Note Title 2/10/2009

Harmonic oscillator:

$$\psi(n) = \langle n | \psi \rangle$$

Compare: $\psi_n(x) = \mathcal{N}_n H_n(x/\lambda_{osc})\, e^{-\frac{1}{2}\left(\frac{x}{\lambda_{osc}}\right)^2}$
$= \langle x | \psi_n \rangle$

completeness of

- charge basis

$$\mathbb{1} = \sum_{n=-\infty}^{+\infty} |n\rangle\langle n|$$

- phase basis

$$\mathbb{1} = \int_0^{2\pi} d\varphi\, |\varphi\rangle\langle\varphi|$$

completeness of the position basis

$$\mathbb{1} = \int_{-\infty}^{+\infty} dx\, |x\rangle\langle x|$$

$$\mathbb{1} = \int dp\, |p\rangle\langle p|$$

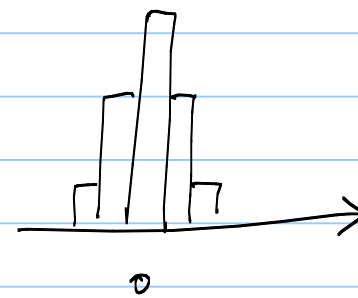
$$\langle x | p \rangle = \frac{1}{\sqrt{\nu}} e^{ipx}$$

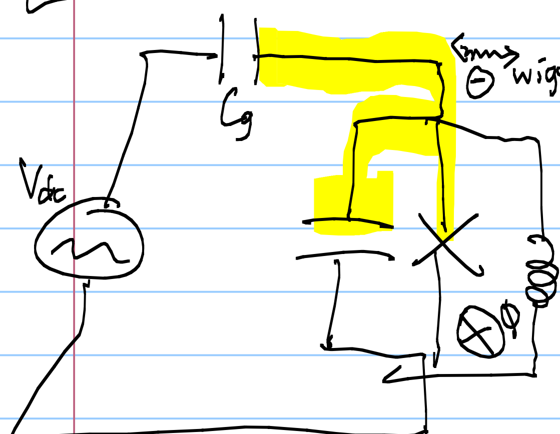
$$\psi(\varphi) = \langle \varphi | \psi \rangle = \langle \varphi | \underbrace{\left(\sum_{n=-\infty}^{+\infty} |n\rangle\langle n| \right)}_{\mathbb{1}} | \psi \rangle = \sum_{n=-\infty}^{+\infty} \underbrace{\underbrace{\langle \varphi | n \rangle}_{\frac{1}{\sqrt{2\pi}} e^{i\varphi n}}}_{f(\varphi)} \underbrace{\langle n | \psi \rangle}_{\psi(n)}$$

To do

* Plot wavefunctions for CPB
* Numerics for the CPBL

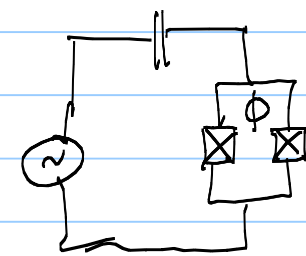
$$-i\frac{d}{d\varphi} f(\varphi) = n f(\varphi)$$





⊖ wiggling charged impurities → charge noise

CPB with 2junctions



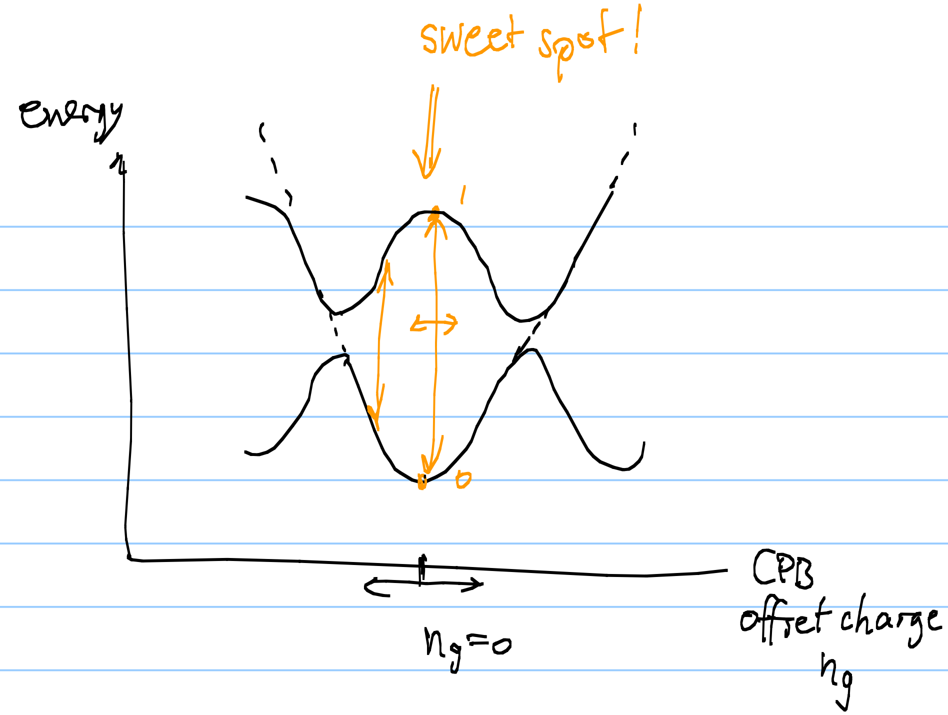
offset charge

$$n_g = \frac{C_g}{2e}\cdot V_{dc}$$

$$H = 4E_C(n-n_g)^2 - E_J\cos\varphi + \frac{1}{2}E_L\left(\varphi - \frac{2\pi\Phi}{\Phi_0}\right)^2$$

skipping n_g

$$= -4E_C\frac{d^2}{d\varphi^2} - E_J\cos\varphi + \frac{1}{2}E_L\left(\varphi - \frac{2\pi\Phi}{\Phi_0}\right)^2$$



