PHGN 462 Final Exam – Fall 2016 Name ________________________

In all parts of every question, explain what you're doing. Draw pictures as appropriate to accompany your explanations. And remember: No matter how hard the problem seems or how lost you think you are, you can always do *something*, so do it.

Conceptual short answer: 20 points total

1) What are the retarded potentials, physically? What is the retarded time?

2) What is gauge freedom, and what is a gauge?

Relativity (30 points total)

In 1952 Einstein wrote: “What led me more or less directly to the special theory of relativity was the conviction that the electromotive force acting on a body in motion in a magnetic field was nothing else but an electric field.”

Suppose that in some inertial reference frame F there is magnetic field $\vec{B} = b\hat{\jmath}$, and no electric field. If a charge q moves with velocity $\vec{u} = u\hat{\imath}$ in F, it’ll feel some force $qub\hat{}$. Now let F' be the rest frame of q.

3a) Find E' and B', the fields in F'.

b) Find the force on q in F'.

c) How do the results of (a) & (b) relate to what Einstein wrote?

Radiation: 30 points (10 per part)

3) Let's suppose you have a charge q with mass m moving with some initial velocity $\vec{v} = v_0\hat{\imath}$ in a uniform magnetic field $\vec{B} = B_0\hat{k}$.

a) Describe and sketch the motion of the charge. Be specific about the direction(s) in which it'll move.

b) Find the magnitude of the power radiated by this charge, written only in terms of given quantities and fundamental constants. Then describe how that radiation will be distributed – in what directions will it be strongest, and in what directions will it be weakest?

c) The fact that this charge is radiating means that it's losing energy – it's slowing down. That the particle is slowing down suggests a force antiparallel to the particle's direction of motion. Deduce an analytic expression for this slowing force in terms of given quantities and constants. Don't strain yourself trying to figure out *where* this force comes from – simply infer that it exists and write down its form.

Note: This retarding force is sometimes called the radiation reaction force and tends to cause theoretical problems if you poke at it too closely, as you might in a grad E&M class.

Also note: If you don't think you did the first parts of this problem right, define some placeholder quantity X (or whatever) that would have come from those parts and use it to move forward.

One point

Sketch a picture of a bunny.

Physics 200: Fundamental Equations

Maxwell's Equations

Gauss's Law for Electric Fields: $\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \frac{Q_{encl}}{\epsilon_0} = 4\pi k Q_{encl}$
Gauss's Law for Magnetic Fields: $\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$
Electric Flux: $\boldsymbol{\Phi_E} = \int \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}$; Magnetic Flux: $\boldsymbol{\Phi_B} = \int \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}}$
Ampère/Maxwell: $\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$
Faraday's Law: $\varepsilon_{ind} = \oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\Phi_B}{dt}$

Fields, Forces and Energy

Electric Field: $d\vec{\boldsymbol{E}} = \frac{kdQ}{r^2}\hat{r} = \frac{kdQ}{r^3}\vec{r}$; $\vec{\boldsymbol{F}}_E = q\vec{\boldsymbol{E}}_{at\ q}$
Electric Potential (Voltage): $V_a - V_b = \int_a^b \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}}$; $E_x = -\frac{dV}{dx}$; $dV = \frac{kdQ}{r}$
Electrostatic Energy: $U_{of\ q} = qV$
Dielectrics: $\epsilon = \kappa_E \epsilon_0$
Magnetic Field: $d\vec{\boldsymbol{B}} = \frac{\mu_0 I d\vec{\boldsymbol{\ell}} \times \hat{\boldsymbol{r}}}{4\pi r^2} = \frac{\mu_0 I d\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{r}}}{4\pi r^3}$
Magnetic Force: $d\vec{\boldsymbol{F}} = I d\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{B}}$; $\vec{\boldsymbol{F}}_B = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$
Magnetic Dipole: $\vec{\boldsymbol{\mu}} = NI\vec{\boldsymbol{A}}$; $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$

