Revitalize or Relocate:

Optimal Place-based Transfers for Local Recessions

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Many regions in the US experience depressed labor demand and high unemployment, even when the rest of the United States does not. How should the US government respond? In this paper, I characterize optimal place-based transfers in a dynamic economic geography model with nominal wage rigidity and compare them to observed government transfers. I show that transfers not only have a stimulus effect—by boosting local demand—but also a migration effect—by encouraging local residents to stay. Analytically, I provide optimal transfer formulas that capture this trade-off and show, perhaps surprisingly, that the optimal transfer to a distressed region may be a tax due to the migration effect. All else equal, transfers should be larger in the short-run and when there are distressed regions nearby. Quantitatively, I find that observed transfers are both too small in the short-run and too large in the medium-run, achieving just over half of the gains from the fully optimal response to idiosyncratic local shocks. I conclude by exploring how the US government could have responded to the China trade shock in the 2000s.

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1 Introduction

The Janesville Assembly Plant produced its final car for GM on December 23, 2008.[1] In the following months and years, large numbers of workers lost their jobs. Though a large factory stood empty and many people were willing to work for low wages, no new company moved in to offer lower wages and employment opportunities. Instead, the area experienced high unemployment and growing poverty for years afterwards. This is not an isolated case. Autor et al. (2013) report widespread declines in employment, often larger than the declines in manufacturing employment, in regions of the United States that compete directly with Chinese goods. North Hickory, NC, Durham, NC, Charleston, SC, and many other cities all saw a steady surge of Chinese imports that directly competed with their own traded output for more than a decade and suffered for it.

In the case of the China shock, there is now a large literature that has shown how it affected, not only labor markets, but also mortality (Pierce and Schott, 2020), political beliefs (Autor et al., 2020; Che et al., 2022), marriage rates (Autor et al., 2019), and many other outcomes. Sophisticated trade and geography models have been developed to evaluate the average incidence of the shock as well (Galle et al., 2023; Caliendo et al., 2019). But surprisingly little has been said about how the national government should have responded. Should the government encourage people to leave, to find jobs elsewhere? Should the government provide funds to help reinvigorate the region? Do the answers differ in the short- and long-run? Does it depend on the nature of the shock? The goal of this paper is to provide a normative framework to help address these questions.

To set the stage, I first show that the government does not sit idle in response to regional increases in unemployment. Instead, national and state governments transfer money to regions after a shock through a variety of tax and transfer programs, including unemployment insurance and a progressive income tax. I then turn to assess how these transfers may improve welfare, both analytically and quantitatively. Analytically, I provide a sufficient statistic for the optimal place-based transfers in response to a shock that is heterogeneous across regions and then derive qualitative results on how the transfers change with time and the nature of the shock. Quantitatively, I calibrate a dynamic economic geography model to the United States and compare the fully optimal transfers in my model to the observed policies.

The starting point of my analysis is that wages may not fully adjust after demand for labor in a region goes down, leading to involuntary underemployment. Since economic conditions may vary across regions, monetary policy is not sufficient to put everyone back to work, but

[1] See Goldstein (2017) for a moving account of what happened to Janesville, Wisconsin after the factory closed.

place-based transfers may help. I first formalize this idea in the context of a two-period economic geography model with fully rigid wages. I model all transfers as explicitly place-based to capture the key trade-off in designing policy, but I discuss how the results apply more generally to place-biased policies as well. I set up the second-best planner's problem where workers are free to live where they would like (subject to migration frictions) and the planner can tax or subsidize certain areas. While the planner cannot directly move people, it can indirectly influence where people want to live by making certain regions more or less attractive with transfers.

In addition to their direct redistributional effects, place-based transfers have two macroeconomic effects: a stimulus effect and a migration effect.[2] The stimulus effect comes from the fact that people spend disproportionately on goods and services near them, and so giving a region money will increase demand in the local area. When wages are rigid, there will be an aggregate demand externality leading to first-order welfare benefits, as emphasized by Kenen (1969) and formalized by Farhi and Werning (2017). Other things equal, transferring money from a booming area to a busting area will cool down the booming economy while heating up the area in a recession, efficiently putting people back to work.

The migration effect emerges because transfers influence where people want to live. If the government gives tax breaks to people living in an area, other people will be more likely to move there, and people already living there will be less likely to move out. When output is demand-determined because wages are sticky, this movement of people will have an important impact on underemployment. Each region produces some traded goods for the country and the amount demanded is independent of local spending and population. Consider the GM factory in Janesville. With sticky prices, it needs to build a certain number of cars to meet the demand of the outside world. It only needs a certain number of man-hours to do that. In the short run, that will not adjust so movement of people in and out of the region will change the population without affecting employment in the traded sector. This force implies that, if anything, the federal government should tax hard-hit areas to encourage people to find jobs somewhere else.

I derive three analytical results that demonstrate how the migration and stimulus effects interact to shape optimal place-based policy. First, I consider what fiscal transfers should be in a small region that just had a negative shock to the demand of its traded output, like Janesville. Starting from a point with no transfers, a transfer to Janesville improves macroeconomic stability if and only if the local multiplier is larger than per capita earnings multiplied by the semi-elasticity of population to a transfer (holding fixed labor supply);

[2] The redistribution effect will not be the focus of my analysis here. See Gaubert et al. (2021) and Donald et al. (2023) for in-depth discussions of how place-based policy can be used for redistribution.

thus, the optimal transfer could be a tax. This might seem counterintuitive since, when there is no migration, transferring money to a region in a recession always helps stimulate the economy, improving welfare. One might have thought that allowing migration would simply mute that effect. In fact, the migration effect can overturn that result, making a place-based transfer counterproductive. This is because government transfers directly increase the utility of living in a location, independent of the stimulus effect, and that increase in utility leads to migration which reduces the employment rate. Therefore, the fully optimal transfer could be positive or negative, depending on the local multiplier and the migration semi-elasticity.

While the previous result provides a clear cut-off to weigh the relative strength of the migration effect versus the stimulus effect, in practice many demand shocks do not hit only one region. Instead, they are spatially correlated. My next result considers what the spatial nature of the shock implies for the optimal transfer. I find that if migrants to and from Janesville disproportionately come from and to areas that are in a recession, then the optimal transfer is larger than that suggested by the local multiplier and the migration semi-elasticity. That is due to the migration effect. If workers disproportionately leave areas in a recession to go to Janesville, that might hurt the recession in Janesville, but it will help the areas that those workers left. Therefore, considering Janesville in a vacuum misses an important effect. The national government might want to transfer money towards near where the shock hit.

My final analytical result considers the effects of dynamics on the optimal place-based transfers. In particular, I show that the transfer to Janesville in period 2 is lower than that suggested by the local multiplier and the migration semi-elasticity. This is due to a dynamic migration effect. One might have thought that transfers in the second period would have the same trade-off between the stimulus effect and the migration effect, but because people have more time to move, the migration effect is stronger and so the optimal transfer is smaller. That is not the full story because period 2 transfers not only affect where people live in period 2, but also period 1. If the government has made it clear that it will tax households that are in Janesville in period 2, households that have the opportunity to leave in period 1 will do so. Thus, the planner can encourage out-migration in period 1 without losing stimulus.

In the quantitative portion of the paper, I develop a dynamic New Keynesian economic geography model to derive the quantitative implications for optimal transfers in response to the China trade shock. To do so, I move to a continuous time, parametric version of my theoretical model where wages are only partially rigid, due to a standard Calvo friction, and there are finite trade costs in the traded sector, in order to capture realistic geographic features of the US economy. In contrast to the leading dynamic economic geography models studying the response to the China trade shock, I calibrate the model to the 722 commuting zones in the continental United States rather than the states to assess the effectiveness of transfers for fighting the local recessions that arise in each of the distinct labor markets of the US. I then match observed trade flows between states, observed migration flows between commuting zones, and economic activity at the commuting level. Despite the rich geography, large number of locations, along with the forward looking migration and wage dynamics, I am able to solve for the optimal time-varying spatial policy using a quadratic approximation to the social welfare function and linear approximation to the constraints.

To illustrate the new dynamic features of the model, I then consider what optimal fiscal transfers look like in the aftermath of a one time, idiosyncratic demand shock. The optimal policy has a distinctive shape featuring very generous transfers immediately after the shock that efficiently put households back to work before anyone has the chance to move. The optimal transfers then fall significantly, becoming a tax 12 years after the shock, to encourage households to leave and find work somewhere else. The optimal transfers then recover to offer redistributive transfers to those still in the region 25 years later. By comparison, the observed transfers are not generous enough immediately after the shock, and are too generous 10 years afterwards. I find that generous policies targeting regions with high unemployment are especially effective at replicating the optimal policy.

Finally, I revisit how the national government could have used place-based policy to fight against the local recessions that resulted from the China shock. If the planner had anticipated how bad the China shock was going to be, the planner should have gradually ramped up transfers towards those region directly affected until the peak of the China shock. That is because, before the China shock peaks, the planner wants to decrease the population so that they are not around when the worst of the recession happens. Balancing that against the stimulus effects leads to slowly increasing transfers. After the peak of the China shock, the optimal transfers fall quickly and, similar to the idiosyncratic case, fall below the redistributive level of transfers for a number of years. Instead of directing transfers to the commuting zones affected, the planner transfers money to households in commuting zones just next to those regions that were hit by the China shock. That is because those transfers have both a stimulus and migration effect. They disproportionately spend on regions hurt as households spend money on nearby regions, but households in the areas affected want to move out to take advantage of the transfers they can get in those CZs.

The rest of the paper is structured as follows. There is a short Related Literature section below where I mention a number of papers related to the current study. In section 2, I present descriptive facts about how government transfers in the United States respond to unexpected increases in local unemployment. I present a two-period economic geography model with wage rigidity in section 3, before analytically characterizing the optimal policy and teasing out the implications in section 4. The continuous time version of this model used for quantification is in section 5. I show what the model implies for optimal policy in response to an idiosyncratic demand shock and the China trade shock in section 6. I give some concluding remarks in section 7. All proofs of propositions are in the appendix.

Related Literature

This paper most directly contributes to the literature on placed-based policy. The literature has identified two motives for place-based policy: redistribution and efficiency. Gaubert et al. (2021) and Donald et al. (2023) both discuss the redistributive reasons for policy. On the efficiency side for policy, Abdel-Rahman and Anas (2004), Wildasin (1980), Fajgelbaum and Gaubert (2020) and Kline and Moretti (2014) all study how optimal spatial policy could correct for agglomeration externalities. More closely related to this paper are those studying labor market distortions. Austin et al. (2018) shows that if the employment elasticity differs between regions, government policy should vary across the US. Kline and Moretti (2013) find optimal place-based policy when finding a job is subject to search and matching frictions, and Bilal (2023a) considers a similar setting where heterogeneous firms sort across markets. I contribute to this literature by considering how place-based policy can fix distortions in the local labor market when wage rigidity prevents workers from working as much as they would like. I show that the implications for optimal policy are different and that the timing of the transfers plays an important role.

My paper also contributes to a large literature studying how regions respond to idiosyncratic shocks. Blanchard and Katz (1992) and Yagan (2019) study how states respond to shocks that are not uniform across the US. Autor et al. (2013), Topalova (2010), and Dix-Carneiro (2014) all study how regions respond to trade shocks. A growing dynamic trade and economic geography literature tries to quantify the welfare impacts of such trade shocks. Galle et al. (2023) and Caliendo et al. (2019) are two such neoclassical examples. Lyon and Waugh (2019) consider the welfare implications when households have imperfect savings tools. My paper differs primarily in focus. I am mostly interested in the normative question: what should the government do to fight the local recessions that arise from the shock? Thus, I differ from much of the literature by modeling more granular geography (commuting zones) and modeling sticky wages so that I can consider optimal policy.

In emphasizing the role of wage rigidity, I also relate to a growing literature studying the role of wage rigidity in regional responses to trade shocks. Rodríguez-Clare et al. (2020) show that downward wage rigidity can account for the employment response to the China trade shock. Kim et al. (2023) show that downward wage rigidity plus currency pegs play a key role in explaining the large impact of the China shock. Costinot et al. (2022) studies the effect of the collapse of trade between Finland and the USSR on worker outcomes and rationalizes some of the results with a model of wage rigidity. This builds on a large macro literature that has found significant evidence for sticky wages for both employed workers (Grigsby et al., 2021) and new hires (Hazell and Taska, 2020).

In focusing on using place-based policy to fight local recessions, I build on the themes and ideas in the Optimal Currency Area (OCA) literature. This literature has emphasized several important features of successful currency unions, such as factor mobility (Mundell, 1961), trade openness (Mundell, 1961), fiscal integration (Kenen, 1969), and financial integration (Mundell, 1973). My paper can be viewed as formalizing the results of Kenen (1969) when there is significant factor mobility as expressed by Mundell (1961). In a more recent contribution, House et al. (2018) quantifies the benefits of labor mobility in the US and the EU in responding to business cycle shocks across states and countries, respectively. I differ from them in exploring what migration implies for optimal place-based transfers.

Within this literature, my paper is most closely related to Farhi and Werning (2014, 2017). Farhi and Werning (2017) consider what optimal fiscal policy should look like in a currency unions when people are stuck in a location. I show that some of the results are overturned when there is significant factor mobility. Like the present paper, Farhi and Werning (2014) allows for factor mobility in a currency union. However, Farhi and Werning (2014) compares equilibrium migration to the migration a planner would enact if the planner could directly control where people live, hence they have nothing to say about place-based policy. My paper takes as given that people can live where they want and then solves an optimal reallocation of funds exercise.

2 Local Recessions & Transfers: Motivating Facts

I start by reviewing the key tax and transfer programs in the United States that work as automatic stabilizers—transferring money towards regions that are in a local recession—to show that the United States currently uses transfers to help hurting regions. For the most part, these are are not explicitly place-based. Instead, they end up biased towards regions in a recession because what they target varies with local recessions. However, they still play an important role in regional macroeconomic stability.

I then describe how these programs respond to the typical local recession. To do that, I plot the impulse response function of each program to an innovation in unemployment in a commuting zone using local projection methods.[3] In the theory and quantitative section, I then describe the features of the fully optimal transfers so that I can assess how the various programs measure up to the fully optimal transfers.

2.1 Data

I gather data on the important tax and transfer programs within the US from various sources. All data are reported at the county level which I then aggregate up to the 1990 commuting zones of Tolbert and Sizer (1996) following Autor and Dorn (2013). Additional details are in Appendix A.

Local Area Unemployment Statistics (LAUS). I use data on local unemployment from the Local Area Unemployment Statistics, managed by the US Bureau of Labor Statistics (BLS). The LAUS provides unemployment and labor force counts by county every year from 1990 to 2022. For large counties,[4] the BLS construct employment estimates by smoothing out responses from the Current Population Survey (CPS). For smaller counties, the BLS uses an approach known as the Handbook method which combines the Current Employment Statistics and the Quarterly Census of Employment and Wages to find non-farm employment with the American Community Survey to capture other employment. Unemployment combines information from the Unemployment Insurance system with BLS estimates of the number of unemployed who no longer qualify for benefits.

Regional Economics Accounts (REA). I use data on government transfers for the years 1990 to 2022 by county from the Regional Economic Accounts, maintained by the Bureau of Economic Analysis (BEA). It provides information on total current transfer receipts from the government to each county. I then consider four important sub-categories of transfers: unemployment insurance (UI) transfers, income maintenance (IM) transfers, retirement and disability (ret+dis) transfers, and medical (med) transfers. Together these programs account for 92% of the government transfers.

UI transfers include the state run unemployment insurance payments, the special benefits from the national government, and trade adjustment assistance, along with other unemployment programs. The state run unemployment insurance programs typically offer unemployment insurance payments for the first 26 weeks after being laid off though exact

[3] These methods were pioneered by Jordà (2005) and have become a standard tool for macroeconomists looking to describe impulse response functions. See Jordà and Taylor (2024) for a review.

[4] Los Angeles County, New York City, Chicago-Naperville-Arlington Heights, Cleveland-Elria, Detroit-Warren-Dearborn, Miami-Miami Beach-Kendall, and Seattle-Bellevue-Everett

eligibility depends on the state and person. The special benefits from the national government are explicitly place-based. In particular, when states see elevated unemployment, the federal government Extended Benefits program can offer an additional 13 weeks of insurance payments to workers.

IM transfers consist primarily of the Earned Income Tax Credit (EITC), Supplemental Nutrition Assistance Program (SNAP) benefits, and other benefits programs. These are programs targeted towards families in need. SNAP benefits are available for low income households to help afford food. Exact benefits and eligibility depends on the state. EITC benefits go to low- to moderate-income working households. The other benefits programs include the Special Supplemental Nutrition program for Women, Infants, and Children, family assistance, and other tax credits.

I include Supplemental Security Income (SSI), Social Security benefits, and other retirement and disability payments like pensions and workers' compensation in the ret+dis transfers category. The SSI program is available to anyone who is disabled and falls below a certain level of income, while a person qualifies for Social Security Disability Insurance only if they have worked for long enough. If going on disability responds to job opportunities, these programs could respond to job opportunities in a commuting zone. Workers could also take an early retirement if they lose a job to start taking retirement payments through pensions or Social Security retirement benefits.

Finally, med transfers include Medicare payments, Medicaid payments, and military medical insurance benefits. Medicaid benefits can respond to local conditions if a worker loses income and starts to qualify for Medicaid. One can also qualify for Medicare early with a disability so that medicare transfers can also respond to local conditions, even independent of changing demographics.

Statistics of Income (SOI). The Statistics of Income is managed by the Internal Revenue Services (IRS). It reports national income tax, state and local taxes, and tax credits by county using the address reported on the individual income tax returns. This is available by county for the years 2010-2022. These taxes also can serve as automatic stabilizers as when earnings decrease, households also have to pay less in taxes. This means that net income does not fall as much as earnings do. In the following regressions, I remove tax credits as they are included in the government transfer programs.

2.2 Transfer Impulse Response

Having summarized the various tax and transfer programs, I next plot how the government responds to unemployment innovations in a commuting zone. I model income of

(a) Public Assistance Programs (b) Less Taxes Paid

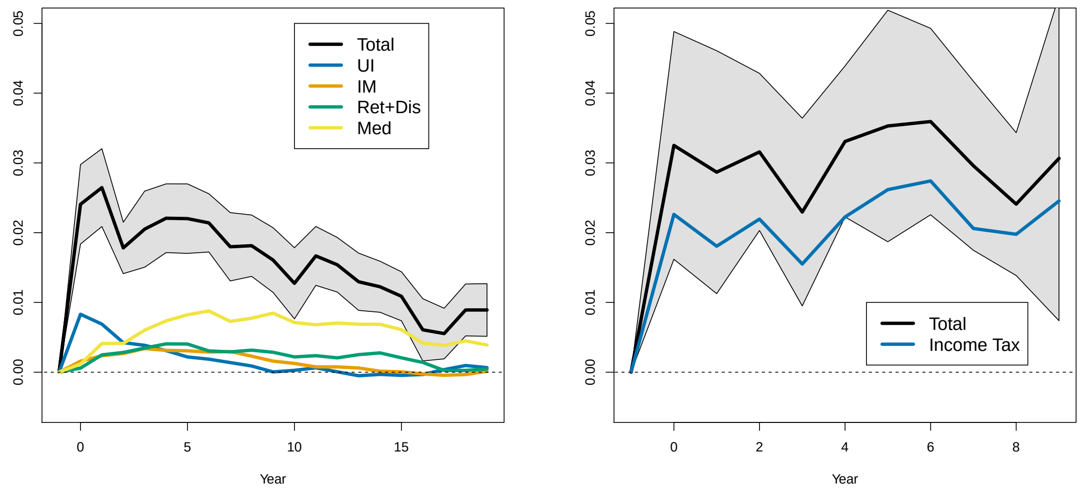


Figure 1: Government Transfer Impact on Log Income

Note: Panel a and b plot local Jorda projections of log public assistance programs and log decrease in taxes paid in a commuting zone on innovations in local unemployment, respectively. Results are normalized to correspond to a jump in unemployment of 10 percentage points and the share of income that comes from the respective program. Bands indicate 95% confidence intervals clustering on state.

households in commuting zone n at year t, I_{nt}, as

$$I_{nt} = X_{nt} + \tau^G_{nt} - \tau^T_{nt},$$

where X_{nt} is earnings per capita, τ^G_{nt} is total government transfers per capita, and τ^T_{nt} is total paid in taxes per capita. Then, to log first order around a steady state with no net transfer,

$$\hat{I}_{nt} = \hat{X}_{nt} + \mu^G \hat{\tau}^G_{nt} - \mu^T \hat{\tau}^T_{nt},$$

where hats denote log deviations from steady state and μ^G and μ^T denote the average value of government transfers relative to earnings and total taxes relative to earnings across commuting zones. In the following regressions, I plot all estimates normalized by μ.

I first analyze how much money the government sends to the region through various public assistance programs in response to an innovation in local unemployment. I then turn to income tax payments to see how much less money the government collects in taxes from the region. Throughout, I will normalize the results to correspond to a 10 percentage point jump in unemployment.

Using τ_{nt} to denote the size of the transfer per capita in commuting zone n year t, my main specification is:

$$\log \tau_{n,t+h} = \beta_h u_{nt} + \gamma_n^h + \gamma_{s(n)t}^h + \sum_{L=1}^{\overline{L}} \gamma_{uL}^h u_{n,t-L} + \gamma_O^h \log \text{OldShare}_{n,t+h} + \varepsilon_{nth}^c,$$

where γ_n^h and γ_{st}^h are commuting zone and state-year fixed effects respectively, u_{nt} is the unemployment in commuting zones n year t, and OldShare_{nt} is the share of adults in the commuting zone over 65. Since retirement makes up a large component of the transfers, controlling for the share of people over 65 removes the mechanical increase in transfers that would occur as working age people leave the commuting zone to find work elsewhere and retired people stay.[5] Controlling for lagged unemployment $u_{n,t-L}$ controls for the expected path of unemployment, so that β_h identifies the impact of an innovation in unemployment at time t on the outcome h years later. I use $\overline{L} = 2$, though including more (or less) lags does not materially affect the results. I include impulse response functions for employment, unemployment, earnings, wages, and population in Appendix A.2.

I plot the estimates of β_h for current government transfers in Figure 1a. I find that on impact, these transfers spike to increase total take home pay by almost 3% of original earnings. The size of the transfers then slowly decrease over the next 15 years. I run the same regression for the four subcategories of current government transfers and plot them on the same figure. UI transfers jump immediately after an increase in unemployment and slowly fall back towards their steady state level in less than 10 years. IM transfers build slowly, peaking 5 years after the sudden increase in unemployment, though they are never quantitatively important. The ret+dis transfers follow a similar time path as the IM transfers. By contrast, most of the increase in transfers, especially in the later years is driven by an increase in med transfers. In Appendix A.3, I show that most of the early increase in medical transfers is driven by medicaid while medicare and medicaid contribute evenly in the long run. In Appendix A.4, I plot the public assistance transfers not including controls for old age and find, consistent with Autor et al. (2021), a much larger portion of income is made up by ret+dis transfers. These transfers suggest that, over the years I have data on taxes, government transfers make up 14% of the lost earnings.

I plot the Jorda projection for less taxes paid in Figure 1b, controlling for only one lag of unemployment so that I can get a longer time horizon. I find that taxes increase by around 0.03 log points immediately after the shock and remain there for all years I have data. This is primarily driven by changes in the national income tax, as shown, though local and state

[5] I include a graph of the change in public assistance programs without controlling for the old age share in Appendix A.4.

taxes play a small role. This decline in taxes makes up around 15% of the lost income. Thus, in total, taxes and transfers make up around 30% of income lost in a local recession over the first 10 years. This is consistent with Feyrer and Sacerdote (2013) who find that current transfers between states make up 25 cents of every dollar of state-wide income shock.

2.3 Taking Stock

In line with what Autor et al. (2021) find in response to the China trade shock, the government transfers large amounts of money to regions that experience a large increase in unemployment through a variety of tax and transfer programs. While these programs are primarily targeted to provide redistribution for individuals, they could also play a key role in helping to stabilize the local business cycle that arises in response to idiosyncratic shocks. Yet, we have no framework to think about what makes transfers effective at fighting local recessions within a country where households are free to move. Thus, it is not clear if these transfers are beneficial or harmful. I turn to explore this issue theoretically in the next section.

3 A Two Period Model of Local Recessions

In this section, I present a dynamic model of economic geography with local recessions. I propose as a starting point that local underemployment may arise from the inability of wages to adjust. To capture the economic forces in the most transparent way, I assume that wages are perfectly rigid, workers are hand-to-mouth, and goods are either freely traded with no trade costs or non-traded. In this setting, I can fully characterize the solution to a second best planner's problem choosing place-based transfers to fight the local recessions. In Section 5, I will introduce a continuous time version of the same model and relax the assumptions on fully rigid wages and no trade costs. This will allow me to quantify optimal place-based transfers in response to the China shock (in Section 6).

For expositional purposes, I model all fiscal transfers as explicitly place-based to illustrate the key mechanism in this section, however, as shown above, most transfers to regions in a recession are facially place-neutral. They only end up place-biased because what they target correlates with local recessions. I will return to the distinction between the two in Section 4.5.

3.1 Environment

Consider an economy with N regions indexed by $n, m \in \mathcal{N} = \{1, \ldots, N\}$ and two periods indexed by $t \in \{1, 2\}$. Throughout, I will use subscripts to index values and superscripts to index functions. I will then use subscripts on functions to denote partial derivatives.

Households. There is a continuum of households that I index by $i \in \mathcal{I}$. I let $n_t(i)$ denote the region where i lives at time t. Each household starts in a region $n_0(i)$. Then, at the beginning of period $t \in \{1, 2\}$, each household observes preference shocks for every region, $\boldsymbol{\varepsilon}_t(i) = (\varepsilon_{1t}(i), \ldots, \varepsilon_{Nt}(i)) \in \mathbb{R}^N$. These shocks are distributed according to a continuous cumulative distribution function that may depend on household i's location at time $t - 1$, $G_{n_{t-1}(i)}(\cdot)$. Thus, these preference shocks can include migration costs or idiosyncratic preferences for location. The utility that household i gets from living in region n at time 1 and region m at time 2 is given by

$$U_{n1} + \varepsilon_{n1}(i) + \beta \left(U_{m2} + \varepsilon_{m2}(i) \right),$$

where U_{nt} is the fundamental utility of region n, and $\beta \in [0, 1]$ is the discount rate.[6]

Then the population of region n at time t, ℓ_{nt}, is given by

$$\ell_{nt} = \int_{\mathcal{I}} \mathbb{1}_{n_t(i)=n} di, \tag{1}$$

where I have normalized the total population to measure 1.

Households agree on the fundamental utility of a location. This fundamental utility in region n period t is determined by a nested set of functions

$$\begin{aligned} U_{nt} &= U^n(C_{nt}, H_{nt}), \\ C_{nt} &= C^n(C_{Tnt}, C_{NTnt}), \\ C_{Tnt} &= C^T(\{C_{Tmnt}\}), \end{aligned}$$

where C_{nt} is the sub-utility that a household in location n derives from consuming goods, H_{nt} is her per capita hours of labor supply, C_{Tnt} is the consumption of a freely traded aggregate,

[6] This general set up nests much of the economic geography literature that puts particular distributional restrictions on $\boldsymbol{\varepsilon}$. The assumption of additive shocks distributed according to a Gumbel distribution as used in Caliendo et al. (2019) is an explicit special case of the model. For the economic geography models that use multiplicative shocks distributed Fréchet as in Fajgelbaum and Gaubert (2020), one can simply define a new utility as log of the old utility. The set of Pareto optimal allocations will be the same in this transformed economy and it will fall under my assumptions. This setup also nests the Calvo friction to migration used by Peters (2022) as a limit case.

C_{NTnt} is the consumption of the non-traded good produced in location n, and C_{Tmnt} is the consumption of the traded good produced in location m. I assume that $U^n(C, H)$ is twice continuously differentiable, quasi-concave, strictly increasing in C, and decreasing in H. The consumption sub-utilities $C^n(C_{Tn}, C_{NTn})$ and $C^T(\{C_{Tmn}\})$ are both homogeneous of degree 1 and strictly quasi-concave.

Firms. In both the freely traded and non-traded sector, a representative firm produces using technology linear in labor. That is,

$$Y_{snt} = A_{sn} H_{snt} \ell_{nt},$$

where Y_{snt} is the production of location n in sector $s \in \{T, NT\}$, A_{sn} is the productivity, and H_{snt} is hours per worker in sector s, region n at time t.[7]

Market Clearing. For the labor market to clear in each location, total labor supply needs to equal the labor used by the freely traded sector and the non-traded sector,

$$H_{nt} \ell_{nt} = H_{Tnt} \ell_{nt} + H_{NTnt} \ell_{nt}, \text{ for all } n, t. \tag{2}$$

The market for the non-traded good needs to clear market-by-market,

$$Y_{NTnt} = C_{NTnt} \ell_{nt}, \text{ for all } n, t. \tag{3}$$

And demand for the freely traded good produced in location i needs to equal production,

$$Y_{Tnt} = \sum_m C_{Tmnt} \ell_{nt}, \text{ for all } n, t. \tag{4}$$

Wage Rigidity. Nominal wages in each location W_n are sticky; they are therefore parameters of the model rather than equilibrium objects. The inefficiencies in the model arise because wages are either too high or too low given the realized demand for labor, given preferences and technology. When wages are too high, the quantity of labor demanded of households in a location will be below what the households would like to supply. Therefore, those households will be underemployed relative to the first best and policy can play some role in correcting that distortion.[8]

[7] Note that I assume that all workers in region n work the same number of hours. Employment will be demand determined, it will therefore lead to underemployment of all workers, not unemployment of some.

[8] I write the model here as one with wage rigidities that are exogenously set. I could also consider a more standard macro model with monopolistic firms that set prices of goods (or unions that set wages) before the

3.2 Decentralized equilibrium

Profit Maximization. Firms are perfectly competitive. They choose production to maximize profits taking as given wages and prices:

$$Y_{snt} \in \underset{Y'_s}{\operatorname{argmax}} \quad \left\{ \left(P_{snt} - \frac{W_n}{A_{sn}} \right) Y'_s \right\}, \text{ for all } s, n, t. \tag{5}$$

Thus, $P_{snt} = W_n/A_{sn}$ for all t. Without risk of confusion, I drop the t index on prices from now on.

Utility Maximization. I start by taking as given utility in each location and characterize the household's dynamic optimization problem. I then return to characterize the intratemporal problem.

Households are free to live wherever they would like. Thus, they move to the location that provides them the most utility, however they do not know their utility shocks for period 2 when choosing their first location. Therefore, I characterize the household migration problem using backward induction. In period 2, household i observes her utility shocks $\boldsymbol{\varepsilon}_2$ and chooses

$$n_2(i) \in \underset{m}{\operatorname{argmax}} \quad U_{m2} + \varepsilon_{m2}(i). \tag{6}$$

Denote by $\overline{U}_{n2} \equiv \mathbb{E}[\max_m U_{m2} + \varepsilon_{m2} | n_1(i) = n]$ the expected utility in period 2 of a household who lives in location n at the end of period 1, before the idiosyncratic utility shocks $\boldsymbol{\varepsilon}_2$ are revealed. This is a function of the vector of fundamental utility levels in period 2. Then in period 1, the household chooses her location to maximize expected utility,

$$n_1(i) \in \underset{m}{\operatorname{argmax}} \quad U_{m1} + \beta \overline{U}_{m2} + \varepsilon_{m1}(i). \tag{7}$$

Conditional on living in location n at time t, households choose consumption to maximize utility subject to a single period budget constraint as they cannot save,

$$\sum_m P_{Tm} C_{Tmnt} + P_{NTn} C_{NTn} \leqslant W_n H_{nt} + T_{nt},$$

where W_n is the wage paid in location n and T_{nt} is the per capita transfer from the government

realization of some state of the world, but cannot change them in the ex-post stage when the state of the word is realized. I will do this in the quantitative section. For now, note that at this ex-post stage, wages are fixed so there is no difference between my analysis and this alternative approach.

to people in location n at time t. That is,

$$
\begin{aligned}
\{C_{nt}, C_{NTnt}, C_{Tnt}, \{C_{Tmnt}\}\} \in \underset{C, C_{NT}, C_T, \{C_{Tm}\}}{\operatorname{argmax}} \Bigg\{ U^n(C, H_{nt}) \Bigg| \\
& C = C^n(C_T, C_{NT}), \\
& C_T = C^T(\{C_{Tm}\}) \\
& \sum_m P_{Tm} C_{Tm} + P_{NTn} C_{NT} \leqslant W_n H_{nt} + T_{nt} \Bigg\}.
\end{aligned} \tag{8}
$$

The nested nature of the preferences allows for the problem to be broken down into subcomponents. First note that $C_T(\cdot)$ is homogeneous of degree 1 and identical across locations. Then, since there are no trade costs within the traded sector, there exists a common aggregate price of the traded good $P_T = \min\{\sum_m P_{Tm} C_{Tm} | C^T(\{C_{Tm}\} \geqslant 1\}$. In turn, the price of the consumption aggregate C_{nt} in each location n is $P_n = \min\{P_{NTn} C_{NT} + P_T C_T | U^n(C_{NT}, C_T) \geqslant 1\}$.

Importantly, households do not choose their hours H_{nt}. Instead, labor is completely demand determined in each location. This creates a wedge since the marginal rate of substitution between consumption and labor may not be equal to the relative price. With flexible wages, the household would choose consumption and labor supply so that $U^n_C / P_n = -U^n_H / W_n$. The labor wedge is a measure of how far this first order condition is from being satisfied. I will denote this wedge as follows:

$$
\tau_{nt} \equiv 1 + \frac{P_n}{W_n} \frac{U^n_H}{U^n_C}.
$$

If a region is in a local recession, then the household is working less than it would like. Therefore, $|U^n_H|$ will be low, leading to a positive labor wedge. On the other hand, the wedge will be negative if the region is going through a local boom.

