PHGN 326 - Advanced Physics Lab II

LAB #2: γ-ray and X-ray Attenuation

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Abstract

The purpose of this lab was to measure the mass absorption coefficients of lead and aluminum. This was accomplished by first collecting radioactive emission data from two sources from behind lead and aluminum shields of varying thicknesses. This data was then used to determine the ratio of detected radiation with each thickness and kind of shield to radiation detected without any shield. From there the data was fit to the theoretical model of mass absorption and a value extracted. The calculated values were:

- Al
    - 31 keV - 0.000758 ± 0.000125 cm^2/mg
    - 662 keV - 0.0000623 ± 0.0000068 cm^2/mg
    - 1137 keV - 0.0004104 ± 0.0000605 cm^2/mg

- Pb
    - 31 keV - 0.001054 ± 0.000109 cm^2/mg
    - 662 keV - 0.0000789 ± 0.0000046 cm^2/mg
    - 1173 keV - 0.00005642 ± 0.0000115 cm^2/mg

Grade	Score	Available
Abstract and Cover Pg.		5
Fig. & Plt.		10
Data & Error Ana.		10
Writing		10
Total		35

1 Introduction

The main purpose of this lab was to calculate the mass absorption of lead and aluminum. Additionally, the use of multiple photo-peaks from multiple sources serves to investigate what materials are better at shielding against certain energy levels of photon radiation.

2 Apparatus

2.1 Layout

The lab's apparatus occupied only two distinct configurations during the course of all data collection, which varied only in the distances between the source and shield and shield and PMT. The actual distances are largely irrelevant. The setup is illustrated in Figure 1, wherein "MCA" refers to the Multichannel Analyzer which is the data collection apparatus for Maestro®
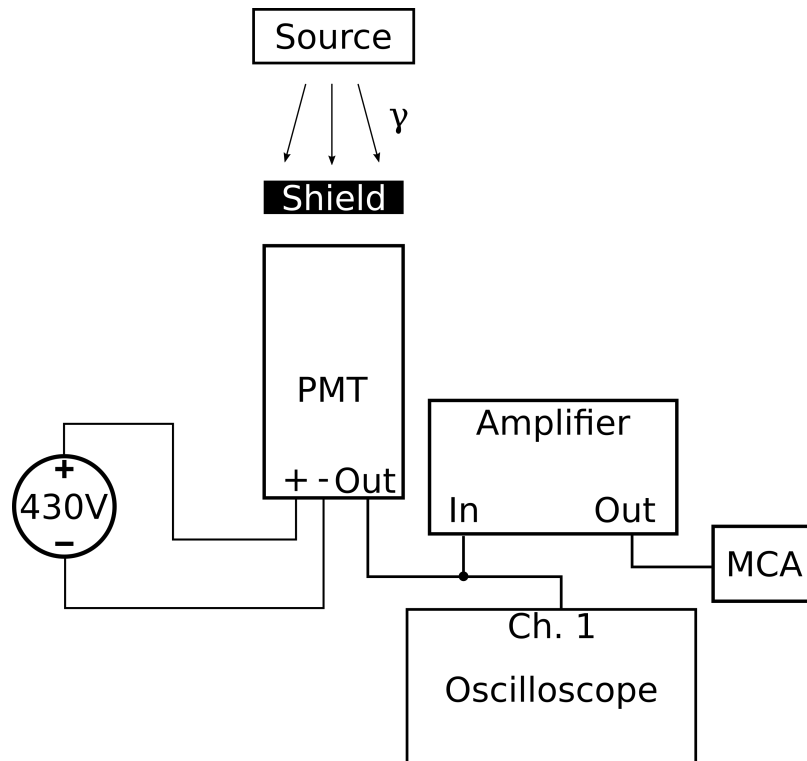

Fig. 1: Basic Illustration of Lab Setup

As shown, the lab utilized a Photomultiplier Tube (PMT) to count incident photons, and generating a signal which was captured both directly and after amplification by the Oscilloscope (or Maestro®software).

2.2 Sources and Shields

The sources used in this lab were as follows:

- ^{137}Cs

- ^{60}Co

Measurements later in the report will indicate which source was used.

The shields used were composed of either aluminum or lead, and were generally shaped like Figure 2. Shields that were of this shape had associated density thickness measurements, and calibration was done to account for the presence of the plastic ring[1].

