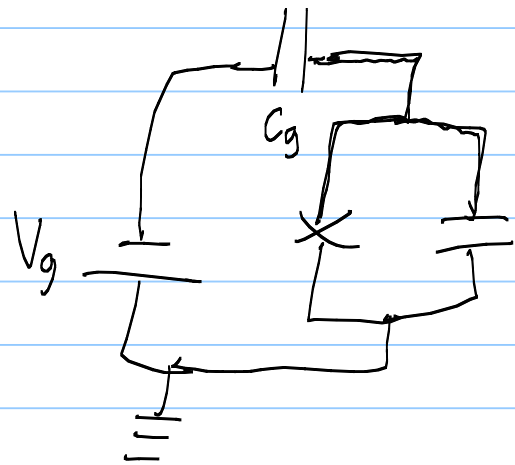
The spectrum of the CPB

Note Title 11/18/2008



$$n_g = C_g V_g / 2e$$

$$H = 4E_c(n-n_g)^2 - E_J \cos\varphi$$

Periodicity of spectrum wrt n_g : $E_m(n_g) = E_m(n_g+1)$

Use phase basis: $H = 4E_c\left(i\frac{d}{d\varphi} - n_g\right)^2 - E_J \cos\varphi$

Hmmm...

Use charge basis:

$$H = 4E_c(n-n_g)^2 - \frac{E_J}{2}\sum_{n=-\infty}^{+\infty}\left[|n\rangle\langle n-1| + |n-1\rangle\langle n|\right]$$

$$n_g \to n_g + 1$$

$$4E_c(n-1-n_g)^2 - \frac{E_J}{2}\sum_{n=-\infty}^{+\infty}\left[|n\rangle\langle n-1| + |n-1\rangle\langle n|\right]$$

From here we see that spectrum is the same!

$\hookrightarrow$ periodicity in general

(1) The charge regime, $E_J \lesssim E_c$

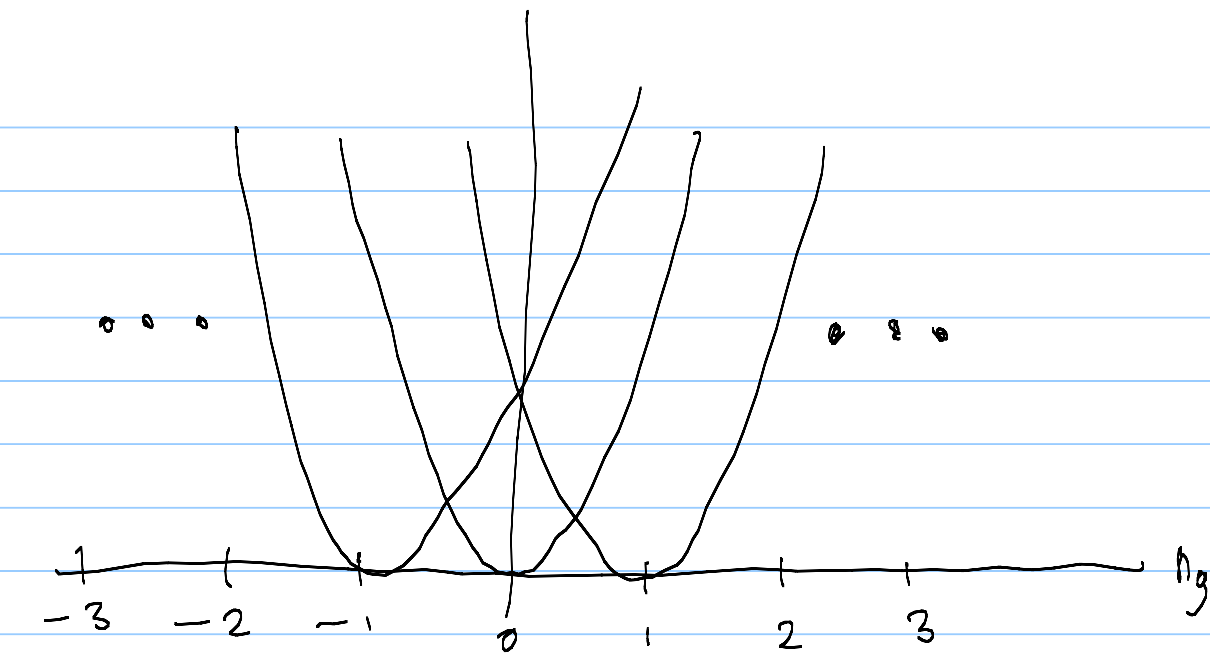
Let's rewrite H in the charge basis:

$$H = 4E_c(n-n_g)^2 - \frac{E_J}{2}\sum_{n=-\infty}^{+\infty}\left[|n\rangle\langle n-1| + |n-1\rangle\langle n|\right]$$

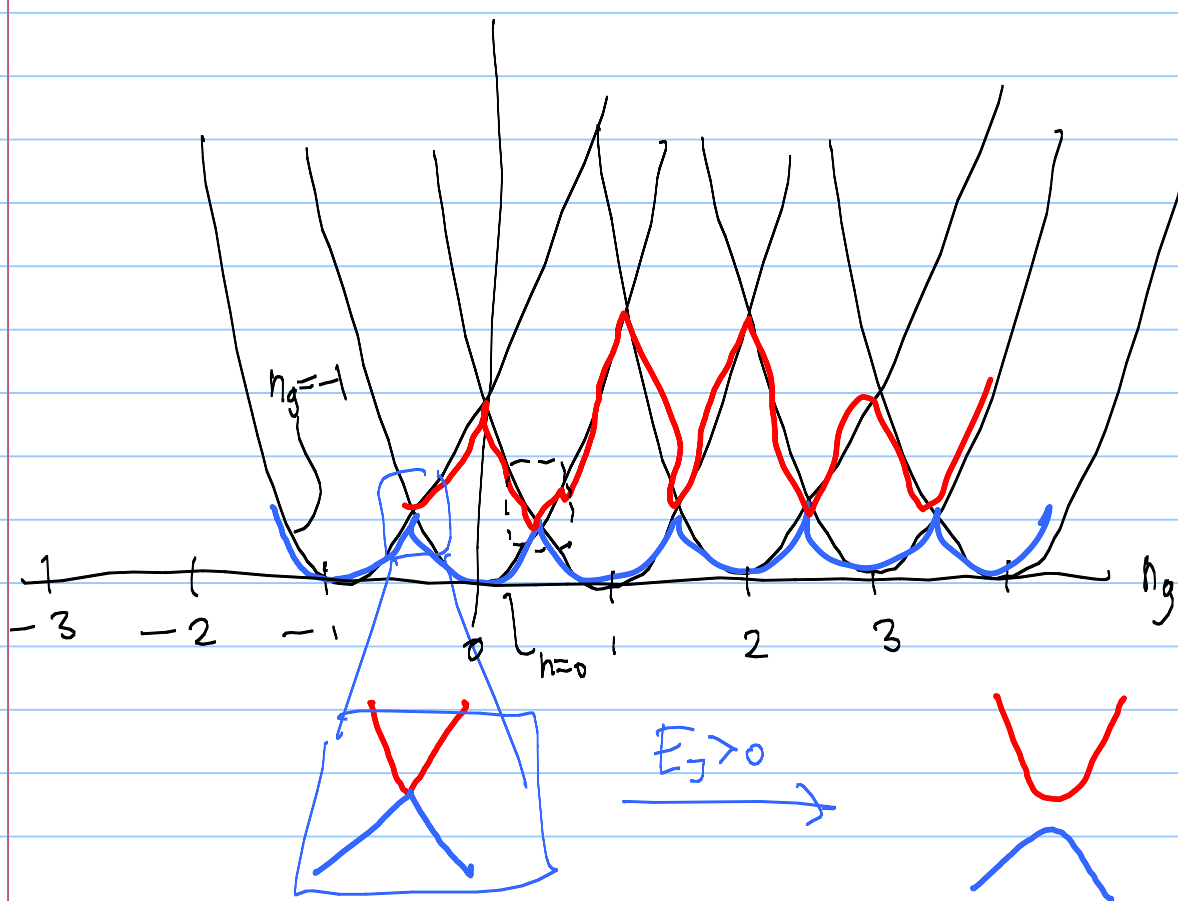
deep in charging regime: $E_J \ll E_c$

$\hookrightarrow E_J/E_c \ll 1$

$$H/E_c = 4(n-n_g)^2 \underbrace{- \frac{1}{2}\frac{E_J}{E_c}\sum_{n=-\infty}^{+\infty}\left[|n\rangle\langle n-1| + |n-1\rangle\langle n|\right]}_{\text{perturbation}}$$



. . .
. . .
n_g
−3
−2
−1
0
1
2
3



$n_g = -1$
n_g
−3
−2
−1
0
$n=0$
1
2
3
$E_J > 0$

For $n=0$ and $n=1$:

$$\frac{H_{eff}}{E_c} = \begin{pmatrix} 4n_g^2 & -\frac{E_J}{2E_c} \\ -\frac{E_J}{2E_c} & 4(1-n_g)^2 \end{pmatrix}$$

Diagonalize:

$$0 = \det\left(\frac{H_{eff}}{E_c} - E_m \mathbb{1}\right) = \det\begin{pmatrix} 4n_g^2 - E_m & -\frac{E_J}{2E_c} \\ -\frac{E_J}{2E} & 4(1-n_g)^2 - E_m \end{pmatrix}$$

at $n_g = 1/2$: $$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

(2) Transmon regime $E_J > E_C$

$$H = 4E_C\left(i\frac{d}{d\varphi} + n_g\right)^2 - E_J \cos\varphi$$

