

$$H = \sum_{n=-\infty}^{+\infty} \frac{-E_J}{2} (|n\rangle \langle n-1| + |n-1\rangle \langle n|) + 4E_C(n - N_g)^2 |n\rangle \langle n|$$

$$H = -4E_C \frac{d^2}{d\varphi^2} - E_J \cos \varphi$$

$$H = -4E_C \frac{d^2}{d\varphi^2} - E_J \cos \varphi + \frac{1}{2}E_L \left(\varphi - \frac{2\pi\phi}{\phi_0} \right)^2$$

$$H = -4E_C \frac{d^2}{d\varphi^2} - E_J \cos \left(\varphi + \frac{2\pi\phi}{\phi_0} \right) + \frac{1}{2}E_L \varphi^2$$

$$H = -4E_C \frac{d^2}{d\varphi^2} - E_J \cos \left(\varphi + \frac{2\pi\phi}{\phi_0} \right) + \frac{1}{2}E_L \varphi^2$$

$$\frac{dH}{dE_C} = -4 \frac{d}{d\varphi^2}$$

$$\frac{dH}{dE_J} = -\cos\left(\varphi + \frac{2\pi\phi}{\phi_0}\right)$$

$$\frac{dH}{dE_L} = \frac{\varphi^2}{2}$$