Several measurements, however, were taken with cylinders of aluminum which were similar in diameter but many times thicker than these standard, circular shields.
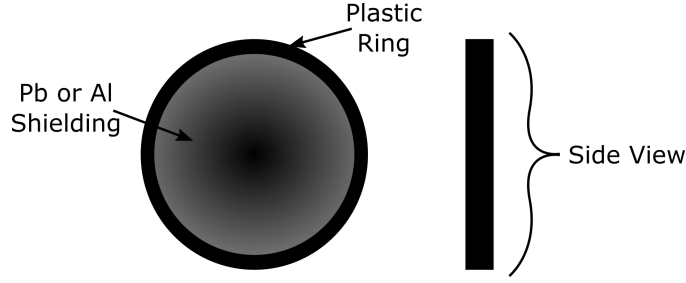
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[1] See section 3

Fig. 2: The Basic Shape of Most Shields

3 Data Collected

Data was collected for various aluminum and lead shields using both sources. All measurements were taken over a period of 60 s (real-time), and the amplifier gain was set to 39 for all ^{137}Cs measurements. Additionally, for measurements on this source, the source was 15.0±0.01 cm from the detector and the shields were mounted 5.2±0.01 cm from the detector. The HV remained at a constant voltage through all measurements.

   Table 1 shows the data collected for a ^{137}Cs source using aluminum shields of varying thicknesses. Data was collected for both the X-ray and γ-ray regimes. The net area of the respective peaks as well as associated uncertainty was also recorded as provided by Maestro®.

Density Thickness (mg/cm^2)	Net Counts (Counts/s)		Net Peak Area	
	γ-ray	X-ray	γ-ray	X-ray
0	787.81	88.48	46213±413	5190±155
6.9	799.47	86.75	46897±390	5098±154
82.2	794.08	82.88	46581±393	4862±153
171.7	787.82	74.91	46229±406	4396±154
491.6	767.86	49.66	45058±408	2914±143
960.3	751.86	33.70	44149±389	1979±136
960.3	742.25	37.63	43570±406	2209±133
1224.8	732.66	30.31	43022±411	1780±135
1604.4	721.46	24.25	42393±385	1425±130
2975±1.35	652.77	14.22	38344±409	835±129
12380±1.35	299.24	2.19	17661±457	129±107
15740±1.35	249.29	-1.51	14723±394	N/A

Tab. 1: Aluminum-Shielded ^{137}Cs Data

   Note that the first entry in Table 1 was made for a density of 0 mg/cm^2. This was done using an empty ring rather than no ring, so that the data would account for any shielding provided by the ring itself. The uncertainties in the measurements provided with each shield are considered negligible in the face of other sources of error. However, the last three entries in Table 1 were not the standard shields, but comparatively large cylinders of aluminum as discussed in Section 2.2. The thicknesses of these were measured using calipers, and the uncertainty shown arises from that measurement (densities were given, and considered much more precise than the caliper measurement). Given a density ρ_{Al} (2.702 g/cm^3[1]) and a measured thickness $t \pm \Delta t$, the density thickness d was calculated[1][2]:

$$d = \rho_{Al}(t \pm \Delta) = \rho_{Al}t + \rho_{Al}\Delta t \tag{1}$$

   Furthermore, the final entry in the table is of particular interest, specifically the X-ray data is incongruous with the rest of the data points. The negative Counts/s measurement indicates that the "peak" was indistinguishable from noise, meaning that at this point X-ray emission from the ^{137}Cs source could no longer reliably be detected. Naturally this implies that there is no real peak to have any area, hence the "N/A" in that field. Finally, notice that by mistake a measurement for a shield of density thickness 960.3 mg/cm^2 was taken twice; that is to say that this is not an error in the table, merely a repeated measurement.

Density Thickness (mg/cm²)	Net Counts (Counts/s)		Net Peak Area	
	γ-ray	X-ray	γ-ray	X-ray
985.3	724.42	7.75	42538±400	455±126
1811.3	675.48	9.66	39732±395	568±123
2650.7	624.79	11.25	36750±409	662±119
4830±5.72	579.88	6.16	30461±378	363±119
7820±5.72	442.74	12.78	26095±373	753±123

Tab. 2: Lead-Shielded ^{137}Cs Data

Table 2 shows data collected from the same ^{137}Cs source, this time from behind various lead shields. The last two entries were non-standard cylinders of lead whose density thicknesses were calculated almost identically to Eqn. 1 but this time using the given density of lead (ρ_{Pb} =11.434 g/cm³[1]).