Circuits

Resistors: $dR = \frac{\rho dL}{A}$; $R_{series} = \sum_i R_i$; $R_{parallel} = (\sum_i R_i^{-1})^{-1}$
Capacitors: $C = \frac{Q}{V}$; $U_C = \frac{1}{2}CV^2$; $C_{series} = (\sum_i C_i^{-1})^{-1}$; $C_{parallel} = \sum_i C_i$; $C = \kappa_E C_0$
Ohm's Law: $V = IR$
Current: $I = \frac{dQ}{dt} = n|q|v_d A$
Power: $P = IV$
Kirchoff's Laws: $\sum_{loop} V_i = 0$; $\sum I_{in} = \sum I_{out}$
RC & LR Circuits: Charging and Discharging equations take the form of $e^{-t/\tau}$ and $1 - e^{-t/\tau}$
$\boldsymbol{\tau}_{RC} = RC$ $\boldsymbol{\tau}_{LR} = \frac{L}{R}$
AC Circuits: $X_C = \frac{1}{\omega C}$; $V_C = IX_C$; $Z = \sqrt{R^2 + X_C^2}$; $V = IZ$; $V_{rms} = \frac{V_{peak}}{\sqrt{2}}$
Inductors: $\varepsilon_{ind} = -L\frac{dI}{dt}$; $L = \frac{N\Phi_{\text{B,1 turn}}}{I}$; $U_L = \frac{1}{2}LI^2$
Inductance: $M_{12} = \frac{N_2 \Phi_{\text{B,1 turn of 2}}}{I_{\text{in 1}}}$; $\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$

Electromagnetic Waves, Optics and Field Energy Density

Field Energy Density: $u_E = \frac{1}{2}\epsilon_0 E^2$; $u_B = \frac{1}{2\mu_0}B^2$
Momentum: $p = \frac{U}{c}$
Wave Properties: $v = \lambda f$; $k = \frac{2\pi}{\lambda}$; $\omega = 2\pi f$; $B = \frac{E}{c}$
Intensity: $I = c\frac{1}{2}\epsilon_0 E_m^2 = \frac{P_{avg}}{A}$
Reflection/Refraction: $c_1 = \frac{c}{n_1}$; $\theta_{in} = \theta_{out}$; Snell-Descartes Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Additional Information/Useful Constants

Common Electric Fields: $E_{inf\ sheet} = \frac{\sigma}{2\epsilon_0}$; $E_{inf\ line} = \frac{2k\lambda}{r}$; $E_{charged\ ring} = \frac{kQx}{(x^2+a^2)^{3/2}}$; $C_{parallel\ plate} = \frac{\epsilon_0 A}{d}$
Common Magnetic Fields: $B_{inf\ wire} = \frac{\mu_0 I}{2\pi r}$; $B_{solenoid} = \mu_0 nI$; $L_{solenoid} = \mu_0 n^2 A\ell$; $B_{current\ loop} = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}}$
Fundamental Charge: $e = 1.602 \times 10^{-19}$C; Electron Mass: $m_e = 9.109 \times 10^{-31}$kg
Proton Mass: $m_p = 1.673 \times 10^{-27}$kg
$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$; $\epsilon_0 = 8.854 \times 10^{-12}\frac{\text{F}}{\text{m}}$
$\mu_0 = 4\pi \times 10^{-7} \approx 12.566 \times 10^{-7}\frac{\text{Tm}}{\text{A}}$
$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3 \times 10^8 \frac{\text{m}}{\text{s}}$

Vector Derivatives

Cartesian Coordinates

$$d\boldsymbol{\ell} = \hat{\mathbf{i}}\,dx + \hat{\mathbf{j}}\,dy + \hat{\mathbf{k}}dz\,, \qquad dV = dx\,dy\,dz$$

$$\nabla f = \hat{\mathbf{i}}\frac{\partial f}{\partial x} + \hat{\mathbf{j}}\frac{\partial f}{\partial y} + \hat{\mathbf{k}}\frac{\partial f}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{\mathbf{i}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{j}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{k}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Cylindrical Coordinates

$$d\boldsymbol{\ell} = \hat{\mathbf{r}}dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{k}}dz\,, \qquad dV = r\,dr\,d\phi\,dz$$

$$\nabla f = \hat{\mathbf{r}}\frac{\partial f}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial f}{\partial \phi} + \hat{\mathbf{k}}\frac{\partial f}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{k}}\left[\frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) - \frac{1}{r}\frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates

$$d\boldsymbol{\ell} = \hat{\mathbf{r}}dr + \hat{\boldsymbol{\theta}}r\,d\theta + \hat{\boldsymbol{\phi}}r\sin\theta\,d\phi\,, \qquad dV = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$\nabla f = \hat{\mathbf{r}}\frac{\partial f}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\,A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{\hat{\mathbf{r}}}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\,A_\phi) - \frac{\partial A_\theta}{\partial \phi}\right] + \frac{\hat{\boldsymbol{\theta}}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)\right] + \frac{\hat{\boldsymbol{\phi}}}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Vector Formulas

