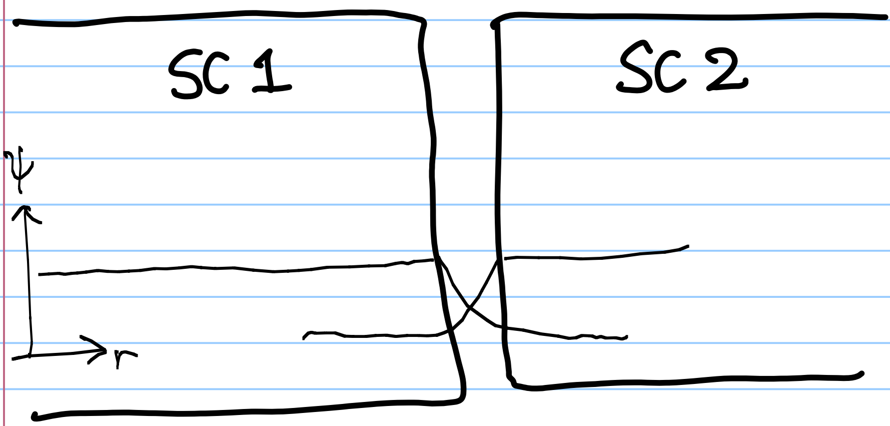
Josephson effect

Note Title 3/31/2009



$\psi_1(r)$ $\psi_2(r)$ $\sim 10^{23}$

"macroscopic wavefunction"

$$(1) \quad i\hbar \frac{\partial}{\partial t}\psi_1 = E_1\psi_1 + \alpha\psi_2$$

$$(2) \quad i\hbar \frac{\partial}{\partial t}\psi_2 = E_2\psi_2 + \alpha\psi_1$$

one and the same quantum phase

$$\psi_j(r,t) = \sqrt{n_j(t)}\, e^{iS_j(r,t)} \qquad j = 1,2$$

$$\bar{j} = \begin{cases} 1 & j=2 \\ 2 & j=1 \end{cases}$$

NOTE: This is a large number of particles which act together coherently

$$i\hbar \frac{1}{2}\frac{1}{\sqrt{n_j}}\dot{n}_j e^{iS_j} + i\hbar\sqrt{n_j}\, i\dot{S}_j e^{iS_j}$$

$$= E_j\sqrt{n_j}\, e^{iS_j} + \alpha\sqrt{n_{\bar{j}}}\, e^{iS_{\bar{j}}}$$

Divide by e^{iS_j} and get:

$$i\hbar\left(\frac{1}{2}\frac{1}{\sqrt{n_j}}\dot{n}_j + \sqrt{n_j}\, i\dot{S}_j\right) = E_j\sqrt{n_j} + \alpha\sqrt{n_{\bar{j}}}\, e^{i(S_{\bar{j}} - S_j)}$$

real part: $$-\hbar\sqrt{n_j}\,\dot{S}_j = E_j\sqrt{n_j} + \alpha\sqrt{n_{\bar{j}}}\cos(S_{\bar{j}} - S_j)$$

If we have same type of SC for $j=1,2$, then

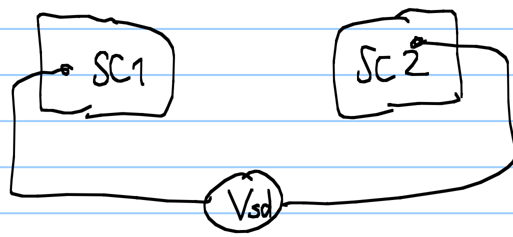
$$-\hbar\dot{S}_j = E_j + \alpha\cos(S_{\bar{j}} - S_j)$$

two equations: one for $j=1$, one for $j=2$

$$\boxed{\begin{aligned}-\hbar(\dot{S}_2 - \dot{S}_1) &= E_2 - E_1 \\ &= (2e)V_{sd}\end{aligned}}$$

the cosine term goes away when taking the difference

2. JOSEPHSON RELATION: PHASE EVOLUTION EQ.



$$i\hbar\left(\frac{1}{2}\frac{1}{\sqrt{n_j}}n_j + \sqrt{n_j}\,i\dot{S}_j\right) = E_j\sqrt{n_j} + \alpha\sqrt{n_{\bar{j}}}\,e^{i(S_{\bar{j}} - S_j)}$$

imaginary part: $$\frac{\hbar}{2}\frac{1}{\sqrt{n_j}}\dot{n}_j = \alpha\sqrt{n_{\bar{j}}}\sin(S_{\bar{j}} - S_j)$$

using $n_j = n_{\bar{j}}$

$$\boxed{\frac{\hbar}{2}\dot{n}_j = \alpha n_j \sin(S_{\bar{j}} - S_j)}$$

1. JOSEPHSON RELATION: CURRENT-PHASE RELATION

$$I \sim \dot{n}_j$$

the current!

CONCLUSION 1: DC Josephson effect

If we take two superconductors with phases S_1 and S_2, there will be a DC (constant) current without any voltage!

$$I = I_c \sin(\underbrace{S_2 - S_1}_{\text{what we call } \varphi \,!})$$

CONCLUSION 2: AC Josephson effect

Now: If we switch on a constant (DC) voltage, we get an AC current:

$$\hbar(\dot{S}_2 - \dot{S}_1) = (2e)V_{sd} \qquad \text{(from 2. JR)}$$

$$\Rightarrow \quad \varphi = S_2 - S_1 = \frac{2e}{\hbar} V_{sd} \cdot t$$

↳ into 1. JR: $I = I_c \sin(\varphi) = I_c \sin\left(\frac{2e}{\hbar} V_{sd} \cdot t\right)$