For data collection from the ^{60}Co source, the first photo-peak - 1173 keV - was chosen, for the reason that radiation at this energy was typically more prevalent in the Maestro®output and therefore could be attenuated more before being lost in noise. Table 5 shows this data.

Density Thickness (mg/cm²)	Net Counts (Counts/s)	Net Peak Area
0	112.66	6674±221
168.4	114.47	6781±213
263.6	108.61	6434±221
482.9	113.66	6733±209
679.4	117.66	5970±66
1005.0	111.17	6588±209
1274.6	107.37	6335±211
1530.0	98.30	5825±225
2980±1.35	96.40	5950±201

Tab. 3: Aluminum-Shielded ^{60}Co Data

Notice that once again a calibration measurement for a 0-density thickness shield (i.e. an empty ring) has been made. Also note another non-standard cylinder used, with density thickness and associated uncertainty again calculated using Eqn. 1.

It bears mentioning that during data collection for this portion of the lab, a bizarre and inexplicable shift in gain occurred which skewed many measurements. Table 5 contains the results of the third set of measurements made using the same shields and source, and is radically different from both previous sets. To facilitate data collection, the amplifier gain was reduced to 32 for all ^{60}Co measurements.

Density Thickness (mg/cm²)	Net Counts (Counts/s)	Net Peak Area
847.8	116.23	6888±204
1959.6	98.80	5855±217
2790.6	99.93	6781±213
4469.4	84.70	5023±208
7829±5.72	75.04	4453±195

Tab. 4: Lead-Shielded ^{60}Co Data

Table 10 shows data collected from the ^{60}Co source from behind lead shields. Once again, a non-standard cylinder was used.

4 Data Analysis

4.1 Model

Lambert's Law gives the intensity of radiation passing through some absorber.[1]

$$I = I_0 e^{-x\mu} \tag{2}$$

where I_0 is the intensity before the absorber, x is the density thickness of the absorber, and μ is the material's mass absorption coefficient. Intensity is a measure of energy flux, and so if any energy must be carried by photons, a direct proportionality relationship can be made between number of photons incident on a detector and the intensity on the detector. If the intensity is set equal to the number of photons incident on the detector multiplied by a proportionality constant, this constant can immediately be reduced from Eqn. 2, leaving

$$n = n_0 e^{-x\mu}. \tag{3}$$

where n is the number of photons incident on the detector and n_0 is the number of photons before absorption. This can be used to model the collected data by applying an integration along the time interval of the window. Assuming a consistent, relatively homogeneous radiation from the source, both n and n_0 ought to be constant at any given time, and neither x nor μ evolve with time, so this integration amounts to multiplication by the size of the time window. However, this is a constant and can therefore be trivially reduced from the equation, so that it applies only a unitary change and gives the following equation to model the collected data:

$$r = r_0^{-x\mu} \tag{4}$$

where r is the measured count rate and r_0 is the count rate before absorption. Now consider the case of the empty ring. In this case $x = 0$ and $\mu = 0$ (considering air as empty space), so immediately it can be seen that r_0 should be the count rate measured for an empty ring.

For every collected data set, a measure of "Net Area" - or total amount of counts in a photo-peak throughout the measurement period - is provided along with associated uncertainty. This allows for determination of the uncertainty in the count rate measurements, because it is related to count rate by

$$r = \frac{c}{t} \tag{5}$$

where c is the total number of counts (net area) and t is the collection period duration. Error in the net area propagates to the count rate in the following manner[2]:

$$\Delta r = \sqrt{\left(\frac{\delta r}{\delta c}\right)^2 (\Delta c)^2} = \frac{\delta r}{\delta c}\Delta c = \frac{\Delta c}{t} \tag{6}$$