Government Policy. The government serves two roles. First, it transfers money between regions. The budget constraint at period t for the national government is

$$
\sum_n \ell_{nt} T_{nt} = 0, \text{ for all } t. \tag{9}
$$

The government also sets aggregate demand through monetary policy. In this simplified setup, I assume that the government can choose nominal GDP directly

$$E_t = \sum_n P_n C_{nt} \ell_{nt}, \text{ for all } t. \tag{10}$$

In a richer, dynamic model, the government would do this by setting the interest rate.

Definition 1. *Given nominal GDP in each period E_t and per capita transfers T_{nt}, an* equilibrium *is a set of location choices $n_t(i)$, utility levels U_{nt}, regional population ℓ_{nt}, prices for freely trade goods P_{Tn}, prices for non-traded goods P_{NTn}, consumption levels C_{Tmnt}, C_{NTnt}, labor supplies H_{nt}, and output Y_{NTnt}, Y_{Tnt}, such that:*

- *Households choose consumption and their location to maximize utility,* (6)*,* (7)*,* (8)*;*
- *Population is consistent with location choices,* (1)*;*
- *Firms maximize profits taking prices as given,* (5)*;*
- *The government's budget constraints hold,* (9)*;*
- *The total value of consumption is equal to nominal GDP* (10)*; and*
- *Markets clear,* (2)*,* (3)*,* (4)*.*

3.3 The Planner's Problem

The planner chooses monetary policy E_t, place-based transfers T_{nt}, and associated expected utilities $U(i) \equiv \max_n U_{n1} + \varepsilon_{n1}(i) + \beta \overline{U}_{n2}$ to maximize social welfare. I assume that social welfare is a weighted sum of utility with weight $\lambda(i)$ on household i. Formally, the planner's problem (PP) is,

$$\max_{E_t, \{T_{nt}\}, \mathcal{W}, \{U(i)\} \in \mathcal{E}} \mathcal{W}, \tag{PP}$$

where $\mathcal{W} \equiv \int_{\mathcal{I}} \lambda(i) U(i) di$ and $\mathcal{E}$ is the set of utility profiles attainable in a competitive equilibrium, as described in Definition 1.

4 Optimal Place-based Transfers

In this section, I derive the implications for optimal place-based transfers. Before I do that, I characterize the economy of a region n at time t as a function of monetary policy, the population ℓ_{nt}, and the transfer from the government T_{nt}. This will provide intuition for how government policies can affect regions in a recession, and also simplify the planner's problem. In setting this up, it will be easier to think of monetary policy as choosing the national spending on the traded sector, E_{Tt} where $E_{Tt} \equiv \sum_m P_T C_{Tmt} \ell_{mt}$, rather than total spending. I show these are equivalent, and provide all of the proofs for this section, in appendix B.

For all the analytical results in this section, I focus on the limit as the discount factor $\beta \to 0$. This allows me to focus on the static implications for policy in the first period without worrying about the second period. Then, in the second period, I illustrate the dynamic implications of policy while ignoring feedback effects of the first period back on the second period. I will dispense with this limit assumption in my quantitative analysis.

4.1 Preliminary: Characterizing Hours & Utility in Equilibria

In this section, I characterize hours and utility in a location m period t as a function of monetary policy E_{Tt}, the transfer T_{mt}, and population ℓ_{mt}. The characterization proceeds in two steps. I start by solving the consumption decision of households in each location summarized in equation (8). I then find what hours worked is consistent with those consumption choices and government policy.

Since prices are fixed and the consumption aggregator over the traded output of each location is homothetic, the consumption decision (8) implies that households spend a fixed proportion ϕ_m of their traded expenditures on the output of location m, i.e.

$$P_{Tm}C_{Tmnt} = \phi_m P_T C_{Tnt}.$$

Multiplying by the population in location n, ℓ_{nt}, and summing across all locations we find, from the market clearing condition for traded production (4), that total spending on the traded output of location m is a fixed share of national spending on the traded sector,

$$P_{Tm}Y_{mt} = \phi_m E_{Tt}.$$

Total labor earnings in location m, $W_j H_{mt}\ell_{mt}$, is then that spending on traded output plus spending on the non-traded good. Again from the consumption decision (8), spending on the non-traded good is simply a fixed share of total income α_m, and total income is labor earnings $W_m H_{mt}\ell_{mt}$, plus the transfer from the government $T_{mt}\ell_{mt}$, therefore, by the market clearing for non-traded goods (3),

$$P_{NTm}Y_{NTmt} = \alpha_m \left(W_m H_{mt}\ell_{mt} + T_{mt}\ell_{mt}\right).$$

Then using the market clearing condition for labor in region m (2),

$$W_m H_{mt}\ell_{mt} = \phi_m E_{Tt} + \alpha_m \left(W_m H_{mt}\ell_{mt} + T_{mt}\ell_{mt}\right).$$

This defines hours worked as a function of monetary policy E_{Tt}, population ℓ_{mt}, and the transfer from the government T_{mt}. In what follows, I define this function as,

$$H^m(E_T, \ell, T) \equiv \frac{1}{W_m}\left(\frac{\phi_m E_T}{1-\alpha_m}\frac{1}{\ell} + \frac{\alpha_m}{1-\alpha_m}T\right). \tag{11}$$

I also define an indirect utility function for households in location m as a function only of the transfer T_{mt} and hours worked H_{mt}. Substituting in that real consumption is total earnings $W_m H$ plus the transfer T divided by the price level P_m, I find that

$$V^m(H, T) \equiv U^m\left(\frac{W_m}{P_m}H + \frac{T}{P_m}, H\right). \tag{12}$$

The derivatives of the two previous functions, H^m and V^m, will play a crucial role in my characterization of optimal place-based transfers. I formally describe them in the lemma below.

Lemma 1. *The derivatives of the hours worked function are*

$$\frac{\partial H^n}{\partial \log E_T} = \frac{1}{W_n}\frac{\phi_n E_T}{1-\alpha_n}\frac{1}{\ell}; \quad \frac{\partial H^n}{\partial \log \ell} = -\frac{1}{W_n}\frac{\phi_n E_T}{1-\alpha_n}\frac{1}{\ell}; \quad \frac{\partial H^n}{\partial T} = \frac{1}{W_n}\frac{\alpha_n}{1-\alpha_n}. \tag{13}$$

The derivatives of the indirect utility function are

$$\frac{\partial V^n}{\partial H} = W_n \frac{U^n_C}{P_n}\tau_{nt}; \quad \frac{\partial V^n}{\partial T} = \frac{U^n_C}{P_n}. \tag{14}$$

First, consider how E_T shapes the hours worked H^m, as described in equation (13). When the central government heats up the entire economy by increasing spending in the freely traded sector, the households in each location will work more in the freely traded sector, $\frac{\partial H^m}{\partial \log E_T} > 0$. However, at the same time, they will get more money, and they will want to spend that money on traded and non-traded goods. This will increase demand for the local non-traded good, increasing the labor supplied to that sector leading to a feedback loop. The size of that feedback loop is summarized by the proportion of spending on the non-traded good, α_n. What this means for the utility V^n of households in region n depends on whether the location is in a boom or bust. If it is in a bust ($\tau_{nt} > 0$), then the households there value the opportunity to work more and earn more money, $\frac{\partial V^n}{\partial H} > 0$, as shown in (14). On the other hand, if the labor market is already hot, household utility will decrease from having to work even harder.

In this model, migration ends up having a similar effect on hours worked and utility as does an increase in the level of expenditures on freely traded goods, as can also be seen from equations (13) and (14). Suppose that more people move to location n. The demand for the traded output of the location remains the same, which means that they cannot start producing more. Instead, every household needs to reduce the number of hours they are working so that the total number of hours worked at the location remains the same when including the extra workers. Then the feedback loop leads to reduced hours per household in the non-traded sector as well, $\frac{\partial H^m}{\partial \log \ell} < 0$. The effect on utility then depends on the labor wedge of (14). If the area is in a recession, workers leaving will increase the utility of those left behind because those left behind can work and earn more, since $\frac{\partial V^n}{\partial H} > 0$.

Direct monetary transfers from the government behave very differently. In particular, they provide a direct utility benefit by increasing consumption of the traded goods (14) on top of the stimulus effect (13). Whether the increase in hours increases utility depends again on the state of the economy. If the economy is in a recession ($\tau_{nt} > 0$), then the social value of an extra dollar is higher than the marginal utility of income, $\frac{V^n}{\partial T} + \frac{\partial V^n}{\partial H}\frac{\partial H^n}{\partial T} > \frac{U_C^n}{P_n}$, and there are positive externalities from spending more. If the economy is already booming ($\tau_{nt} < 0$), then working more will hurt the residents and the total benefit from a transfer is smaller than the private internalized benefit.

4.2 The Simplified Planner's Problem

Having characterized hours and utility as a function of monetary policy E_{Tt}, transfers T_{nt}, and population ℓ_{nt}, I now restate the planner's problem in a simplified form that only includes the government's policy (E_{Tt} and T_{nt}), and the fundamental utilities and population in each region (U_{nt} and ℓ_{nt}). To do that, I need to bring in what fundamental utility means for households' migration decisions and include the government budget constraint.

I define the expected utility in period 2 of living in location n in period 1 as $\overline{U}^{n2}(\{U_{m2}\}) = \mathbb{E}[\max_m U_{m2} + \varepsilon_{m2} | n_1(i) = n]$. Then, to make the formula slightly more compact, I introduce the notation $\overline{U}_{n1}(i) \equiv U_{n1} + \varepsilon_{n1}(i) + \beta \overline{U}^{n2}(\{U_{m2}\})$. The simplified planner's problem is as follows:

$$\max_{\{\overline{U}_{n1}(i)\}, E_{Tt}, \{T_{nt}\}, \{U_{nt}\}, \{\ell_{nt}\}} \int_{\mathcal{I}} \lambda(i) \sum_n \mathbb{1}_{n \in \arg\max_m \overline{U}_{m1}(i)} \overline{U}_{n1}(i) di, \tag{SPP}$$

such that utility is given by the indirect utility functions derived in section 4.1,

$$U_{mt} = V^m \left(T_{mt}, H^m \left(E_{Tt}, \ell_{mt}, T_{mt}\right)\right) \text{ for all } m, t; \tag{15}$$

population in period 1 is consistent with free mobility, (1), (7),

$$\ell_{n1} = \ell^{n1} \left(\{\overline{U}_{m1}\}\right) \text{ for all } n, \tag{16}$$

where $\ell^{n1}(\{\overline{U}_{m1}\}) \equiv \mathbb{P}[n \in \arg\max_m \overline{U}_{m1}]$; population in period 2 is consistent with location choice in period 1 and free mobility (1), (6),

$$\ell_{m2} = \sum_n \ell_{n1}\mu^{nm}\left(\{U_{k2}\}\right) \text{ for all } m, \tag{17}$$

where $\mu^{nm}\left(\{U_{k2}\}\right) \equiv \mathbb{P}[m \in \operatorname{argmax}_k U_{k2} + \varepsilon_{k2}|n_1(i) = n]$; and the government budget constraints hold, (9).

4.3 Optimal Short-Run Transfers

To characterize the optimal short-run transfers, I focus on the first order necessary conditions of (SPP). I start by summarizing how monetary policy adjusts in the background to ensure that the average labor wedge across locations is zero.

Lemma 2. *In any interior solution to* (SPP)*,*

$$\sum_n \frac{W_n H_{Tn1}}{1-\alpha_n}\ell_{n1}\frac{\tau_{n1}}{1+\frac{\alpha_n}{1-\alpha_n}\tau_{n1}} = 0.$$

By increasing the overall spending in the entire economy, the planner can stimulate all regions. Thus, the planner sets the average labor wedge to zero, properly weighting each region according to its economic importance. Before I show the relevant first order condition for place-based transfers, I introduce a variable ζ_{n1} to denote the social marginal utility of income in region n period 1. It is defined as $\zeta_{n1} \equiv \frac{\overline{\lambda}_{n1}U_C^n}{P_n}$, where $\overline{\lambda}_{nt} = \mathbb{E}[\lambda(i)|n_t(i) = n]$ is the average Pareto weight on households in location n at time t. This measures how much social welfare increases if the income of the average household in location n increases slightly, holding all else fixed. The household's utility increase depends on the price index in the location P_n and her marginal utility of consumption U_C^n. What that means for social welfare then depends on the average weight the planner puts on those in the location, $\overline{\lambda}_{nt}$.

The first order condition for a transfer to location n implies the next lemma.

Lemma 3. *In any interior solution to* (SPP)*, first period transfers must satisfy*

$$\underbrace{\sum_m \ell_{m1}T_{m1}\nu_{n1}^{m1}}_{\textit{fiscal externality}} = \ell_{n1}\left[\underbrace{\frac{\zeta_{n1}}{\lambda_{G1}} - 1}_{\textit{redistribution}} + \underbrace{\frac{\zeta_{n1}}{\lambda_{G1}}\frac{\alpha_n}{1-\alpha_n}\tau_{n1}}_{\textit{stimulus}}\right] - \underbrace{\sum_m \frac{W_m H_{Tm1}}{1-\alpha_m}\ell_{m1}\frac{\tau_{m1}}{1+\frac{\alpha_m}{1-\alpha_m}\tau_{m1}}\nu_{n1}^{m1}}_{\textit{migration}},$$

where $\nu_{n1}^{m1} \equiv \frac{\partial \log \ell^{m1}}{\partial U_{n1}}\left(\frac{\partial V^n}{\partial T_{n1}} + \frac{\partial V^n}{\partial H_{n1}}\frac{\partial H^{n1}}{\partial T_{n1}}\right)$ *is the migration semi-elasticity of population in location m to a transfer in location n holding fixed utility in locations other than n, and $\lambda_{G1} > 0$ is the social value of the government having another dollar in period 1.*

Increasing the transfer to location n has four effects, each labeled in Lemma 3. The first effect is a fiscal externality. By increasing the transfer to location n, households move away from other locations and into location n. The extent to which the planner values this movement depends on how much households were being taxed in their old location versus their tax in their new location. If households were being taxed in their previous location m but gaining a transfer in their new location n, this will hurt the government's ability to raise money.

The next effect is a direct redistributive effect.[9] Ignoring any effect on labor demand, giving a transfer to households in location n increases utility. The amount that that improves social welfare depends on the social marginal utility of consumption divided by the value of an extra dollar to the government ζ_{n1}/λ_{G1}.

The final two effects are the macroeconomic effects that are the focus of this paper. First, there is the stimulus effect. When the government increases transfers to a location n, utility increases over and above the direct utility benefit when n is in a recession (i.e. $\tau_{n1} > 0$) because total work hours demanded increases by a factor of $\frac{\alpha_n}{1-\alpha_n}$ as discussed in Lemma 1. Whether or not the government values that stimulus depends on the labor wedge τ_{n1}. Second, there is the migration effect. Providing a transfer to location n will increase the population in location n and decrease the population in every other location m. If the regions households leave are in a recession, the out-migration improves social welfare, while if those regions are in a boom, that will be harmful as discussed in Lemma 1. The total migration effect of a transfer then depends on the distribution of recessions τ_{m1} and the matrix of migration semi-elasticities ν_{n1}^{m1}.

Localized Shock. Specializing these equations to the case of Janesville, where there is one small region in a recession within the US, I find the following.

Proposition 1. *Suppose that there are two locations, j (Janesville) and u (Rest of the US), location j is arbitrarily small, $\ell_{jt} \to 0$, and there are no redistributive reasons for policy, i.e. $\zeta_{nt} = 1$. Then in any interior solution to* (SPP)*, the optimal period 1 transfer to location j must satisfy*

$$T_{j1} = \frac{1}{\nu_{j1}^{j1}} \left(\frac{\alpha_j}{1-\alpha_j} - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}} \right) \tau_{j1},$$

where $\frac{\partial \log \ell^{j1}}{\partial T_{j1}} \equiv \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \frac{\partial V^j}{\partial T}$ is the semi-elasticity of location 1 population to a transfer, holding fixed hours worked, and $\nu_{j1}^{j1} \equiv \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \left(\frac{\partial V^j}{\partial T_{j1}} + \frac{\partial V^j}{\partial H_{j1}} \frac{\partial H^{j1}}{\partial T_{j1}} \right)$ is the semi-elasticity of location 1

[9] This can also be thought of as an insurance effect from the perspective of a household before her utility draws are revealed. Mongey and Waugh (2024) discuss this perspective in the context of a discrete choice model similar to mine.

population to a transfer, allowing hours to vary.[10]

Proposition 1 shows that the optimal transfer depends on five statistics: the labor wedge τ_{j1}, the local multiplier $\frac{\alpha_j}{1-\alpha_j}$, the micro migration semi-elasticity $\frac{\partial \log \ell^{j1}}{\partial T_{j1}}$, the per capita wage earnings in the traded sector divided by the traded share of consumption $\frac{W_j H_{Tj1}}{1-\alpha_j}$, and the macro migration semi-elasticity ν_{j1}^{j1}. I will take each of these in turn.

The first statistic is the labor wedge τ_{j1}. This determines if the region is in a recession or not and so whether the planner wants to stimulate the economy or cool it down. In the following discussion, I assume that Janesville is in a recession, so that $\tau_{j1} > 0$.

The sign of the optimal transfer to Janesville then depends on the relative size of the local multiplier $\frac{\alpha_j}{1-\alpha_j}$ and the micro migration semi-elasticity with the traded sector adjustment $\frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}}$. In particular, the optimal transfer could actually be a tax on Janesville if households are sufficiently mobile. Why? Because a transfer to Janesville has a direct effect on both the demand and supply for total labor.

To demonstrate this, suppose that, starting from an equilibrium with no transfers, the national government gives a small transfer to Janesville, $dT_{j1} > 0$, paid for with a small tax on the rest of the US, $dT_{u1} = -\frac{\ell_{j1}}{\ell_{u1}} dT_{j1}$ in an attempt to stimulate Janesville since households in Janesville are working less than they would like. I assume that monetary policy sets the labor wedge in u to 0. Then the total effect on social welfare, when there are no redistributive reasons for policy ($\zeta_{j1} = \zeta_{u1} = 1$), is given by

$$\begin{aligned}
d\mathcal{W} &= \overline{\lambda}_{j1} \ell_{j1} dU_{j1} + \overline{\lambda}_{u1} \ell_{u1} dU_{u1} \\
&= \overline{\lambda}_{j1} \ell_{j1} \left(\frac{U_C^j}{P_j} dT_{j1} + W_j \frac{U_C^j}{P_j} \tau_{j1} dH_{j1} \right) + \overline{\lambda}_{u1} \ell_{u1} \frac{U_C^u}{P_u} dT_{u1} \\
&= \ell_{j1} dT_{j1} + \ell_{j1} W_j \tau_{j1} dH_{j1} - \ell_{u1} \frac{\ell_{j1}}{\ell_{u1}} dT_{j1} \\
&= \ell_{j1} W_j \tau_{j1} dH_{j1},
\end{aligned}$$

using the indirect utility function derivatives from Lemma 1. Therefore, since Janesville is in a recession, $\tau_{j1} > 0$, the transfer increases social welfare if and only if it increases per capita hours worked in Janesville, $dH_{j1} > 0$. The direct effect of the transfer will increase the number of hours worked per capita because households spend some of their money on non-traded goods. However, the transfer will also have an indirect effect because it will affect how many people would like to live there which will also affect hours.

[10] In the limit where households do not move across locations the planner will use transfers to set the labor wedge in Janesville to 0 since the planner has no redistributive reasons for policy. In Farhi and Werning (2017), the optimal stimulus transfers are weighed against the redistributive reasons for policy.

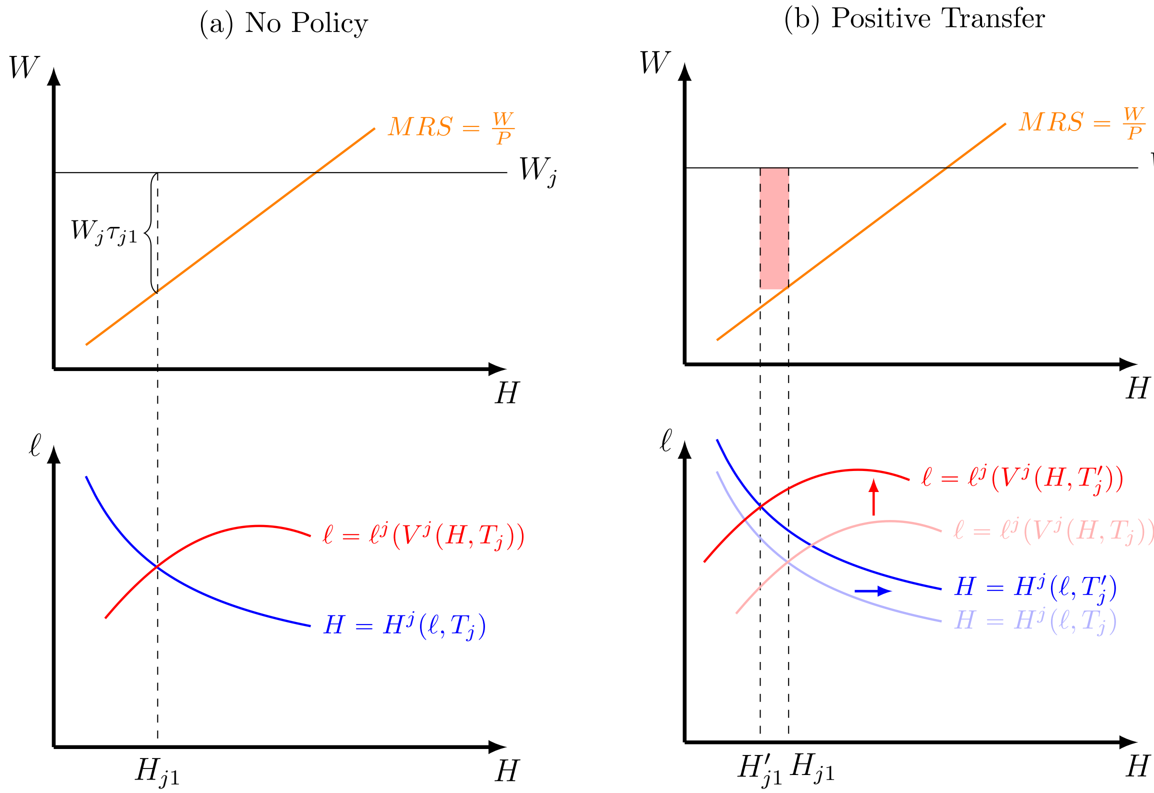


Figure 2: Illustration of Stimulus and Migration Effect of a Transfer

Notes: The top panel of (a) plots the per capita hours of work demanded and the first best level of hours supplied in Janesville holding fixed population with no government transfer. Population is endogenously determined by the population supply and the hours demanded curves in the bottom panel which both take as given the transfer. (b) plots the comparative static with respect to a small increase in the transfer to Janesville. The top panel shades the welfare loss due to the decrease in hours worked per capita.

I graph the equilibrium in Figure 2a in order to illustrate the comparative static. For notational convenience, I omit the dependence on monetary policy E_{Tt} and variables in the rest of the United States u since Janesville is infinitesimal and so has no effect on those aggregates. The top panel plots the optimal number of hours the households would like to supply, holding fixed the transfer from the government and total population. Distinct from the usual supply and demand framework, wages are rigid at W_j and so do not clear the market. I have therefore left off the labor demand curve and simply take as given the number of hours the households are working in that panel. $W_j \tau_{j1}$ then measures how far households in location j are from their ideal labor supply.

To complete the description of equilibrium, I endogenize ℓ_{j1} and H_{j1} in the bottom panel of Figure 2a. I plot the population supply curve in red. This curve shows how many households would like to live in location j as a function of the hours worked per capita. It is increasing for most H_{j1} because the region is in a recession and fundamental utility increases in hours. I also plot the hours demanded curve as a function of population. Where they cross determines the equilibrium population and hours.

I plot how the equilibrium changes when the national government gives a small transfer to Janesville in Figure 2b. The stimulus effect leads to the hours demanded curve in the bottom panel shifting to the right by $\frac{1}{W_j}\frac{\alpha_j}{1-\alpha}$ as shown in Lemma 1. That is, for a given population, if those households have extra income, there will be more demand for their labor because there is home bias in consumption. If this were the only direct effect of a transfer, then the transfer might affect total population, but the inflow of population would only come from a shift along the population supply curve and could not decrease hours demanded.

However, that is not the case here because transfers directly increase utility independent of the stimulus effect. Therefore, the population supply curve also shifts up by $\frac{\partial \log \ell^j}{\partial U_{j1}}\frac{\partial V^j}{\partial T_j}$. It is this shift that determines whether or not the migration effect can dominate the stimulus effect, which is why the migration semi-elasticity that matters for the migration effect is this micro semi-elasticity, holding fixed hours worked, rather than the macro semi-elasticity which would take into account moves along the supply curve.

The curve that shifts up the most dominates. That is, if the hours demanded curve shifts up more, then hours worked will increase and welfare will improve from a transfer. If the population supply curve shifts more, then hours will decrease since too many people move in. I note that the slope of the hours demanded curve is $-\frac{W_j}{\frac{W_j H_{Tj1}}{1-\alpha_j}}$ so that a shift to the right of $\frac{1}{W_j}\frac{\alpha_j}{1-\alpha}$ corresponds to a shift up of $\frac{1}{\frac{W_j H_{Tj1}}{1-\alpha_j}}\frac{\alpha_j}{1-\alpha}$. Thus, the stimulus effect dominates, and the optimal policy features a positive transfer to Janesville, if and only if

$$\frac{\alpha_j}{1-\alpha_j} > \frac{W_j H_{Tj1}}{1-\alpha_j}\frac{\partial \log \ell^j}{\partial U_{j1}}\frac{\partial V^j}{\partial T_j}.$$

Not drawn is the fiscal externality that comes from increasing or decreasing a transfer to Janesville. The ultimate size of the transfer balances the direct stimulus and migration effects on the labor wedge against that force. Therefore, the formula is divided by the migration semi-elasticity ν_{j1}^{j1}. Importantly, this elasticity takes into account the effect on hours worked of a transfer since that determines the total utility effect of the transfer, which determines the total increase in the population and thereby the fiscal cost of the transfer.

Spatially Correlated Shock. In practice, many labor demand shocks do not hit only one small region. Instead, they hit whole industries, as is the case with the China trade shock. In that case, the migration effect of a transfer can have more complicated effects. If giving a transfer to a region in a recession causes households to leave a region that is in a worse recession, the migration effect will be a net positive. The next proposition makes precise how the spatial distribution of shocks interacts with migration patterns to shape optimal spatial policy.

Proposition 2. *Suppose that there are two large locations, s (southern US) and n (northern US), and one small location, j (Janesville). Then, if there are no redistributive reasons for transfers $\zeta_{nt} = \zeta_{st} = \zeta_{jt} = 1$, in any interior solution to* (SPP)*,*

$$T_{j1} > \frac{1}{\nu_{j1}^{j1}} \left(\left(\frac{1}{\lambda_{G1}} - 1 \right) + \frac{1}{\lambda_{G1}} \frac{\alpha_j}{1-\alpha_j} - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}} \right) \tau_{j1},$$

if and only if migrants to j disproportionately come from the region in a recession, i.e. $\mathrm{Cov}_{k \neq j}(|\nu_{j1}^{k1}|, \tau_{k1}) > 0$.

Proposition 2 says that if migrants to location j disproportionately come from parts of the US which are in a recession, the national government should give more money to location j than that suggested by the local multiplier and migration semi-elasticity, taking into account the social marginal value of a government dollar λ_{G1}. Importantly, the formula in Proposition 2 looks slightly different than that in Proposition 1 because, in Proposition 1, the social value of another government dollar is 1. When there are two large locations, and one is in a recession, that is no longer the case because the value of another dollar is not simply the marginal value of consumption (which is 1 when there are no redistributive reasons for transfers). That extra dollar can now also be used to stimulate the region in a recession.

Above and beyond that difference, the transfer to Janesville is larger than that suggested by the local trade-off if and only if migrants to Janesville disproportionately come from the region in a recession. To see why that is, suppose that migrants to Janesville came proportionately from the north and the south, i.e., $\nu_{j1}^{n1} = \nu_{j1}^{s1}$. In that case, increasing the transfer to Janesville will have the effects on Janesville discussed in Proposition 1, but it will also have an impact on the fiscal externality and the migration effect in the north and the south. In particular, households will leave the south and the north proportionately to their population. However, the average labor wedge and the average transfer across the two locations are zero due to monetary policy and budget balance, respectively. Therefore, the net effect on the fiscal externality and the net migration effect are both zero.

By contrast, if migrants to Janesville disproportionately come from the region in a recession, increasing the transfer to Janesville will have a net migration effect and an effect on the fiscal externality that depends on the social value of having slightly fewer households in the recessionary region. Importantly, the fact that the planner has the optimal transfer on the north and south already tells us something about the combined fiscal externality and migration effect. The combined fiscal externality and migration effect that come from migration into the region in a recession in response to the transfer are balanced against the stimulus effect of the transfer. However, because the region is in a recession, the stimulus effect must be positive, and therefore the combined fiscal externality and migration effect must be negative. Thus, the planner values encouraging households to disproportionately leave the region in a recession by giving extra money to those in Janesville.

This implies that the nature of the demand shock matters for the optimal policy. If the shock is very correlated, then regions that are in recessions will be near other regions in recessions. Therefore, a transfer to one of those regions will not have a large net migration effect since all migrants in response to the transfer will come from other areas also in a recession. I will return to this quantitatively in Section 6.

4.4 Optimal Long-Run Transfers

Having shown that fiscal transfers to a region in the immediate aftermath of a factory closure have competing stimulus and migration effects, I next turn to the effects of a transfer in the long run. One might think that the same basic trade-off between the migration effect and the stimulus effect apply in the second period as it did in period 1. The only difference is that people have more time to move so that the migration effect will likely be stronger. But that intuition turns out to be incomplete, as I now discuss.

I start by stating the first order necessary condition for a transfer to location n in period 2. I define the social marginal utility of income in region n period 2, $\zeta_{n2} \equiv \frac{\beta \overline{\lambda}_{n2} U_C^n}{P_n}$.

Lemma 4. *In any interior solution to* (SPP)*, second period transfers must satisfy*

$$\sum_t \frac{\lambda_{Gt}}{\lambda_{G2}} \sum_m \ell_{mt} T_{mt} \nu_{n2}^{mt} = \ell_{n2} \left[\frac{\zeta_{n2}}{\lambda_{G2}} - 1 + \frac{\zeta_{n2}}{\lambda_{G2}} \frac{\alpha_n}{1-\alpha_n} \tau_{n2} \right] - \sum_t \frac{\lambda_{Gt}}{\lambda_{G2}} \sum_m \frac{W_m H_{Tmt}}{1-\alpha_m} \ell_{mt} \frac{\tau_{mt}}{1 + \frac{\alpha_m}{1-\alpha_m} \tau_{mt}} \nu_{n2}^{mt},$$

where $\lambda_{G2} > 0$ is the social value of the government having another dollar in period 2, and ν_{n2}^{mt} is the elasticity of population in location m at time t to a transfer to location i at time 2.

Lemma 4 shows the same four effects of a transfer from the period 1 first order condition shown in Lemma 3: fiscal externality, redistribution, stimulus, and migration. The redistribution and stimulus effects remain the same as before. Transferring an extra dollar to households in location n improves social welfare by ζ_{n2} directly through consumption. The planner weights that use of the money against the marginal value of a dollar in period 2, λ_{G2}. Similarly, the transfer leads to a stimulus of $\frac{\alpha_n}{1-\alpha_n}$. The only difference is that real consumption and the labor wedge might be different in period 2 as compared to period 1.[11]

Both the fiscal externality and the migration effect now have dynamic components. That is because a promise to tax certain locations in period 2 will affect where households decide to live at time 1. Therefore, the planner has to take into account how that movement in the first period will affect the fiscal externality and recessions in the first period. Under the limit $\beta \to 0$, this effect is infinitesimal. However, the value of an extra dollar in period 2 (λ_{G2}) is also infinitesimal, so the effect still shapes the optimal policy.

In the next proposition, I consider what this implies for optimal policy in Janesville in period 2.

Proposition 3. *Suppose that there are two locations, j (Janesville) and u (Rest of the US), location j is arbitrarily small, $\ell_{jt} \to 0$, there are no redistributive reasons for policy, $\zeta_{nt} = 1$, and j is in a recession, $\tau_{jt} > 0$. Then in any interior solution to* (SPP)*, the optimal period 2 transfer to location j satisfies*

$$T_{j2} < \frac{1}{\nu_{j2}^{j2}} \left(\frac{\alpha_j}{1-\alpha_j} - \frac{W_j N_{Tj2}}{1-\alpha_j} \frac{\partial \log \ell^{j2}}{\partial T_{j2}} \right) \tau_{j2},$$

when the share of workers in location j in period 1 who stay in location j in period 2 is greater than zero.

Comparing Proposition 3 to Proposition 1 reveals that in period 2, the optimal transfer to a region in a recession is always lower than that implied by the simple static trade-off between the stimulus effect and the migration effect.

A transfer in the second period has the same stimulus, migration, and fiscal externality effects on period 2 as first period transfers did in period 1. However, giving a transfer to households in Janesville in period 2 also increases the expected utility of living in Janesville in period 1 if those who live in Janesville in period 1 are likely to live there in period 2 (due to moving costs). Therefore, if the planner promises to give a transfer to everyone who is in Janesville in period 2, households that would have left in period 1 because they had a job opportunity somewhere else will be less likely to leave. So the period 2 transfer will increase population in period 1 Janesville, impacting the first period fiscal externality and migration effect.

[11] These have no dynamic components as all households are hand-to-mouth and so cannot spend income they earn in period 2 in period 1. While useful for illustrating the mechanism, this stark assumption is not necessary to imply the next result. I will return to the necessary ingredients below.

What is the net effect on social welfare? To answer that, we need to know the signs and relative strength of those two forces. The key is to note that, just as in Proposition 2, period 1 transfers already reveal something about their combined effect. Period 1 transfers optimally trade off those exact forces that come from an increase in population against the positive stimulus effect of giving a little extra money to people in location 1. Therefore, the net effect of increasing population in period 1 Janesville must be negative, and a transfer in period 2 makes that worse. Thus, transfers in period 2 should be lower than what would be suggested by the static trade-off since taxes in period 2 allow the planner to encourage out migration without decreasing stimulus in the first period.