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C} = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}) = (\mathbf{C}\times\mathbf{A})\cdot\mathbf{B} = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A})$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\,(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}\,(\mathbf{A}\cdot\mathbf{B})$$

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})\,(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})\,(\mathbf{B}\cdot\mathbf{C})$$

Derivatives of Sums

$$\nabla(f+g) = \nabla f + \nabla g$$

$$\nabla\cdot(\mathbf{A}+\mathbf{B}) = \nabla\cdot\mathbf{A} + \nabla\cdot\mathbf{B}$$

$$\nabla\times(\mathbf{A}+\mathbf{B}) = \nabla\times\mathbf{A} + \nabla\times\mathbf{B}$$

Derivatives of Products

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\,\mathbf{B} + (\mathbf{B}\cdot\nabla)\,\mathbf{A}$$

$$\nabla\cdot(f\mathbf{A}) = f\,(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$$

$$\nabla\times(f\mathbf{A}) = f\,(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$$

$$\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\,(\nabla\cdot\mathbf{B}) - \mathbf{B}\,(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\,\mathbf{A} - (\mathbf{A}\cdot\nabla)\,\mathbf{B}$$

Second Derivatives

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

$$\nabla\times(\nabla f) = 0$$

Integral Theorems

$$\int_V (\nabla\cdot\mathbf{A})\,dV = \oint_S \mathbf{A}\cdot\hat{\mathbf{n}}\,dS$$ Gauss's (divergence) Theorem

$$\int_S (\nabla\times\mathbf{A})\cdot\hat{\mathbf{n}}\,dS = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell}$$ Stokes's (curl) Theorem

$$\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f)\cdot d\boldsymbol{\ell} = f(\mathbf{b}) - f(\mathbf{a})$$

$$\int_V \left(f\nabla^2 g - g\nabla^2 f\right)dV = \oint_S (f\nabla g - g\nabla f)\cdot\hat{\mathbf{n}}\,dS$$ Green's Theorem

Chapter 5, 6, 7, 8 useful relationships

Separation of variables general solutions

Cartesian: $V(x, y, z)$ as combinations of $cos(kx) + sin(kx)$ or $cosh(kx) + sinh(kx)$
or $e^{kx} + e^{-kx}$

Spherical: $V(r, \theta) = \sum_{l=0}^{\infty}(A_l r^l + B_l r^{-l-1})P_l(cos\ (\theta))$

Cylindrical: $V(r, \varphi) = Aln(r) + B + \sum_{n=0}^{\infty}(A_n r^n + B_n r^{-n})(C_n \cos(n\varphi) + D_n \sin(n\varphi))$

***Legendre polynomials*: $P_0(x) = 1$; $P_1(x) = x$; $P_2(x) = \frac{3}{2}x^2 - 1$; $P_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$**

Legendre orthogonality relationship: $\int_{-1}^{1} P_n(x)P_l(x)\, dx = \delta_{nl}\frac{2}{2n+1}$

Polarization relationships

$\vec{p} = \alpha\vec{E}$ $\quad \vec{P} = \chi_e\varepsilon_0\vec{E}$ $\quad \varepsilon = \varepsilon_0(1 + \chi_e)$ $\quad \kappa = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_0}$

$\sigma_b = \hat{n} \cdot \vec{P}$ $\qquad \rho_b = -\vec{\nabla} \cdot \vec{P}$

$\vec{P} = n\vec{p}$ $\qquad \alpha = \frac{3\varepsilon_0}{n}\frac{\kappa-1}{\kappa+2}$

$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon\vec{E}$ $\qquad \vec{\nabla} \cdot \vec{D} = \rho_f$

Current and current densities

$I = \frac{dQ}{dt} = qn_L v$ $\qquad dI = \vec{J} \cdot \overrightarrow{dA}$ $\qquad \vec{J} = qn\vec{v}$ $\qquad \vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$

$\vec{J} = \sigma\vec{E}$ $\qquad dI = \vec{K} \cdot \widehat{e_\perp} dl$ $\qquad RC = \frac{\varepsilon}{\sigma}$