4.2   Aluminum

The data collected using aluminum shielding conforms much better to the model when the source is ^{137}Cs. This is made obvious by the contrast in curve fits between Figures 4 and 5 and Figure 3.
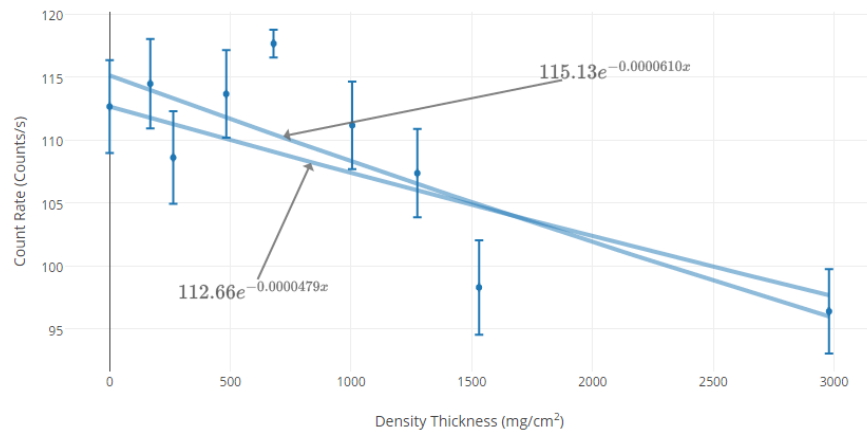

Fig. 3: Aluminum Count Rate using ^{60}Co Source

There are two primary reasons for this, the first is strange gain fluctuations in the measurement equipment, as previously stated in section 3, and the second reason has to do with the choice of photo-peak. Unfortunately, it just so happens that the 1332 keV γ-Ray emission's Compton Edge occurs in the midst of the 1173 keV photo-peak. This means that the region's count rate was affected not only by a decrease in incident photons actually at those

energies, but by a decrease in scattered photons from higher energies. Therefore, the model described in section 4.1 is insufficient to describe the behaviour of the system around the first photo-peak. To further complicate matters, the much higher energies of photons emitted from ^{60}Co photo-peaks as compared to those of ^{137}Cs mean that the photo-peaks are much less attenuated by light metals such as aluminum. The calculated values of the mass absorption coefficient of aluminum from these measurements are shown in 5

168.4	-0.0001439±0.000184
482.9	-0.00008243727049±0.00006346440001
679.4	-0.0003735967511±0.0000137606294
1005	0.0001149694563±0.00003117750577
1247.6	0.0004429626736±0.00002625263411
1530	0.001381665194±0.00002493367642
263.6	0.000332181488±0.0001286547436
2980	0.001611364254±0.00001166144584

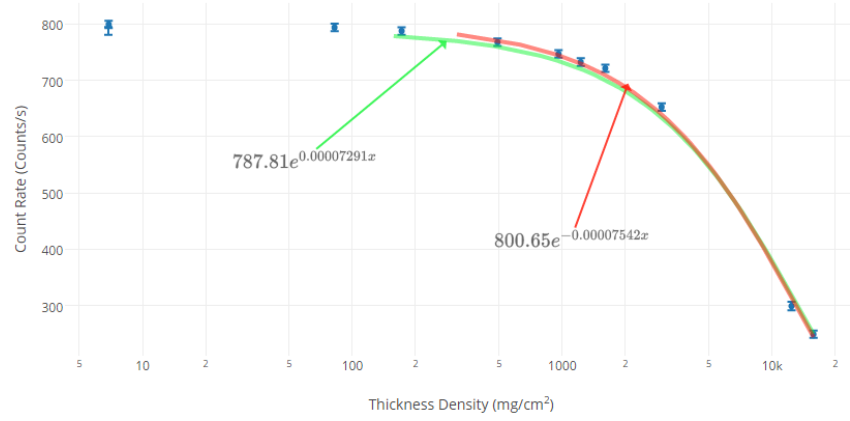Tab. 5: μ_{Al} Values Predicted from γ-Ray Data

Fig. 4: Aluminum Count Rate using ^{137}Cs Source - γ-Ray Regime

The γ-ray data from Table 1 and plotted in Figure 4[2] gives the following predicted values for μ_{Al}, calculated by solving Eqn. 4 for μ