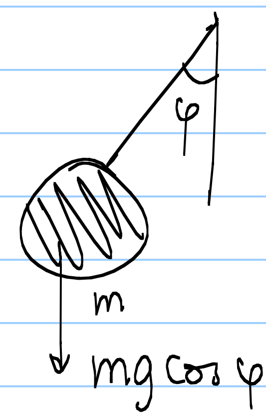
b.c. $\psi(0) = \psi(2\pi)$

$$-4E_C\frac{d^2}{d\varphi^2} - E_J\cos\varphi$$

▷ think about this as of a rotor

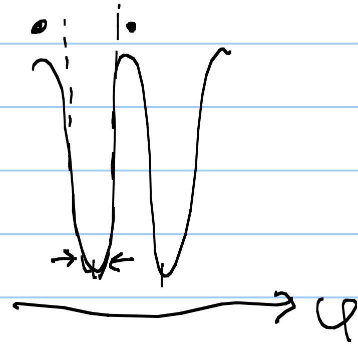
For $E_J \gg E_C$
only small φ are important!

$$\cos\varphi \approx 1 - \frac{1}{2}\varphi^2 + \dots$$



$$H \simeq 4E_C\left(i\frac{d}{d\varphi} + n_g\right)^2 - E_J + \frac{1}{2}E_J\varphi^2$$

b.c. $\int_{-\infty}^{+\infty} d\varphi\, |\psi(\varphi)|^2 = 1$



±?

$$\overline{\psi}(\varphi) = \psi(\varphi)\, e^{i n_g \varphi}$$

$$H\psi(\varphi) = E\psi(\varphi)$$

$$\left[4E_C\left(i\frac{d}{d\varphi}+n_g\right)^2 - E_J\cos\varphi\right] \underbrace{e^{i n_g\varphi}\,\overline{\psi}(\varphi)} = E\,\overline{\psi}(\varphi)\,e^{i n_g\varphi}$$

⋮ for Jack to check

$$\left[-4E_C\frac{d^2}{d\varphi^2} - E_J\cos\varphi\right]\overline{\psi}(\varphi) = E\,\overline{\psi}(\varphi)$$

$$\underbrace{\overline{\psi}(0) = \psi(0) = \psi(2\pi) = \overline{\psi}(2\pi)\, e^{i n_g 2\pi}}_{\text{quasiperiodic b.c.}}$$