The fact that households are all hand-to-mouth delivers this clean result, but it is not necessary for this mechanism to matter. If households can borrow against their future income to some extent, the transfer in period 2 would also affect consumption in period 1. Therefore, the transfer in period 2 would have some stimulus effect on period 1 which would need to be balanced against the period 1 migration effect. The key asymmetry that delivers the result is that, compared to transfers in period 1, transfers in period 2 have a greater effect on period 1 utility than on period 1 consumption. Then increasing the transfer in period 2 has a relatively larger migration effect and, since the period 1 transfer optimally balances the migration effect against the transfer effect, this loads onto the wedge in period 1.

This anticipation effect of the transfer relies on the commitment power of the planner. The planner can commit to taxing households in period 2 who are still in Janesville so as to encourage households to find jobs somewhere else.

The actual size of the transfer then depends on these anticipation effects and how the static trade-off changes. For most models, the migration semi-elasticity in period 2 will be larger than the semi-elasticity in period 1, suggesting the transfer should be lower. However the labor wedge in period 2, τ_{j2}, will often be closer to 0 than the labor wedge in period 1 τ_{j1}, shrinking the transfer towards 0. I will demonstrate how this plays out quantitatively in Section 6.

4.5 Extensions and Robustness

So far, I have characterized optimal transfers in response to local recessions in the baseline model. The model is intentionally stylized to illustrate the key forces shaping policy in the most transparent way possible. It features perfect wage rigidity, no trade costs in the traded sector, explicitly place-based policies, no agglomeration externalities, and labor that is freely mobile between sectors. In the quantitative model presented in section 5, I relax the assumption of no trade costs and perfect wage rigidity, but keep the other assumptions for computational reasons. Here I summarize how my results change when I enrich the model along a number of other dimensions and discuss how they affect my results. Formal derivations are in Appendix C.

Downward Wage Rigidity and Costly Price Adjustments. This model features perfect wage rigidity, but empirical evidence suggests that wages are more rigid going downwards. In appendix C.1, I consider a variant of this model with 2 locations, downward wage rigidity, and costly upward price adjustments. In that case, I derive a new version of Proposition 1. The formula is largely unchanged because I consider a small region in a recession, where wages are rigid in both the downwardly rigid case and the completely rigid case.

Place-biased Policy. In appendix C.2, I consider an extension of the model with multiple types and transfers that can be partially targeted towards those types and locations. I show that starting from an equilibrium with no taxes, whether or not a place-biased transfer helps with macroeconomic stability still depends on the same sufficient conditions: the local multiplier and the migration semi-elasticity. Importantly, the stimulus effect depends on the observed place-biased nature of the transfer, while the migration effect is determined by how place-biased the transfer is within a type.

This extension has important implications for automatic stabilizers. In particular, the extension finds the conditions under which making a particular place-biased program more generous helps macroeconomic stability. Consider the income tax. The income tax will have stimulus effects if income decreases in a recession. But also, because the tax rate is progressive, higher paying jobs are less attractive. Therefore, households have less incentive to take a higher paying job in a region with higher demand. Similarly, unemployment benefits will stimulate the region, but it will reduce the incentive to find a job. Assuming that it is easier to find a job in a low unemployment area, this reduces the attractiveness of other regions that are not in a recession.

Another interpretation of this extension is as transfers that can be targeted based on starting location. The type is then the starting location. In that case, this extension says that the planner would target money towards types who tend to be in recessionary regions, that is, those who were there before. However, the migration effect still operates within the group, so that the planner might want to offer households more money to go somewhere else if the migration semi-elasticity is high enough.

Households Affect Demand. In appendix C.3, I consider an extension of the model to have multiple household types who can affect demand for a particular region. These could represent entrepreneurs, for example. When they move into a region, they open up new businesses that export new products to the rest of the country, but this could also represent agglomeration externalities more generally. I find an adjusted version of Proposition 1 in that case. The migration effect then also has an effect on demand that depends on the covariance between the household type's effect on demand and their migration semi-elasticity to the transfer. In practice, this covariance is likely small since entrepreneurs likely move to areas with good economic conditions, regardless of the government transfers, though this force could suggest other place-based policies to fight local recessions.

Wage Stickiness Only in Traded Goods. While Autor et al. (2013) found that earnings decreased significantly, they found no evidence that average wages decreased in the manufacturing sector. Instead, all of the wage movement was in services. In appendix C.4, I consider an extension of the model where labor is imperfectly substitutable across the traded and non-traded sector, and wages are not sticky in the non-traded sector. In that case, there is no stimulus effect of a transfer because there is no wedge on the non-traded labor. Instead, there is only a migration effect, so I show that in an adjusted version of Proposition 1, the optimal transfer to Janesville is always negative.

5 A Quantitative Model of Local Recessions

The two-period model with fully sticky wages and freely traded and non-traded goods in section 3 reveals the key forces in a transparent manner, but it is too stylized to bring to the data to quantify how large place-based transfers should be. Thus, in this section, I formulate and calibrate a quantitative model. Section 6 then uses that model to illustrate how the US government could use place-based transfers to combat localized recessions caused by the China trade shock. To this end, I enrich the baseline model of Section 3 by moving to continuous time, introducing a continuum of unions in each commuting zone who can adjust their wages subject to a Calvo friction, and allowing for finite trade costs within the traded sector.

To focus on the distinct US labor markets, I will interpret each of the 722 commuting zones in the contiguous United States as a location. This geographic richness comes at a computational cost, and I am forced to abstract from a number of important features like a marginal propensity to consume less than 1 and imperfect labor mobility across sectors. I will discuss how including these features would affect my results below.

5.1 Quantitative Model

In this subsection, I provide a brief overview of the parametric assumptions I impose and the new features of the quantitative model. A more formal description of the environment and decentralized equilibrium is in Appendix D.

5.1.1 Households

Each household starts in some location n and receives the opportunity to move at a Poisson rate $\delta_\ell > 0$.[12] At that point, the household observes migration costs to each location $\tau_{\ell nm}$ and idiosyncratic utility shocks for living in every location ε_m that are independent and distributed according to a Gumbel distribution with shape parameter $\nu > 0$. The household then chooses the location that maximizes her utility.

Denoting by $v_n(t)$ the expected utility of living in location n at time t and $U_n(t)$ the flow utility from living in location n, the Bellman equation for a household in location n is

$$\rho v_n(t) - \dot{v}_n(t) = U_n(t) + \delta_\ell(V_n(t) - v_n(t)), \tag{18}$$

where

$$V_n(t) = \frac{1}{\nu} \log \left(\sum_m \exp(\nu(v_m(t) - \tau_{\ell nm})) \right), \tag{19}$$

is the expected utility of a household in n that gets an opportunity to move before realizing her idiosyncratic preference shocks. This implies that $\exp(\nu(v_m(t) - \tau_{\ell nm} - V_n(t))$ share of households in location n who have the chance to move will move to location m. Therefore, population changes according to,

$$\dot{\ell}_m(t) = \delta_\ell \left[\sum_n \exp\left(\nu(v_m(t) - \tau_{\ell nm} - V_n(t)\right) \ell_n(t) - \ell_m(t) \right]. \tag{20}$$

Conditional on living in location n, the flow utility of a household is a function of consumption and labor supply,

$$U_n(t) = \frac{C_n(t)^{1-\theta}}{1-\theta} - \frac{H_n(t)^{1+\eta}}{1+\eta},$$

where $C_n(t)$ is the consumption aggregate, θ is the elasticity of intertemporal substitution,

[12] Bilal (2023b) uses the same migration restriction in a continuous time model and Peters (2022) also uses this migration restriction in a discrete time model. In a continuous time model, this is necessary to transform population in a region into a state variable. To first order, making the arrival rate lower is similar to raising moving costs to all other locations.

$H_n(t)$ is hours supplied, and η is the Frisch labor elasticity. The consumption aggregate is a Cobb-Douglas aggregation of consumption of the traded good and the non-traded good,

$$C_n(t) = C_{NTn}(t)^{\alpha} C_{Tn}(t)^{1-\alpha},$$

where $C_{sn}(t)$ is consumption of the sector s good and $\alpha \in (0,1)$ is the share of spending on non-traded goods. The traded good is an aggregation of the varieties produced in each location,

$$C_{Tn}(t) = \left(\sum_{m \in \mathcal{N}} \phi_m(t)^{\frac{1}{\sigma}} C_{Tmn}(t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $\phi_m(t)$ is the consumption weight on the variety produced by location m, $C_{Tmn}(t)$ is consumption of the traded good produced in location m by a household in n, and σ is the elasticity of substitution between varieties produced by the locations.

Households are hand-to-mouth so they choose consumption of traded and non-traded goods to maximize utility, taking prices as given, subject to the period t budget constraint,

$$\sum_m p_{Tmn}(t) C_{Tmn}(t) + p_{NTn}(t) C_{NTn}(t) \leqslant W_n(t) H_n(t) + T_n(t),$$

where $p_{Tmn}(t)$ is the price of traded good produced in m in location n, $p_{NTn}(t)$ is the price of the non-traded good in n, $W_n(t)$ is the average wage paid in n, and $T_n(t)$ is the per capita transfer to households in location n. Just as in Section 3, households cannot choose their labor supply and instead supply the necessary labor to meet demand.

5.1.2 Production

In each location, there is a continuum of competitive producers $\omega \in [0,1]$ who produce differentiated products but maximize profits taking prices and wages set by their union as given using a linear technology. Therefore, the price of the intermediate is simply the wage $p_n(\omega, t) = W_n(\omega, t)$.

A final producer in each location then combines those intermediates according to a CES aggregator

$$Y_n(t) = A_n \left[\int_0^1 Y_n(\omega, t)^{\frac{\epsilon-1}{\epsilon}} d\omega \right]^{\frac{\epsilon}{\epsilon-1}},$$

where $Y_n(t)$ is the aggregate production of location n and $\epsilon > 1$ is the elasticity of substitution across intermediates. The final producer is competitive and so maximizes profits taking the price of the final good and each of the intermediates as given. I denote by $p_n(t)$ the price of the final good.

The final good can then be consumed as a non-traded good or a traded good. There are iceberg trade costs so for a household in m to consume a single unit of the good produced in n, $\tau_{nm} \geqslant 1$ needs to be sent. Trade is competitive so that $p_{Tnm}(t) = \tau_{nm} p_n(t)$ and $p_{NTn}(t) = p_n(t)$.

5.1.3 Labor Unions

For each intermediate ω in location n, there is a union that can unilaterally set the wage it demands. Wages are sticky, and the union only gets the chance to change the wage demanded at a Poisson rate $\delta_w > 0$.

Given wages, the union supplies the labor necessary to meet demand for intermediate ω. I assume that there is efficient rationing. When a union gets the chance to change its wage, it sets the wage to maximize utility of the average household in its location. As is standard in this literature, I assume the local government has a wage subsidy κ to undo the monopoly distortion, funded by a tax on the residents. That is, the unions who can change their wage at time t choose a new wage $\tilde{W}_n(t)$ that solves

$$\tilde{W}_n(t) \in \underset{W'}{\operatorname{argmax}} \int_t^\infty e^{-(\rho+\delta_w)(t'-t)} \left[\kappa \frac{C_n(t')^{-\theta}}{P_n(t')} (W')^{1-\epsilon} - H_n(t')^\eta (W')^{-\epsilon} \right] A_n^{\epsilon-1} P_n(t')^\epsilon Y_n(t') dt'.$$

5.1.4 Government

The government sets aggregate spending $E(t)$, such that

$$E(t) = \sum_n E_n(t) \ell_n(t), \text{ for all } t, \tag{21}$$

and also chooses the place specific transfers between locations. The government budget constraint then must hold in each period,

$$\sum_n \ell_n(t) T_n(t) = 0, \text{ for all } t. \tag{22}$$

5.2 The Planner's Problem

The government chooses monetary policy $E(t)$, place-based transfers $T_n(t)$ and associated flow utilities $U(i, t)$ to maximize social welfare. Following Dávila and Schaab (2022), I allow the government to have a time-varying Pareto weight $\lambda(i, t)$ on households. That is, the planner could care about the consumption of a household more at some time t than another time t'. I include these weights as, in order to do a linear-quadratic approximation to the planner's problem, the original equilibrium needs to be efficient. However, in this dynamic setting where households are hand-to-mouth and so do not have access to complete markets, this is only possible if the planner cares less about a household's consumption if they live in a location that earns less. Thus, I vary the weights to rationalize the observed patterns.[13]

Formally, the planner faces the problem

$$\max_{E(t),\{T_n(t)\},\{U(i,t)\}\in\mathcal{E}} \int_{\mathcal{I}} \int_0^\infty e^{-\rho t}\lambda(i,t)U(i,t)dtdi, \tag{23}$$

where $\mathcal{E}$ is the set of utility profiles attainable in equilibrium, as described in Appendix D.

5.3 Computation

This is a non-linear model with state variables utility $v_n(t)$, population $\ell_n(t)$, along with wages $W_n(\omega,t)$ for each intermediate. Solving the optimal planner's problem with the 722 commuting zones of the United States would be infeasible. Therefore, I follow the macro literature in doing a log-quadratic approximation to the social welfare function and a log-linear approximation to all of the constraints around a no-inflation, no-transfer steady state, where Pareto weights $\lambda(i,t)$ are such that it is optimal before any shocks. Details of how I derive the loss function including distortions in migration, trade, inflation, and output along with the final linearized constraints are in appendix E. I use $\hat{x}$ to denote log deviations from that steady state, and I consider idiosyncratic demand shocks to the traded output of specific regions $\phi_m(t)$.

The final linearized model features four state variables for each commuting zone: population $\hat{\ell}_n(t)$, utility $\hat{v}_n(t)$, wage $\hat{w}_n(t)$, and inflation $\hat{\pi}_n(t)$, for a total of 2,888. When solving the planner's problem, I also need to keep track of the 2,888 co-state variables. I give details of how I compute the optimal policy for time-varying shocks in Appendix G.

5.4 Calibration & Estimation

In this section, I provide an overview of how I calibrate the model in Section 5.1 to match the United States in 2000. Additional details can be found Appendix F. I interpret a local labor market in the model as a commuting zone (CZ), as developed by Tolbert and Sizer (1996). My analysis will focus on the 722 commuting zones of the contiguous United States, as in Autor et al. (2013).

[13]This could be the case if the political power of a household depends on his location because of the electoral college or the power of the local representative.

Table 1: Calibration Summary

Parameter	Value	Description	Source
Panel A. Stimulus effects			
Parameter	Value	Description	Source
α	0.61	Non-traded share	Moretti (2010)
σ	4.5	Trade EoS	Head and Mayer (2014)
τ_{nm}		Trade costs	CFS state trade flows
Panel B. Migration effects			
Parameter	Value	Description	Source
ν	0.49	Migration shape parameter	Hornbeck and Moretti (2024)
δ_ℓ	0.171	Migration calvo friction	ACS migration flows
$\tau_{\ell nm}$		Migration costs	
Panel C. Policy Function			
Parameter	Value	Description	Source
γ^w	-0.158	Wage effect on transfers	Estimates on taxes and transfers
γ^H	-0.390	Hours effect on transfers	
Panel D. Other Parameters			
Parameter	Value	Description	Source
ρ	0.02	Patience	2% Real Interest Rate
ϵ	11	Intermediate EoS	Farhi and Werning (2017)
η	2	Frisch labor supply elasticity	Peterman (2016)
δ_w	0.189	Wage Calvo friction	China Shock Unemployment
θ	1	Intertemporal EoS	log preferences
A_n		Productivity	REA earnings

As discussed in Section 4.5, the migration effect of a transfers depends on the transfer a household would expect to get in each location. While, in principle, one could introduce many types and infer the transfer to each type in each location, this would not be computationally feasible. Therefore, instead, I follow Fajgelbaum and Gaubert (2020) in adjusting earnings and transfers per capita to each commuting zone by the demographic characteristics. Thus, in all of the following, all earnings, employment rates, taxes, and transfers are net of the demographic characteristics.

Stimulus Effects. As I show in Appendix H, for a small open region, the stimulus effect of a transfer depends on the local multiplier $\frac{\alpha}{1-\alpha}$ when wages are perfectly rigid. While it does not estimate the local multiplier in response to a government transfer, Moretti (2010) measures the next best thing: how many jobs in the non-traded sectors are created in response to the creation of a new manufacturing job, 1.6. I set α to rationalize what he finds. This gives a value of 0.61, similar to the 0.62 in Diamond (2016).

With a finite number of regions, the stimulus effect also depends on trade flows between commuting zones. I set the elasticity of substitution across varieties produced by different commuting zones to be 4.5, taking the mean estimate from Head and Mayer (2014). The Commodity Flow Survey (CFS) reports the flow of manufacturing goods shipped between states for the year 2002. I disaggregate this data to infer trade flows between commuting zones by adapting the approach in Allen and Arkolakis (2014) and Fajgelbaum and Gaubert (2020). In my context, I use the gravity equation predicted by the model to find the unique trade flows between CZs that are consistent with the distance between CZs, my own estimates of how trade frictions change with distance using state trade flows, and observed earnings in each CZ.

Migration Effects. As I show in Appendix H, the migration effect depends on the long-run migration elasticity and the speed of transition δ_ℓ when wages are perfectly rigid. I set ν so that the average long-run migration elasticity across commuting zones in the model matches the elasticity of Metropolitan Statistical Area (MSA) population to earnings found in Hornbeck and Moretti (2024), 2.87.[14] This is not ideal as it is the elasticity in response to earnings rather than a transfer, but under the envelope theorem, the elasticities are the same at the point $T_n = 0$, which determines the sign of the transfer.

The speed of transition is then jointly determined by δ_ℓ and the matrix of migration costs $\tau_{\ell nm}$. I calibrate these parameters using migration reported in the American Community Survey (ACS). In particular, I set $\delta_\ell = 0.171$ to match the average number of households moving every year. I then construct yearly CZ-to-CZ migration flows conditional on moving. This matrix has many zeros, so I assume that migration costs take the form

$$\tau_{\ell nm} = \delta_{\ell D} \log \text{distance}_{nm} + \delta_{\ell H} + \hat{e}_m.$$

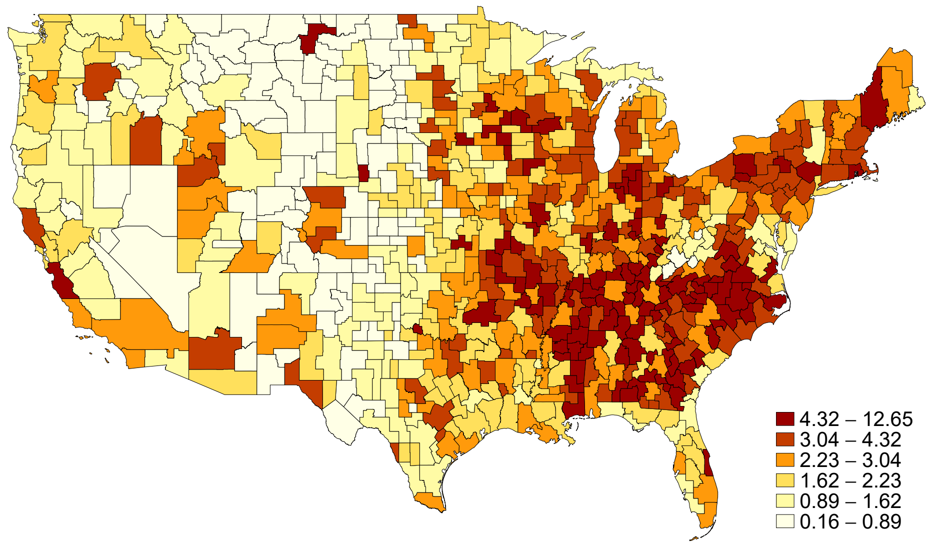
I estimate $\delta_{\ell D}$ by regressing log migration flows on log distance with fixed effects for origin and destination. I then choose $\delta_{\ell H}$ to match the probability a household moves but stays in the same commuting zone where $\hat{e}_m$ is set so that observed working age population in the year 2000 is a steady state.

Policy Function. To simulate the model with current policy, I need an estimate of how the US government currently uses transfers to respond to local conditions. I follow Beraja (2022) in assuming that both government transfers and taxes to a region are a function of current wages and employment:

$$\hat{T}_n(t) = \gamma^w \hat{w}_n(t) + \gamma^H \hat{H}_n(t) + \hat{T}(t), \tag{24}$$

[14] As opposed to CZs, MSAs do not cover all of the United States, leaving off rural areas. However, they are similar-sized: some CZs fully encompass an MSA and some MSAs encompass a CZ. Bryan and Morten (2019) find a value of 2.7 for the US and 3.2 in Indonesia, and Hsieh and Moretti (2019) find a value of 3.3. Other papers studying the effect of the China trade shock like Artuç et al. (2010), Caliendo et al. (2019), and Rodríguez-Clare et al. (2020) consider the elasticity across sectors and/or states.

Figure 3: 100× China Shock to Trade Demand



Note: This figure plots the incidence of the China Trade shock across the 722 commuting zones of the contiguous United States. The shock is constructed instrumenting for the increase in Chinese import penetration in 4 digit industries to the United States from 2000 to 2012 with the increase in export penetration to a group of other developed countries, as in Autor et al. (2021). The impact on each commuting zone is determined by share of commuting zone employment in the sector in year 2000.

where $\hat{T}(t)$ adjusts so that the federal budget constraint holds. I estimate (24) through OLS first differences and aggregate them up to find the size of the transfer relative to earnings.[15]

China Trade Shock. I model the trade shock as a uniformly increasing demand shock for traded production of commuting zones starting in the year 2000 and ending at the beginning of year 2012 following Autor et al. (2021). I follow Autor et al. (2021) in constructing the China Trade shock to each commuting zone. In particular, I use the notion of average change in import penetration across industries, weighted by industry shares in initial CZ employment:

$$\Delta IP_n = \sum_s \frac{\ell_{n,s,2000}}{\ell_{n,2000}} \Delta IP_s^{US},$$

where $\Delta IP_s^{US} = \Delta M_{china,US,s}/(Y_{US,s,2000} + M_{US,s,2000} - X_{US,s,2000})$ is the growth of Chinese import penetration for U.S. manufacturing industry s over the period 2000 to 2012,[16] $\frac{\ell_{n,s,2000}}{\ell_{n,2000}}$ is the share of industry s in CZ n's total employment in the year 2000, and $Y_{US,s,2000} + M_{US,s,2000} - X_{US,s,2000}$ is total US absorption of industry s production in the year 2000.

[15] Beraja (2022) estimates the policy functions using both OLS and instrumenting for wages and hours using model implied shocks. He finds similar results using both methods.

[16] The authors use that time frame as 2000 is the year before China enters the WTO and 2012 is sufficiently after the 2008 financial crisis that the volatility in global trade has subsided.

I then instrument for that import penetration using import penetration of China to eight other developed countries[17] and the share of employment in industry s commuting zone n in the year 1990,

$$\Delta IP_n^{IV} = \sum_s \frac{\ell_{n,s,1990}}{\ell_{n,1990}} \Delta IP_s^{oc},$$

where $\Delta IP_s^{oc} = \Delta M_{china,oc,s}/(Y_{US,s,1997} + M_{US,s,1997} - X_{US,s,1997})$ following Autor et al. (2021). Then I interpret the predicted exposure as the negative demand shock to the traded output of CZ n, normalized by the traded share, $-\hat{\phi}_n(1-\alpha)$. I plot the distribution of shocks in Figure 3.

I ignore the exposure of the US to Chinese imports as the increase in US exports to China are much smaller than the growth in China's exports to the US, and Adao et al. (2019) find negligible effects of import exposure on wages and the employment rate.

Wage Rigidity. I calibrate the degree of wage rigidity to match the impulse response function of working age population and unemployment to the China shock with population and the labor wedge in the model. I use the controls in Autor et al. (2021) for all regressions. In the model, I assume that the households are continuously surprised by the China shock, though the results do not materially depend on that assumption.

I find a wage Calvo friction of 0.189, which is very sticky wages. This is the case for a few reasons. Wage rigidity is the only source of nominal rigidity in the model, so this rigidity stands in for other rigidities like price rigidity and complementarities in price adjustments. There is also reason to believe that relative wages across commuting zones are stickier than absolute wages since many firms set national wages (Hazell et al., 2022). I discuss how optimal transfers vary with different levels of wage stickiness below.

Other Parameters. I set ρ to imply a real interest rate of 2%. I set the elasticity of substitution across intermediates ϵ to the value in Farhi and Werning (2017). I take a value of 2 for the Frisch labor supply elasticity η to be closer to the macro estimates of Peterman (2016). And finally, I set $\theta = 1$ implying log preferences.

6 Quantifying Optimal Place-based Transfers

I presented a New Keynesian economic geography model and calibrated it to the continental US in Section 5. In this section, I use this model to illustrate what the model implies

[17]The eight other countries are Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland.

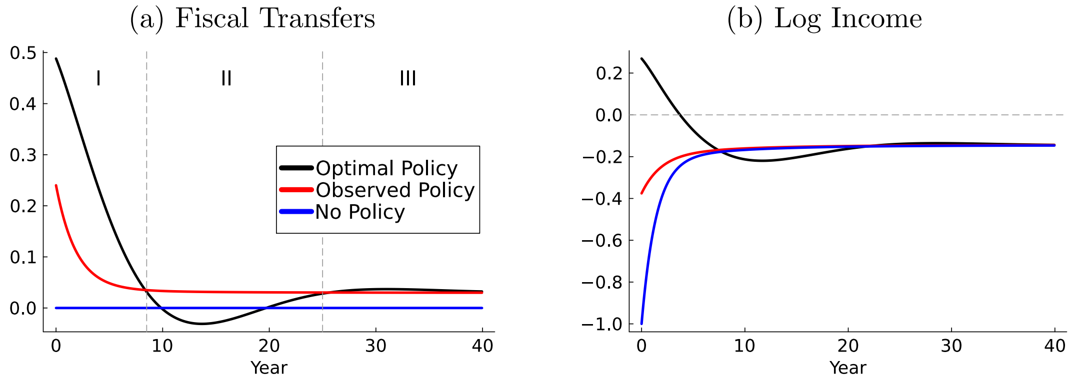


Figure 4: Policy Response

Note: This figure shows transfers and income in an average commuting zone after a shock to demand for tradable output under various policies. This is calculated assuming the rest of the country remains unchanged. All values are in log differences from the steady state except transfers which are relative to original earnings. Phase I corresponds to the stimulus effect dominating. Phase II corresponds to the migration effect dominating. In Phase III, only the redistribution effect matters for the transfer.

for optimal policy in response to a demand shock. Section 6.1 considers a one time idiosyncratic demand shock to a single commuting zone, like the one that Janesville experienced, to illustrate how the stimulus effect and migration effect interact in this dynamic setting. Sections 6.2 and 6.3 then explore the implications for policy after the China trade shock.

6.1 Idiosyncratic Demand Shock

I consider a commuting zone with the average amount of home bias in consumption and in migration. Larger locations will have stronger stimulus effects and weaker migration effects on average while smaller locations will have weaker stimulus effects and stronger migration effects. I then simulate a local recession by considering a drop in demand for traded output of 1 log point, $\hat{\phi}_m = -1$, assuming that every other location in the United States is unaffected. In the absence of any transfers, this implies a 1 log point drop in earnings from the traded and non-traded sectors on impact, though after wages adjust, income will recover. The model is log linear, so all results can be scaled up or down to consider a different sized recession.

I plot the model implied optimal policy in Figure 4. Figure 4a plots the time path of the optimal transfers relative to earnings in the steady state. There are three distinct phases to the optimal transfer labeled in the figure that roughly correspond to each of the three effects transfers have: stimulus, migration, and finally, redistribution.

I will start by discussing Phase III. In phase III, wages and population have mostly adjusted and transfers do not have a large impact on the period when the region is in a recession. Therefore, there is no longer any macroeconomic reason for policy. Instead, this

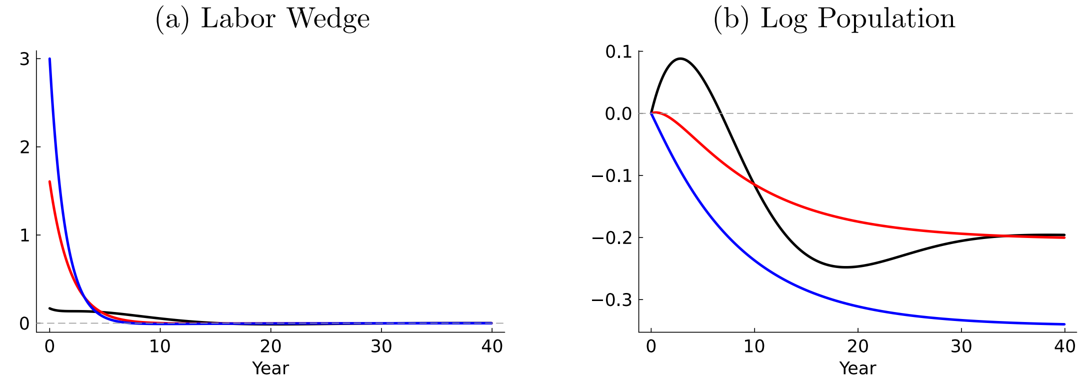


Figure 5: Effect of Policy Response

Note: This figure shows the labor wedge and log population in an average commuting zone after a shock to demand for tradable output under various policies. This is calculated assuming the rest of the country remains unchanged. All values are in log differences from the steady state.

transfer optimally trades off redistribution to households that are now poorer, due to the shock, against the misallocation that comes from worker migration response as explored in Gaubert et al. (2021).

Phase I lasts for the first 9 years after the shock. In this phase, the stimulus effects of the transfer dominate. Immediately after the demand shock, there is a large amount of underemployment, but people do not have time to move in response to government policy, so the government can get free stimulus by giving people a check immediately upon being laid off. Thus, optimal transfers jump to around 0.5 log points of original income. In fact, the transfers are so large, one can see in Figure 4b that total income of the region actually increases. That is because, immediately after the shock, migration cannot respond. Therefore, transfers only have two effects: redistribution and stimulus. Redistribution would suggest that the planner should exactly make up for the lost income so that the marginal utility of consumption remains the same. However, at that level of spending, the household is still working less than he would like as he is not working as much in the traded sector. Therefore, the planner would like to give extra money for the added stimulus. As shown in Figure 5, these generous transfers greatly reduce the labor wedge and actually lead to an increase in population at the location. Thus, the optimal transfers initially slow the transition after the shock. Optimal transfers then taper in size as the migration effect of the transfer becomes more important.

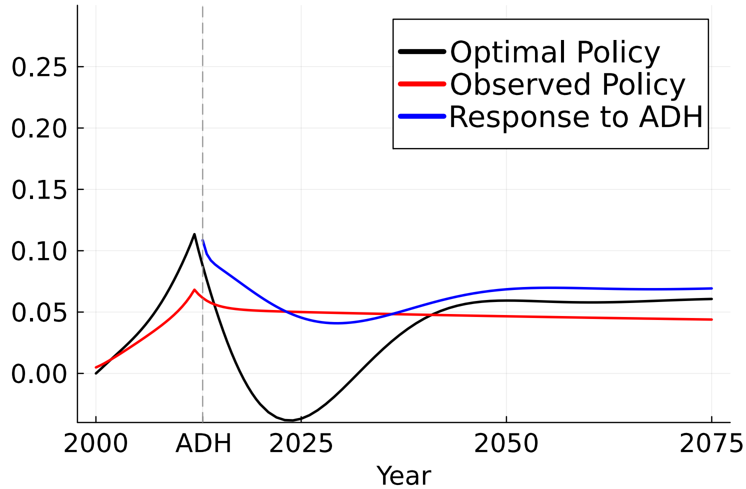
In phase II, the migration effect of the transfer dominates, consistent with Proposition 3. 9 to 25 years after the shock, the optimal transfer falls below the long-run redistributive level. That is because, after the demand shock, the planner commits to an entire time path of fiscal transfers. The planner promises very generous transfers in the immediate aftermath of the shock, but she also includes a promise to tax people who stay in the commuting zone in the medium run. This limits the increase in population in response to the stimulus transfers, and further, speeds up the transition out of the region 10 years after the shock. Thus, the planner can have her cake and eat it too. She can get the immediate stimulus with the front loaded transfers while still encouraging workers to find work elsewhere through the promise of less generous transfers in the medium run.

By comparison, the observed transfers are not generous enough immediately after the shock and are too generous 10 to 25 years afterwards. Thus, underemployment jumps much higher than under the optimal policy, and population slowly adjusts over the following 40 years. The policies only achieve 59% of the welfare gains that the fully optimal policies achieve over no policy at all. However, a planner restricted to policies that depend only on the hours employed and wages could achieve up to 95.6% of the welfare gains by having transfers depend very little on wages, $\gamma^w = -0.05$, and more than making up for lost income due to decreased hours, $\gamma^H = -1.7$. This set of transfers mostly sacrifices long-run redistribution because hours mostly recover in the model, but achieves most of the optimal gains by offering generous transfers that fade out over the next 10 years. Making unemployment insurance that generous would have other distortionary effects, but this suggests that transferring money to regions with high unemployment could be helpful in fighting local recessions.

Robustness. The key insights about optimal policy are robust to alternate parameters and assumptions. I show how the optimal policy varies with the share of non-traded goods, the long-run migration elasticity, the Poisson arrival rate of moving opportunities, and the wage Calvo friction in Appendix I.1. I will briefly discuss how changing the degree of wage rigidity affects the results here. Increasing the speed of wage adjustment decreases the size and importance of phase II. When $\delta_w = 0.5$, the transfer drops only slightly below the long-run level around year 10, while if $\delta_w = 1$, phase II disappears entirely. However, the basic insight that optimal transfers should be generous immediately after the shock and decrease in size quickly remains robust.