Density Thickness (mg/cm^2)	μ_{Al} (cm^2/mg)
6.9	-0.0021±0.00117
82.2	-0.0000964± 0.0001003
171.7	-0.00000007393±0.00005005
491.6	0.00005218±0.00001801
960.3	0.00005531±0.00000924
1224.8	0.000059255±0.000007633
1604.4	0.000054837±0.000005546
2975±1.35	0.00006320±0.000003512
12380±1.35	0.000078192±0.000002057
15740±1.35	0.000073103±0.000001674

Tab. 6: μ_{Al} Values Predicted from γ-Ray Data

Error was calculated using

$$\Delta\mu = \sqrt{\left(\frac{\delta\mu}{\delta r}\right)\Delta r^2 + \left(\frac{\delta\mu}{\delta x}\right)\Delta x^2}$$

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[2] A logarithmic x-axis was used to prevent mangling of y-axis values; purely for aesthetic.

$$\frac{\delta\mu}{\delta r} = -\frac{1}{rx} \qquad\qquad \frac{\delta\mu}{\delta x} = \frac{\ln\left[\frac{r}{r_0}\right]}{x^2}$$

The negative values are obvious outliers, as that would imply aluminum is florescent, which it is not. Discounting those, the average value for μ_{Al} is 0.0000623±0.0000068 cm^2/mg. The fit curve values from Figure 4 both fit within 12% of calculated error for this average. The red curve was calculated by running a least-squares algorithm on the data set, allowing the curve to take the form $ae^{-\mu x}$, where a and μ are allowed to vary, which resulted in $a = 800.65$ and $\mu = 0.00007291$. The green curve was obtained by the same method, but with a fixed at $r_0 = 787.81$, which resulted in $\mu = 0.00007542$.
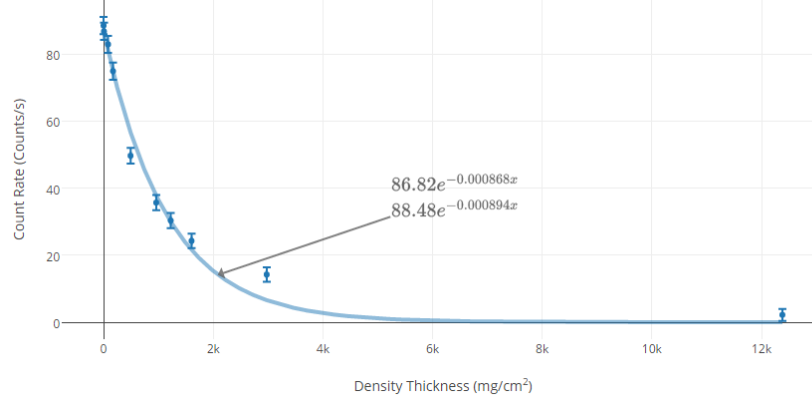

The figure shows a plot with y-axis "Count Rate (Counts/s)" and x-axis "Density Thickness (mg/cm²)" with labeled curves $86.82e^{-0.000868x}$ and $88.48e^{-0.000894x}$.

Fig. 5: Aluminum Count Rate using ^{137}Cs Source - X-ray Regime

The accepted value for the mass absorption coefficient for photons at 600keV passing through aluminum is 0.00007802 cm^2/mg[4], which is within 25.3% of the calculated value, and within 7.01% of both fit curve values. Not too shabby.

The data plotted in Figure 5 tells a very different story, resulting in the following values for μ_{Al}:

Density Thickness (mg/cm^2)	μ_{Al} (cm^2/mg)
6.9	0.0029±0.0043
82.2	0.000795± 0.000374
171.7	0.0009697±0.0001998
491.6	0.001175±0.0000975
960.3	0.0009460±0.000066
1224.8	0.00087467±0.0000606
1604.4	0.00080676±0.00005577
2975±1.35	0.0006145±0.0000508
12380±1.35	0.00029878±0.00006565

Tab. 7: μ_{Al} Values Predicted from X-Ray Data

Disregarding the first and fourth values, which are an order of magnitude off from the others, this gives an average $\mu_{Al} = 0.000758 \pm 0.000125$ cm^2/mg. The fit curve values are similar, 0.000868 when a is allowed to vary, 0.000894 when it is held fixed.

The accepted value for aluminum's mass absorption coefficient for photons in the neighborhood of 30 keV, the X-ray regime[3], is about 0.001128 cm^2/mg, which is within a pretty dismal 48.9% of the calculated value and within 30% of both fit curve values.

