Function Definitions

I'm setting λ=1, w_0=1, k=1, ω=1 and E[n,m]=1, since the shape of the oscillations is really what's important here.

Also, I'm assuming that r^2=x^2+y^2 since idk what else it could be.

I know the actual prompt said "Cross Section" which, strictly speaking, these 3D Plots are not, but I think they better illustrate what's going on and I've always found heat maps difficult to interpret.

Furthermore: I've decided that for all "cross-sections", I'll take a snapshot in time at t=0 rather than mess with any time averages.

Also, some of the x-y "cross-sections" had color issues near z=z_r so I took to using z=10z_r instead- (which wound up plotting much nicer anyway).

In case it's not obvious, the first plot in each section is the x-y "cross

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w[z_]:= Sqrt[1+(z/Pi)^2]
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R[z_]:=z(1+(Pi/z)^2)
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TEM[x_,y_,z_,t_,n_,m_]:= HermiteH[n,Sqrt[2]*x/w[z]] \
HermiteH[m,Sqrt[2]y/w[z]] 1/w[z] Exp[-(x^2+y^2)/(w[z]^2)]\
 Exp[-ⅈ(x^2+y^2)/(2 R[z])] Exp[ⅈ(1+n+m)ArcTan[z]] Exp[ⅈ(z-t)]
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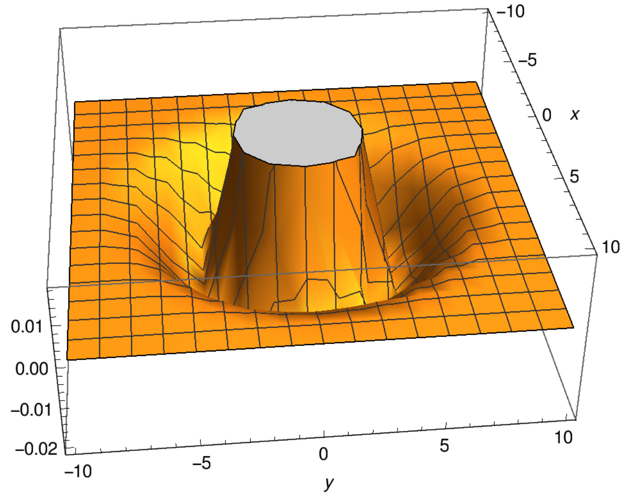
Mode : n = 0, m = 0

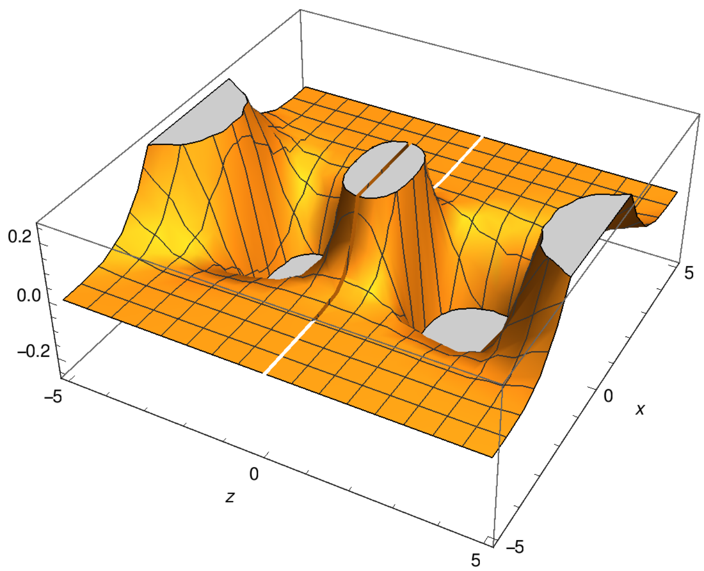
This is the field magnitude in the x-y plane at around z = 10 z_r. It looks pretty basic, a simple circular wave that shows high intensity focused in a small area. Very beamlike.

Out[44]=



It’s a bit difficult to tell this far in (but if you zoom out it becomes completely unintelligible), but the oscillations in z gradually grow wider as you move further from x=0, showing that the tightest focus for the beam occurs near the center of the beam, which makes sense. Oscillations along the x axis are of a simple trigonometric (specifically cosine) form.