Including imperfect labor mobility across sectors would weaken the stimulus effect of the transfer as the planner would not be interested in putting traded workers back to work in the non-traded sector. However, on impact, non-traded workers end up unemployed because of demand spillovers, so the planner would always have reason to at least make up the lost income. If workers are partially mobile between sectors, the planner would want to more than make up for the lost income. Thus, the observed transfers immediately after a shock are still too small. In the medium-run, weakening the stimulus effect would increase the relative importance of the migration effect, making the optimal policy drop more in Phase

Figure 6: Optimal Policy Response to China Trade Shock



Note: This figure plots the coefficients of a regression of transfers relative to original earnings on the size of the China shock for each time t, weighting by pre-shock population, just as in Autor et al. (2021). I plot the optimal transfers when the planner knows the entire China shock starting in the year 2000 in blue. In red, I plot the optimal transfers when they follow the estimated policy rule. And in blue I plot the optimal policy if transfers were to follow the estimated policy rule before 2013 and then set optimal place-based transfers afterwards.

II.

Including optimal savers would also weaken the stimulus effect, but in a slightly different way. In Phase II, the relative strength of the migration effect becomes stronger, leading to a larger drop in the optimal transfer. However, on impact, the planner still wants to put people back to work as much as possible. Therefore, the optimal transfer actually becomes even higher immediately after the shock as the planner needs to give more money to get the same stimulus, though the transfer will scale back quickly.

6.2 Average Policy for China Trade Shock

Having provided some intuition for how transfers should respond to a one-time, idiosyncratic shock, I next turn to assess how policy could have responded to the China trade shock. I calculate the optimal policy assuming that starting in the year 2000, everyone knows the entire time path of the shock, and the planner announces the entire time path of all transfers. As this is somewhat unrealistic, I consider an alternate exercise where the planner does not implement the optimal policy until 2013, when Autor et al. (2013) first came out.

I start by plotting the average optimal policy directed at commuting zones affected by the China trade shock. I regress the optimal transfer, as a share of initial earnings, to each CZ on the size of the shock to the region's demand for traded output to make the results comparable to Section 6.1. I weight each CZ by initial population before the shock, just as Autor et al. (2021) do. I plot the results in Figure 6.

The time path of the transfers is significantly different from that found in Section 6.1 because starting in the year 2000, the planner expects future shocks. Therefore, the planner does not want to encourage too many households to enter the CZs hit by the China shock before the worst of the shock hits. Thus, the optimal transfer starts small and slowly builds until 2012 when the China shock stops intensifying. At that point, the optimal transfers start to fall. Just as in the idiosyncratic case, the optimal transfers fall below their long-run redistributive value before recovering.

Alternatively, if the government responds after Autor et al. (2013) is published, transfers jump in 2013. The planner then reduces transfers relatively slowly as he does not need to worry about the effect of these transfers on migration decisions before 2013. The optimal transfers fall only slightly below optimal redistributive transfers before recovering, in contrast to Section 6.1 and the fully optimal transfers.

Compared to the optimal transfers, observed transfers are not generous enough on impact of the shock. Too many households end up underemployed without much impact on their incentive to go find a job in another location. Furthermore, the observed transfers do not scale back enough around the year 2025. Thus, the increase in disability insurance that Autor et al. (2013) find likely hurt the macroeconomic stability of the regions hit by the China shock.

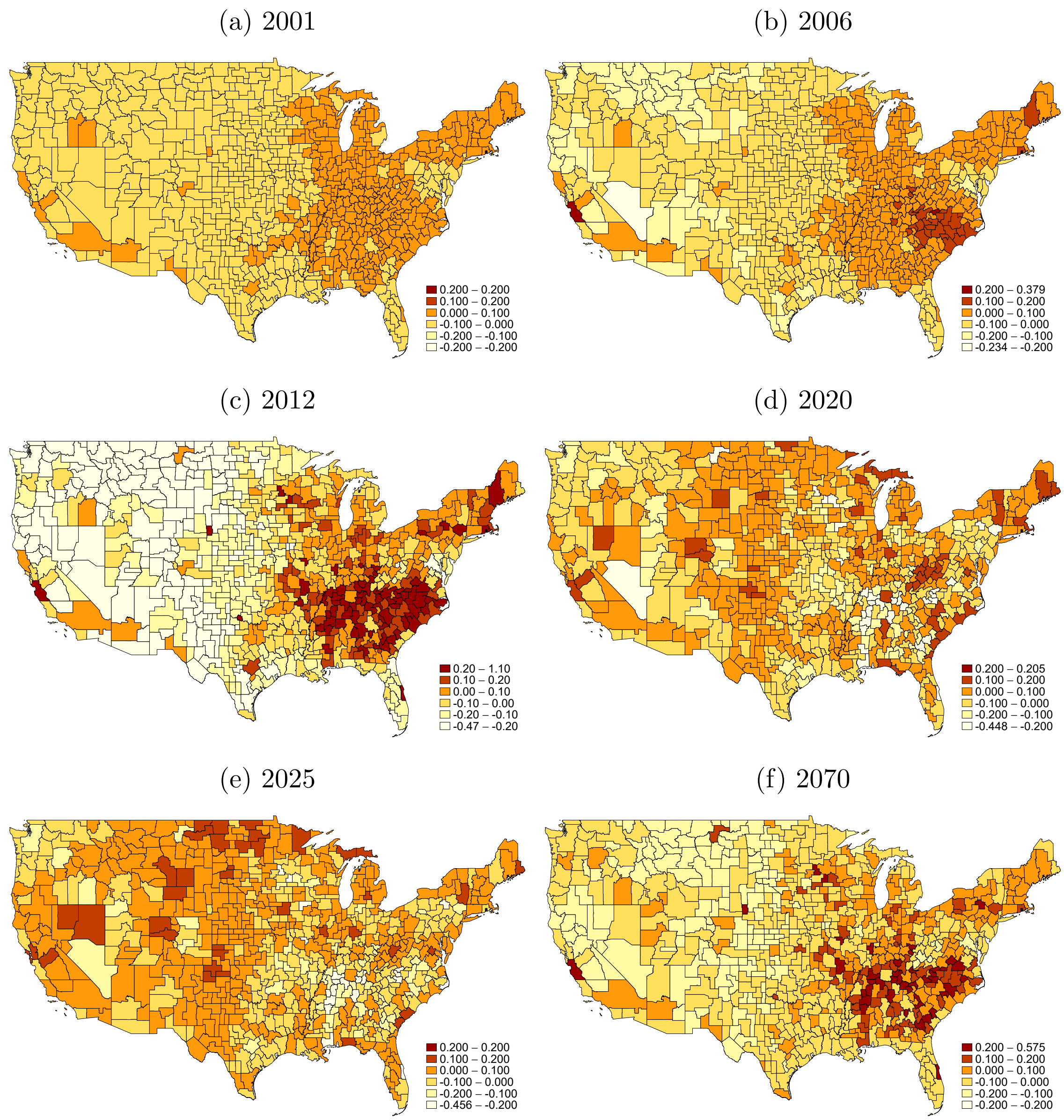
6.3 The Geography of Optimal Policy

The average policy hides a significant amount of spatial heterogeneity in the optimal policy, as suggested by Proposition 2. I plot some of that heterogeneity in Figure 7.

In the year 2001, as Figure 7a shows, the planner transfers money towards CZs that are either directly impacted by the China shock or are nearby. Thus, most of the eastern seaboard sees transfers from the national government. Even West Virgina, though it was not hit hard by the shock, gets relatively large transfers. That is because West Virginians disproportionately spend money on nearby regions that were hit by the shock and so a transfer has a sitmulus effect. At the same time, those transfers encourage workers to leave the areas that are directly hit to move to West Virginia before the peak of the shock. Those transfers then become more generous by 2006, especially in the Carolinas where a large number of CZs were hit.

By 2012, optimal transfers are narrowly targeted towards those CZs directly hit by the shock as the stimulus effect is at its strongest. Those transfers then scale back relatively

Figure 7: 100× Optimal Transfer Relative to Original Income



Note: This figure shows the geography of optimal transfers in response to the China trade shock for commuting zones at various years.

quickly, but the planner does not abandon the shocked regions. Instead, the planner sends generous transfers to nearby regions like West Virginia, North Michigan, Northern Minnoesota, Upstate New York, and the East coast of South Carolina in 2020 and 2025. All of these regions are just outside of the worst of the China shock and so have the dual migration and stimulus effect. By 2070, the planner offers redistributive transfers towards the regions hit by the China.

The observed policy achieves only 13.4% of the welfare gains granted by the fully optimal policy. Just as with the the one location case, this is partially because the transfers are not sufficiently generous during and immediately after the shock while also being too generous ten years afterwards. However, implementing the optimal policy rule that depends only on hours and wages from Section 6.1 achieves only 35.4% of the welfare gains of the optimal policy while being sufficiently generous during the shock, as I show in Appendix I.2. Much of the gains from the optimal policy comes from targeting those regions around where the shock hit. Thus, a large correlated shock might require a more coordinated policy.

7 Concluding Remarks

Regions are subject to idiosyncratic shocks. Changes in trade policies can lead to large shifts in demand. Economic structural change can make the product one location produces less enticing. And idiosyncratic shocks to individual firms can end up greatly hurting a town. Central governments cannot use monetary policy to fight the resulting local recessions, but it can use other policies.

In this paper, I focused on one key market failure that shapes how regions respond to these shocks: wage rigidity. In such a case, I have shown that place-based transfers can be used to fight the resulting local recession. The resulting optimal transfers should be aggressive, but short lived. For idiosyncratic shocks, more generous unemployment insurance could provide the necessary stimulus without distorting location choice greatly. More aggregate shocks likely call for a more coordinated response across space and time.

My analysis leaves many questions unanswered. Are there other tools available to a central government for fighting local recessions? What if households lose skills from not working? Can retraining programs work to stimulate the local economy without distorting migration decisions? When can a commuting zone reinvent itself and rebuild demand for its traded output? I hope to address these topics in future research.

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A Empirical Details

A.1 Data Details

Local Area Unemployment Statistics (LAUS). The Local Area Unemployment Statistics are maintained by the Bureau of Labor Statistics (BLS) and provide counts of the labor force, the number of employed workers, and the number of unemployed workers by county in the United States for the years 1990-2023.

For Los Angeles County, New York City, Chicago-Naperville-Arlington Heights, Cleveland-Elyria, Detroit-Warren-Dearborn, Miami-Miami Beach-Kendall, and Seattle-Bellevue-Everett, the BLS constructs the counts by smoothing out the responses from the Current Population Survey (CPS). They assume that in any given month, the reported unemployment in the CPS has some measurement error. They then model how the true values move around with some autocorrelation and back out an estimate.

For every other county, the BLS use an approach known as the Handbook method. The total employment estimate comes from the Current Employment Statistics (CES) survey and the Quarterly Census of Employment and Wages (QCEW) which are designed to find non-farm employment. For the remaining employment, they use CPS estimates combined with ACS estimates. The count of unemployment primarily comes from the Unemployment Insurance system. Those covered by the UI system are counted. The BLS then includes estimates of how many who are still unemployed but no longer qualify for benefits. For those who are never covered, the BLS uses the CPS. All series are then adjusted so that they sum up to be consistent with the state-level data.

Details can be found at https://www.bls.gov/opub/hom/lau/calculation.htm. The data can be downloaded from https://www.bls.gov/lau/data.htm.

Regional Economic Accounts (REA). The Regional Economic Accounts (REA) are maintained by the Bureau of Economic Analysis (BEA). I use information on earnings, population, employment, and government transfers from the REA. Estimates of total population come from the Census Bureau. For earnings, I use net earnings by place of residence plus dividends, interest, and rent. Net earnings by place of residence includes all compensation of employees and proprietor's income in a county less the employer contribution to government social insurance with an adjustment for where people live rather than work. I use their count of total employment for employment. The government transfers are discussed in the text.

See https://www.bea.gov/resources/methodologies/local-area-personal-income-employment for details.

Current Population Survey (CPS). I use data on wage earnings from the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS) which I download from IPUMS. I use the variable INCWAGE where respondents are asked their pre-tax wage and salary income in the previous year. I use the variable WKSWORK1 where respondents are asked how many weeks they worked in the previous year.

Internal Revenue Service (IRS) Statistics of Income (SOI). The IRS creates the Statistics of Income (SOI) by county based on the addresses reported on the individual income tax returns field. Data on income is available for the years 1989-2021. Data on total income tax paid by county begins in 2010. I use the variables A06500 for total income tax and I add in the variables for the tax credis included in government transfers so as to not double count them. I use data on total state and local taxes paid from the variables A18425, A18450, and A18500. The data can be easily downloaded from https://www.irs.gov/statistics/soi-tax-stats-county-data.

US County Population Data. I download data on US population by county broken up by age from National Cancer Institute Surveillance, Epidemiology, and End Results Program. It is available at https://seer.cancer.gov/popdata/.

Aggregation All data is aggregated up to the 1990 commuting zone level following Tolbert and Sizer (1996) and Autor et al. (2013). The crosswalks I use are available at https://www.ddorn.net/data.htm.

A.2 Other Variable Responses to Unemployment Innovations

Here I include the impulse response functions for wages and population after an innovation in unemployment. As in the main text, I normalize all variables to correspond to a 10 percentage point jump in the unemployment rate.

I start by looking at how wage earnings adjust to the shock. My main regression specification is:

$$\log E^w_{i,t+h} = \delta^h \log \text{weeks}_{i,t+h} + \beta_h u_{n(i)t} + \gamma^h_{n(i)} + \gamma^h_t + \sum_{L=1}^{\overline{L}} \gamma^h_{uL} u_{n(i),t-L} + \Gamma^h X_{ith} + \varepsilon^w_{ith},$$

where E^w_{it} is the wage earnings of individual i in year t, weeks_{it} is the number of weeks that individual worked, $u_{n(i)t}$ is the unemployment in i's commuting zone in year t, γ^h_n and γ^h_t are commuting zone and year fixed effects respectively, and X_{ith} is a vector of individual level controls for education, race, sex, industry, age, and age^2. CPS is relatively small, so I lack power to include state year fixed effects as I did in the main text. Therefore, I include commuting zone fixed effect and year fixed effects. The detailed demographic controls should control for differences across agents. Controlling for log weeks worked leaves the impulse response of weekly wage earnings.

I plot the estimates of β_h in Figure A1. I find that weekly wage earnings do not move at all the year of the increase in unemployment. Instead, weekly wage earnings in the commuting zone slowly decreases relative to earning in the rest of the US over the following 4 years before leveling off and recovering.

I next consider how population ℓ_{nt}, unemployment u_{nt}, earnings per capita X_{nt}, and employment per capita H_{nt} adjust in response to an innovation in unemployment. My main specification is

$$\log y_{n,t+h} = \beta_h u_{nt} + \gamma^h_n + \gamma^h_{s(n)t} + \sum_{L=1}^{\overline{L}} \gamma^h_{uL} u_{n,t-L} + \varepsilon^\ell_{nth},$$

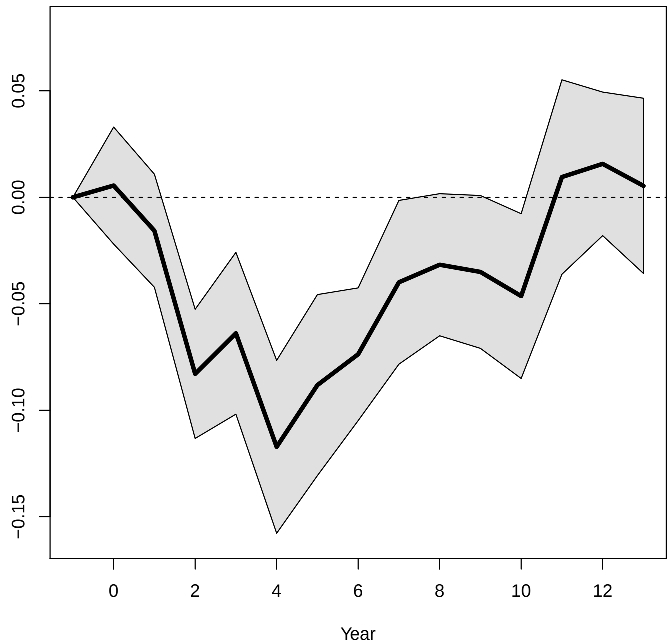


Figure A1: Log Wage Response

Note: This figure plots local Jorda projections of log wages in a commuting zone on innovations in local unemployment, respectively. Results are normalized to correspond to a jump in unemployment of 10 percentage points. Bands indicate 95% confidence intervals clustering on commuting zone.

for outcome $y_{n,t+h}$, γ_n^h is a commuting zone fixed effect, and $\gamma_{s(n)t}^h$ is a state-year fixed effect. Just as before, in my main specification I include 2 years of lagged unemployment ($\overline{L} = 2$), though the main results remain robust including more.

I plot the estimates of β_h in Figure A2.

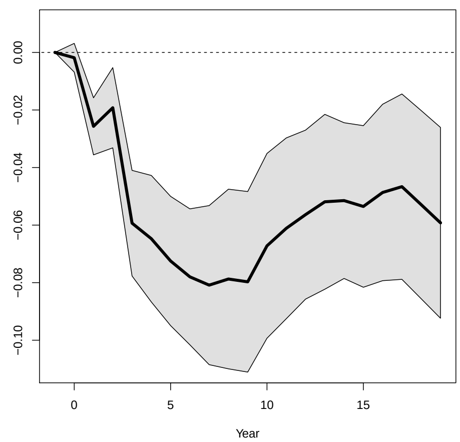
A.3 Medical Details

I include a graph showing the details of how medical transfers respond to an innovation in unemployment. The normalized transfers from Medicaid and Medicare are plotted in Figure A3.

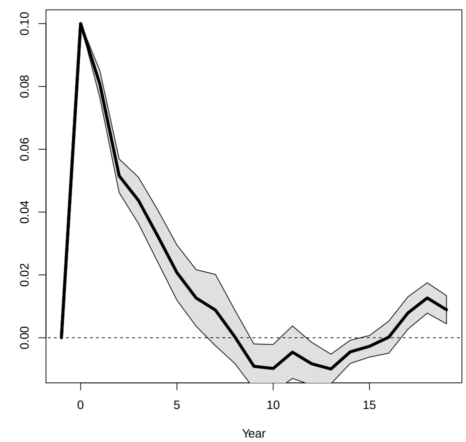
A.4 Not Controlling for Old Age

In this subsection, I present the details of how the Public Assistance Programs respond to a shock without controlling for the old age share of the region. The results are plotted in Figure A4. As one might expect, the size of the government transfers are larger, especially for later years. Furthermore, a much larger share of the transfers are explained by an increase in ret+dis transfers from the government. This is consistent with the findings of Autor et al.

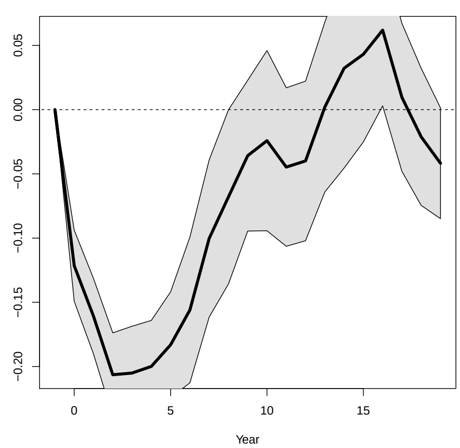
Figure A2: Impulse Response to an Innovation in Unemployment



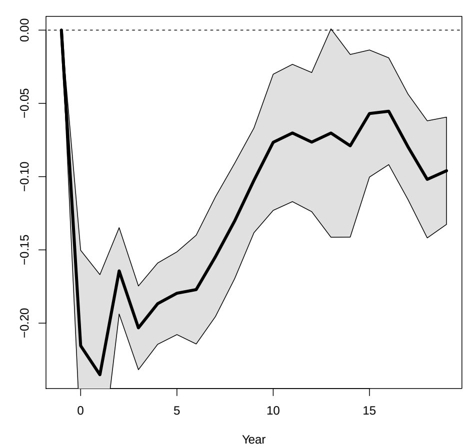
(a) Population



(b) Unemployment



(c) Earnings Per Capita



(d) Employment Per Capita

Note: This plots local Jorda projections of various economic variables in a commuting zone on innovations in local unemployment. Results are normalized to correspond to a jump in unemployment of 10 percentage points. Bands indicate 95% confidence interval clustering on state.

(2021) who find that most of the transfers from the government come through disability, retirement, and Medicare payments.

B Proofs for Section 4

Throughout, I make a few technical assumptions to ensure that the limits I take are well-defined. First, I assume that the labor wedge is bounded away from infinity and away

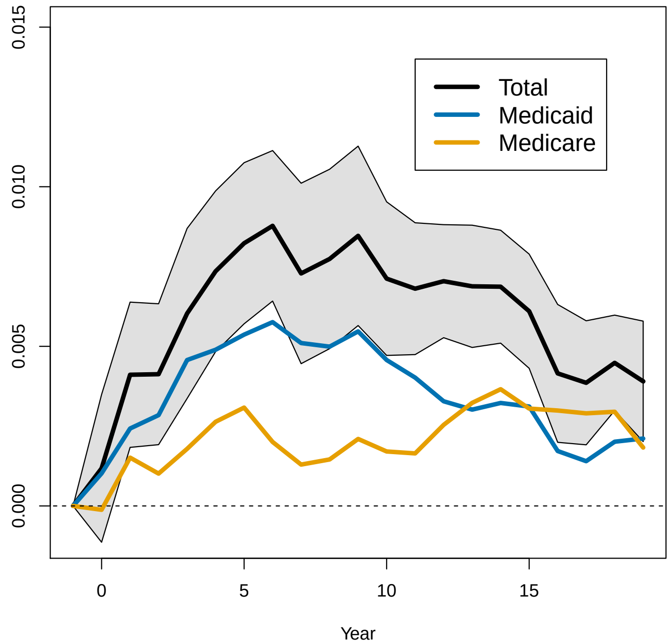


Figure A3: Medical Details

Note: This figure plots local Jorda projections of medical transfers in a commuting zone on innovations in local unemployment, respectively. Results are normalized to correspond to a jump in unemployment of 10 percentage points. Bands indicate 95% confidence intervals clustering on state.

from being too negative.

Assumption 1. *There exists a $\varepsilon_\tau > 0$ and $\overline{B}_\tau > 0$ such that, in any interior solution to* (SPP),

$$\tau_{nt} < \overline{B}_\tau$$

and

$$1 + \frac{\alpha_n}{1-\alpha_n}\tau_{nt} > \varepsilon_\tau.$$

This restricts my analysis to equilibria where regions are not booming or busting too much. The assumption that $1 + \frac{\alpha_n}{1-\alpha_n}\tau_{nt}$ is bounded away from 0 also implies that utility in each location is increasing in a transfer as $\frac{\partial V^n}{\partial T} + \frac{\partial V^n}{\partial H}\frac{\partial H^n}{\partial T} = \frac{U^n_C}{P_n}\left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{nt}\right)$.

The next assumption is that the migration semi-elasticities are all bounded away from infinity.

Assumption 2. *There exists a $\overline{B}_\ell > 0$ such that*

$$\left|\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\right| \leqslant \overline{B}_\ell; \quad \left|\frac{\partial \log \mu^{mn}}{\partial U_{n1}}\right| \leqslant \overline{B}_\ell.$$

I also assume that H_{Tnt} is bounded above and away from 0.

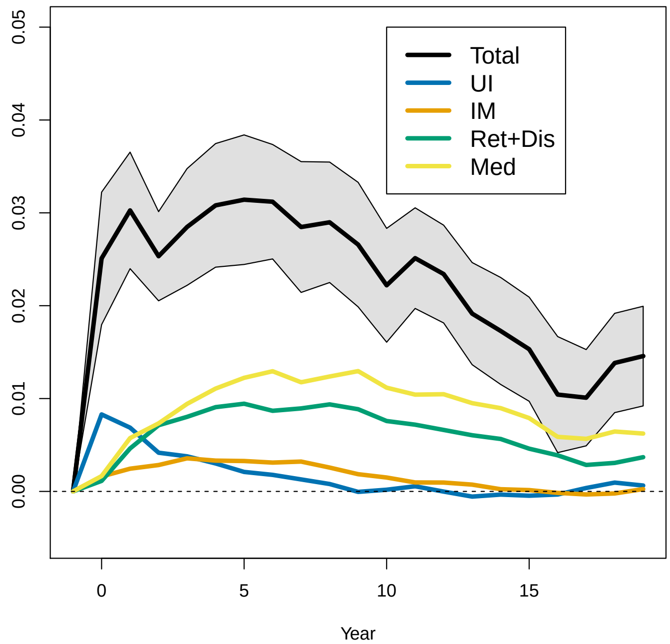


Figure A4: Government Transfer Impact on Log Income

Note:This figure plots the Jorda projections of log public assistance programs in a commuting zone on innovations in local unemployment without controlling for the old age share. Results are normalized to correspond to a jump in unemployment of 10 percentage points and the share of income that comes from the respective program. Bands indicate 95% confidence intervals clustering on state.

Assumption 3. *There exists a* $\overline{B}_H > 0$ *and* $\varepsilon_H > 0$ *such that, in any interior solution to* (SPP),

$$\varepsilon_H < H_{Tnt} < \overline{B}_H.$$

I assume that the optimal transfers are also bounded.

Assumption 4. *There exists a* $\overline{B}_T > 0$ *such that*

$$|T_{nt}| \leqslant \overline{B}_T,$$

in any interior solution to (SPP).

And I also assume that the marginal utility of expenditures is bounded away from 0 and infinity.

Assumption 5. *There exists* $\varepsilon_C > 0$ *and* $\overline{B}_C$ *such that*

$$\varepsilon_C \leqslant \frac{U_C^n}{P_n} \leqslant \overline{B}_C,$$

in all interior solution to (SPP).

I start by proving Lemma 1.

Lemma 1. *The derivatives of the indirect utility function are*

$$\frac{\partial V^n}{\partial T} = \frac{U_C^n}{P_n}; \qquad \frac{\partial V^n}{\partial H} = W_n \frac{U_C^n}{P_n} \tau_{nt}.$$

The derivatives of the hours worked function are

$$\frac{\partial H^n}{\partial T} = \frac{1}{W_n} \frac{\alpha_n}{1-\alpha_n}; \quad \frac{\partial H^n}{\partial \log E_T} = \frac{1}{W_n} \frac{\phi_n E_T}{1-\alpha_n} \frac{1}{\ell}; \quad \frac{\partial H^n}{\partial \log \ell} = -\frac{1}{W_n} \frac{\phi_n E_T}{1-\alpha_n} \frac{1}{\ell}.$$

Proof. The derivatives of the hours are trivial. Recall that

$$H^m(E_T, \ell, T) = \frac{1}{W_m} \left(\frac{\phi_m E_T}{1-\alpha_m} \frac{1}{\ell} + \frac{\alpha_m}{1-\alpha_m} T \right).$$

Then

$$\begin{aligned}
\frac{\partial H^m}{\partial T} &= \frac{1}{W_m} \frac{\alpha_m}{1-\alpha_m} \\
\frac{\partial H^m}{\partial \ell} &= -\frac{1}{W_m} \frac{\phi_m E_T}{1-\alpha_m} \frac{1}{\ell^2} \\
\frac{\partial H^m}{\partial E_T} &= \frac{1}{W_m} \frac{\phi_m}{1-\alpha_m} \frac{1}{\ell}.
\end{aligned}$$

These can then be rewritten to get the derivatives.

The derivatives for the indirect utility function are

$$\begin{aligned}
\frac{\partial V^n}{\partial T} &= \frac{d}{dT} \left[U^n \left(\frac{W_n}{P_n} H + \frac{T}{P_n}, H \right) \right] \\
&= \frac{U_C^n}{P_n} \\
\frac{\partial V^n}{\partial H} &= \frac{d}{dH} \left[U^n \left(\frac{W_n}{P_n} H + \frac{T}{P_n}, H \right) \right] \\
&= \frac{W_n}{P_n} U_C^n + U_H^n \\
&= W_n \frac{U_C^n}{P_n} \left(1 + \frac{P_n}{W_n} \frac{U_H^n}{U_C^n} \right) \\
&= W_n \frac{U_C^n}{P_n} \tau_{nt}.
\end{aligned}$$

□

Next, I confirm that choosing traded GDP is equivalent to choosing total GDP. In particular, I show that for any equilibrium, there is a one-to-one mapping from E_t to E_{Tt}. Therefore, choosing E_t is equivalent to choosing E_{Tt}.

Lemma 6. *In any equilibrium,*

$$E_t = \left[1 + \sum_n \frac{\alpha_n}{1-\alpha_n}\phi_n\right] E_{Tt} + \sum_n \frac{\alpha_n}{1-\alpha_n} T_{nt}\ell_{nt}.$$

Proof. Note that total spending on the non-traded sector in location n, E_{NTnt}, is α_n share of per capita income times the population. That is

$$\begin{aligned} E_{NTnt} &= \alpha_n \left(W_n H^n \left(E_{Tt}, \ell_{nt}, T_{nt}\right) + T_{nt}\right) \ell_{nt} \\ &= \alpha_n \frac{\phi_n E_{Tt}}{1-\alpha_n} + \frac{\alpha_n}{1-\alpha_n} T_{nt}\ell_{nt}, \end{aligned}$$

using the expression for hours, (11). Then total nominal GDP is,

$$\begin{aligned} E_t &= E_{Tt} + \sum_n E_{NTnt} \\ &= E_{Tt} + \sum_n \frac{\alpha_n}{1-\alpha_n}\phi_n E_{Tt} + \sum_n \frac{\alpha_n}{1-\alpha_n} T_{nt}\ell_{nt} \\ &= \left[1 + \sum_n \frac{\alpha_n}{1-\alpha_n}\phi_n\right] E_{Tt} + \sum_n \frac{\alpha_n}{1-\alpha_n} T_{nt}\ell_{nt}. \end{aligned}$$

□

I next turn to prove the lemmas associated with the first order necessary conditions.

Lemma 2. *In any interior solution to* (SPP)*,*

$$\sum_n \frac{W_n H_{Tn1}}{1-\alpha_n} \ell_{n1} \frac{\tau_{n1}}{1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1}} = 0.$$

Lemma 3. *In any interior solution to* (SPP)*, first period transfers must satisfy*

$$\underbrace{\sum_m \ell_{m1} T_{m1} \nu_{n1}^{m1}}_{\textit{fiscal externality}} = \ell_{n1}\left[\underbrace{\frac{\zeta_{n1}}{\lambda_{G1}} - 1}_{\textit{redistribution}} + \underbrace{\frac{\zeta_{n1}}{\lambda_{G1}} \frac{\alpha_n}{1-\alpha_n}\tau_{n1}}_{\textit{stimulus}}\right] - \underbrace{\sum_m \frac{W_m H_{Tm1}}{1-\alpha_m} \ell_{m1} \frac{\tau_{m1}}{1 + \frac{\alpha_m}{1-\alpha_m}\tau_{m1}} \nu_{n1}^{m1}}_{\textit{migration}},$$

where $\nu_{n1}^{m1} \equiv \frac{\partial \log \ell^{m1}}{\partial U_{n1}} \left(\frac{\partial V^n}{\partial T_{n1}} + \frac{\partial V^n}{\partial H_{n1}} \frac{\partial H^{n1}}{\partial T_{n1}}\right)$ *is the migration semi-elasticity of population in location m to a transfer in location n holding fixed utility in locations other than n, and $\lambda_1^G > 0$ is the social value of the government having another dollar.*

Lemma 4. *In any interior solution to* (SPP)*, second period transfers must satisfy*

$$\begin{aligned} \sum_t \frac{\lambda_{Gt}}{\lambda_{G2}} \sum_m \ell_{mt} T_{mt} \nu_{n2}^{mt} &= \ell_{n2}\left[\frac{\zeta_{n2}}{\lambda_{G2}} - 1 + \frac{\zeta_{n2}}{\lambda_{G2}} \frac{\alpha_n}{1-\alpha_n}\tau_{n2}\right] \\ &\quad - \sum_t \frac{\lambda_{Gt}}{\lambda_{G2}} \sum_m \frac{W_m H_{Tmt}}{1-\alpha_m} \ell_{mt} \frac{\tau_{mt}}{1 + \frac{\alpha_m}{1-\alpha_m}\tau_{mt}} \nu_{n2}^{mt}, \end{aligned}$$

where λ_{G2} is the social value value of the government having another dollar in period 2, and ν_{n2}^{mt} is the elasticity of population in location m at time t to a transfer to location i at time 2.