4.3   Lead

For the limited sample size, Figure 6 does fit exponential curves similar to those found for γ-ray radiation with aluminum shields.
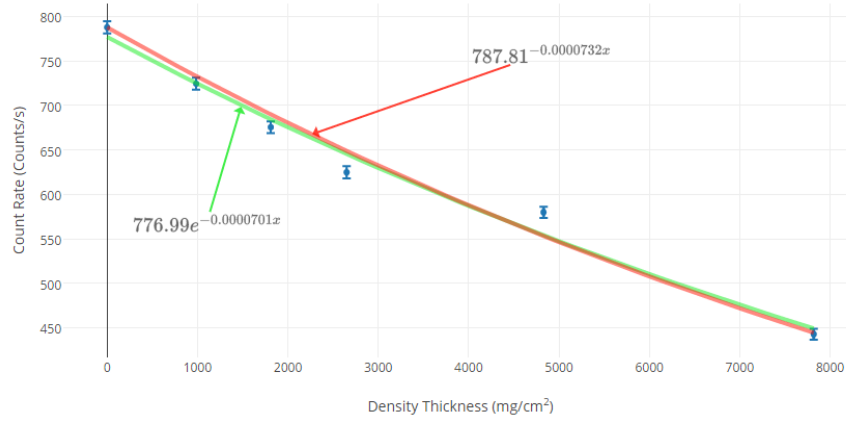
Fig. 6: Lead Count Rate using ^{137}Cs Source - γ-Ray Regime

The μ_{Pb} values calculated from this data are shown in Table 8, and the average value is $\mu_{\text{Pb}} = 0.0000789 \pm 0.0000046$ cm^2/mg. The accepted value for the mass absorption of lead is 0.00007102 cm^2/mg at 1000 keV[4]; just about 10% under the calculated value.

Density Thickness (mg/cm^2)	μ_{Pb} (cm^2/mg)
985.3	0.00008514±0.00000934
1811.3	0.000084930±0.00000538
2650.7	0.000087464±0.000004116
4830	0.0000634±0.0000023
7820	0.0000737±0.0000018

Tab. 8: μ_{Pb} Values Calculated from ^{137}Cs γ-Ray radiation

The data shown in Figure 7 fits the model pretty terribly. The best This is most likely due to the data collected being at density thicknesses far greater than the curve in the exponential, the part where it tapers off to nearly a straight line. An offset factor could make this fit better - and bring the mass absorption coefficient closer to the accepted value - but there's no reason I can think of for that to appear in the model. The only things that could reasonably shift it upward is from Compton scattering of higher-energy photons (which is certainly occurring) or from some other source of X-rays at that energy. Other samples of Cesium in the room should never be significant disruptions, but the Compton Angle is almost zero for photons at the energy of Cesium's γ-ray photo-peak, meaning the photon would need to go right through the electron. Normally, a direct collision would result in absorption though, so that's not really a good explanation either. Most likely, it's merely Compton scattering from random background radiation.
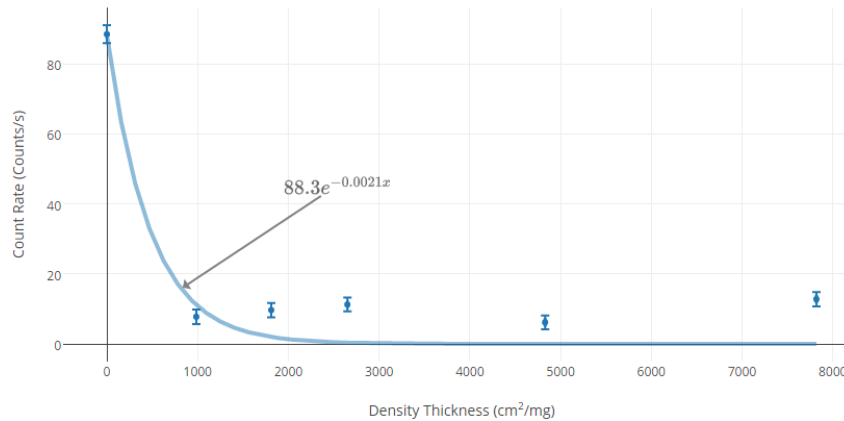

Fig. 7: Lead Count Rate using ^{137}Cs Source - X-Ray Regime

The calculated values for μ_{Pb} at this energy are shown in Table 9, and their average is 0.001054 ± 0.000109. The accepted value is 0.03032 cm^2/mg, which is way outside error at 188% of the calculated value.