Variations in transition freq. (E_{01}) result in "dephasing"

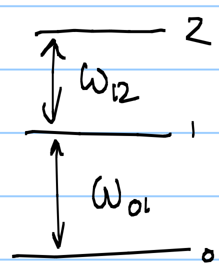
$$H|\psi\rangle = E|\psi\rangle$$

$$i\hbar\frac{d}{dt}|\psi\rangle = H|\psi\rangle$$

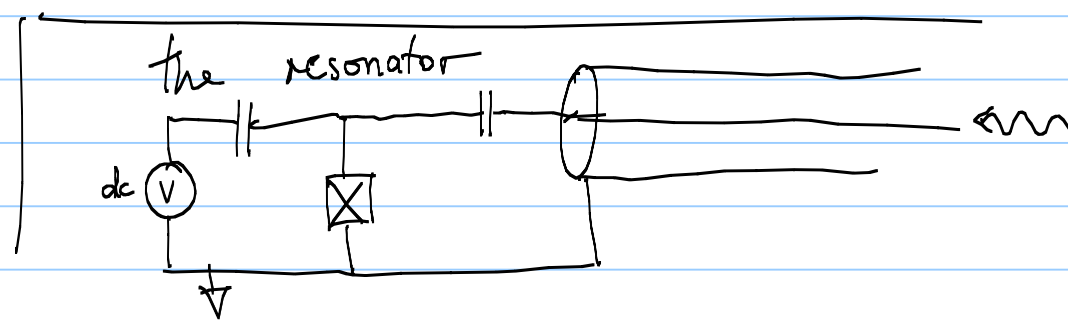
$$|\psi(t)\rangle = e^{-iEt/\hbar}|\psi\rangle$$

this phase factor becomes unpredictable

Anharmonicity



$$\alpha = \omega_{12} - \omega_{01}$$



$$H = 4E_C(n-n_g)^2 - E_J\cos\varphi \quad + \frac{1}{2}E_L\left(\varphi - \frac{2\pi\Phi}{\Phi_0}\right)^2$$

skipping n_g

$$= -4E_C\frac{d^2}{d\varphi^2} - E_J\cos\varphi + \frac{1}{2}E_L\left(\varphi - \frac{2\pi\Phi}{\Phi_0}\right)^2$$

Step 0 : Shift φ: $\varphi' = \varphi - \frac{2\pi\Phi}{\Phi_0}$

call it φ...

Step 1 : Harmonic oscillator

$$H_0 = -4E_C\frac{d^2}{d\varphi^2} + \frac{1}{2}E_L\left(\varphi - \cancel{\frac{2\pi\Phi}{\Phi_0}}\right)^2$$

for later: $V = -E_J\cos\left(\varphi + \frac{2\pi\Phi}{\Phi_0}\right)$

$a \sim ?\left(\frac{d}{d\varphi} + \varphi\right)$

diagonalize H_0

$\hookrightarrow \quad H_0 = \hbar\omega_0(a^\dagger a + \frac{1}{2})$

you should get: $\hbar\omega_0 = \sqrt{8E_LE_C}$, I hope!

In the h.o. basis:

$$H_0 = \hbar\omega_0 \begin{pmatrix} \frac{1}{2} & & & & 0 \\ & \frac{3}{2} & & & \\ & & \frac{5}{2} & & \\ & & & \ddots & \\ 0 & & & & \end{pmatrix} \qquad H_0 = \hbar\omega_0 \left(a^\dagger a + \frac{1}{2}\right)$$

<u>Step 2</u>: What to do with V?

Let's call the eigenstates of the harm. osc. $|m\rangle$

so $H_0 |m\rangle = E_m |m\rangle = \hbar\omega_0 \left(m + \frac{1}{2}\right) |m\rangle$

$$(H_0)_{mm'} = \langle m | H_0 | m' \rangle = \delta_{mm'} \, \hbar\omega_0 \left(m + \frac{1}{2}\right)$$

$$(H)_{mm'} = (H_0 + V)_{mm'} = \underbrace{\langle m | H_0 | m' \rangle}_{\text{this we know!}} + \langle m | V | m' \rangle$$

$$\langle m|V|m'\rangle = -E_J \int_{-\infty}^{+\infty} d\varphi\, \mathcal{N}_m \mathcal{N}_{m'} \cos\left(\varphi + \frac{2\pi\Phi}{\Phi_0}\right) H_m(\varphi/\varphi_{osc}) H_{m'}(\varphi/\varphi_{osc})$$
$$\times \exp\left[-\left(\frac{\varphi}{\varphi_{osc}}\right)^2\right]$$

like m, m'

$\varphi_0 = \varphi_{osc} = (8E_C/E_L)^{1/4}$

generalized Laguerre polynomial

$$\langle\, i \,|\, \cos\varphi \,|\, i + 2j \,\rangle \tag{A2}$$
$$= (-2)^m \left[\frac{n!}{(n+2m)!}\right]^{1/2} \varphi_0^{2m} e^{-\varphi_0^2/4} \mathrm{L}_n^{2m}(\varphi_0^2/2)$$
$$\langle\, i \,|\, \sin\varphi \,|\, i + 2j + 1 \,\rangle \tag{A3}$$
$$= (-2)^m \left[\frac{n!}{2(n+2m+1)!}\right]^{1/2} \varphi_0^{2m+1} e^{-\varphi_0^2/4} \mathrm{L}_n^{2m+1}(\varphi_0^2/2),$$