Lemma 5. *In any interior solution to* (SPP),

$$\sum_n \frac{W_n H_{Tn2}}{1-\alpha_n}\ell_{n2}\frac{\tau_{n2}}{1+\frac{\alpha_n}{1-\alpha_n}\tau_{n2}} = 0.$$

Proof. The planners problem is

$$\max_{E_{Tt},\{T_{nt}\},\{U_{nt}\},\{\ell_{nt}\}} \int_{\mathcal{I}} \lambda(i) \sum_n \mathbb{1}_{n\in\arg\max U_{n'1}+\varepsilon_{n'1}(i)+\beta\overline{U}_{n'2}} \left[U_{n1}+\varepsilon_{n1}(i)+\beta\overline{U}_{n2}\right] di$$

subject to the constraints

$$\begin{aligned}
\ell_{n1} &= \ell^{n1}\left(\left\{U_{n1}+\beta\overline{U}_{n2}\right\}\right), \\
\ell_{n2} &= \sum_m \ell_{m1}\mu^{mn}\left(\{U_{k2}\}\right), \\
U_{nt} &= \tilde{V}^n\left(T_{nt}, E_{Tt}, \ell_{nt}\right), \\
\sum_n \ell_{nt}T_{nt} &= 0,
\end{aligned}$$

where $\tilde{V}^n(T, E_T, \ell) = V^n(T, H(T, \ell, E_T))$. I can then take the first order conditions. This gives

$$\begin{aligned}
U_{n1} &: 0 = \overline{\lambda}_{n1}\ell_{n1} + \sum_m \lambda^{\ell}_{m1}\frac{\partial \ell^{m1}}{\partial U_{n1}} - \lambda^V_{n1} \\
U_{n2} &: 0 = \beta\overline{\lambda}_{n2}\ell_{n2} + \sum_m \lambda^{\ell}_{m1}\beta\frac{\partial \ell^{m1}}{\partial U_{n1}}\mu_{mn} + \sum_m\sum_k \lambda^{\ell}_{m2}\ell_{k1}\frac{\partial \mu^{kj}}{\partial U_{n2}} - \lambda^V_{n2} \\
\ell_{n1} &: 0 = -\lambda^{\ell}_{n1} + \sum_m \lambda^{\ell}_{m2}\mu_{nm} + \lambda^V_{n1}\frac{\partial \tilde{V}^n}{\partial \ell_{n1}} - \lambda_{G1}T_{n1} \\
\ell_{n2} &: 0 = -\lambda^{\ell}_{n2} + \lambda^V_{n2}\frac{\partial \tilde{V}^n}{\partial \ell_{n2}} - \lambda_{G2}T_{n2} \\
T_{n1} &: 0 = \lambda^V_{n1}\frac{\partial \tilde{V}^n}{\partial T_{n1}} - \lambda_{G1}\ell_{n1} \\
T_{n2} &: 0 = \lambda^V_{n2}\frac{\partial \tilde{V}^n}{\partial T_{n2}} - \lambda_{G2}\ell_{n2},
\end{aligned}$$

where λ^V_{nt} is the Lagrange multiplier on the utility constraint, λ^{ℓ}_{nt} is the Lagrange multiplier on the population constraint, λ_{Gt} is the Lagrange multiplier on the government budget constraint, and $\mu_{nm} \equiv \mu^{mn}\left(\{U_{k2}\}\right)$. Then in the first order conditions for the transfers, we can solve for λ_{nt}^V and then substitute in for it in the other equations. That gives

$$\begin{aligned}
U_{n1} : 0 &= \overline{\lambda}_{n1}\ell_{n1} + \sum_m \lambda_{m1}^\ell \frac{\partial \ell^{m1}}{\partial U_{n1}} - \frac{\lambda_{G1}\ell_{n1}}{\frac{\partial \tilde{V}^n}{\partial T_{n1}}} \\
U_{n2} : 0 &= \beta\overline{\lambda}_{n2}\ell_{n2} + \sum_m \lambda_{m1}^\ell \beta \frac{\partial \ell^{m1}}{\partial U_{n1}}\mu_{mn} + \sum_m\sum_k \lambda_{m2}^\ell \ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}} - \frac{\lambda_{G2}\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}} \\
\ell_{n1} : 0 &= -\lambda_{n1}^\ell + \sum_m \lambda_{m2}^\ell \mu_{nm} + \frac{\lambda_1^G \ell_{n1}}{\frac{\partial \tilde{V}^n}{\partial T_{n1}}}\frac{\partial \tilde{V}^n}{\partial \ell_{n1}} - \lambda_{G1}T_{n1} \\
\ell_{n2} : 0 &= -\lambda_{n2}^\ell + \frac{\lambda_{G2}\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}\frac{\partial \tilde{V}^n}{\partial \ell_{n2}} - \lambda_{G2}T_{n2}.
\end{aligned}$$

Then we look to take the limit $\beta \to 0$. Turning to the first order conditions for the second period, note that the first order condition for ℓ_{n2} is,

$$0 = -\lambda_{n2}^\ell + \frac{\lambda_{G2}\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}\frac{\partial \tilde{V}^n}{\partial \ell_{n2}} - \lambda_{G2}T_{n2}$$

and the first order condition for U_{n2},

$$0 = o(\beta) + \sum_m\sum_k \lambda_{m2}^\ell \ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}} - \frac{\lambda_{G2}\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}},$$

since the migration semi-elasticity is bounded by assumption 2. Then substituting out λ_{n2}^ℓ, we find that

$$0 = o(\beta) + \sum_m\sum_k \left[\frac{\lambda_{G2}\ell_{m2}}{\frac{\partial \tilde{V}^m}{\partial T_{m2}}}\frac{\partial \tilde{V}^m}{\partial \ell_{m2}} - \lambda_{G2}T_{m2}\right]\ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}} - \frac{\lambda_{G2}\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}.$$

Summing across all n implies that

$$\begin{aligned}
0 &= o(\beta) + \sum_n \left\{\sum_m\sum_k \left[\frac{\lambda_{G2}\ell_{m2}}{\frac{\partial \tilde{V}^m}{\partial T_{m2}}}\frac{\partial \tilde{V}^m}{\partial \ell_{m2}} - \lambda_{G2}T_{m2}\right]\ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}} - \lambda_{G2}\ell_{n2}\frac{1}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}\right\} \\
&= o(\beta) + \sum_m \left[\frac{\lambda_{G2}\ell_{m2}}{\frac{\partial \tilde{V}^m}{\partial T_{m2}}}\frac{\partial \tilde{V}^m}{\partial \ell_{m2}} - \lambda_{G2}T_{m2}\right]\left(\sum_n\sum_k \ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}}\right) - \lambda_{G2}\sum_n \ell_{n2}\frac{1}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}.
\end{aligned}$$

Then note that because idiosyncratic utility shocks are additive, a uniform increase in utility across all locations does not change population, $\left(\sum_n\sum_k \ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}}\right) = 0$. Therefore,

$$0 = o(\beta) - \lambda_{G2}\sum_n \ell_{n2}\frac{1}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}.$$

And since $\frac{\partial \tilde{V}^n}{\partial T_{n2}} = \frac{U_C^n}{P_n}\left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1}\right)$ is bounded away from infinity and zero by assumptions 1 and 5, $\lambda_{G2} \in o(\beta)$. And therefore, $\lambda_{n2}^{\ell} = \frac{\lambda_{G2}\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}\frac{\partial \tilde{V}^n}{\partial \ell_{n2}} - \lambda_{G2}T_{n2} \in o(\beta)$.

Therefore, focusing on the first period we can solve for the Lagrange multiplier on population in the first period

$$\lambda_{n1}^{\ell} = \frac{\lambda_{G1}\ell_{n1}}{\frac{\partial \tilde{V}^n}{\partial T_{n1}}}\frac{\partial \tilde{V}^n}{\partial \ell_{n1}} - \lambda_{G1}T_{n1} + o(\beta).$$

Plugging this into the first order condition for utility gives

$$\frac{\lambda_1^G \ell_{n1}}{\frac{\partial \tilde{V}^n}{\partial T_{n1}}} = \overline{\lambda}_{n1}\ell_{n1} + \sum_m \left[\frac{\lambda_{G1}\ell_{m1}}{\frac{\partial \tilde{V}^m}{\partial T_{m1}}}\frac{\partial \tilde{V}^m}{\partial \ell_{m1}} - \lambda_{G1}T_{m1} + o(\beta)\right]\frac{\partial \ell^{m1}}{\partial U_{n1}}.$$

Rewriting slightly, taking as given that $\lambda_{G1} > 0$ for now,

$$\sum_m \ell_{m1}T_{m1}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\frac{\partial \tilde{V}^n}{\partial T_{n1}} = \ell_{n1}\left(\frac{\overline{\lambda}_{n1}}{\lambda_{G1}}\frac{\partial \tilde{V}^n}{\partial T_{n1}} - 1\right) + \sum_m \ell_{m1}\frac{\frac{\partial \tilde{V}^m}{\partial \log \ell_{m1}}}{\frac{\partial \tilde{V}^m}{\partial T_{m1}}}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\frac{\partial \tilde{V}^n}{\partial T_{n1}} + o(\beta).$$

Substituting in,

$$\frac{\partial \tilde{V}^n}{\partial T} = \frac{U_C^n}{P_n}\left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{nt}\right)$$
$$\frac{\partial \tilde{V}^n}{\partial \log \ell} = -\frac{\phi_n E_T}{1-\alpha_n}\frac{1}{\ell_n}\frac{U_C^n}{P_n}\tau_{nt},$$

and $\nu_{n1}^{m1} \equiv \frac{\partial \log \ell^{m1}}{\partial U_{n1}}\frac{\partial \tilde{V}^n}{\partial T_{n1}}$ and taking the limit as $\beta \to 0$ then completes the proof of the formula in Lemma 3. Next, I note that if the government were to increase transfers to location n by $\frac{\overline{T}}{\frac{\partial \tilde{V}^n}{\partial T}} > 0$, then utility in every location increases by $\frac{\partial \tilde{V}^n}{\partial T}\frac{\overline{T}}{\frac{\partial \tilde{V}^n}{\partial T}} = \overline{T}$. Then since utility shocks are additive, no one changes where they live and utility increases in every location. Therefore, the Lagrange multiplier on the budget constraint must be positive, i.e. $\lambda_{G1} > 0$.

Next I take the first order condition with respect to E_{T1}. That gives

$$E_{T1} : 0 = \sum_n \lambda_{n1}^V \frac{\partial \tilde{V}^n}{\partial E_T}.$$

Substituting in for λ_{n1}^V then implies

$$\sum_n \lambda_{G1}\ell_{n1}\frac{\frac{\partial \tilde{V}^n}{\partial E_T}}{\frac{\partial \tilde{V}^n}{\partial T_{n1}}} = 0.$$

Plugging in the derivatives then completes the proof of Lemma 2.

Next we return to the second period first order conditions. Dividing those equations by

β gives

$$0 = \overline{\lambda}_{n2}\ell_{n2} + \sum_m \lambda^{\ell}_{m1}\ell_{m1}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\mu_{mn} + \sum_m\sum_k \frac{\lambda^{\ell}_{m2}}{\beta}\ell_{k1}\mu_{km}\frac{\partial \log \mu^{km}}{\partial U_{n2}} - \frac{\lambda_{G2}}{\beta}\frac{\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}$$

$$\frac{\lambda^{\ell}_{n2}}{\beta} = \frac{\lambda_{G2}}{\beta}\frac{\frac{\partial \tilde{V}^n}{\partial \log \ell_{n2}}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}} - \frac{\lambda_{G2}}{\beta}T_{n2}.$$

We can then plug in for $\frac{\lambda^{\ell}_{n2}}{\beta}$ and λ^{ℓ}_{m1}. Therefore

$$\frac{\lambda_{G2}}{\beta}\frac{\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}} = \overline{\lambda}_{n2}\ell_{n2} + \sum_m \left[\frac{\lambda_{G1}}{\frac{\partial V^m}{\partial T_{m1}}}\frac{\partial V^m}{\partial \log \ell_{m1}} - \lambda_{G1}T_{m1} + o(\beta)\right]\ell_{m1}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\mu_{mn}$$
$$+ \sum_m\sum_k\left[\frac{\lambda_{G2}}{\beta}\frac{\frac{\partial \tilde{V}^m}{\partial \log \ell_{m2}}}{\frac{\partial \tilde{V}^m}{\partial T_{m2}}} - \frac{\lambda_{G2}}{\beta}T_{m2}\right]\ell_{k1}\mu_{km}\frac{\partial \log \mu^{km}}{\partial U_{n2}}.$$

Then rewriting slightly

$$\sum_m \frac{\lambda_{G1}}{\lambda_{G2}/\beta}T_{m1}\ell_{m1}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\mu_{mn}\frac{\partial \tilde{V}_n}{\partial T_{n2}}$$
$$+\sum_m T_{m2}\ell_{m2}\left(\sum_k \frac{\ell_{k1}\mu_{km}}{\ell_{m1}}\frac{\partial \log \mu^{km}}{\partial U_{n2}}\frac{\partial \tilde{V}^n}{\partial T_{n2}}\right) = \ell_{n2}\left(\frac{\overline{\lambda}_{n2}}{\lambda_{G2}/\beta}\frac{\partial \tilde{V}^n}{\partial T_{n2}} - 1\right) + o(\beta)$$
$$+\sum_m \frac{\lambda_{G1}}{\lambda_{G2}/\beta}\frac{\frac{\partial V^m}{\partial \log \ell_{m1}}}{\frac{\partial V^m}{\partial T_{m1}}}\ell_{m1}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\mu_{mn}\frac{\partial \tilde{V}^n}{\partial T_{n2}}$$
$$+\sum_m \frac{\frac{\partial V^m}{\partial \log \ell_{m2}}}{\frac{\partial V^m}{\partial T_{m2}}}\ell_{m2}\left(\sum_k \frac{\ell_{k1}\mu_{km}}{\ell_{m1}}\frac{\partial \log \mu^{km}}{\partial U_{n2}}\frac{\partial \tilde{V}^n}{\partial T_{n2}}\right).$$

Then to complete the proof, we plug in the derivative values and note that

$$\nu^{m1}_{n2} \equiv \beta\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\mu_{mn}\frac{\partial \tilde{V}^n}{\partial T_{n2}},$$

and

$$\nu^{m2}_{n2} = \sum_k \frac{\ell_{k1}\mu_{km}}{\ell_{m1}}\frac{\partial \log \mu^{km}}{\partial U_{n2}}\frac{\partial \tilde{V}^n}{\partial T_{n2}}.$$

Just as before, I can consider a deviation where the planner increases the transfers to every location by $\frac{\overline{T}}{\frac{\partial \tilde{V}^n}{\partial T}}$ in period 2 to conclude that $\lambda_{G2}/\beta > 0$.

Finally we turn to the first order condition for the monetary policy in period 2. We have

$$E_{T2} : 0 = \sum_n \lambda^V_{n2}\frac{\partial \tilde{V}^n}{\partial E_{T2}}.$$

Then we can plug in for the λ^V_{n2},

$$0 = \sum_n \ell_{n2} \frac{\frac{\partial \tilde{V}^n}{\partial E_{T2}}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}.$$

Plugging in the values for the derivatives proves the result. □

Next I turn to proving the propositions.

Proposition 1. *Suppose that there are two locations, j (Janesville) and u (Rest of the US), location j is arbitrarily small compared to location u, $\ell_{jt} \to 0$, and there are no redistributive reasons for policy, $\zeta_{nt} = 1$. Then in any interior solution to* (SPP)*, the optimal period 1 transfer to location j must satisfy*

$$T_{j1} = \frac{1}{\nu^{j1}_{j1}} \left(\frac{\alpha_j}{1-\alpha_j} - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}} \right) \tau_{j1},$$

where $\frac{\partial \log \ell^{j1}}{\partial T_{j1}} \equiv \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \frac{\partial V^j}{\partial T}$ is the semi-elasticity of location 1 population to a transfer, holding fixed hours worked, and $\nu^{j1}_{j1} \equiv \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \left(\frac{\partial V^j}{\partial T_{j1}} + \frac{\partial V^j}{\partial H_{j1}} \frac{\partial H^{j1}}{\partial T_{j1}} \right)$ is the semi-elasticity of location 1 population to a transfer, allowing hours to vary.

Proof. With no redistributive reasons for policy, $\frac{\overline{\lambda}_{n1} U^n_C}{P_n} = 1$. We then have that the budget constraint is

$$T_{u1}\ell_{u1} + T_{j1}\ell_{j1} = 0.$$

By Assumption 4, transfers are bounded, so that $T_{u1}\ell_{u1} + o(\ell_{j1}) = 0$, i.e. $T_{u1} \in o(\ell_{j1})$. Similarly, monetary policy is

$$\frac{W_j H_{Tj1}}{1-\alpha_j} \ell_{j1} \frac{\tau_{j1}}{1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1}} + \frac{W_u H_{Tu1}}{1-\alpha_u} \ell_{u1} \frac{\tau_{u1}}{1 + \frac{\alpha_u}{1-\alpha_u}\tau_{u1}} = 0.$$

By Assumptions 1 and 2, wedges and elasticities are bounded, so that

$$\frac{W_j H_{Tj1}}{1-\alpha_j} \ell_{j1} \frac{\tau_{j1}}{1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1}} \in o(\ell_{j1}),$$

and thereby, $\tau_{u1} \in o(\ell_{j1})$.

I next turn to find λ_{G1}. The key is to realize that taking the first order condition associated with the transfer to location n, multiplying by $\frac{1}{\frac{U^n_C}{P_n}\left(1+\frac{\alpha_n}{1-\alpha_n}\tau_{n1}\right)}$ and summing across all locations n cancels out the fiscal externality and migration effect. That is because, it leads to a uniform increase in utility across all locations so no one moves. It follows that

$$0 = \sum_n \ell_{n1} \frac{\overline{\lambda}_{n1}}{\lambda_{G1}} - \sum_n \frac{\ell_{n1}}{\frac{U^n_C}{P_n}\left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1}\right)}.$$

It follows that, with two locations,

$$\lambda_{G1}\left(\frac{\ell_{u1}}{\frac{U_C^n}{P_n}\left(1+\frac{\alpha_u}{1-\alpha_u}\tau_{u1}\right)}+o(\ell_{j1})\right)=\overline{\lambda}_{u1}\ell_{u1}+o(\ell_{j1}).$$

Since $\tau_{u1}\in o(\ell_{j1})$ and $\overline{\lambda}_{u1}\frac{U_C^n}{P_n}=1$, it follows that $\lambda_{G1}=1+o(\ell_{j1})$.

Next note that, with 2 locations

$$\begin{aligned}\nu_{j1}^{j1}&=\frac{1}{\ell_{j1}}\frac{\partial\ell^{j1}}{\partial U_{j1}}\left(\frac{\partial V^j}{\partial T_{j1}}+\frac{\partial V^j}{\partial H_{j1}}\frac{\partial H_{j1}}{\partial T_{j1}}\right)\\&=-\frac{1}{\ell_{j1}}\frac{\partial\ell^{u1}}{\partial U_{j1}}\left(\frac{\partial V^j}{\partial T_{j1}}+\frac{\partial V^j}{\partial H_{j1}}\frac{\partial H_{j1}}{\partial T_{j1}}\right)\\&=-\frac{\ell_{u1}}{\ell_{j1}}\nu_{j1}^{u1}.\end{aligned}$$

Then I can return to the optimal transfer to Janesville,

$$\begin{aligned}\ell_{j1}T_{j1}\nu_{j1}^{j1}+\ell_{u1}T_{u1}\nu_{j1}^{u1}=\ell_{j1}&\left[\frac{1}{\lambda_{G1}}-1+\frac{1}{\lambda_{G1}}\frac{\alpha_j}{1-\alpha_j}\tau_{j1}\right]\\&-\frac{W_uH_{Tu1}}{1-\alpha_u}\ell_{u1}\frac{\tau_{u1}}{1+\frac{\alpha_u}{1-\alpha_u}\tau_{u1}}\nu_{j1}^{u1}\\&-\frac{W_jH_{Tj1}}{1-\alpha_j}\ell_{j1}\frac{\tau_{j1}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}}\nu_{j1}^{j1}.\end{aligned}$$

Dividing by ℓ_{j1}, and substituting in $\nu_{j1}^{u1}=-\frac{\ell_{j1}}{\ell_{u1}}\nu_{j1}^{j1}$, I find

$$\begin{aligned}T_{j1}\nu_{j1}^{j1}-T_{u1}\nu_{j1}^{j1}=&\left[\frac{1}{\lambda_{G1}}-1+\frac{1}{\lambda_{G1}}\frac{\alpha_j}{1-\alpha_j}\tau_{j1}\right]\\&+\frac{W_uH_{Tu1}}{1-\alpha_u}\frac{\tau_{u1}}{1+\frac{\alpha_u}{1-\alpha_u}\tau_{u1}}\nu_{j1}^{j1}\\&-\frac{W_jH_{Tj1}}{1-\alpha_j}\frac{\tau_{j1}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}}\nu_{j1}^{j1}.\end{aligned}$$

Then substituting in $T_{u1},\lambda_{G1}-1,\tau_{u1}\in o(\ell_{j1})$, and taking the limit gives

$$T_{j1}\nu_{j1}^{j1}=\frac{\alpha_j}{1-\alpha_j}\tau_{j1}-\frac{W_jH_{Tj1}}{1-\alpha_j}\frac{\tau_{j1}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}}\nu_{j1}^{j1}.$$

I complete the proof by noting that

$$\frac{\partial\log\ell^{j1}}{\partial T_{j1}}=\frac{1}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}}\nu_{j1}^{j1}.$$

$\square$

Next I move on to proposition 2.

Proposition 2. *Suppose that there are two large locations, s (southern US) and n (northern US), and one small location, j (Janesville). Then, if there are no redistributive reasons for transfers $\zeta_{nt} = \zeta_{st} = \zeta_{jt} = 1$, in any interior solution to* (SPP),

$$T_{j1} > \frac{1}{\nu_{j1}^{j1}}\left(\left(\frac{1}{\lambda_{G1}} - 1\right) + \frac{1}{\lambda_{G1}}\frac{\alpha_j}{1-\alpha_j} - \frac{W_j H_{Tj1}}{1-\alpha_j}\frac{\partial \log \ell^{j1}}{\partial T_{j1}}\right)\tau_{j1},$$

if and only if migrants to j disproportionately come from the region in a recession, i.e. $\mathrm{Cov}_{k\neq j}(|\nu_{j1}^{k1}|, \tau_{k1}) > 0$.

Proof. Without loss of generality, I am going to assume that the north is in a recession. The budget constraint is

$$o(\ell_{j1}) + T_{n1}\ell_{n1} + T_{s1}\ell_{s1} = 0,$$

since, by assumption 4, transfers to Janesville are bounded. The first order condition for monetary policy is

$$o(\ell_{j1}) + \frac{W_s H_{Ts1}}{1-\alpha_s}\ell_{s1}\frac{\tau_{s1}}{1+\frac{\alpha_s}{1-\alpha_s}\tau_{s1}} + \frac{W_n H_{Tn1}}{1-\alpha_n}\ell_{n1}\frac{\tau_{n1}}{1+\frac{\alpha_n}{1-\alpha_n}\tau_{n1}} = 0,$$

where I use the fact that hours and the labor wedges are bounded by assumptions 1 and 3. Then the first order condition for transfers is

$$\sum_m \ell_{m1}T_{m1}\nu_{k1}^{m1} = \ell_{k1}\left[\frac{\overline{\lambda}_{k1}U_C^k}{\lambda_{G1}P_k}\left(1+\frac{\alpha_k}{1-\alpha_k}\tau_{k1}\right) - 1\right] - \sum_m \frac{W_m H_{Tm1}}{1-\alpha_m}\ell_{m1}\frac{\tau_{m1}}{1+\frac{\alpha_m}{1-\alpha_m}\tau_{m1}}\nu_{k1}^{m1}.$$

Then solving for the transfers in the north and south gives

$$T_{n1} = \frac{\ell_{n1}\ell_{s1}}{\nu\bar{\ell}}\frac{1}{\frac{\partial \tilde{V}^{n1}}{\partial T_{n1}}}\left[\frac{1}{\lambda_{G1}}\left(1+\frac{\alpha_n}{1-\alpha_n}\tau_{n1}\right) - 1\right] - \frac{W_n H_{Tn1}}{1-\alpha_n}\frac{\tau_{n1}}{1+\frac{\alpha_n}{1-\alpha_n}\tau_{n1}} + o(\ell_{j1}),$$

and

$$T_{s1} = \frac{\ell_{s1}\ell_{n1}}{\nu\bar{\ell}}\frac{1}{\frac{\partial \tilde{V}^{s1}}{\partial T_{s1}}}\left[\frac{1}{\lambda_{G1}}\left(1+\frac{\alpha_s}{1-\alpha_s}\tau_{s1}\right) - 1\right] - \frac{W_s H_{Ts1}}{1-\alpha_s}\frac{\tau_{s1}}{1+\frac{\alpha_s}{1-\alpha_s}\tau_{s1}} + o(\ell_{j1}),$$

where $\nu = \frac{\partial \ell^{s1}}{\partial U_{s1}}$. The transfer to Janesville must satisfy

$$\begin{aligned} T_{j1}\nu_{j1}^{j1} &= \frac{1}{\lambda_{G1}}\left(1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}\right) - 1 - \frac{W_j H_{Tj1}}{1-\alpha_j}\frac{\tau_{j1}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}}\nu_{j1}^{j1} \\ &\quad - \frac{W_n H_{Tn1}}{1-\alpha_n}\ell_{n1}\frac{\tau_{n1}}{1+\frac{\alpha_n}{1-\alpha_n}\tau_{n1}}\frac{\nu_{j1}^{n1}}{\ell_{j1}} - \frac{W_s H_{Ts1}}{1-\alpha_s}\ell_{s1}\frac{\tau_{s1}}{1+\frac{\alpha_s}{1-\alpha_s}\tau_{s1}}\frac{\nu_{j1}^{s1}}{\ell_{j1}} \\ &\quad - T_{s1}\ell_{s1}\frac{\nu_{j1}^{s1}}{\ell_{j1}} - T_{n1}\ell_{n1}\frac{\nu_{j1}^{n1}}{\ell_{j1}}. \end{aligned}$$

So that

$$T_{j1} = \frac{1}{\nu_{j1}^{j1}} \left[\frac{1}{\lambda_{G1}} \left(1 + \frac{\alpha_j}{1-\alpha_j} \tau_{j1} \right) - 1 - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\tau_{j1}}{1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1}} \nu_{j1}^{j} \right] + \frac{1}{\nu_{j1}^{j1}} X + o(\ell_{j1})$$

where X is the weighted average of $\ell_{n1} \frac{W_n H_{Tn1}}{1-\alpha_n} \frac{\tau_{n1}}{1+\frac{\alpha_n}{1-\alpha_n}\tau_{n1}} + \ell_{n1} T_{n1}$ and $\ell_{s1} \frac{W_s H_{Ts1}}{1-\alpha_s} \frac{\tau_{s1}}{1+\frac{\alpha_s}{1-\alpha_s}\tau_{s1}} + \ell_{s1} T_{s1}$ with weights $-\frac{\nu_{j1}^{n1}}{\ell_{j1}}$ and $-\frac{\nu_{j1}^{s1}}{\ell_{j1}}$ respectively. Note from the expressions for T_{n1} and T_{s1}, we have

$$\ell_{n1} T_{n1} + \ell_{n1} \frac{W_n H_{Tn1}}{1-\alpha_n} \frac{\tau_{n1}}{1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1}} = \ell_{n1} \frac{\ell_{n1}\ell_{s1}}{\nu \bar{\ell}} \frac{1}{\frac{\partial \tilde{V}^{n1}}{\partial T_{n1}}} \left[\frac{1}{\lambda_{G1}} \left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1} \right) - 1 \right],$$

and

$$\ell_{s1} T_{s1} + \ell_{s1} \frac{W_s H_{Ts1}}{1-\alpha_s} \frac{\tau_{s1}}{1 + \frac{\alpha_s}{1-\alpha_s}\tau_{s1}} = \ell_{s1} \frac{\ell_{s1}\ell_{n1}}{\nu \bar{\ell}} \frac{1}{\frac{\partial \tilde{V}^{s1}}{\partial T_{s1}}} \left[\frac{1}{\lambda_{G1}} \left(1 + \frac{\alpha_s}{1-\alpha_s}\tau_{s1} \right) - 1 \right].$$

Furthermore, adding together equals zero. Since n is in a recession so that $\tau_{n1} > 0$ and s is in a boom $\tau_{s1} < 0$, it must be the case that

$$\frac{\ell_{n1}\ell_{s1}}{\nu \bar{\ell}} \frac{1}{\frac{\partial \tilde{V}^{n1}}{\partial T_{n1}}} \left[\frac{1}{\lambda_{G1}} \left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1} \right) - 1 \right] > 0 > \frac{\ell_{s1}\ell_{n1}}{\nu \bar{\ell}} \frac{1}{\frac{\partial \tilde{V}^{s1}}{\partial T_{s1}}} \left[\frac{1}{\lambda_{G1}} \left(1 + \frac{\alpha_s}{1-\alpha_s}\tau_{s1} \right) - 1 \right].$$

Therefore, if $|\nu_{j1}^{n}| > |\nu_{j1}^{s}|$, $X > 0$ proving the result when I take the limit as $\ell_{j1} \to 0$. □

Proposition 3. *Suppose that there are two locations, j (Janesville) and u (Rest of the US), location j is arbitrarily small, $\ell_{jt} \to 0$, there are no redistributive reasons for policy, $\beta_{nt} = 1$, and j is in a recession, $\tau_{jt} > 0$. Then in any interior solution to* (SPP)*, the optimal period 2 transfer to location j satisfies*

$$T_{j2} < \frac{1}{\nu_{j2}^{j2}} \left(\frac{\alpha_j}{1-\alpha_j} - \frac{W_j N_{Tj2}}{1-\alpha_j} \frac{\partial \log \ell^{j2}}{\partial T_{j2}} \right) \tau_{j2},$$

when the share of workers in location j in period 1 who stay in location j in period 2 is greater than zero.

Proof. In period 1, we have that

$$\begin{aligned} T_{j1} &= \frac{1}{\nu_{j1}^{j1}} \left(\frac{\alpha_j}{1-\alpha_j} - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}} \right) \tau_{j1} + o(\ell_{j1}) \\ T_{u1} &= o(\ell_{j1}) \\ \lambda_{G1} &= 1 + o(\ell_{j1}). \end{aligned}$$

Recall that

$$0 = \beta \overline{\lambda}_{n2} \ell_{n2} + \sum_m \lambda_{m1}^{\ell} \beta \frac{\partial \ell^{m1}}{\partial U_{n1}} \mu_{mn} + \sum_m \sum_k \lambda_{m2}^{\ell} \ell_{k1} \frac{\partial \mu^{km}}{\partial U_{n2}} - \frac{\lambda_{G2} \ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}.$$

Then summing across all n implies that

$$\lambda_{G2} \sum_n \frac{\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}} = \beta \sum_n \overline{\lambda}_{n2} \ell_{n2},$$

since no one moves from uniform increases in utility. Therefore,

$$\frac{\lambda_{G2}}{\beta} = \overline{\lambda}_{u2} \frac{\partial \tilde{V}^u}{\partial T_{u2}} + o(\ell_{j2}).$$

Meanwhile, the monetary policy is

$$0 = \frac{W_u H_{Tu1}}{1-\alpha_u} \ell_{u1} \frac{\tau_{u1}}{1 + \frac{\alpha_u}{1-\alpha_u} \tau_{u1}} + o(\ell_{j2}).$$

Therefore, $\tau_{u1} \in o(\ell_{j2})$ and $\overline{\lambda}_{u2} \frac{\partial \tilde{V}^u}{\partial T_{u2}} = \overline{\lambda}_{u2} \frac{U_C^u}{P_u} \left(1 + \frac{\alpha_u}{1-\alpha_u} o(\ell_{j2})\right) = 1 + o(\ell_{j2})$. Then we can turn to the first order conditions in Janesville in the second period. They are,

$$\sum_t \frac{\lambda_{Gt}}{\lambda_{G2}} \sum_m \ell_{mt} T_{mt} \nu_{n2}^{mt} = \ell_{n2} \left[\frac{\beta \overline{\lambda}_{n2} U_C^n}{\lambda_{G2} P_n} \left(1 + \frac{\alpha_n}{1-\alpha_n} \tau_{n2}\right) - 1 \right] - \sum_t \frac{\lambda_{Gt}}{\lambda_{G2}} \sum_m \frac{W_m H_{Tmt}}{1-\alpha_m} \ell_{mt} \frac{\tau_{mt}}{1 + \frac{\alpha_m}{1-\alpha_m} \tau_{mt}} \nu_{n2}^{mt}.$$

Note that,

$$\begin{aligned} \nu_{j2}^{j1} &= \beta \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \mu_{jj} \frac{\partial \tilde{V}^j}{\partial T_{j2}} \\ &= \beta \mu_{jj} \nu_{j1}^{j1} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}}. \end{aligned}$$

And we have $\frac{\lambda_1^G}{\lambda_2^G} = \frac{1+o(\ell_{j1})}{\beta(1+o(\ell_{j2}))}$. Transfers to the rest of the United States are $T_{u2} \in o(\ell_{j2})$ as transfers are bounded. Then the transfers to Janesville satisfy

$$
\begin{aligned}
&o(\ell_{j1}) + o(\ell_{j2}) + \\
\ell_{j1} T_{j1} \mu_{jj} \nu_{j1}^{j1} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}} \frac{1 + o(\ell_{j1})}{1 + o(\ell_{j2})} + \ell_{j2} T_{j2} \nu_{j2}^{j2} &= \ell_{j2} \left[\frac{1}{1 + o(\ell_{j2})} - 1 + \frac{1}{1 + o(\ell_{j2})} \frac{\alpha_j}{1 - \alpha_j} \tau_{j2} \right] \\
&\quad - \frac{W_j H_{Tj2}}{1 - \alpha_j} \ell_{j2} \frac{\tau_{j2}}{1 + \frac{\alpha_j}{1-\alpha_j} \tau_{j2}} \nu_{j2}^{j2} \\
&\quad - \frac{W_j H_{Tj1}}{1 - \alpha_j} \ell_{j1} \frac{\tau_{j1}}{1 + \frac{\alpha_j}{1-\alpha_j} \tau_{j1}} \frac{\nu_{j2}^{j1}}{\beta} \\
\ell_{j2} T_{j2} \nu_{j2}^{j2} &= \ell_{j2} \left[\frac{1}{1 + o(\ell_{j2})} - 1 + \frac{1}{1 + o(\ell_{j2})} \frac{\alpha_j}{1 - \alpha_j} \tau_{j2} \right] \\
&\quad - \frac{W_j H_{Tj2}}{1 - \alpha_j} \ell_{j2} \frac{\tau_{j2}}{1 + \frac{\alpha_j}{1-\alpha_j} \tau_{j2}} \nu_{j2}^{j2} \\
&\quad - \frac{W_j H_{Tj1}}{1 - \alpha_j} \ell_{j1} \frac{\tau_{j1}}{1 + \frac{\alpha_j}{1-\alpha_j} \tau_{j1}} \mu_{jj} \nu_{j1}^{j} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}} \\
&\quad - \ell_{j1} \mu_{jj} \frac{\alpha_j}{1 - \alpha_j} \tau_{j1} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}} \frac{1 + o(\ell_{j1})}{1 + o(\ell_{j2})} \\
&\quad + \ell_{j1} \mu_{jj} \frac{W_j H_{Tj1}}{1 - \alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}} \tau_{j1} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}} \frac{1 + o(\ell_{j1})}{1 + o(\ell_{j2})} \\
&\quad - o(\ell_{j1}) - o(\ell_{j2}) \\
T_{j2} \nu_{j2}^{j2} &= \frac{\alpha_j}{1 - \alpha_j} \tau_{j2} - \frac{W_j H_{Tj2}}{1 - \alpha_j} \frac{\tau_{j2}}{1 + \frac{\alpha_j}{1-\alpha_j} \tau_{j2}} \nu_{j2}^{j2} \\
&\quad - \frac{\mu_{jj} \ell_{j1}}{\ell_{j2}} \frac{\alpha_j}{1 - \alpha_j} \tau_{j1} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}},
\end{aligned}
$$

taking the limit as $\ell_{j1} \to 0$ and $\ell_{j2} \to 0$. Noting that $\tau_{j1}, \mu_{jj} > 0$ completes the proof. □

C Extensions of the Simple Model

Next, I consider how the model results change under a variety of assumptions. In all of the extensions, I only focus on period 1, so I drop the t subscript for notational simplicity.

