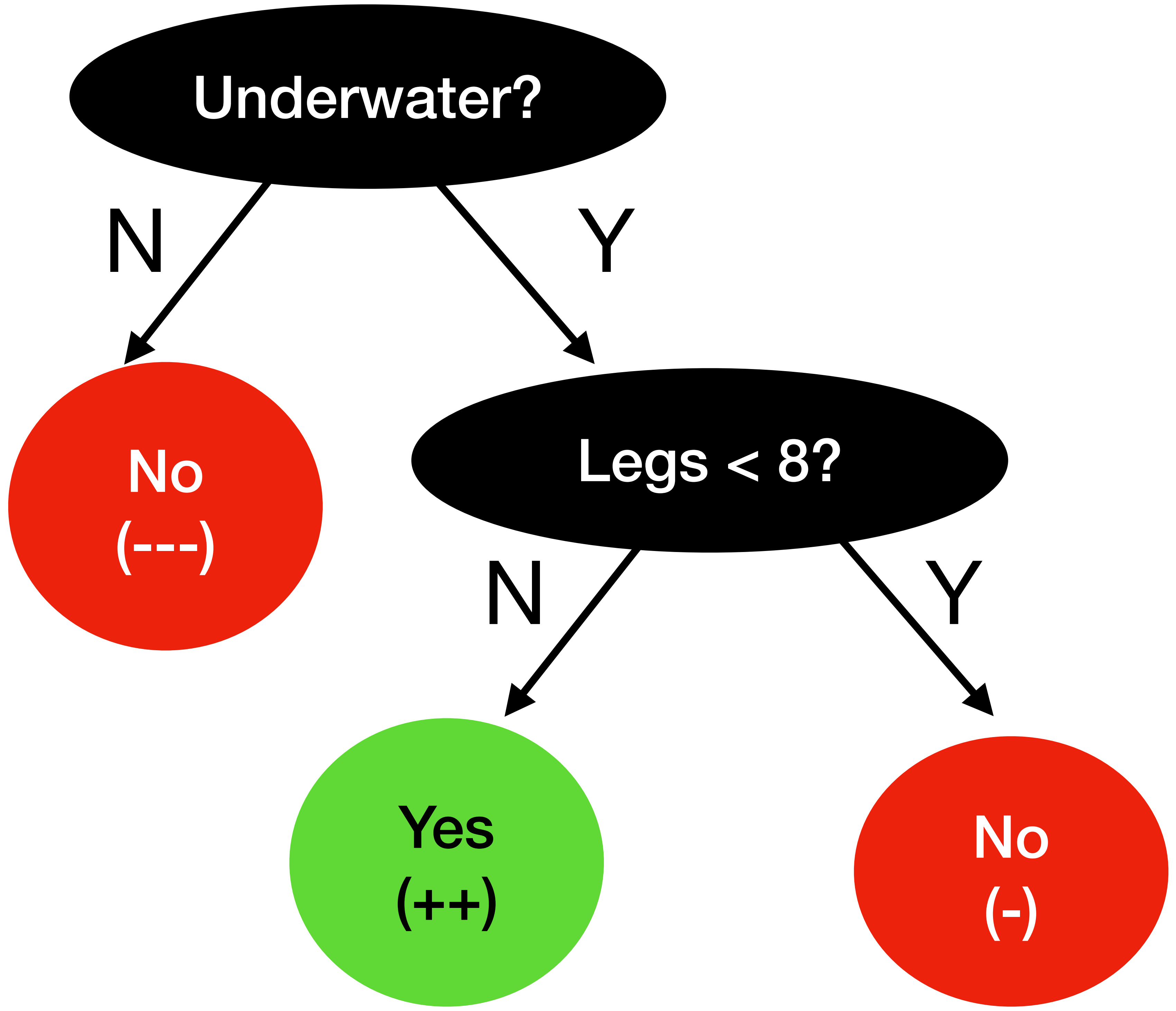
Underwater	Legs	Octopus?
No	8	-
No	8	-
No	4	-
Yes	0	-
Yes	8	+
Yes	8	+



- Nothing more to do for the branch where all agree;
  set this to be a leaf where tree returns "No"
- For remaining branch, try the different legs features that made it to this side:
  legs < 0 has nothing on one side, entropy remains 0.459
  legs < 8 gets entropy 0 on both sides of the split (++, -), expected entropy 0