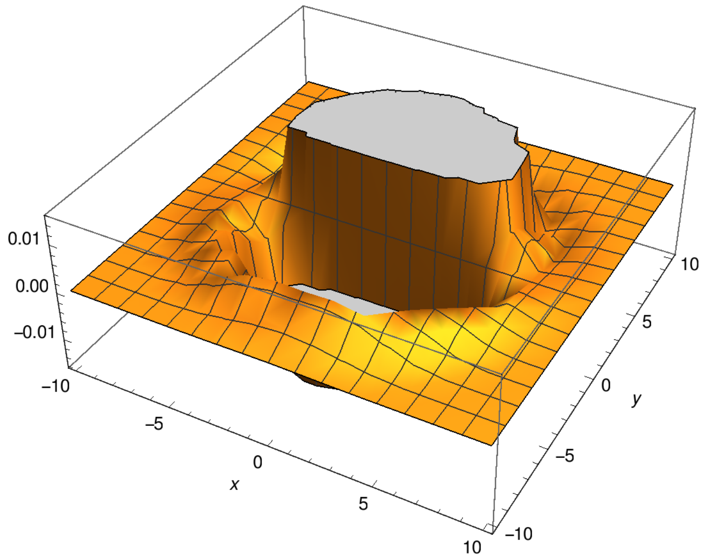
Out[61]=



Mode : n = 0, m = 1

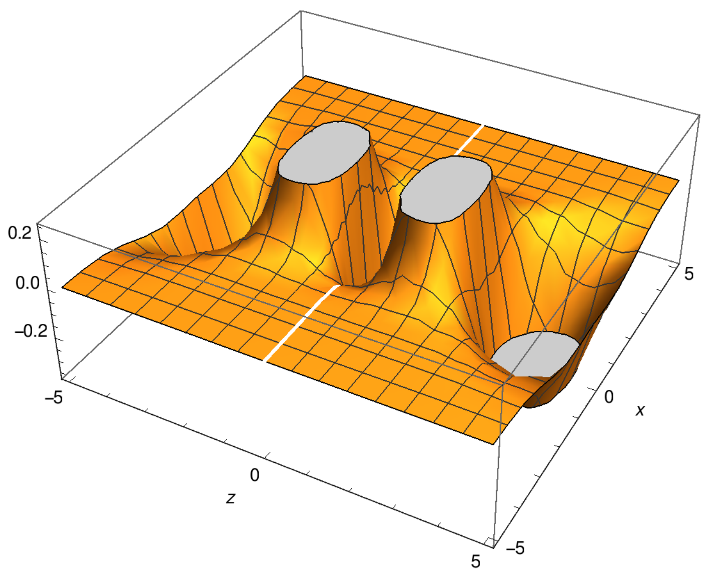
Like the TEM_ 00 mode, this exhibits a concentration of intensity within a narrow circle of the x - y plane. However, there' s an oscillation within this beam itself, meaning that there' s a node along the x - axis at y = 0, where there' s zero intensity even within the beam, which is odd.

Out[45]=



This exhibits almost exactly the same behavior as the TEM_ 00 mode, exactly opposite, in fact. It looks like maybe what happened is the modes introduce a phase shift of π to the oscillations in the x direction.