C.1 Downward wage rigidity and costly price adjustments

C.1.1 Environment

There are two locations $n \in \{j, u\}$.

Households. The migration set-up is exactly the same as in the main text. But now, each household is endowed with 1 unit of labor supply that the household supplies in elastically with no utility cost. Therefore $H_n \leqslant 1$. Utility of a household living in location n is

$$U_n = U^n(C_n)$$
$$C_n = (C_{NTn})^{\alpha}(C_{Tn})^{1-\alpha}$$

where α is the weight put on the non-traded sector. The households then choose non-traded consumption and traded consumption to maximize utility subject to the budget constraint taking prices as given,

$$p_n C_{NTn} + P_T C_{Tn} \leqslant W_n H_n + T_n + \Pi,$$

where p_n is the price of the local good, P_T is the price of the traded good, and Π are the profits.

Production. In each location, there is a continuum of firms that choose prices to maximize profits. A representative firm competitively produces a final good with the varieties produced by the firms, with a CES aggregator with elasticity of substitution ϵ.

The firms compete monopolistically. Changing the price requires a Rotemberg real cost in the freely traded good. That is, the firm that produces variety ω solves the problem

$$\max_{p_n(\omega), y_n(\omega), H_n(\omega)} \tau p_n(\omega) y_n(\omega) - W_n H_n(\omega)\ell_n - \psi \left(\frac{p_n(\omega) - p_{n0}}{p_{n0}}\right)^2 P_T Y_n$$

where Y_n is total production of the region, and p_{n0} is the previous price, subject to the the technology constraint

$$Y_n(\omega) = A_n H_n(\omega)\ell_n$$

and demand

$$p_n(\omega) y_n(\omega) = \left(\frac{p_n(\omega)}{p_n}\right)^{1-\epsilon} p_n Y_n.$$

I further include τ which is a subsidy on prices to undo the monopoly distortion. I assume that this is paid for by the local workers. Assuming that all the firms are symmetric, we get that prices solve

$$\frac{p_n - p_{n0}}{p_{n0}} \frac{P_T}{p_{n0}} = \frac{\epsilon}{\psi}\left(\frac{W_n}{p_n A_n}\right).$$

Then the profit losses of the firm are

$$\Pi = -\sum_n \psi \left(\frac{p_n - p_{n0}}{p_{n0}}\right) P_T Y_n$$

Wages are downwardly rigid so

$$W_n \geqslant W_{n0}, H_n \leqslant 1,$$

where W_{n0} is the previously set wage.

A single firm aggregates up the goods produced by each location

$$Y_T = \left[\sum_n \phi_n^{\frac{1}{\sigma}} (Y_{Tn})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

This firm is competitive so therefore

$$P_T = \left[\sum_n p_n^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

Market clearing. Market clearing implies that

$$Y_n = A_n H_n \ell_n.$$

Demand for the good produced in the location is

$$Y_n = C_{NTn} \ell_n + Y_{Tn}.$$

Finally, demand for the traded good comes from consumption and the goods required for Rotemberg price adjustment

$$Y_T = \sum_n C_{Tn} \ell_n + \sum_n \psi \left(\frac{p_n - p_{n0}}{p_{n0}} \right) Y_n.$$

C.1.2 Adjusted Proposition

Proposition A2. *Suppose that location j is arbitrarily small compared to location u, location j is in a recession, there are no redistributive reasons for policy $\frac{\overline{\lambda}_n U_C^n}{P_n} = 1$, and monetary policy is such that there is no inflation in u. Then in any interior equilibrium, the optimal period 1 transfer to location j must satisfy*

$$T_j = \frac{1}{\frac{d \log \ell_j}{dT_j}} \frac{\frac{\alpha}{1-\alpha} - \frac{\phi_j E_T}{1-\alpha} \frac{1}{\ell_j} \frac{\partial \log \ell^j}{\partial T_j}}{1 + \frac{\phi_j E_T}{1-\alpha} \frac{1}{\ell_j} \frac{\partial \log \ell^j}{\partial T_j}}.$$

Proof. The monetary policy ensures that there is full employment in u. Therefore, wages in j are downwardly rigid, no prices change, and $\Pi = 0$. Defining $\tilde{\phi}_j = \phi_j \left(\frac{p_j}{P_T} \right)^{1-\sigma}$, hours worked in j is given by

$$W_j H_j = \frac{\alpha}{1-\alpha} T_j + \frac{\tilde{\phi}_j E_T}{1-\alpha} \frac{1}{\ell_j}.$$

The utility in location j is

$$U_j = U^j \left(\frac{W_j H_j + T_j}{P_j} \right)$$

while utility in location u is

$$U_u = U^u\left(\frac{W_u + T_u}{P_u}\right).$$

Furthermore, population remains the same so that

$$\ell_j = \ell^j\left(U_j - U_u\right).$$

Then I will consider a change in taxes

$$\ell_j T_j d\log\ell_j + \ell_j dT_j + \ell_u T_u d\log\ell_u + \ell_u dT_u = 0.$$

Therefore,

$$dT_u = -\frac{1}{\ell_u}\left(\ell_j T_j d\log\ell_j + \ell_j dT_j + \ell_u T_u d\log\ell_u\right).$$

The total change in welfare is

$$\begin{aligned}
d\mathcal{W} &= \overline{\lambda}_j \ell_j dU_j + \overline{\lambda}_u \ell_u dU_u \\
&= \overline{\lambda}_j \ell_j U_C^j\left(\frac{W_j}{P_j} dH_j + \frac{1}{P_j} dT_j\right) + \overline{\lambda}_u \ell_u U_C^u \frac{1}{P_u} dT_u \\
&= \overline{\lambda}_j \ell_j \frac{U_C^j}{P_j} W_j dH_j + \overline{\lambda}_j \ell_j \frac{U_C^j}{P_j} dT_j - \overline{\lambda}_u \frac{U_C^u}{P_u}\left(\ell_j T_j d\log\ell_j + \ell_j dT_j + \ell_u T_u d\log\ell_u\right) \\
&= \ell_j W_j dH_j - T_j \ell_j d\log\ell_j - T_u \ell_u d\log\ell_u,
\end{aligned}$$

where we use the fact that there is no insurance reason for policy,

$$\overline{\lambda}_j \frac{U_C^j}{P_j} = \overline{\lambda}_u \frac{U_C^u}{P_u} = 1.$$

Then population changes according to

$$\begin{aligned}
d\log\ell_j &= \frac{\partial\log\ell^j}{\partial U}\left(dU_j - dU_u\right) \\
&= \frac{\partial\log\ell^j}{\partial U}\left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j - \frac{U_C^u}{P_u} dT_u\right)
\end{aligned}$$

Meanwhile, hours change according to

$$W_j dH_j = \frac{\alpha}{1-\alpha} dT_j - \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j} d\log\ell_j.$$

Next, I note that $d\ell_j = -d\ell_u$ so that $-\frac{\ell_j}{\ell_u} d\log\ell_j = d\log\ell_u$. Therefore, the change in total welfare, normalized by the population in Janesville, is given by

$$\frac{d\mathcal{W}}{\ell_j} = W_j dH_j - T_j d\log\ell_j - T_u d\log\ell_j.$$

Then taking the limit as $\ell_j \to 0$ holding fixed the $\frac{\partial \log \ell^j}{\partial U}$,

$$\frac{d\mathcal{W}}{\ell_j} = W_j dH_j - T_j d\log \ell_j,$$

since $T_u \to 0$. In the limit,

$$\begin{aligned}
d\log \ell_j &= \frac{\partial \log \ell^j}{\partial U}\left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j - \frac{U_C^u}{P_u} dT_u\right) \\
&= \frac{\partial \log \ell^j}{\partial U}\left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j + \frac{U_C^u}{P_u}\left(\ell_j T_j d\log \ell_j + \ell_j dT_j + \ell_u T_u d\log ell_u\right)\right) \\
&= \frac{\partial \log \ell^j}{\partial U}\left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j\right).
\end{aligned}$$

I then turn to solving for the change in hours. This is

$$\begin{aligned}
W_j dH_j &= \frac{\alpha}{1-\alpha} dT_j - \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j} d\log \ell_j \\
&= \frac{\alpha}{1-\alpha} dT_j - \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j\right) \\
W_j\left(1 + \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\frac{U_C^j}{P_j}\right) dH_j &= \left(\frac{\alpha}{1-\alpha} - \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\frac{U_C^j}{P_j}\right) dT_j
\end{aligned}$$

Then plugging into the welfare equation,

$$\frac{d\mathcal{W}}{\ell_j} = \frac{\frac{\alpha}{1-\alpha} - \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\frac{U_C^j}{P_j}}{1 + \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\frac{U_C^j}{P_j}} dT_j - T_j d\log \ell_j.$$

In the optimum, this must be zero. Rearranging gives the expression. □

C.2 Place-biased policy

C.2.1 Environment

The environment is the same as in the text, but now there are types $\theta \in \Theta$ and I assume that there are only two locations j and u. These types can represent a wide variety of categories. If a policy can target households based on where they started, then θ can denote starting location. It also could denote people over 65.

I will use a superscript θ for functions and subscript θ for values just as in the main text. I assume that households have Greenwood-Hercowitz-Huffman preferences, i.e.

$$U^{\theta n}(C, H) = u(C - v(H)).$$

This ensures that even if agents of different types earn different amounts in the same location, they still supply the same number of hours worked when hours are efficiently rationed. Also, each type θ might have a different distribution of idiosyncratic utilities $G_{\theta n}(\cdot)$.

I also suppose that there is some policy parametrized by a parameter κ. Agents of type θ get a transfer of $T_{\theta n}\kappa$ for living in location n when there is κ amount of the policy. This captures the place-biased nature of the policy. It can be place-biased for two reasons. One is that within a type θ, it is biased toward a single location. Alternatively, the policy is biased toward a type θ that is disproportionately in one particular location. Then earnings of type θ in location n is

$$E_{\theta n} = W_n H_n + T_{\theta n}\kappa + T,$$

where T is the lump sum transfer from the government. The government budget constraint is

$$\sum_{\theta}\sum_{n} T_{\theta n}\ell_{\theta n}\kappa + T = 0.$$

C.2.2 Adjusted Proposition

Proposition A3. *Suppose that location j is arbitrarily small compared to location u, location j is in a recession, there are no redistributive reasons for policy $\frac{\overline{\lambda}_{nt}U_C^n}{P_{nt}} = 1$, and monetary policy is such that there is no labor wedge in u. Then starting from an equilibrium with no transfers, using a place-biased policy paid for with a lump-sum transfer increases welfare if and only if*

$$\frac{\alpha_j}{1-\alpha_j}\left(\sum_{\theta}\frac{T_{\theta j}\ell_{\theta j}}{\ell_j} - \sum_{\theta}\sum_{n}\ell_{\theta n}T_{\theta n}\right) > \frac{\phi_j E_T/\ell_j}{1-\alpha_j}\frac{d\log\ell_j}{d\kappa}.$$

Proof. I will start by characterizing the equilibrium. Spending on the traded output in j is given by

$$P_{Tj}Y_{Tj} = \phi_j E_T.$$

Spending on the non-traded goods is

$$P_{NTj}Y_{NTj} = \alpha_n\left[\sum_{\theta}\left(W_j H_j + T_{\theta j}\kappa + T\right)\ell_{\theta j}\right].$$

Summing together and solving for hours worked I get

$$H_j = \frac{1}{W_j}\left[\frac{\phi_j E_T/\ell_j}{1-\alpha_j} + \frac{\alpha_j}{1-\alpha_j}\left(\sum_{\theta}\frac{T_{\theta j}\ell_{\theta j}\kappa}{\ell_j} + T\right)\right],$$

where $\ell_n = \sum_{\theta}\ell_{\theta n}$. Therefore, the utility a type θ household receives from living in Janesville is

$$V^{\theta j}(\kappa, T, H) = U^{\theta j}\left(\frac{W_j H + T_{\theta j}\kappa + T}{P_j}, H\right).$$

With that, I can then turn to find what happens with a small increase in the place-biased policy to a region, and the lump sum transfer changes to maintain budget balance. Budget balance implies

$$\sum_\theta T_{\theta j}\ell_{\theta j}d\kappa + \sum_\theta T_{\theta u}\ell_{\theta u}d\kappa + dT = 0.$$

Then the change in total welfare is

$$\begin{aligned}
d\mathcal{W} &= \sum_\theta \overline{\lambda}_{\theta j}\ell_{\theta j}dU_{\theta j} + \sum_\theta \overline{\lambda}_{\theta u}\ell_{\theta u}dU_{\theta u} \\
&= \sum_\theta \overline{\lambda}_{\theta j}\ell_{\theta j}\left(U_C^{\theta j}\frac{T_{\theta j}}{P_j}d\kappa + \frac{U_C^{\theta j}}{P_j}dT + W_j\frac{U_C^{\theta j}}{P_j}\tau_j dH_j\right) \\
&\quad + \sum_\theta \overline{\lambda}_{\theta u}\ell_{\theta u}\left(U_C^{\theta u}\frac{T_{\theta u}}{P_u}d\kappa + \frac{U_C^{\theta u}}{P_u}dT\right) \\
&= \ell_j W_j \tau_j dH_j.
\end{aligned}$$

Taking the derivative of the hours function, I find that

$$\begin{aligned}
W_j dH_j = &-\frac{\phi_j E_T/\ell_j}{1-\alpha_j}\left(\sum_\theta \frac{\ell_{\theta j}}{\ell_j}d\log\ell_{\theta j}\right) + \frac{\alpha_j}{1-\alpha_j}\left(\sum_\theta \frac{T_{\theta j}\ell_{\theta j}}{\ell_j}\right)d\kappa \\
&+ \frac{\alpha_j}{1-\alpha_j}dT.
\end{aligned}$$

Then plugging in for transfers and noting that $d\log\ell_j = \sum_\theta \frac{\ell_{\theta j}}{\ell_j}d\log\ell_{\theta j}$ we get that welfare increases from an increase in the policy if and only if

$$\frac{\alpha_j}{1-\alpha_j}\left(\sum_\theta \frac{T_{\theta j}\ell_{\theta j}}{\ell_j} - \sum_\theta\sum_n \ell_{\theta n}T_{\theta n}\right)d\kappa > \frac{\phi_j E_T/\ell_j}{1-\alpha_j}d\log\ell_j.$$

□

Importantly, the stimulus effect depends on the observed place bias. The migration effect depends on the place bias within a single type. To see that, note that

$$\begin{aligned}
d\log\ell_j &= \sum_\theta \frac{\ell_{\theta j}}{\ell_j}d\log\ell_{\theta j} \\
&= \sum_\theta \frac{\ell_{\theta j}}{\ell_j}\frac{\partial\log\ell^{\theta j}}{\partial U_j}\left(\frac{U_C^{\theta j}}{P_j}\left[T_{\theta j} - \frac{\sum_\theta\sum_n \ell_{\theta n}T_{\theta n}}{\overline{\ell}}\right]d\kappa\right. \\
&\qquad - \frac{U^{\theta u}}{P_u}\left[T_{\theta u} - \frac{\sum_\theta\sum_n \ell_{\theta n}T_{\theta n}}{\overline{\ell}}\right]d\kappa \\
&\qquad \left. + W_j\frac{U_C^j}{P_j}\tau_j dH_j\right).
\end{aligned}$$

C.3 Households affect demand

C.3.1 Environment

Next I suppose that households affect demand for a particular region. In particular, assume that there are types $\theta \in \Theta$ and that the share of spending on the traded good is a function of the people living in the location. That is,

$$\phi_n = \phi^n\left(\{\ell_{\theta n}\}\right).$$

I will continue to focus on the two location version of the model. These types have the same fundamental utility across all locations but might have different distributions of idiosyncratic shocks.

C.3.2 Adjusted Proposition

Proposition A4. *Suppose that location j is arbitrarily small compared to location u, location j is in a recession, there are no redistributive reasons for policy $\frac{\overline{\lambda}_n U_C^n}{P_n} = 1$, and monetary policy is such that there is no labor wedge in u. Then in any interior equilibrium, the optimal period 1 transfer to location j must satisfy*

$$T_j = \frac{1}{\frac{d\log \ell_j}{dT_j}} \frac{\frac{\alpha_j}{1-\alpha_j} - \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[\frac{\ell_{\theta j}}{\ell_j} - \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right]\frac{\partial \log \ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}}{1+\Omega}\tau_j,$$

where

$$\Omega \equiv \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[\frac{\ell_{\theta j}}{\ell_j} - \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right]\frac{\partial \log \ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}.$$

Proof. In this slightly adjusted setting, hours worked is now given by

$$H^n(E_T, \{\ell_\theta\}, T) = \frac{1}{W_n}\left(\frac{\phi_n(\{\ell_\theta\})E_T}{1-\alpha_n}\frac{1}{\sum_\theta \ell_\theta} + \frac{\alpha_n}{1-\alpha_n}T\right).$$

Then I will consider a change in the transfer to j paid for with a small tax on u. Budget balance implies that

$$\ell_j dT_j + T_j \ell_j d\log \ell_j + \ell_u dT_u + T_u \ell_u d\log \ell_u = 0.$$

The change in total welfare is

$$\begin{aligned}
d\mathcal{W} &= \overline{\lambda}_j \ell_j dU_j + \overline{\lambda}_u \ell_u dU_u \\
&= \overline{\lambda}_j \ell_j U_C^j\left(\frac{W_j}{P_j}\tau_j dH_j + \frac{1}{P_j}dT_j\right) + \overline{\lambda}_u \ell_u U_C^u \frac{1}{P_u}dT_u \\
&= \ell_j W_j \tau_j dH_j - T_j \ell_j d\log \ell_j - T_u \ell_u d\log \ell_u,
\end{aligned}$$

where I use the fact that there is no insurance reason for policy. Then population of each type changes according to

$$\begin{aligned} d\log\ell_{\theta j} &= \frac{\partial \log \ell^{\theta j}}{\partial U}\left(dU_j - dU_u\right) \\ &= \frac{\partial \log \ell^{\theta j}}{\partial U}\left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j - \frac{U_C^u}{P_u} dT_u\right). \end{aligned}$$

Meanwhile, hours change according to

$$W_j dH_j = \frac{\alpha_j}{1-\alpha_j} dT_j - \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j} d\log\ell_j + \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}} d\log\ell_{\theta j}.$$

I will then take the limit as $\ell_{\theta j} \to 0$ holding fixed the migration semi-elasticities to j. Then the change in welfare, normalized by the population is

$$\frac{d\mathcal{W}}{\ell_j} = W_j \tau_j dH_j - T_1 d\log\ell_j$$

since $T_u \to 0$. In the limit,

$$d\log\ell_{\theta j} = \frac{\partial \log \ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}\left(W_j dH_j + dT_j\right).$$

I then turn to solving for the change in hours. This is

$$\begin{aligned} W_j dH_j &= \frac{\alpha_j}{1-\alpha_j} dT_j - \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j} d\log\ell_j + \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}} d\log\ell_{\theta j} \\ &= \frac{\alpha_j}{1-\alpha_j} dT_j + \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[-\frac{\ell_{\theta j}}{\ell_j} + \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right] d\log\ell_{\theta j} \\ &= \frac{\alpha_j}{1-\alpha_j} dT_j + \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[-\frac{\ell_{\theta j}}{\ell_j} + \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right]\frac{\partial \log \ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}\left(W_j dH_j + dT_j\right) \end{aligned}$$

Which we can rearrange

$$W_j\left(1+\Omega\right) = \left[\frac{\alpha_j}{1-\alpha_j} + \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[-\frac{\ell_{\theta j}}{\ell_j} + \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right]\frac{\partial \log \ell^{j\theta}}{\partial U}\frac{U_C^j}{P_j}\right] dT_j$$

where

$$\Omega \equiv \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[-\frac{\ell_{\theta j}}{\ell_j} + \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right]\frac{\partial \log \ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}.$$

Therefore, plugging this into the expression for welfare change, the optimal transfer needs to satisfy

$$T_j = \frac{1}{\frac{d\log\ell_j}{dT_j}}\frac{\frac{\alpha_j}{1-\alpha_j} + \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[-\frac{\ell_{\theta j}}{\ell_j} + \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right]\frac{\partial \log \ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}}{1+\Omega}\tau_j.$$

$\square$

C.4 Wage stickiness only in traded goods

C.4.1 Environment

The final extension I consider is when wages are only sticky in the traded sector. Suppose that supplying labor to the non-traded sector and the traded are not perfect substitutes so that the fundamental utility of living in location n is

$$U_n = U^n(C_n, H_{Tn}, H_{NTn}).$$

I then assume that wages in the traded sector are completely sticky, just as before, and that wages in the non-traded sector are free to adjust. I will further assume Cobb-Douglas preferences so that

$$C_n = (C_{Tn})^{1-\alpha}(C_{NTn})^{\alpha}.$$

C.4.2 Adjusted Proposition

I will take this in two steps. First, I will define an indirect utility function

$$v^n\left(T_n, H_{Tn}, W_{NTn}\right) = U^n\left(\frac{W_{Tn}H_{Tn} + \frac{\alpha}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right) + T_n}{P_n\left(W_{NTn}\right)}, H_{Tn}, \frac{\frac{\alpha}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right)}{W_{NTn}}\right)$$

where $P_n(W_{NTn})$ is the local price index as a function of the non-traded wages, and I have already substituted in for earnings in the non-traded sector, using the equation for non-traded demand,

$$W_{NT}H_{NTn}\ell_n = \alpha\left(W_{Tn}H_{Tn}\ell_n + W_{NTn}H_{NTn}\ell_n + T_n\ell_n\right).$$

I then define another indirect utility function

$$V^n(T, H_T) = \max_{W_{NT}} v^n\left(T_n, H_T, W_{NT}\right).$$

The derivatives are then

$$\begin{aligned}
\frac{\partial v^n}{\partial T_n} &= \frac{1}{1-\alpha}\frac{U_C^n}{P_n} + \frac{1}{W_{NTn}}U_{HNT}^n\frac{\alpha}{1-\alpha} \\
&= \frac{U_C^n}{P_n}\left(1 + \frac{\alpha}{1-\alpha}\tau_{NTn}\right) \\
\frac{\partial v^n}{\partial H_{Tn}} &= W_{Tn}\frac{U_C^n}{P_n}\tau_{Tn}.
\end{aligned}$$

The derivative with respect to the non-tradable wage is slightly more complicated

$$\begin{aligned}
\frac{\partial v^n}{\partial W_{NTn}} = &-\frac{U_C^n}{P_n}\frac{W_{TnH_{Tn}} + \frac{\alpha}{1-\alpha}(W_{Tn}H_{Tn} + T_n) + T_n}{P_n}\frac{\partial P_n}{\partial W_{NTn}} \\
&- U_{HNT}^n\frac{\alpha}{1-\alpha}\frac{W_{Tn}H_{Tn} + T_n}{W_{NTn}^2}
\end{aligned}$$

By the envelope theorem,

$$\frac{\partial \log P_n}{\partial \log W_{NTn}} = \alpha.$$

Therefore, we can write

$$\begin{aligned}
\frac{\partial v^n}{\partial W_{NTn}} &= -\frac{U_C^n}{P_n}\frac{W_{Tn}H_{Tn} + \frac{\alpha}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right) + T_n}{P_n}\frac{\partial P_n}{\partial W_{NTn}} \\
&\quad - U_{HNT}^n \frac{\frac{\alpha}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right)}{W_{NTn}^2} \\
&= -\frac{U_C^n}{P_n}\frac{\frac{1}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right)}{W_{NTn}}\frac{\partial \log P_n}{\partial \log W_{NTn}} \\
&\quad - U_{HNT}^n \frac{\frac{\alpha}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right)}{W_{NTn}^2} \\
&= -\frac{\alpha}{1-\alpha}\frac{U_C^n}{P_n}\frac{W_{Tn}H_{Tn} + T_n}{W_{NTn}}\left[1 + \frac{P_n}{W_{NTnt}}\frac{U_{HNT}^n}{U_C^n}\right] \\
&= -\frac{\alpha}{1-\alpha}\frac{U_C^n}{P_n}\frac{W_{Tn}H_{Tn} + T_n}{W_{NTn}}\tau_{NTnt}.
\end{aligned}$$

That means that under V^n and the market allocation $\tau_{NTnt} = 0$. Meanwhile, by the envelope theorem,

$$\begin{aligned}
\frac{\partial V^n}{\partial T} &= \frac{U_C^n}{P_n} \\
\frac{\partial V^n}{\partial H_{Tn}} &= W_n \frac{U_C^n}{P_n}\tau_{Tn}.
\end{aligned}$$

Finally I note that traded hours are given by

$$H_{Tn} = \frac{\phi_n E_T}{W_{Tn}}\frac{1}{\ell_n}.$$

Then I can state the adjusted proposition.

Proposition A5. *Suppose that location j is arbitrarily small compared to location u, location j is in a recession, there are no redistributive reasons for policy $\frac{\bar{\lambda}_n U_C^n}{P_n} = 1$, and monetary policy is such that there is no labor wedge in u. Then in any interior equilibrium, the optimal period 1 transfer to location j must satisfy*

$$T_j = -\frac{1}{\frac{d\log \ell_j}{dT_j}}\frac{\phi_j E_T/\ell_j \frac{\partial \log \ell^j}{\partial T_j}}{1 + \frac{\phi_j E_T}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\frac{U_C^j}{P_j}\tau_{Tj}}\tau_{Tj}.$$

Proof. Just as before, I will consider a change in taxes. By budget balance, we have

$$\ell_j T_j d\log \ell_j + \ell_j dT_j + \ell_u T_u d\log \ell_u + \ell_u dT_u = 0.$$

Therefore,

$$dT_u = -\frac{1}{\ell_u}\left(\ell_j T_j d\log\ell_j + \ell_j dT_j + \ell_u T_u d\log\ell_u\right).$$

The total change in welfare is then

$$\begin{aligned}
d\mathcal{W} &= \overline{\lambda}_j \ell_j dU_j + \overline{\lambda}_u \ell_u dU_u \\
&= \overline{\lambda}_j \ell_j U_C^j \left(\frac{W_{Tj}}{P_j}\tau_{Tj} dH_{Tj} + \frac{1}{P_j} dT_j\right) + \overline{\lambda}_u \ell_u U_C^u \frac{1}{P_u} dT_u \\
&= \ell_j W_{Tj}\tau_{Tj} dH_{Tj} - T_j \ell_j d\log\ell_j - T_u \ell_u d\log\ell_u,
\end{aligned}$$

where we use the fact that there is no insurance reason for policy. Then population changes according to

$$\begin{aligned}
d\log\ell_j &= \frac{\partial\log\ell^j}{\partial U}(dU_j - dU_u) \\
&= \frac{\partial\log\ell^j}{\partial U}\left(U_C^j \frac{W_{Tj}}{P_j} dH_{Tj} + \frac{U_C^j}{P_j} dT_j - \frac{U_C^u}{P_u} dT_u\right).
\end{aligned}$$

Meanwhile, traded hours change according to

$$W_{Tj} dH_{Tj} = -\frac{\phi_j E_T}{\ell_j} d\log\ell_j.$$

Next I note that $d\ell_j = -d\ell_u$ so that $\frac{\ell_j}{\ell_u} d\log\ell_j = d\log\ell_u$. Therefore, the change in total welfare, normalized by the population in Janesville is given by

$$\frac{d\mathcal{W}}{\ell_j} = W_{Tj}\tau_{Tj} dH_{Tj} - T_j d\log\ell_j - T_u d\log\ell_j.$$

Then taking the limit as $\ell_j \to 0$ holding fixed the $\frac{\partial\log\ell^j}{\partial U}$,

$$\frac{d\mathcal{W}}{\ell_j} = W_{Tj}\tau_{Tj} dH_{Tj} - T_j d\log\ell_j,$$

since $T_u \to 0$. In the limit,

$$d\log\ell_j = \frac{\partial\log\ell^j}{\partial U}\left(U_C^j \frac{W_j}{P_j}\tau_{Tj} dH_{Tj} + \frac{U_C^j}{P_j} dT_j\right).$$

I then turn to solving for the change in hours. This is

$$\begin{aligned}
W_{Tj} dH_{Tj} &= -\frac{\phi_j E_T}{\ell_j} d\log \ell_j \\
&= -\frac{\phi_j E_T}{\ell_j} \frac{\partial \log \ell^j}{\partial U} \left(U_C^j \frac{W_j}{P_j} \tau_{Tj} dH_{Tj} + \frac{U_C^j}{P_j} dT_j \right) \\
W_{Tj} \left(1 + \frac{\phi_j E_T}{\ell_j} \frac{\partial \log \ell^j}{\partial U} \frac{U_C^j}{P_j} \tau_{Tj} \right) dH_{Tj} &= -\frac{\phi_j E_T}{\ell_j} \frac{\partial \log \ell^j}{\partial U} \frac{U_C^j}{P_j} dT_j.
\end{aligned}$$

Putting it together, the optimal transfer needs to satisfy

$$T_j = -\frac{1}{\frac{d\log \ell_j}{dT_j}} \frac{\phi_j E_T / \ell_j \frac{\partial \log \ell^j}{\partial T_j}}{1 + \frac{\phi_j E_T}{\ell_j} \frac{\partial \log \ell^j}{\partial U} \frac{U_C^j}{P_j} \tau_{Tj}} \tau_{Tj}.$$

□

D Quantitative Model of Local Recessions - Details

In this section, I provide a formal description of the quantitative model presented in Section 5 and characterize the labor union problem and the intratemporal consumption decision.

D.1 Environment

There are N regions indexed by $n, m \in \mathcal{N} = \{1, \ldots, N\}$, one non-traded sector and one traded sector, and continuous time indexed by $t \in [0, \infty)$.

Households. There is a continuum of households that I index by $i \in \mathcal{I}$. I will start by describing the dynamic welfare taking as given flow utility before returning to describe the flow utility.

I denote the location of agent i at time t by $n(i, t)$. Then each household starts in some location $n(i, 0)$ and it gets the opportunity to move at a Poisson rate $\delta_\ell > 0$. At that point, the household observes additive utility shocks of moving to every location m, $\varepsilon_m(i, t)$. The utility shocks are distributed according to a Gumbel distribution with shape parameter ν. The household can then move subject to an additive migration cost of moving to a location m, $\tau_{\ell nm}$.

Denoting the set of all times where household i moves from location n to m by $\mathcal{M}_{nm}(i) \subset [0, \infty)$, realized utility of household i is

$$\int_0^\infty e^{-\rho t} \left[U_{n(i,t)}(t) + \sum_{n,m} \delta_{t \in \mathcal{M}_{nm}(i)} [-\tau_{\ell nm} + \varepsilon_m(i, t)] \right] dt,$$

where $U_n(t)$ is the flow utility of living in location n, $\rho > 0$ is household's discount rate, and $\delta_{t\in\mathcal{M}_{nm}(i)}$ is the dirac delta function.

The immediate flow utility of a household in location n at time t, $U_n(t)$ is a function of consumption and labor supply,

$$U_n(t) = \frac{C_n(t)^{1-\theta}}{1-\theta} - \frac{H_n(t)^{1+\eta}}{1+\eta},$$

where $C_n(t)$ is the consumption aggregate, θ is the elasticity of intertemporal substitution, $H_n(t)$ is hours supplied, and η is the Frisch labor elasticity. The consumption aggregate is a Cobb-Douglas aggregation of consumption of the traded good and the non-traded good,

$$C_n(t) = C_{NTn}(t)^{\alpha} C_{Tn}(t)^{1-\alpha},$$

where $C_{sn}(t)$ is consumption of the sector s good and $\alpha \in (0,1)$ is the share of spending on non-traded goods. The traded good is an aggregation of the varieties produced in each location,

$$C_{Tn}(t) = \left(\sum_{m\in\mathcal{N}} \phi_m^{\frac{1}{\sigma}} C_{Tmn}(t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where ϕ_m is the consumption weight on the variety produced by location m, which I normalize so that $\sum_m \phi_m = 1$, $C_{Tmn}(t)$ is consumption of the traded good produced in location m by the consumer in n, and σ is the elasticity of substitution between varieties produced by the locations.

Firms. In each location n, there is a continuum of intermediate producers $\omega \in [0,1]$ who produce an intermediate using labor. Firm ω produces

$$Y_n(\omega, t) = H_n(\omega, t)\ell_n(t),$$

where $Y_n(\omega, t)$ is production and $H_n(\omega, t)$ is the amount of per capita labor supplied to intermediate ω.

A final producer then combines those intermediates according to a CES aggregator

$$Y_n(t) = A_n \left[\int_0^1 Y_n(\omega, t)^{\frac{\epsilon-1}{\epsilon}} d\omega\right]^{\frac{\epsilon}{\epsilon-1}},$$

where $Y_n(t)$ is the aggregate production of location n and $\epsilon > 1$ is the elasticity of substitution across intermediates. This final good can then be consumed as a non-traded or traded good.