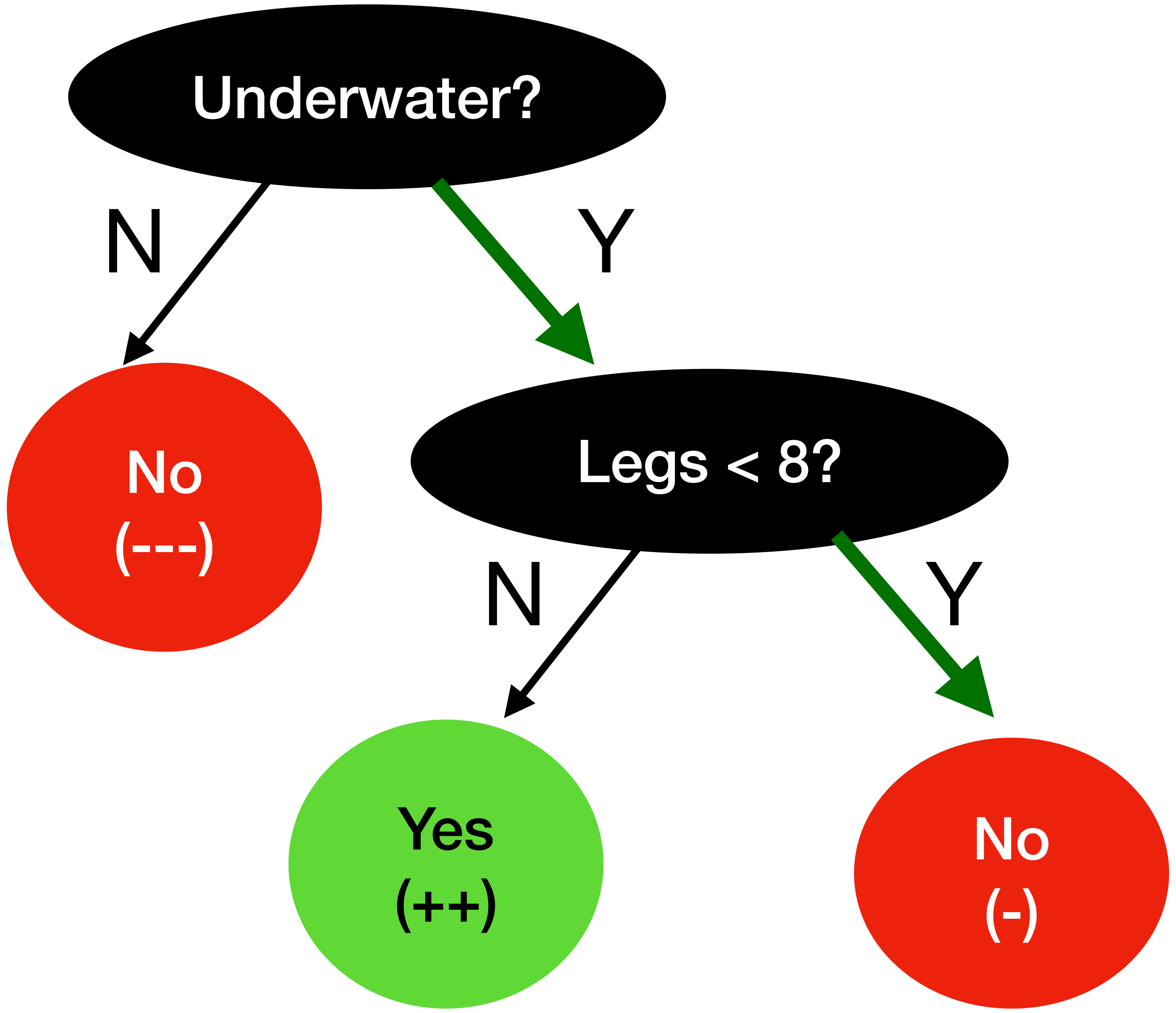
Sample run - the completed octopus-classifying tree

Underwater	Legs	Octopus?
No	8	-
No	8	-
No	4	-
Yes	0	-
Yes	8	+
Yes	8	+



- The tree is now ready to classify new instances it hasn't seen before

Sample run - the completed octopus-classifying tree



- The tree is now ready to classify new instances it hasn't seen before

What if the data labels are inconsistent?