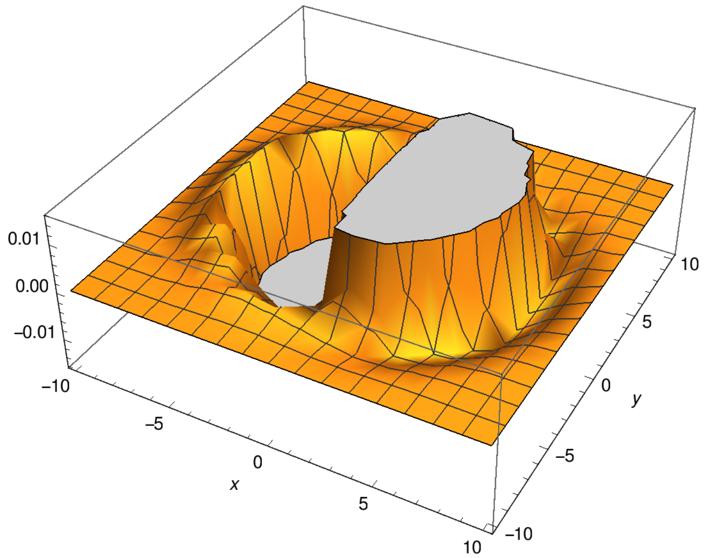
Out[59]=



Mode : n = 1, m = 0

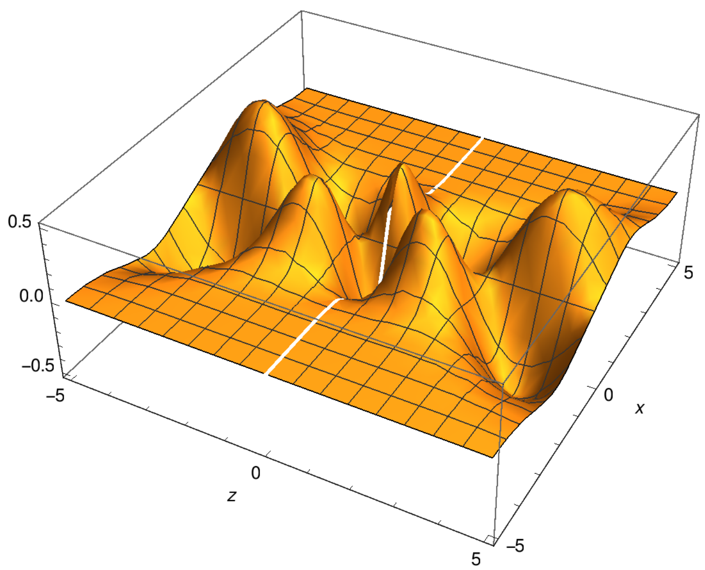
In contrast to the n = 0, m = 1 mode, this beam oscillates such that a node forms as a line along the y-axis at x=0. Come to think of it, the Hermite polynomial corresponding to n deals with x and the one corresponding to m deals with y, so it's starting to seem reasonable that there's an oscillation in x depending on n and likewise for y and m.

Out[42]=



Based on my previous observations, I'd say this is probably highly similar to what we' d observe for a y-z cross-section of n = 0, m = 1. Here we have a new oscillation along the x axis, owing to the non-trivial Hermite Polynomial.

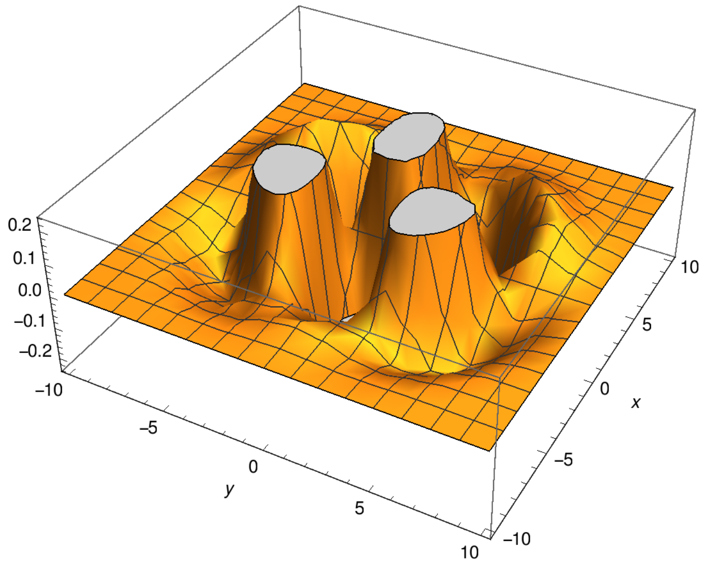
Out[57]=



Mode : n = 1, m = 2

Consistent with my theory, this shows one oscillation on the x-axis, and two on the y-axis. Since looking at the last mode, I've done what I should've done during part b of this problem and actually googled what a "Hermite Polynomial" is, and I'm pleased to report that my guess was on right track, since the Hermite Polynomials give rise to the eigenstates of a quantum harmonic oscillator for HermiteH[n,x] where n is the number of oscillations along x.

Out[54]=



Our Hermite Polynomials dictate one oscillation on x, and in fact that occurs, and we can see the usual beam spreading as we diverge from z = 0, so I feel pretty good about my handle on these.

Out[56]=