Market Clearing. For the labor market to clear, labor supplied equals the sum of labor demand by each intermediate producer,

$$H_n(t) = \int_0^1 H_n(\omega, t) d\omega, \text{ for all } n, t. \tag{A1}$$

Aggregate production of location n is consumed as a traded good and non-traded good. The non-traded good is only consumed by the local households. Trade is subject to iceberg trade costs. Therefore, goods market clearing requires production in location n is equal to consumption of non-traded goods in the location plus consumption of its produce as a traded good across all locations,

$$Y_n(t) = C_{NTn}(t)\ell_n(t) + \sum_m \tau_{nm} C_{Tnm}(t)\ell_m(t), \text{ for all } n, t, \tag{A2}$$

where $\tau_{nm} \geqslant 1$ is the iceberg trade costs of delivering a good from location n to location m.

D.2 Decentralized equilibrium

D.2.1 Utility Maximization

I start by characterizing the household's migration decision taking as given flow utility in location n at time t, $U_n(t)$. I then turn to the consumption decision. Just as before, workers do not choose labor and instead supply the labor demanded.

Migration Decision. The Bellman equation for a household in location n is

$$\rho v_n(t) - \dot{v}_n(t) = U_n(t) + \delta_\ell \left(V_n(t) - v_n(t)\right), \tag{A3}$$

where $v_n(t)$ is the expected lifetime utility of a household in location n at time t and $V_n(t)$ is the expected utility if that households gets the opportunity to move. Because the utility shocks are distributed Gumbel,

$$V_n(t) = \frac{1}{\nu} \log\left(\sum_m \exp(\nu(v_m(t) - \tau_{\ell nm}))\right). \tag{A4}$$

This implies that a $\exp\left(\nu(v_m(t) - \tau_{\ell nm} - V_n(t)\right)$ share of households in location n who have the chance to move will move to location m. The population in location m changes according to

$$\dot{\ell}_m(t) = \delta_\ell \left[\sum_n \exp\left(\nu(v_m(t) - \tau_{\ell nm} - V_n(t)\right)\ell_n(t) - \ell_m(t)\right]. \tag{A5}$$

Intratemporal Consumption Decision. Given expenditures $E_n(t)$, households in location n at time t choose consumption to maximize utility taking prices as given. In particular,

$$\{C_{NTn}(t), C_{Tn}(t), \{C_{Tmn}(t)\}\} \in \underset{C_{NT}, C_T\{C_{Tm}\}}{\operatorname{argmax}} \Bigg\{ (C_{NT})^\alpha (C_T)^{1-\alpha} \Bigg| \; C_T = \left(\sum_m \phi_m^{\frac{1}{\sigma}} (C_{Tm})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \; \sum_m p_{Tmn}(t) C_{Tm} + p_{NTn} C_{NT} \leqslant E_n(t) \Bigg\}. \tag{A6}$$

This problem is standard so the characterization is left for the Appendix D.3. I denote by $P_n(t)$ the perfect price index so that $E_n(t) = P_n(t)C_n(t)$.

Households are hand-to-mouth so they spend all of their income in each period. Income comes from two different sources: labor earnings and government transfers. That is,

$$E_n(t) = \left(\int_0^1 W_n(\omega, t) H_n(\omega, t) d\omega \right) + T_n(t), \tag{A7}$$

where $W_n(\omega, t)$ is the wage offered by intermediate producer ω in location n and $T_n(t)$ is the transfer to households in location n.

D.2.2 Production

Profit Maximization. A competitive, representative firm for each intermediate ω in location n maximizes profits taking prices and wages set by the union as given using a linear technology. Therefore, the price of the intermediate is simply the wage $p_n(\omega, t) = W_n(\omega, t)$.

The final producer is competitive and so maximizes profits taking as given the price of the final good $p_n(t)$ and intermediates $W_n(\omega, t)$. That is,

$$Y_n(t), \{Y_n(\omega, t)\} \in \underset{Y, Y(\omega)}{\operatorname{argmax}} \left\{ p_n(t) Y - \int_0^1 W_n(\omega, t) Y(\omega) d\omega \,\middle|\, Y = A_n \left[\int_0^1 Y(\omega)^{\frac{\epsilon-1}{\epsilon}} d\omega \right]^{\frac{\epsilon}{\epsilon-1}} \right\}. \tag{A8}$$

Trade is also competitive so that $p_{Tnm}(t) = \tau_{nm} p_n(t)$ and $p_{NTn}(t) = p_n(t)$.

Labor Unions. For each intermediate ω in location n, there is a union that can unilaterally set the wage it demands. Wages are sticky, and the union only gets the chance to change the wage demanded at a poisson rate δ_w.

Given wages, the union supplies the labor necessary to meet demand for intermediate ω. I assume that there is efficient rationing. When a union gets the chance to change its wage, it sets the wage to maximize utility of the average household in its location. As is standard in this literature, I assume the local government has a wage subsidy κ to undo the monopoly distortion, funded by a tax on the residents. That is, the unions who can change their wage at time t choose a new wage $\tilde{W}_n(t)$ that solves

$$\tilde{W}_n(t) \in \underset{W'}{\operatorname{argmax}} \int_t^\infty e^{-(\rho+\delta_w)(t'-t)} \left[\kappa \frac{C_n(t')^{-\theta}}{P_n(t')} (W')^{1-\epsilon} - H_n(t')^\eta (W')^{-\epsilon} \right] A_n^{\epsilon-1} P_n(t')^\epsilon Y_n(t') dt'. \tag{A9}$$

Appendix D.4 describes further details.

D.2.3 Government

The government sets aggregate spending $E(t)$, such that

$$E(t) = \sum_n E_n(t) \ell_n(t), \text{ for all } t, \tag{A10}$$

and also chooses the place specific transfers between locations. The government budget constraint then must hold in each period,

$$\sum_n \ell_n(t) T_n(t) = 0, \text{ for all } t. \tag{A11}$$

Definition 2. *Given monetary policy $E(t)$ and per capita transfers $T_n(t)$, an* equilibrium *is a set of location choices $n(i,t)$, utility levels $U_n(t)$, regional population $\ell_n(t)$, prices $P_n(t)$, wages $W_n(\omega, t)$, consumption levels $C_{Tmn}(t)$, $C_{NT}(t)$, labor supplies $H_n(t), H_n(\omega, t)$, and output $Y_n(t)$, such that:*

- *Households choose consumption and their location to maximize utility* (A3), (A4), (A5), (A6), (A7)*;*
- *Firms maximize profits taking prices as given,* (A8)*;*
- *Unions set wages to maximize expected utility of the local households,* (A9)*;*
- *The government's budget constraints hold,* (A11)*;*
- *Total spending is equal to nominal GDP* (A10)*; and*
- *Markets clear* (A1), (A2).

D.3 Intratemporal Consumption Decision

Given expenditures $E_n(t)$, households in location n at time t choose consumption to maximize utility taking prices as given. In particular,

$$\{C_{NTn}(t), C_{Tn}(t), \{C_{Tmn}(t)\}\} \in \underset{C_{NT}, C_T\{C_{Tm}\}}{\operatorname{argmax}} \Bigg\{ (C_{NT})^{\alpha} (C_T)^{1-\alpha} \Bigg| \\ C_T = \left(\sum_m \phi_m^{\frac{1}{\sigma}} (C_{Tm})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \\ \sum_m p_{Tmn}(t) C_{Tm} + p_{NTn} C_{NT} \leqslant E_n(t) \Bigg\}.$$

I further break this problem down into a traded consumption problem and then an aggregated consumption problem. Suppose that the household is spending E_T on trade goods. Then the household solves the problem

$$\max_{C_{tm}} \left(\sum_m \phi_m^{\frac{1}{\sigma}} (C_{Tm})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

such that

$$\sum_m p_{Tmn}(t) C_{Tm} \leqslant E_T.$$

Raising the maximand to $\frac{\sigma-1}{\sigma}$ and taking the first order condition with respect to C_{Tm} gives

$$\lambda p_{Tmn}(t) = \frac{\sigma - 1}{\sigma} \phi_m^{\frac{1}{\sigma}} (C_{Tm})^{-\frac{1}{\sigma}}.$$

Defining $\mu = (\lambda\frac{\sigma}{\sigma-1})^{-\sigma}$,$C_{Tm} = \mu\phi_m p_{Tmn}(t)^{-\sigma}$. Then the budget constraint is

$$\begin{aligned} E_T &= \sum_m p_{Tmn}(t) C_{Tm} \\ &= \sum_m \mu p_{Tmn}(t)^{1-\sigma}\phi_m \\ \mu &= \frac{E_T}{\sum_m \phi_m p_{Tmn}(t)^{1-\sigma}} \end{aligned}$$

Therefore, traded consumption is

$$\begin{aligned} C_T &= \left(\sum_m \phi_m^{\frac{1}{\sigma}} (C_{Tm})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(\sum_m \phi_m^{\frac{1}{\sigma}} (\mu\phi_m p_{Tmn}(t)^{-\sigma})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\ &= \mu\left(\sum_m \phi_m p_{Tmn}(t)^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}} \\ &= E_T\left(\sum_m \phi_m p_{Tmn}(t)^{1-\sigma}\right)^{\frac{1}{\sigma-1}}. \end{aligned}$$

Defining $p_{Tn}(t) \equiv (\sum_m \phi_m p_{Tmn}(t)^{1-\sigma})^{\frac{1}{1-\sigma}}$, $C_T = \frac{E_T}{p_{Tn}}$. Furthermore,

$$p_{Tmn}(t)C_{Tmn}(t) = \phi_m\left(\frac{p_{Tmn}(t)}{p_{Tn}}\right)^{1-\sigma} p_{Tn}(t)C_{Tn}(t). \tag{A12}$$

Then choosing between traded and non-traded, the household solves the problem

$$\max_{C_{NT},C_T} (C_{NT})^{\alpha}(C_T)^{1-\alpha}$$

such that

$$p_{Tn}C_T + p_{NTn}C_{NT} \leqslant E_n(t).$$

Taking the first order conditions,

$$p_{Tn}(t)C_{Tn}(t) = (1-\alpha)E_n(t) \tag{A13}$$

and

$$p_{NTn}(t)C_{NTn}(t) = \alpha E_n(t). \tag{A14}$$

D.4 Labor Unions

In this subsection, I derive the key equations describing how unions operate in this model. I start by taking as given wages of each union and characterizing labor supply and production. I then turn to the maximization problem of the unions and derive the equations describing how wages move.

D.4.1 Labor Demand

The final, competitive producer looks to maximize profits taking as given wages of each of the unions. That is

$$Y_n(t), \{Y_n(\omega,t)\} \in \underset{Y,Y(\omega)}{\operatorname{argmax}} \left\{ p_n(t)Y - \int_0^1 W_n(\omega,t)Y(\omega)d\omega \,\middle|\, Y = A_n \left[\int_0^1 Y(\omega)^{\frac{\epsilon-1}{\epsilon}} d\omega\right]^{\frac{\epsilon}{\epsilon-1}} \right\}.$$

The usual CES algebra, reviewed above in the consumer maximization problem, implies that,

$$p_n(t) = \frac{1}{A_n}\left[\int_0^1 W_n(\omega,t)^{1-\epsilon} d\omega\right]^{\frac{1}{1-\epsilon}}.$$

Furthermore, demand for the labor of union ω is

$$Y_n(\omega,t) = \frac{1}{A_n^{1-\epsilon}}\left(\frac{W_n(\omega,t)}{p_n(t)}\right)^{-\epsilon} Y_n(t).$$

Production is $Y_n(\omega,t) = H_n(\omega,t)\ell_n(t)$. Therefore, the total amount of labor demanded is given by

$$\begin{aligned}
H_n(t) &= \int_0^1 H_n(\omega,t)d\omega \\
&= \int_0^1 \frac{Y_n(\omega,t)}{\ell_n(t)} d\omega \\
&= \frac{1}{\ell_n(t)A_n(t)^{1-\epsilon}} \int_0^1 \left(\frac{W_n(\omega,t)}{p_n(t)}\right)^{-\epsilon} Y_n(t) d\omega \\
&= \frac{Y_n(t)}{\ell_n(t)A_n(t)^{1-\epsilon}} p_n(t)^{\epsilon} \int_0^1 W_n(\omega,t)^{-\epsilon} d\omega.
\end{aligned}$$

Then I solve for wage earnings. That is

$$\begin{aligned}
\int_0^1 W_n(\omega,t)H_n(\omega,t)d\omega &= \int_0^1 W_n(\omega,t)\frac{Y_n(\omega,t)}{\ell_n(t)}d\omega \\
&= \frac{1}{\ell_n(t)A_n(t)^{1-\epsilon}}\int_0^1 W_n(\omega,t)\left(\frac{W_n(\omega,t)}{p_n(t)}\right)^{-\epsilon} Y_n(t)d\omega \\
&= \frac{Y_n(t)p_n(t)^{\epsilon}}{\ell_n(t)A_n(t)^{1-\epsilon}}\int_0^1 W_n(\omega,t)^{1-\epsilon}d\omega \\
&= \frac{Y_n(t)p_n(t)^{\epsilon}}{\ell_n(t)A_n(t)^{1-\epsilon}}(A_np_n(t))^{1-\epsilon} \\
&= \frac{p_n(t)Y_n(t)}{\ell_n(t)}.
\end{aligned}$$

Defining

$$\begin{aligned}
W_n(t) &\equiv \frac{\int_0^1 W_n(\omega,t)H_n(\omega,t)d\omega}{H_n(t)} \\
&= \frac{p_n(t)Y_n(t)/\ell_n(t)}{\frac{Y_n(t)}{\ell_n(t)A_n(t)^{1-\epsilon}}p_n(t)^{\epsilon}\int_0^1 W_n(\omega,t)^{-\epsilon}d\omega} \\
&= \frac{A_n^{1-\epsilon}p_n(t)^{1-\epsilon}}{\int_0^1 W_n(\omega,t)^{-\epsilon}d\omega} \\
&= \frac{A_np_n(t)}{\int_0^1\left(\frac{W_n(\omega,t)}{A_np_n(t)}\right)^{-\epsilon}d\omega},
\end{aligned}$$

we get an expression for wages as a function of $v_n^p(t) \equiv \int_0^1 \left(\frac{W_n(\omega,t)}{A_np_n(t)}\right)^{-\epsilon} d\omega$. Then we can also write hours

$$H_n(t) = \frac{\int_0^1 W_n(\omega,t)H_n(\omega,t)d\omega}{W_n(t)} = v_n^p(t)\frac{Y_n(t)}{A_n(t)}. \tag{A15}$$

I can further solve for prices. $p_n(t) = \frac{1}{A_n}v_n^p(t)W_n(t)$.

D.4.2 The Union Problem

Next I characterize the labor union's problem. A union that gets an opportunity to choose wages at time t looks to maximize welfare of the workers there. The utility that the households get from more earnings at time s is $\frac{C_n(s)^{-\theta}}{P_n(s)}$. Meanwhile, the utility loss from working more is $-H_n(s)^{\eta}$.

Setting a wage of $\tilde{W}$ leads to hours demanded H at time s given by

$$\begin{aligned} H(\tilde{W}) &= \frac{Y_n(\tilde{W})}{\ell_n(t)} \\ &= \frac{1}{A_n^{1-\epsilon}}\left(\frac{\tilde{W}}{p_n(t)}\right)^{-\epsilon} Y_n(t)\frac{1}{\ell_n(t)} \\ &= \tilde{W}^{-\epsilon}A_n^{\epsilon-1}p_n(t)^{\epsilon}Y_n(t)\frac{1}{\ell_n(t)}. \end{aligned}$$

Therefore, the flow utility at time s is

$$\tau\frac{C_n(s)^{-\theta}}{P_n(s)}\tilde{W}^{1-\epsilon}A_n^{\epsilon-1}p_n(s)^{\epsilon}Y_n(s) - H_n(s)^{\eta}\tilde{W}^{-\epsilon}A_n^{\epsilon-1}p_n(s)^{\epsilon}Y_n(s),$$

when there is a subsidy of κ on wage earnings. The union then chooses $\tilde{W}$ to maximize

$$\tilde{W}_n(t) = \underset{\tilde{W}}{\operatorname{argmax}}\int_t^{\infty} e^{-(\rho+\delta_w)(s-t)}\left[\kappa\frac{C_n(s)^{-\theta}}{P_n(s)}\tilde{W}^{1-\epsilon} - H_n(s)^{\eta}\tilde{W}^{-\epsilon}\right]A_n^{\epsilon-1}p_n(s)^{\epsilon}Y_n(s)ds.$$

To undo the monopoly distortion, $\kappa = \frac{\epsilon}{\epsilon-1}$. Taking the first order condition with respect to $\tilde{W}$ and rearranging I get

$$\tilde{W}_n(t) = \frac{\int_t^{\infty} e^{-(\rho+\delta_w)(s-t)}H_n(s)^{\eta}A_n(s)^{\epsilon-1}p_n(s)^{\epsilon}Y_n(s)ds}{\int_t^{\infty} e^{-(\rho+\delta_w)(s-t)}\frac{C_n(s)^{-\theta}}{P_n(s)}A_n(s)^{\epsilon-1}P_n(s0Y_n(s)ds}.$$

I then define a variable $X_{1n}(t)$ as the numerator and $X_{2n}(t)$ as $W_n(t)$ times the denominator. Then these variables change according to

$$\dot{X}_{1n}(t) = -H_n(t)^{\eta}A_n(t)^{\epsilon-1}P_n(t)^{\epsilon}Y_n(t) + (\rho+\delta_w)X_{1n}(t), \tag{A16}$$

and

$$\dot{X}_{2n}(t) = -W_n(t)\frac{C_n(t)^{-\theta}}{P_n(t)}A_n(t)^{\epsilon-1}P_n(t)^{\epsilon}Y_n(t) + (\rho+\delta_w+\pi_n^w(t))X_{2n}(t), \tag{A17}$$

where $\pi_n^w(t) \equiv \frac{\dot{W}_n(t)}{W_n(t)}$. Then to describe how wages change, note that

$$W_n(t)^{1-\epsilon} = \int_{-\infty}^{t} \delta_w e^{-\delta_w(t-\tau)}\tilde{W}_n(\tau)^{1-\epsilon}d\tau.$$

Taking the derivative with respect to time I find that

$$\pi_n^w(t) = \frac{\delta_w}{1-\epsilon}\left[\left(\frac{\tilde{W}_n(t)}{W_n(t)}\right)^{1-\epsilon} - 1\right],$$

or

$$\pi_n^w(t) = \frac{\delta_w}{1-\epsilon}\left[\left(\frac{X_{1n}(t)}{X_{2n}(t)}\right)^{1-\epsilon} - 1\right]. \tag{A18}$$

Next we need to describe how the misallocation term changes. rewriting in terms of when wages were set,

$$v_n^p(t) = \int_{-\infty}^{\tau} \delta_w e^{-\delta(t-\tau)} \left(\frac{\tilde{W}_n(\tau)}{W_n(t)}\right)^{-\epsilon} d\tau.$$

Taking the derivative with respect to time I find that

$$\dot{v}_n^p(t) = \delta_w \left(\frac{X_{1n}(t)}{X_{2n}(t)}\right)^{-\epsilon} + (\epsilon\pi_n^w(t) - \delta_w)v_n^p(t). \tag{A19}$$

E Linear-Quadratic Approximation

E.1 Summarizing Equations

Summarizing the equations describing equilibrium, I have the following. For total welfare,

$$\mathcal{W} = \sum_\gamma \int_0^\infty e^{-\rho t}\left[\sum_n \lambda_n(t)U_n(t)\ell_n(t) - \delta_\ell \sum_n\sum_m \tilde{\lambda}_{nm}(t)\ell_n(t)\exp\left(\nu(v_m(t) - \tau_{\ell nm} - V_n(t))\right)(v_m(t) - V_n(t))\right] dt,$$

where $\lambda_n(t)$ is the planner's average weight on households living in location n, and $\tilde{\lambda}_{nm}(t)$ is the planner's average weight on households moving from location n to location m at time t. The constraints can be written as follows:

$$\dot{v}_n(t) = -U_n(t) - \delta_\ell V_n(t) + (\rho + \delta_\ell)v_n(t)$$

$$\exp\left(\nu V_n(t)\right) = \sum_m \exp\left(\nu\left(v_m(t) - \tau_{\ell nm}\right)\right)$$

$$\dot{\ell}_n(t) = \delta_\ell\left(\sum_m \exp\left(\nu(v_n(t) - \tau_{\ell mn} - V_m(t))\right)\ell_m(t) - \ell_n(t)\right)$$

$$U_n(t) = \log E_n(t) - \log P_n(t) - \frac{H_n(t)^{1+\eta}}{1+\eta}$$

$$P_{Tn}(t) = \left(\sum_{m\in\mathcal{N}} \phi_m\left(\tau_{mn}p_m(t)\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

$$P_n(t) = p_n(t)^\alpha P_{Tn}(t)^{1-\alpha}$$

$$E_n(t) = W_n(t)H_n(t) + T_n(t)$$

$$\frac{\dot{W}_n(t)}{W_n(t)} = \pi_n(t)$$

$$y_n(t) = \frac{A_n}{v_n^p(t)} H_n(t) \ell_n(t)$$

$$p_n(t) y_n(t) = \alpha E_n(t) \ell_n(t) + p_n(t) y_{Tn}(t)$$

$$p_m(t) y_{Tm}(t) = \sum_n \phi_m \left(\frac{\tau_{mn} p_m(t)}{P_{Tn}(t)} \right)^{1-\sigma} (1-\alpha) E_n(t) \ell_n(t)$$

$$\dot{X}_{1n}(t) = -H_n(t)^\eta A_n^{\epsilon-1} p_n(t)^\epsilon Y_n(t) + (\rho + \delta_w) X_{1n}(t)$$

$$\dot{X}_{2n}(t) = -W_n(t) \frac{C_n(s)^{-\theta}}{P_n(s)} A_n^{\epsilon-1} p_n(t)^\epsilon Y_n(t) + (\pi_n^w(t) + \rho + \delta_w) X_{2n}(t)$$

$$\pi_n(t) = \frac{\delta_w}{1-\epsilon} \left[\left(\frac{X_{1n}(t)}{X_{2n}(t)} \right)^{1-\epsilon} - 1 \right]$$

$$\dot{v}_n^p(t) = \delta_w \left(\frac{X_{1n}(t)}{X_{2n}(t)} \right)^{-\varepsilon} + \left(\epsilon \pi_n^w(t) - \delta_w \right) v_n^p(t)$$

$$\sum_n \ell_n(t) T_n(t) = 0.$$

E.2 Loss Function

I do a second order approximation to the welfare function. I then do a second order approximation to the constraints and make substitutions until the welfare function is only second order. Derivations are available upon request.

In doing this, I need to make two assumptions so that the observed equilibrium is efficient. First, I assume that $\theta = 1$ and $\overline{\lambda}_n = E_n$. This guarantees that the planner has no redistributive reasons to transfer money across locations in the steady state. The other assumption I make is related. I need that there is a ψ_1 such that

$$\psi_1 = \overline{\lambda}_n U_n - \delta_\ell \sum_m \tilde{\lambda}_{nm} \pi_{nm} (v_m - V_n).$$

I further assume that

$$\tilde{\lambda}_{nm}[1 + \nu(v_m - V_n)] = \psi_{n2},$$

for some ψ_{n2}. I set ψ_1 so that the average weight on households moving is the same as the average weight on households in a location. This guarantees that there are no first order reasons for the planner to distort household's migration decisions. Then the loss function is,

$$\begin{aligned}
\tilde{\mathcal{W}}(t) = &-\frac{1}{2}\sum_n\sum_m \nu\delta_\ell\tilde{\lambda}_{nm}\ell_n\mu_{nm}\left(\hat{v}_m(t)-\hat{V}_n(t)\right)^2 \\
&+\sum_n\overline{\lambda}_n\ell_n\hat{U}_n(t)\hat{\ell}_n(t) \\
&-\frac{1+\eta}{2}\sum_n\overline{\lambda}_n\ell_n\hat{H}_n(t)^2 \\
&+\frac{1}{2}\sum_n\overline{\lambda}_n\ell_n\left(\hat{w}_n(t)+\hat{H}_n(t)+\hat{\ell}_n(t)\right)^2 \\
&-\frac{\alpha}{2}\sum_n\overline{\lambda}_n\ell_n\left(\hat{E}_n(t)+\hat{\ell}_n(t)\right)^2 \\
&-\frac{1-\alpha}{2}\sum_m\sum_n\overline{\lambda}_n\ell_n\phi_{mn}\left(\hat{\phi}_m(t)+(1-\sigma)\hat{w}_m(t)-(1-\sigma)\hat{P}_{Tn}(t)+\hat{E}_n(t)+\hat{\ell}_n(t)\right)^2 \\
&-\frac{1-\alpha}{2}\frac{\sigma}{1-\sigma}\sum_m\overline{\lambda}_m\ell_m\left(\hat{\phi}_m(t)+(1-\sigma)\hat{w}_m(t)\right)^2 \\
&+\frac{1-\alpha}{2}\sigma(1-\sigma)\sum_n\overline{\lambda}_n\ell_n\hat{P}_{Tn}(t)^2 \\
&-\frac{1}{2}\frac{\epsilon}{\delta_w(\rho+\delta_w)}\sum_n\overline{\lambda}_n\ell_n\hat{\pi}_n(t)^2 \\
&+\text{other terms independent of policy,}
\end{aligned}$$

where

$$\phi_{mn} \equiv \frac{p_{Tmn}C_{Tmn}}{\sum_k p_{Tkn}C_{Tkn}},$$

is the share of tradable spending on location m for a household in n, and

$$\mu_{nm} \equiv \exp\left(\nu(v_m-\tau_{\ell nm}-V_n)\right)$$

is the share of households in location n who have the opportunity to move that decide to go to location m.

E.3 Linearized Constraints

The constraints can then be linearized. Derivations are available upon request.

$$\hat{U}_n(t) = \hat{E}_n(t) - \hat{P}_n(t) - \hat{H}_n(t)$$

$$\hat{P}_n(t) = \alpha\hat{w}_n(t) + (1-\alpha)\hat{P}_{Tn}(t)$$

$$\hat{P}_{Tn}(t) = \sum_m \phi_{mn}\left[\frac{1}{1-\sigma}\hat{\phi}_m + \hat{w}_m(t)\right]$$

$$\hat{V}_m(t) = \sum_n \mu_{mn} \hat{v}_n(t)$$

$$\dot{\hat{w}}_n = \hat{\pi}_n(t)$$

$$\hat{E}_n(t) = \hat{w}_n(t) + \hat{H}_n(t) + \hat{T}_n(t)$$

$$\sigma \hat{w}_m(t) + \hat{y}_{Tm}(t) = \hat{\phi}_m + \sum_n \frac{\phi_{mn} E_n}{E_m} \left[(\sigma - 1)\hat{P}_{Tn}(t) + \hat{E}_n(t) + \hat{\ell}_n(t) \right]$$

$$\hat{H}_n(t) + \hat{\ell}_n(t) = \alpha \left(\hat{E}_n(t) + \hat{\ell}_n(t) - \hat{w}_n(t) \right) + (1 - \alpha)\hat{y}_{Tn}(t)$$

$$\dot{\hat{\pi}}_n(t) = \delta_w(\rho + \delta_w) \left[\hat{w}_n(t) - \hat{E}_n(t) - \eta \hat{H}_n(t) \right] + \rho \hat{\pi}_n(t)$$

$$\dot{\hat{\ell}}_n(t) = \delta_\ell \left[\sum_m \frac{\mu_{mn} \ell_m}{\ell_n} \left[\nu \left(\hat{v}_n(t) - \hat{V}_m(t) \right) + \hat{\ell}_m(t) \right] - \hat{\ell}_n(t) \right]$$

$$\dot{\hat{v}}_n(t) = -\hat{U}_n(t) - \delta_\ell \hat{V}_n(t) + (\delta_\ell + \rho)\hat{v}_n(t)$$

F Calibration Details

In this appendix, I go through the details of the demographic adjustment along with how I calibrate the trade flows, the migration flows, and the observed policy response.

F.1 Demographic Adjustment

I net out demographic differences across locations following Fajgelbaum and Gaubert (2020). I download demographic information for each commuting zone from the National Cancer Institute Surveillance, Epidemiology, and End Results Program. I find the share of individuals in each commuting zone with the following characteristics: age by bins: < 20, $20 - 40$, $40 - 65$, > 65; share non-white; and share male. Denoting by x_n the per capita measure in commuting zone n, I run the following CZ level regression:

$$x_n = x_0 + \sum_j \beta_j DEM_{nj} + \varepsilon_n,$$

where DEM_{nj} is the demographic variable j in CZ n. I then adjust the observed x_n from those compositional differences across CZs by expressing it as a deviation from the population mean:

$$\tilde{x}_n \equiv x_n - \sum_j \hat{\beta}_j \left(DEM_{nj} - \overline{DEM}_{nj} \right)$$

where $\hat{\beta}_j$ is the estimated coefficient and $\overline{DEM}_{nj} \equiv \frac{1}{N} \sum_n DEM_{nj}$. I then use $\tilde{x}_n$ in the subsequent analysis for earnings, transfers, and taxes.

F.2 Trade flows

As described in the main text, data on trade between states comes from the 2002 Commodity Flow Survey. I assume that the trade costs between two distinct commuting zones n and m are

$$\log \tau_{nm} = \delta_D \log \text{distance}_{nm} + \delta_H$$

where distance_{nm} is the bilateral distance between the population centroids of CZs n and m. I run a gravity regression of trade flows on bilateral distance controlling for origin and destination. This regression, along with the calibrated value of σ, provides an estimate of δ_D.

To finish calibrating trade costs between locations, I normalize the cost of a commuting zone trading with itself to 1 and guess δ_H. Given trade costs between each commuting zone, I find the unique $\frac{W_m}{A_m}$ and P_{Tn} that are consistent with earnings in each location,

$$W_m H_m \ell_m = \sum_n \frac{(\tau_{mn})^{1-\sigma} \left(\frac{W_m}{A_m}\right)^{1-\sigma}}{(P_{Tn})^{1-\sigma}} (1-\alpha) E_n$$

where

$$(P_{Tn})^{1-\sigma} = \sum_m \tau_{mn}^{1-\sigma} \left(\frac{W_m}{A_m}\right)^{1-\sigma}$$

and $W_m H_m \ell_m$ and E_n are the demographic adjusted variables found in the REA. For earnings, I use net earnings by place of residence. I back out the spending flows between CZs,

$$X_{mn} = \frac{\tau_{mn}^{1-\sigma} \left(\frac{W_m}{A_m}\right)^{1-\sigma}}{P_{Tn}^{1-\sigma}} (1-\alpha) E_n.$$

I then aggregate up to the state level and calculate the square loss of the implied consumptions shares in the model and the implied consumption shares from the data. I optimize over δ_H to minimize that loss.

F.3 Migration flows

I construct CZ-to-CZ migration flows over one year from the American Community Survey (ACS) for the years 2006-2022 leaving out 2020 due to the pandemic. Households report their current Public Use microdata area (PUMA) in the current year and an adjusted Public Use microdata area called the MIGPUMA in the previous year. The MIGPUMAs occassionally include multiple PUMAs. I start by assuming that if a household did not move, they are in the same commuting zone now as they were last year.

For those household who did report moving, I use crosswalks from the census to map the MIGPUMA onto the PUMA, weighted by the relative population. I then use the PUMA commuting zone crosswalk from https://www.ddorn.net/data.htm to map that into commuting zone to commuting zone migration flows. I then run a regression of log migration flows on log distance with origin and destination fixed effects.

	Transfers		Taxes	
	(1)	(2)	(3)	(4)
Earnings	-0.285	-0.188	0.911	0.769
	(0.022)	(0.030)	(0.107)	(0.098)
Employment		-0.490		0.693
		(0.111)		(0.240)
Observations	13,718	13,718	6,498	6,498

Table A1: Policy Function

Notes: The table shows the effects of earnings and employment on the size of government transfers and taxes paid at the commuting zone level. All values are demographically adjusted. All regressions are run in logs. All columns include controls for years. Standard errors clustered on CZ are in parentheses.

I set δ_D to rationalize the results from the regression. I set δ_ℓ to match the fact that 0.157 share of households report moving in the past year. I then guess a value for the cost of moving commuting zones δ_H, normalizing the cost of remaining in one's own commuting zone to $\tau_{\ell nn} = 0$. I find the unique utilities v_m so that the observed population in 2000 is a steady state,

$$\ell_m = \sum_n \exp\left(\nu(v_m - \tau_{\ell nm} - V_n)\right) \ell_n,$$

where

$$V_n = \frac{1}{\nu} \log\left(\sum_n \exp(\nu(v_m - \tau_{\ell nm})\right).$$

This implies a certain number of households staying in their commuting zone each year. I search over δ_H to match the observed share in the data.

F.4 Policy Function

I get data on government transfers from the Regional Economic Accounts and on taxes from IRS Statistics of Income as described in Appendix A. I adjust these transfers and taxes as described above. I then estimate these functions using first differences in logs. The results are Table A1. I use columns (2) and (4) as my main specification. I denote by δ_E^G and δ_H^G the coefficients of government transfers on earnings and employment respectively. Similarly, I denote by δ_E^T and δ_H^T the coefficients of taxes on earnings and employment respectively.

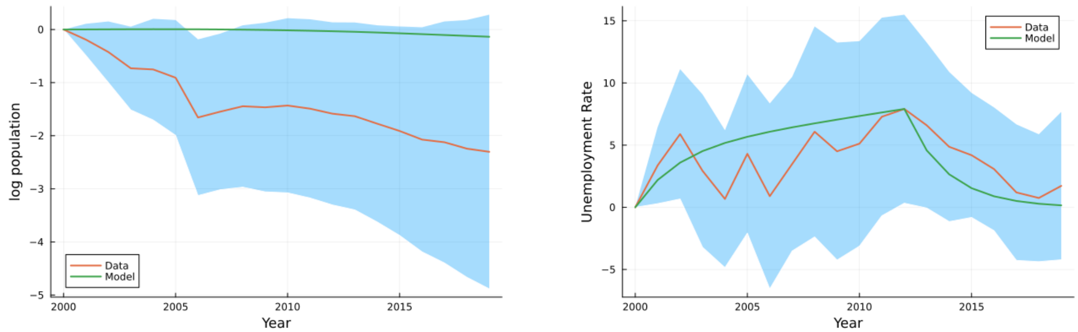
I then get γ^w and γ^H from

$$\gamma^w = \mu_G \delta_E^G - \mu_T \delta_E^T,$$
$$\gamma^H = \gamma^w + \mu_G \delta_H^G - \mu_T \delta_H^T.$$

F.5 Wage Rigidity

As described in the main text, I calibrate the degree of wage rigidity to match the impulse response function of working age population and unemployment to the China shock

Figure A5: Model Fit



(a) Population (b) Unemployment

Note: This plots the fit of the model on unemployment and population in response to the China trade shock. Bands indicate 95% confidence interval clustering on state.

with population and the labor wedge in the model. I use the measure of unemployment from the LAUS and I use the working age population available for download as part of the replication package for Autor et al. (2021). I also use their controls from their replication package.