- The data doesn't necessarily lend itself to perfect classification in this way - the exact same features may be labeled differently for different examples.
  - If the diving bell spider on the right were in the dataset (8 legs, underwater), there would be no way to distinguish it from the octopus

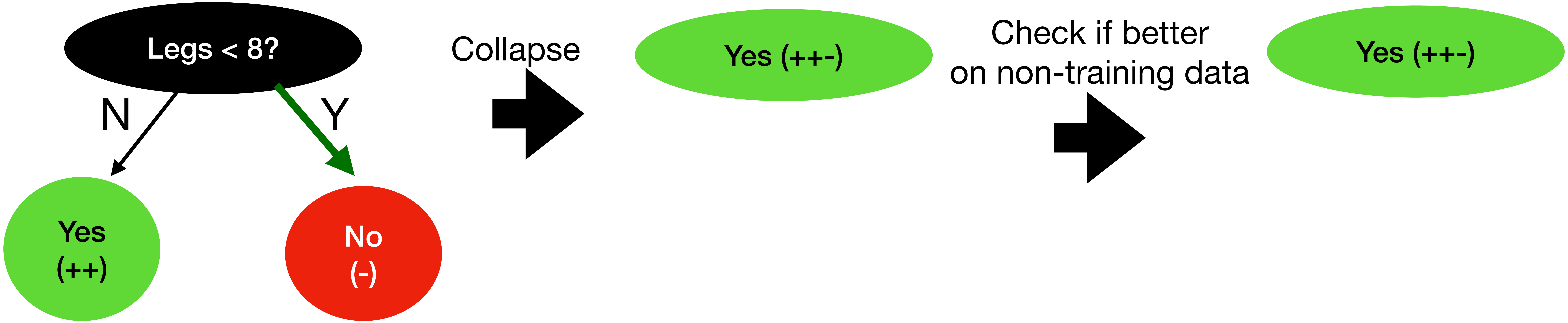
- If the algorithm detects that no feature improves the expected entropy, it stops looking for features and creates a leaf with the majority label of the examples
  - If octopuses are more common than diving bell spiders, the final classification of these will be "octopus"

Overfitting

- It's possible for the learning to create a tree with features that don't generalize well from the training set to unseen examples.
    - Names ("Don't give anyone a loan whose name is Simon!")
    - License plates ("It goes over 200,000 miles if its license plate is 5SG528")
    - Any numerical value that is very specific to an individual, such as an exact height or weight
    - A combination of features that seem harmless in isolation but together uniquely identify an individual ("Blue 2010 Honda Fit garaged in Somerville")
- Overfitting generally results in worse performance on test sets and in deployment

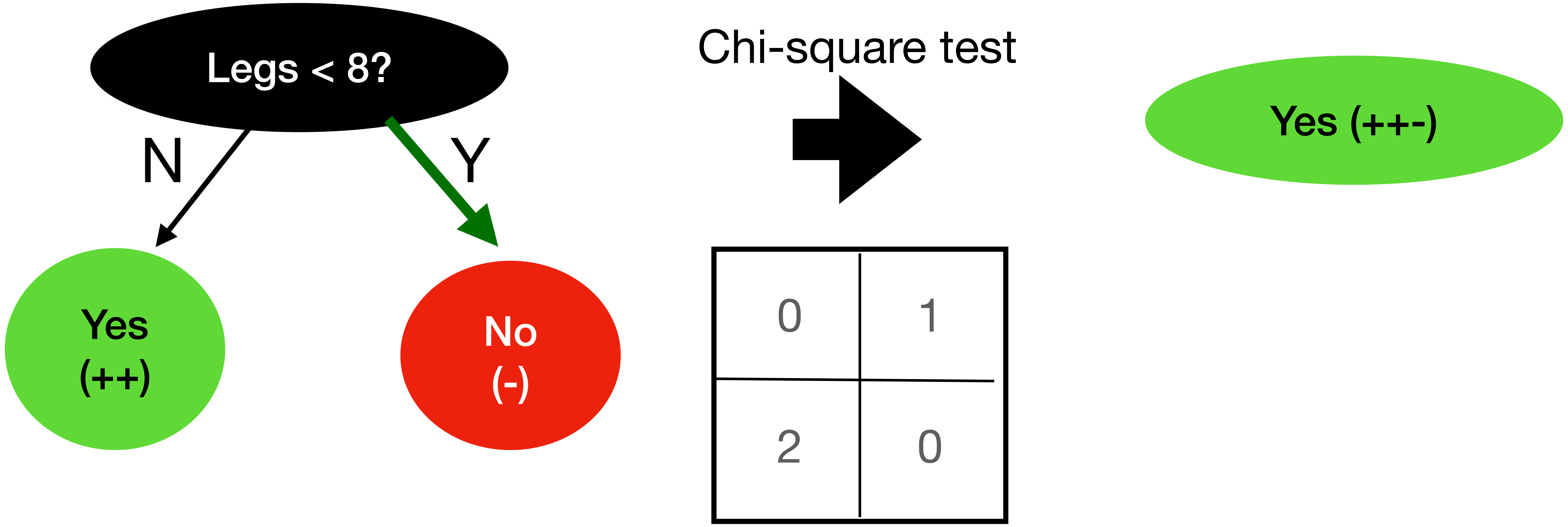
Combatting overfitting

- Various methods exist for **pruning** the decision tree to combat overfitting
  - For each pair of leaves, can check whether tree performs better on a validation set if the leaves are merged back into a single leaf