Magnetic vector potential

$\vec{B} = \nabla \times \vec{A}$ $\qquad \vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{x'})}{|\vec{x} - \vec{x'}|} d^3x$

Chapter 9, 10, 11 useful relationships

Magnetization relationships

$\vec{M} = \chi_m \vec{H} \quad \mu = \mu_0(1 + \chi_m)$

$\vec{J}_b = \nabla \times \vec{M} \qquad \vec{K}_b = \vec{M} \times \hat{n}$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu}$$

Forms of Faraday's Law

$$EMF = -\frac{d\Phi}{dt} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d}{dt} \int \vec{B} \cdot \overrightarrow{dA}$$

$$\oint \vec{E} \cdot \overrightarrow{dl} = -\int \overrightarrow{\frac{\partial B}{\partial t}} \cdot \overrightarrow{dA}$$

Momentum & energy

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} \qquad u_{em} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \qquad \frac{d\vec{P}}{dV} = \mu_0 \varepsilon_0 \vec{S}$$

Boundary conditions in matter

$D_{2,perp} - D_{1,perp} = \sigma_f \qquad B_{1,perp} = B_{2,perp}$

$\vec{E}_{1,parallel} = \vec{E}_{2,parallel} \qquad \vec{H}_{2,parallel} - \vec{H}_{1,parallel} = \overrightarrow{K_f} \times \hat{n}$

Potentials & Gauges

$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \nabla \times \vec{A}$ *Coulomb:* $\nabla \cdot \vec{A} = 0$

Transforms: $V \to V - \frac{\partial f}{\partial t} \qquad \vec{A} \to \vec{A} + \nabla f$ *Lorentz:* $\nabla \cdot \vec{A} = -\varepsilon_0 \mu_0 \frac{\partial V}{\partial t}$

Chapter 13, 14, 15 & optics useful relationships

Optics

Brewster's angle: $\tan\theta_B = \frac{n_2}{n_1}$ $T = \frac{I_{trans}}{I_{incident}}$ $R = \frac{I_{reflect}}{I_{incident}}$

Phase velocity $v_{ph} = \frac{\omega}{k}$ Group velocity $v_g = \frac{d\omega}{dk}$

Retarded potentials and radiation

Lorentz gauge potentials: $-\nabla^2 V + \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} = \frac{\rho}{\varepsilon_0}$ $-\nabla^2 \vec{A} + \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = \mu_0 \vec{J}$

Larmor formula: $P = \frac{1}{4\pi\varepsilon_0}\frac{2q^2a^2}{3c^3}$ Scattering cross-section: $\sigma = \frac{P_{avg}}{S_{inc}}$

Lienard-Wiechert: $V(\vec{x},t) = \frac{q}{4\pi\varepsilon_0 R}\frac{1}{1-\hat{n}\cdot\vec{\beta}}$ $\vec{A}(\vec{x},t) = \frac{q\vec{v}}{4\pi\varepsilon_0 c^2 R}\frac{1}{1-\hat{n}\cdot\vec{\beta}}$

Monopole radiation fields:

$$\vec{B} = -\frac{\mu_0 q}{4\pi c}\frac{\hat{R}\times\vec{a}}{R} \qquad \vec{E} = c\vec{B}\times\hat{R}$$

Chapter 12 relations

$$\Lambda^{\mu}_{\ \nu} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$$

$$E'_{\parallel} = E_{\parallel} \quad \text{and} \quad \mathbf{E}'_{\perp} = \gamma\left(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}_{\perp}\right),$$
$$B'_{\parallel} = B_{\parallel} \quad \text{and} \quad \mathbf{B}'_{\perp} = \gamma\left(\mathbf{B}_{\perp} - \mathbf{v} \times \mathbf{E}_{\perp}/c^2\right)$$

Four vectors

Position/time:

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Energy/momentum:

$$\begin{pmatrix} \gamma E/c \\ \gamma p_x \\ \gamma p_y \\ \gamma p_z \end{pmatrix}$$

Charge/current:

$$\begin{pmatrix} c\rho \\ J_x \\ J_y \\ J_z \end{pmatrix}$$

Potentials:

$$\begin{pmatrix} V/c \\ A_x \\ A_y \\ A_z \end{pmatrix}$$