Density Thickness (mg/cm^2)	μ_{Pb} (cm^2/mg)
6.9	0.0029±0.0043
82.2	0.000795±0.00037
171.7	0.0009697±0.0001998
491.6	0.001175±0.0000975
960.3	0.0009460±0.0000657
1224.8	0.0008747±0.0000606
1604.4	0.0008068±0.0000558
2975	0.0006145±0.00005082
12390	0.0002988±0.0000657

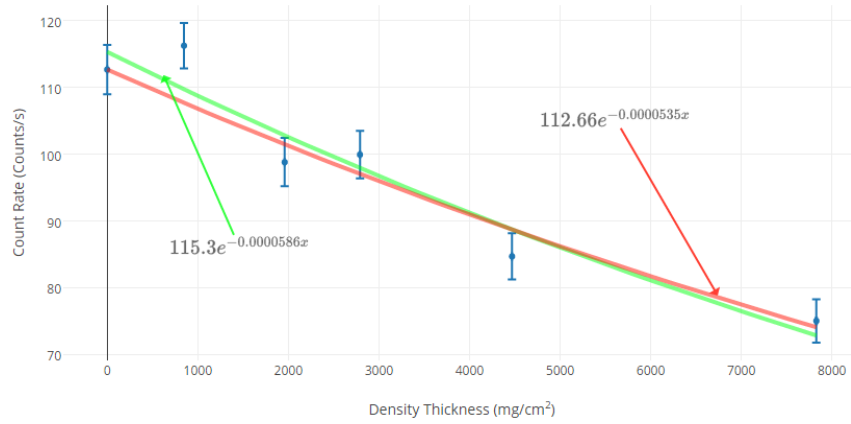Tab. 9: μ_{Pb} Values Calculated from ^{137}Cs X-Ray radiation

Fig. 8: Lead Count Rate using ^{60}Co Source

Figure 8 shows data collected using the 1137 keV photo-peak of ^{60}Co radiation, and the fit curves have μ_{Pb} values of 0.0000535 cm^2/mg when r_0 is fixed to the count rate measured with an empty ring, and 0.0000586 cm^2/mg when it isn't. The calculated values for each density thickness are given in Table 10 and their average is 0.00005642 ± 0.0000115 cm^2/mg.

Density Thickness (mg/cm^2)	μ_{Pb} (cm^2/mg)
847.8	-0.00003680±0.0000345
1959.6	0.00006699±0.00001868
2790.6	0.00004297±0.00001273
4469.4	0.00006382±0.000009158
7829	0.00005190±0.00000553

Tab. 10: μ_{Pb} Values Calculated from ^{60}Co gamma radiation

The accepted value for the mass absorption coefficient of lead for photons at 1250 keV is 0.00005876 cm^2/mg[4], and the calculated value is actually really close, within 4%.

5   Conclusion

According to the data, lead isn't actually a much better shield of radiation than aluminum, despite its reputation. In particular, it's way better than aluminum for X-rays, but the gap quickly closes in the γ region, leading to aluminum actually being better. Further measurements would be needed, but my complete guess is that a lot of the shielding comes from oxidation.

References

[1] Dr. Ed Cecil, Professor for PHGN-326. Instructions for Experiments: Experiment 2: γ-ray and X-ray Attenuation. Colorado School of Mines, Physics Department, 2017. `http://hyperphysics.phy-astr.gsu.edu/HBASE/forces/isq.html`

[2] John R. Taylor, Professor at the University of Colorado Department of Physics."An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements" Second Edition, 1982.

[3] S.Y.F. Chu, L.P. Ekstrm, and R.B. Firestone1. "The Lund/LBNL Nuclear Data Search", Last Modification:February 1999, `http://nucleardata.nuclear.lu.se/nucleardata/toi/perchart.htm`

[4] NIST, "X-Ray Mass Attenuation Coefficients", `http://physics.nist.gov/PhysRefData/XrayMassCoef/tab3.html`

A   Supplemental Figures