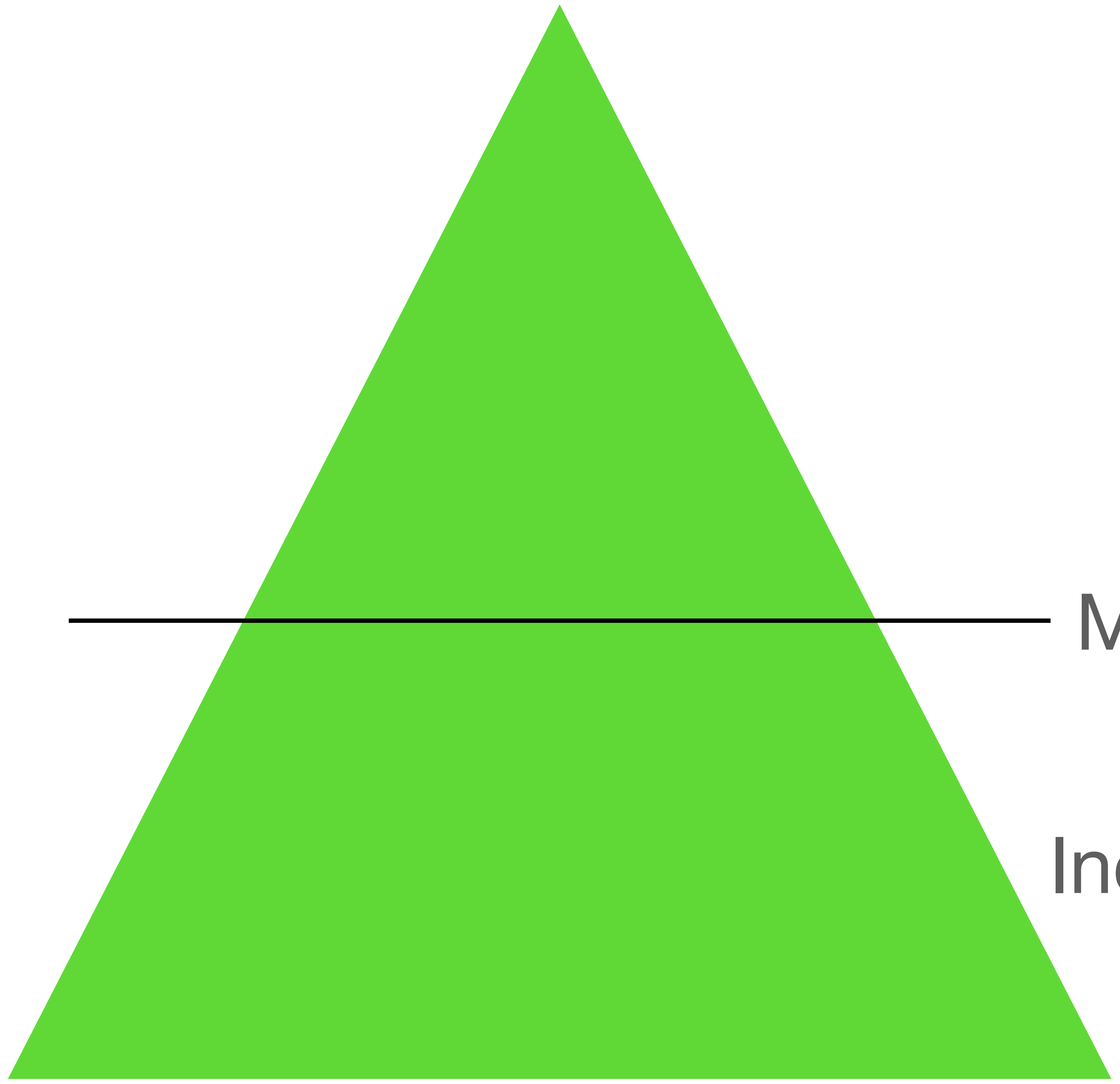
Combatting overfitting

- Various methods exist for **pruning** the decision tree to combat overfitting
  - The leaves could be merged if the relationship between feature and label isn't significant under a chi-square significance test



Combatting overfitting

- Various methods exist for **pruning** the decision tree to combat overfitting
  - A simple method is to just specify the maximum depth of the tree, on the assumption overfitting tends to happen once good features are exhausted



Using Decision Trees in scikit-learn