I plot the fit in Figure A5. I do a decent job of matching the unemployment response to the China shock, but population adjusts too slowly in the model. In figure A6, I show that the long-run change in population is in the ball park of observed population drop in response to the China shock. This fit suggests that, if anything, the calibrated population adjustment is too slow and the migration elasticity is too small. Matching the observed China trade shock working age population response would increase the importance of the migration effect relative to what I found in the paper.

G Computational Algorithm

In this section, I describe the computational algorithm. I stack all of the variables into vectors. I start by describing the state variables $x(t)$. These are stacked so that

$$\begin{aligned} x(t)[n] &= \hat{v}_n(t) \\ x(t)[N+n] &= \hat{\ell}_n(t) \\ x(t)[2N+n] &= \hat{\pi}_n(t) \\ x(t)[3N+n] &= \hat{w}_n(t). \end{aligned}$$

I consider shocks

$$u(t)[n] = \hat{\phi}_n(t).$$

I have the planner directly chooses expenditure at each time t, which is equivalent to choosing transfers,

$$y(t)[n] = \hat{E}_n(t).$$

Figure A6: Population fit, longer time scale

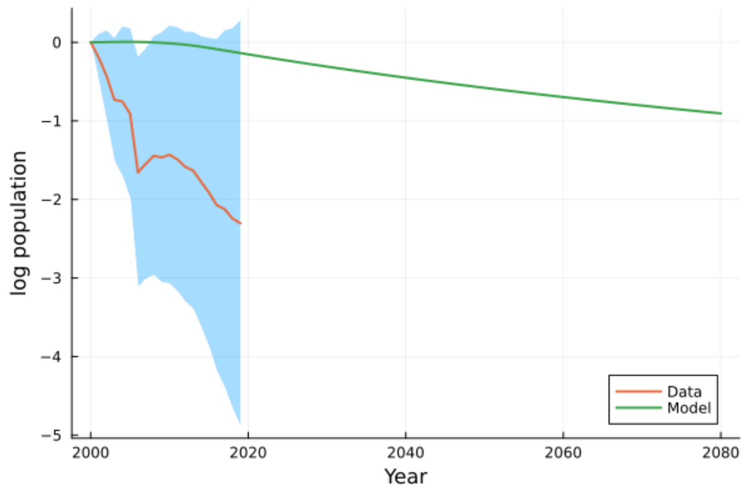


Figure A7: Population

Note: This plots the fit of the model on population in response to the China trade shock in the long run. Bands indicate 95% confidence interval clustering on state.

Then I also include a vector of intermediates variables

$$\begin{aligned} z(t)[n] &= \hat{V}_n(t) \\ z(t)[N+n] &= \hat{U}_n(t) \\ z(t)[2N+n] &= \hat{H}_n(t) \\ z(t)[3N+n] &= \hat{P}_{Tn}(t) \end{aligned}$$

I then put the linearized system into matrix form. I write the loss function as

$$\mathcal{W}(t) = (A_x x(t) + A_y y(t) + A_u u(t) + A_{2z} z(t))^\intercal A_1 (A_x x(t) + A_y y(t) + A_u u(t) + A_z z(t))$$

where A_1 is a diagonal matrix and each entry corresponds to one summand in the expression of the loss function.

The intermediate variables obey equations that I summarize in matrix form,

$$\Omega_z z(t) = \Omega_x x(t) + \Omega_u u(t) + \Omega_y y(t). \tag{A20}$$

And the state variables evolve according to

$$\dot{x}(t) = B_x x(t) + B_u u(t) + B_y y(t) + B_z z(t). \tag{A21}$$

G.1 Simulating Equilibrium

To solve for equilibrium given current policy, I note that given policy, expenditures are a function of state variables, the shocks, and intermediate variables, i.e.

$$y(t) = C_x x(t) + C_u u(t) + C_z z(t). \tag{A22}$$

Then combining this with equation (A20), we find how the intermediate variables vary with the state variable and the shocks:

$$z(t) = \Omega^p_{zx} x(t) + \Omega^p_{zu} u(t),$$

where

$$\Omega^p_{zx} \equiv \left(\Omega_{zz} - \Omega_{zy}\tilde{C}_z\right)^{-1}\left(\Omega_{zx} + \Omega_{zy}C_x\right); \quad \Omega^p_{zu} \equiv \left(\Omega_{zz} - \Omega_{zy}\tilde{C}_z\right)^{-1}\left(\Omega_{zu} + \Omega_{zy}\tilde{C}_u\right).$$

Then combining that expression with how the state variable evolves, we find how state variables evolve over time,

$$\dot{x}(t) = \left(B_x + B_y C_x + \left(B_z + B_y C_z\right)\Omega^p_{zx}\right)x(t) + \left(B_u + B_y C_u + \left(B_z + B_y C_z\right)\Omega^p_{zu}\right)u(t). \tag{A23}$$

This system of differential equations along with the initial conditions on population $\hat{\ell}_n(t)$ and wages $\hat{w}_n(t)$ then determine the equilibrium. Importantly, the matrix in front of $x(t)$ has $2N-1$ negative eigenvalues and one 0 eigenvalue. I describe how to solve a system of this form for an arbitrary time varying shock $u(t)$ below.

G.2 Optimal Policy

To solve for optimal policy, I solve for $z(t)$ as a function of the other variables

$$z(t) = (\Omega_z)^{-1}\Omega_x x(t) + (\Omega_z)^{-1}\Omega_u u(t) + (\Omega_z)^{-1}\Omega_y y(t).$$

I then plug this into the welfare function and how the state variables evolve to get a simplified system

$$\mathcal{W}(t) = \left(\tilde{A}_x x(t) + \tilde{A}_y y(t) + \tilde{A}_u u(t)\right)^{\intercal} A_1 \left(\tilde{A}_x x(t) + \tilde{A}_y y(t) + \tilde{A}_u u(t)\right),$$

with the matrices

$$\begin{aligned}\tilde{A}_x &= A_x + A_z(\Omega_z)^{-1}\Omega_x\\ \tilde{A}_y &= A_y + A_z(\Omega_z)^{-1}\Omega_y\\ \tilde{A}_u &= A_u + A_z(\Omega_z)^{-1}\Omega_u.\end{aligned}$$

I then multiply the matrices out to get

$$\begin{aligned}\mathcal{W} = {} & x^{\intercal}(t)A_{xx}x(t) + u^{\intercal}(t)A_{ux}x(t) + y^{\intercal}(t)A_{yx}x(t) + u^{\intercal}A_{uu}u(t) \\ & + y^{\intercal}(t)A_{yu}u(t) + y^{\intercal}(t)A_{yy}y(t),\end{aligned}$$

where

$$\begin{aligned}A_{xx} &= \tilde{A}_x^{\intercal}A_1\tilde{A}_x \\ A_{ux} &= \tilde{A}_u^{\intercal}A_1\tilde{A}_x \\ A_{yx} &= \tilde{A}_y^{\intercal}A_1\tilde{A}_x \\ A_{uu} &= \tilde{A}_u^{\intercal}A_1\tilde{A}_u \\ A_{yu} &= \tilde{A}_y^{\intercal}A_1\tilde{A}_u \\ A_{yy} &= \tilde{A}_y^{\intercal}A_1\tilde{A}_y.\end{aligned}$$

Computationally, constructing these matrices is not feasible for memory reasons. Instead, I break up the loss function into sub-problems and follow this exact procedure for each sub-problem. Then I add all of the results together to get the final welfare loss function.

The state variables change according to

$$\dot{x}(t) = \tilde{B}_x x(t) + \tilde{B}_u u(t) + \tilde{B}_y y(t),$$

where,

$$\begin{aligned}\tilde{B}_x &= B_x + B_z(\Omega_z)^{-1}\Omega_x \\ \tilde{B}_y &= B_y + B_z(\Omega_z)^{-1}\Omega_y \\ \tilde{B}_u &= B_u + B_z(\Omega_z)^{-1}\Omega_u.\end{aligned}$$

The planner faces the problem of

$$\begin{aligned}\max_{y(t),x(t)} \int_0^{\infty} e^{-\rho t}\Big[& x^{\intercal}(t)A_{xx}x(t) + u^{\intercal}(t)A_{ux}x(t) + y^{\intercal}(t)A_{yx}x(t) \\ & + u^{\intercal}A_{uu}u(t) + y^{\intercal}(t)A_{yu}u(t) + y^{\intercal}(t)A_{yy}y(t)\Big]dt\end{aligned}$$

such that

$$\dot{x}(t) = \tilde{B}_x x(t) + \tilde{B}_u u(t) + \tilde{B}_y y(t).$$

I set up the current value Hamiltonian and then take the first order necessary conditions

$$\begin{aligned}0 &= 2A_{yy}y(t) + A_{yx}x(t) + A_{yu}u(t) + \left(\mu^{\intercal}(t)\tilde{B}_y\right)^{\intercal} \\ \rho\mu(t) - \dot{\mu}(t) &= 2A_{xx}x(t) + (u^{\intercal}(t)A_{ux})^{\intercal} + (y^{\intercal}(t)A_{yx})^{\intercal} + \left(\mu^{\intercal}(t)\tilde{B}_x\right)^{\intercal}.\end{aligned}$$

I rearrange to get

$$\begin{aligned}
2A_{yy}y(t) + A_{yx}x(t) + A_{yu}u(t) + \tilde{B}_y^{\intercal}\mu(t) &= 0 \\
-2A_{xx}x(t) - A_{ux}^{\intercal}u(t) - A_{yx}^{\intercal}y(t) + \left(\rho\mathbb{I} - \tilde{B}_x^{\intercal}\right)\mu(t) &= \dot{\mu}(t) \\
\tilde{B}_x x(t) + \tilde{B}_u u(t) + \tilde{B}_y y(t) &= \dot{x}(t).
\end{aligned}$$

Solving for $y(t)$,

$$y(t) = -\frac{1}{2}A_{yy}^{-1}\left[\begin{bmatrix} A_{yx} & \tilde{B}_y^{\intercal} \end{bmatrix}\begin{bmatrix} x(t) \\ \mu(t) \end{bmatrix} + A_{yu}u(t)\right].$$

I can then set up a matrix that describes how the state and co-state variables develop. This is

$$\begin{aligned}
\begin{bmatrix} \dot{x}(t) \\ \dot{\mu}(t) \end{bmatrix} &= \begin{bmatrix} \tilde{B}_x & 0 \\ -2A_{xx} & \rho\mathbb{I} - \tilde{B}_x^{\intercal} \end{bmatrix}\begin{bmatrix} x(t) \\ \mu(t) \end{bmatrix} + \begin{bmatrix} \tilde{B}_y \\ -A_{yx}^{\intercal} \end{bmatrix} y(t) + \begin{bmatrix} \tilde{B}_u \\ -A_{ux}^{\intercal} \end{bmatrix} u(t) \\
&= \left(\begin{bmatrix} \tilde{B}_x & 0 \\ -2A_{xx} & \rho\mathbb{I} - \tilde{B}_x^{\intercal} \end{bmatrix} - \frac{1}{2}\begin{bmatrix} \tilde{B}_y \\ -A_{yx}^{\intercal} \end{bmatrix} A_{yy}^{-1}\begin{bmatrix} A_{yx} & \tilde{B}_y^{\intercal} \end{bmatrix}\right)\begin{bmatrix} x(t) \\ \mu(t) \end{bmatrix} \\
&\quad + \left(\begin{bmatrix} \tilde{B}_u \\ -A_{ux}^{\intercal} \end{bmatrix} - \frac{1}{2}\begin{bmatrix} \tilde{B}_y \\ -A_{yx}^{\intercal} \end{bmatrix} A_{yy}^{-1}A_{yu}\right) u(t).
\end{aligned}$$

I then define the matrix,

$$\Psi \equiv \begin{bmatrix} \tilde{B}_x & 0 \\ -2A_{xx} & \rho\mathbb{I} - \tilde{B}_x^{\intercal} \end{bmatrix} - \frac{1}{2}\begin{bmatrix} \tilde{B}_y \\ -A_{yx}^{\intercal} \end{bmatrix} A_{yy}^{-1}\begin{bmatrix} A_{yx} & \tilde{B}_y^{\intercal} \end{bmatrix},$$

with shocks

$$\psi = \begin{bmatrix} \tilde{B}_u \\ -A_{ux}^{\intercal} \end{bmatrix} - \frac{1}{2}\begin{bmatrix} \tilde{B}_y \\ -A_{yx}^{\intercal} \end{bmatrix} A_{yy}^{-1}A_{yu}.$$

The solution to the planner's problem can then be described by

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\mu}(t) \end{bmatrix} = \Psi\begin{bmatrix} x(t) \\ \mu(t) \end{bmatrix} + \psi u(t).$$

This system of differential equations along with the initial conditions on population $\hat{\ell}_n(t)$ and wages $\hat{w}_n(t)$ define the equilibrium. For the jump variables $\hat{v}_n(t)$ and $\hat{\pi}_n(t)$, their associated co-state variables must start at 0. Similar to the case for simulating equilibrium, Ψ has $4N-1$ negative eigenvalues and one 0 eigenvalue. I describe how to solve this system below.

G.3 Solving for Equilibrium

In this section, I describe the algorithm for solving an arbitrary system of the form

$$\dot{x}(t) = \Psi x(t) + \psi u(t),$$

with M initial conditions, where Ψ is a $2M \times 2M$ matrix with $M-1$ negative eigenvalues and one 0 eigenvalue. I will denote the eigenvalues by $\zeta_1, \ldots, \zeta_{2M}$ in increasing value with associated eigenvectors $v_1, \ldots, v_{2M}$.

I start by describing how to solve for equilibrium taking as given how $u(t)$ are changing. Suppose that shocks and transfers take the form

$$u(t) = \begin{cases} 0 & t < \tau \\ \overline{u} & t \geqslant \tau. \end{cases}$$

That is, there is a permanent shock starting at time τ. Below, I describe how to solve the model for this type of shock. Then I can approximate many time varying shocks using these results. Since the the model is linear, to get the full equilibrium response to a time varying shock, one simply needs to add together the equilibrium response to the shock at every point.

The computations then proceed in a few steps.

G.3.1 Steady State

I start by finding the steady state, however this is difficult as Ψ has a 0 eigenvalue so that there is a continuum of potential steady states. Denote by C the change of basis so that if $\tilde{x}$ is in eigenvector basis, $x = C\tilde{x}$ is in the usual basis. Then the steady state must solve

$$\begin{aligned} -\psi\overline{u} &= \Psi x^{SS} \\ -B_u\overline{u} &= C\tilde{\Psi}C^{-1}x^{SS} \\ -C^{-1}B_u\overline{u} &= \tilde{\Psi}C^{-1}x^{SS} \end{aligned}$$

where $\tilde{\Psi}$ is diagonal. I then find some vector x_temp that solves this equation. Then

$$x^{SS} = x_{temp} + \alpha_M v_M,$$

as $\Psi_x v_M = 0$.

G.3.2 After $\bar{t}$

I can rewrite the system after $t > \bar{t}$ as

$$\dot{x}(t) = \Psi(x(t) - x^{SS}).$$

Therefore, in order to converge to the steady state, starting at time $\bar{t}$ there exist values $\alpha_1, \ldots \alpha_{M-1}$ with eigenvectors $v_1, \ldots v_{M-1}$ such that

$$x(t) - x^{SS} = \sum_{i=1}^{M-1} \alpha_i v_i e^{\zeta_i t}.$$

G.3.3 Before $\bar{t}$

Before $\bar{t}$, we know that

$$\dot{x}(t) = \Psi x(t).$$

Then the system evolves according to

$$x(t) = \sum_{i=1}^{2M} \beta_i v_i e^{\zeta_i t}$$

G.3.4 Putting it Together

The equilibrium then follows

$$x(t) = \begin{cases} \sum_{i=1}^{2M} \beta_i v_i e^{\zeta_i t} & t < \bar{t} \\ \sum_{i=1}^{M-1} \alpha_i v_i e^{\zeta_i t} + \alpha_M v_M + x_{temp} & t \geqslant \bar{t}. \end{cases}$$

I then solve for α_i and β_i so that

$$\sum_{i=1}^{4N} \beta_i v_i e^{\zeta_i \bar{t}} = \sum_{i=1}^{2N-1} \alpha_i v_i e^{\zeta_i \bar{t}} + \alpha_{2N} v_{2N} + x_{temp}^{SS},$$

and the initial conditions are satisfied.

H Per Capita Labor Demand Effects in Dynamic Model

Proposition A5. *Suppose that there is a continuum of locations with no migration costs and wages are perfectly rigid ($\delta_w = 0$). Then, after a small demand shock $\hat{\phi}_n$, the total effect on per capita labor demand of a small transfer at time t' is*

$$\int_0^\infty e^{-\rho t} \hat{H}_n(t) \frac{d\hat{H}_n(t)}{d\hat{T}_n(t')} dt = \left(\frac{\alpha}{1-\alpha} - \frac{\nu}{\rho + \delta_\ell} \left(1 - e^{-\delta_\ell t'} \right) \right) e^{-\rho t'} \hat{\phi}_n.$$

Proof. In the limit with an infinite number of locations and fully rigid wages ($\delta_w = 0$) the system is described by

$$\dot{\hat{v}}_n(t) = -\hat{U}_n(t) + (\rho + \delta_\ell)\hat{v}_n(t)$$

$$\dot{\hat{\ell}}_n(t) = \delta_\ell(\nu \hat{v}_n(t) - \hat{\ell}_n(t))$$

$$\hat{U}_n(t) = \hat{E}_n(t) - \hat{H}_n(t)$$

$$\hat{E}_n(t) = \hat{H}_n(t) + \hat{T}_n(t)$$

$$\hat{H}_n(t) = (1-\alpha)\hat{\phi}_n - (1-\alpha)\hat{\ell}_n(t) + \alpha \hat{E}_n(t).$$

Combining, the two differential equations are

$$\dot{\hat{v}}_n(t) = -\hat{T}_n(t) + (\rho + \delta_\ell)\hat{v}_n(t)$$

$$\dot{\hat{\ell}}_n(t) = \delta_\ell(\nu\hat{v}_n(t) - \hat{\ell}_n(t)).$$

Integrating up the utility equation

$$\hat{v}_n(t) = \int_t^\infty e^{-(\rho+\delta_\ell)(s-t)}\hat{T}_n(s)ds,$$

Integrating up the labor equation, using $\hat{\ell}_n(0) = 0$,

$$\hat{\ell}_n(t) = \delta_\ell\nu\int_0^t e^{\delta_\ell(s-t)}\hat{v}_n(s)ds.$$

Then solving for $\hat{\ell}_n(t)$ in terms of transfers

$$\begin{aligned}
\hat{\ell}_n(t) &= \delta_\ell\nu\int_0^t e^{\delta_\ell(s-t)}\hat{v}_n(s)ds \\
&= \delta_\ell\nu\int_0^t e^{\delta_\ell(s-t)}\int_s^\infty e^{-(\rho+\delta_\ell)(r-s)}\hat{T}_n(r)drds \\
&= \delta_\ell\nu e^{-\delta_\ell t}\int_0^t\int_s^\infty e^{(\rho+2\delta_\ell)s}e^{-(\rho+\delta_\ell)r}\hat{T}_n(r)drds \\
&= \delta_\ell\nu e^{-\delta_\ell t}\int_0^t\int_0^r e^{(\rho+2\delta_\ell)s}e^{-(\rho+\delta_\ell)r}\hat{T}_n(r)dsdr \\
&\quad + \delta_\ell\nu e^{-\delta_\ell t}\int_t^\infty\int_0^t e^{(\rho+2\delta_\ell)s}e^{-(\rho+\delta_\ell)r}\hat{T}_n(r)dsdr \\
&= \delta_\ell\nu e^{-\delta_\ell t}\int_0^t \frac{e^{(\rho+2\delta_\ell)r}-1}{\rho+2\delta_\ell}e^{-(\rho+\delta_\ell)r}\hat{T}_n(r)dr \\
&\quad + \delta_\ell\nu e^{-\delta_\ell t}\int_t^\infty \frac{e^{(\rho+2\delta_\ell)t}-1}{\rho+2\delta_\ell}e^{-(\rho+\delta_\ell)r}\hat{T}_n(r)dr \\
&= -\frac{\delta_\ell\nu}{\rho+2\delta_\ell}e^{-\delta_\ell t}\int_0^\infty e^{-(\rho+\delta_\ell)r}\hat{T}_n(r)dr \\
&\quad + \frac{\delta_\ell\nu}{\rho+2\delta_\ell}\int_0^t e^{\delta_\ell(r-t)}\hat{T}_n(r)dr + \frac{\delta_\ell\nu}{\rho+2\delta_\ell}\int_t^\infty e^{-(\rho+\delta_\ell)(r-t)}\hat{T}_n(r)dr
\end{aligned}$$

Then taking the derivative with respect to $\hat{T}_n(r)$ for $r < t$ is

$$\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)} = -\frac{\delta_\ell\nu}{\rho+2\delta_\ell}e^{-\delta_\ell t}e^{-(\rho+\delta_\ell)r} + \frac{\delta_\ell\nu}{\rho+2\delta_\ell}e^{\delta_\ell(r-t)} = e^{-\delta_\ell t}\frac{\delta_\ell\nu}{\rho+2\delta_\ell}\left[e^{\delta_\ell r} - e^{-(\rho+\delta_\ell)r}\right].$$

For $r > t$

$$\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)} = -\frac{\delta_\ell\nu}{\rho+2\delta_\ell}e^{-\delta_\ell t}e^{-(\rho+\delta_\ell)r} + \frac{\delta_\ell\nu}{\rho+2\delta_\ell}e^{-(\rho+\delta_\ell)(r-t)} = \frac{\delta_\ell\nu}{\rho+2\delta_\ell}e^{-(\rho+\delta_\ell)r}\left[e^{(\rho+\delta_\ell)t} - e^{-\delta_\ell t}\right].$$

Then I look to find the effect on hours. Hours are given by

$$\hat{H}_n(t) = \hat{\phi}_n - \hat{\ell}_n(t) + \frac{\alpha}{1-\alpha}\hat{T}_n(t).$$

Therefore,

$$\frac{d\hat{H}_n(t)}{d\hat{T}_n(r)} = -\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)} + \frac{\alpha}{1-\alpha}\mathbb{I}_{r=t}.$$

Meanwhile, starting with no transfers, hours are given by $\hat{H}_n(t) = \hat{\phi}_n$. Therefore, defining $X_n = \int_0^\infty e^{-\rho t}\hat{H}_n(t)^2 dt$, I have

$$\begin{aligned}
\frac{dX_n}{d\hat{T}_n(r)} &= \int_0^\infty e^{-\rho t}\hat{H}_n(t)\frac{d\hat{H}_n(t)}{d\hat{T}_n(r)}dt \\
&= \int_0^\infty e^{-\rho t}\hat{\phi}_n(t)\left[-\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)} + \frac{\alpha}{1-\alpha}\mathbb{I}_{r=t}\right]dt \\
&= -\int_0^\infty e^{-\rho t}\hat{\phi}_n(t)\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)}dt + e^{-\rho r}\hat{\phi}_n(r)\frac{\alpha}{1-\alpha}
\end{aligned}$$

Plugging in for how population changes,

$$\begin{aligned}
\frac{dX_n}{d\hat{T}_n(r)} &= -\int_0^\infty e^{-\rho t}\hat{\phi}_n(t)\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)}dt + e^{-\rho r}\hat{\phi}_n(r)\frac{\alpha}{1-\alpha} \\
&= \int_0^r e^{-\rho t}\hat{\phi}_n(t)\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)}dt - e^{-\rho r}\hat{\phi}_n(r)\frac{\alpha}{1-\alpha} + \int_r^\infty e^{-\rho t}\hat{\phi}_n(t)\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)}dt \\
&= \int_0^r e^{-\rho t}\hat{\phi}_n(t)\frac{\delta_\ell \nu}{\rho+2\delta_\ell}e^{-(\rho+\delta_\ell)r}\left[e^{(\rho+\delta_\ell)t} - e^{-\delta_\ell t}\right]dt - e^{-\rho r}\hat{\phi}_n(r)\frac{\alpha}{1-\alpha} \\
&\quad + \int_r^\infty e^{-\rho t}\hat{\phi}_n(t)e^{-\delta_\ell t}\frac{\delta_\ell \nu}{\rho+2\delta_\ell}\left[e^{\delta_\ell r} - e^{-(\rho+\delta_\ell)r}\right]dt
\end{aligned}$$

I then integrate

$$
\begin{aligned}
\frac{dX_n}{d\hat{T}_n(r)} &= \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{-(\rho+\delta_\ell)r} \int_0^r e^{-\rho t} \hat{\phi}_n \left[e^{(\rho+\delta_\ell)t} - e^{-\delta_\ell t} \right] dt - e^{-\rho r} \hat{\phi}_n \frac{\alpha}{1-\alpha} \\
&\quad + \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} \left[e^{\delta_\ell r} - e^{-(\rho+\delta_\ell)r} \right] \int_r^\infty e^{-\rho t} \hat{\phi}_n e^{-\delta_\ell t} dt \\
&= \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{-(\rho+\delta_\ell)r} \hat{\phi}_n \int_0^r e^{\delta_\ell t} dt - \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{-(\rho+\delta_\ell)r} \hat{\phi}_n \int_0^r e^{-(\rho+\delta_\ell)t} dt \\
&\quad - e^{-\rho r} \hat{\phi}_n \frac{\alpha}{1-\alpha} \\
&\quad + \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{\delta_\ell r} \hat{\phi}_n \int_r^\infty e^{-(\rho+\delta_\ell)t} dt - \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} \hat{\phi}_n e^{-(\rho+\delta_\ell)r} \int_r^\infty e^{-(\rho+\delta_\ell)t} dt \\
&= \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{-(\rho+\delta_\ell)r} \hat{\phi}_n \frac{e^{\delta_\ell r} - 1}{\delta_\ell} - \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{-(\rho+\delta_\ell)r} \hat{\phi}_n \frac{e^{-(\rho+\delta_\ell)\infty} - 1}{-(\rho+\delta_\ell)} \\
&\quad - e^{-\rho r} \hat{\phi}_n \frac{\alpha}{1-\alpha} \\
&\quad + \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{\delta_\ell r} \hat{\mu}_1 \frac{e^{-(\rho+\delta_\ell)\infty} - e^{-(\rho+\delta_\ell)r}}{-(\rho+\delta_\ell)} \\
&= \hat{\phi}_n \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} \left[\frac{1}{\delta_\ell} + \frac{1}{\rho + \delta_\ell} \right] \left(e^{-\rho r} - e^{-(\rho+\delta_\ell)r} \right) - e^{-\rho r} \hat{\mu}_1 \frac{\alpha}{1-\alpha}
\end{aligned}
$$

We can then rewrite to get the result.

□

I Computational Appendix

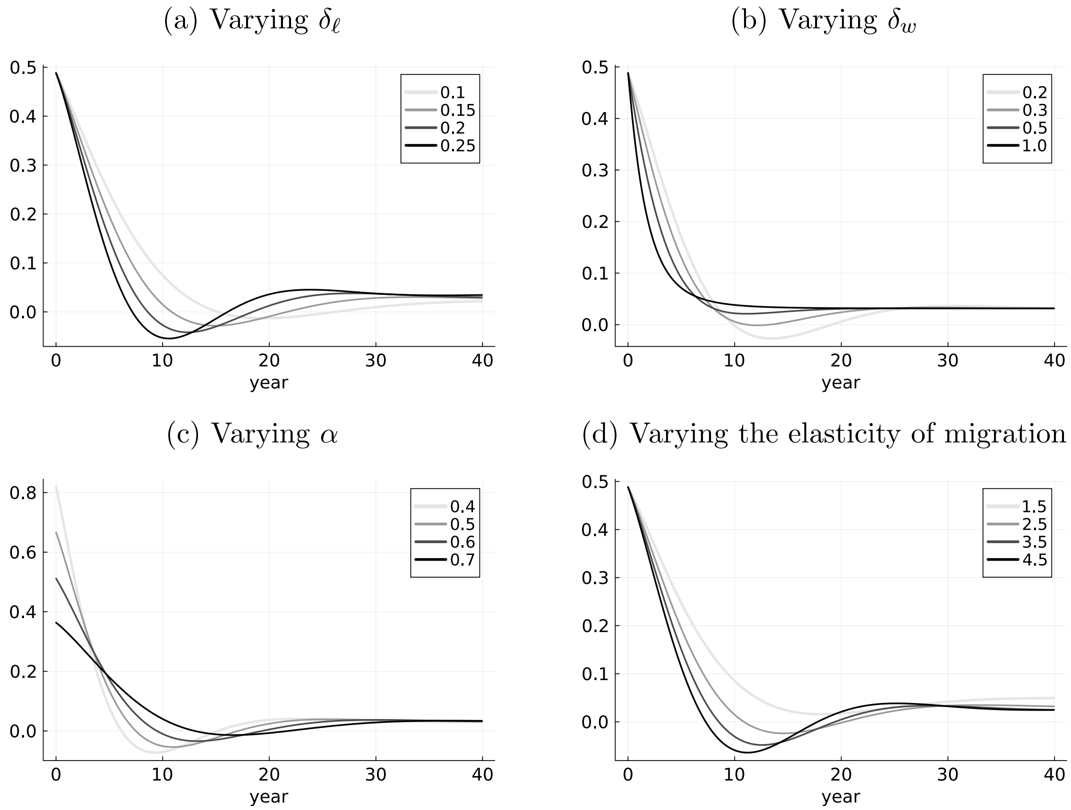
I.1 Idiosyncratic Shock Robustness

In this section, I show how robust the optimal policy time path is to varying key model parameters. In particular, I plot the time path of optimal transfers in response to a demand shock while varying key parameters determining the relative strength of the migration and stimulus effect in Figure A8. In Figure A8a, I vary the speed of migration δ_ℓ, holding fixed the long run migration elasticity. Figure A8b varies the degree of wage rigidity. Figure A8c shows how the policy changes with the local multiplier, and Figure A8d shows how sensitive the policy is to the long run migration elasticity.

I start by discussing how the speed of population change affects the optimal policy in A8a. When population adjusts very slowly (i.e. δ_ℓ is close to 0), the optimal transfer never falls below the long run insurance level. That is because the planner cannot affect population on the time scale necessary to affect the recession. People might be very mobile in the long run, but if they will only move out 10 years after a policy change, there is no macroeconomic benefit because wages will have already adjusted by that point. When people are very quick to move, the migration effect becomes more important because people's migration decision is very responsive to planned taxes.

Next I vary the speed with which wages adjust in A8b. Similar to δ_ℓ, varying δ_w plays

Figure A8: Optimal Policy Robustness



Note: This figure shows how the optimal policy changes with various parameters.

a large role in how important the migration effect is. The main difference is that while increasing δ_ℓ speeds up the movement of households so they can respond while the recession is happening, decreasing δ_w slows down the wages so that the recession is still happening while population slowly adjusts. Thus, as wages become perfectly rigid (i.e δ_w becomes very small), the optimal transfer becomes negative for a large number of years following the demand shock. As wages adjust more quickly, migration cannot react in time so that the transfers never drop below their long run insurance levels. However, the basic structure of generous transfers that quickly fade out remains robust.

Varying the home bias in consumption α has very different impacts on the optimal transfers as seen in A8c. Increasing α makes stimulus payments much more effective. Therefore, as $\alpha \to 1$, the stimulus effect always dominates the migration effect so that there is no large dip in the optimal transfer. However, when transfers are very effective at stimulating the local economy, the government does not need to transfer as much money to a region in a recession to stimulate it. Therefore, at time 0, the optimal transfer is actually decreasing in the degree of home bias.

Finally, I show how the optimal transfer changes with the long run migration elasticity in Figure A8d. Increasing that elasticity changes the insurance effect because it increases the misallocation caused by giving a small transfer to the region. Therefore, the optimal long run transfer decreases in the migration elasticity. This comparative static also changes the migration effect. When households' location choices are more responsive to transfers, the

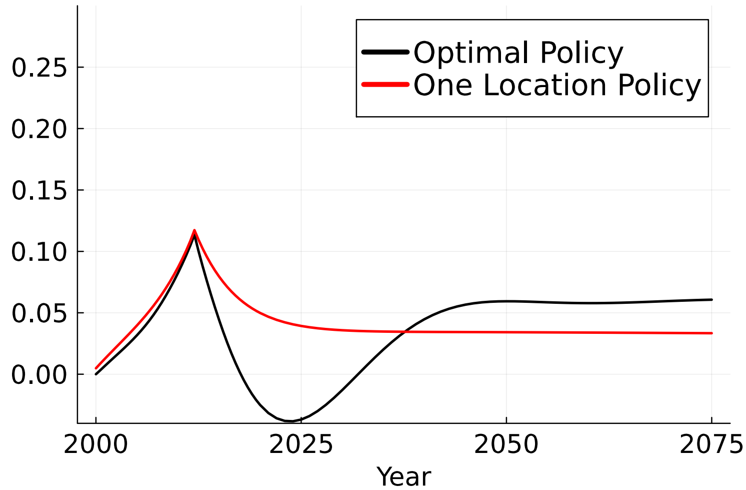


Figure A9: Comparison to one location optimal policy

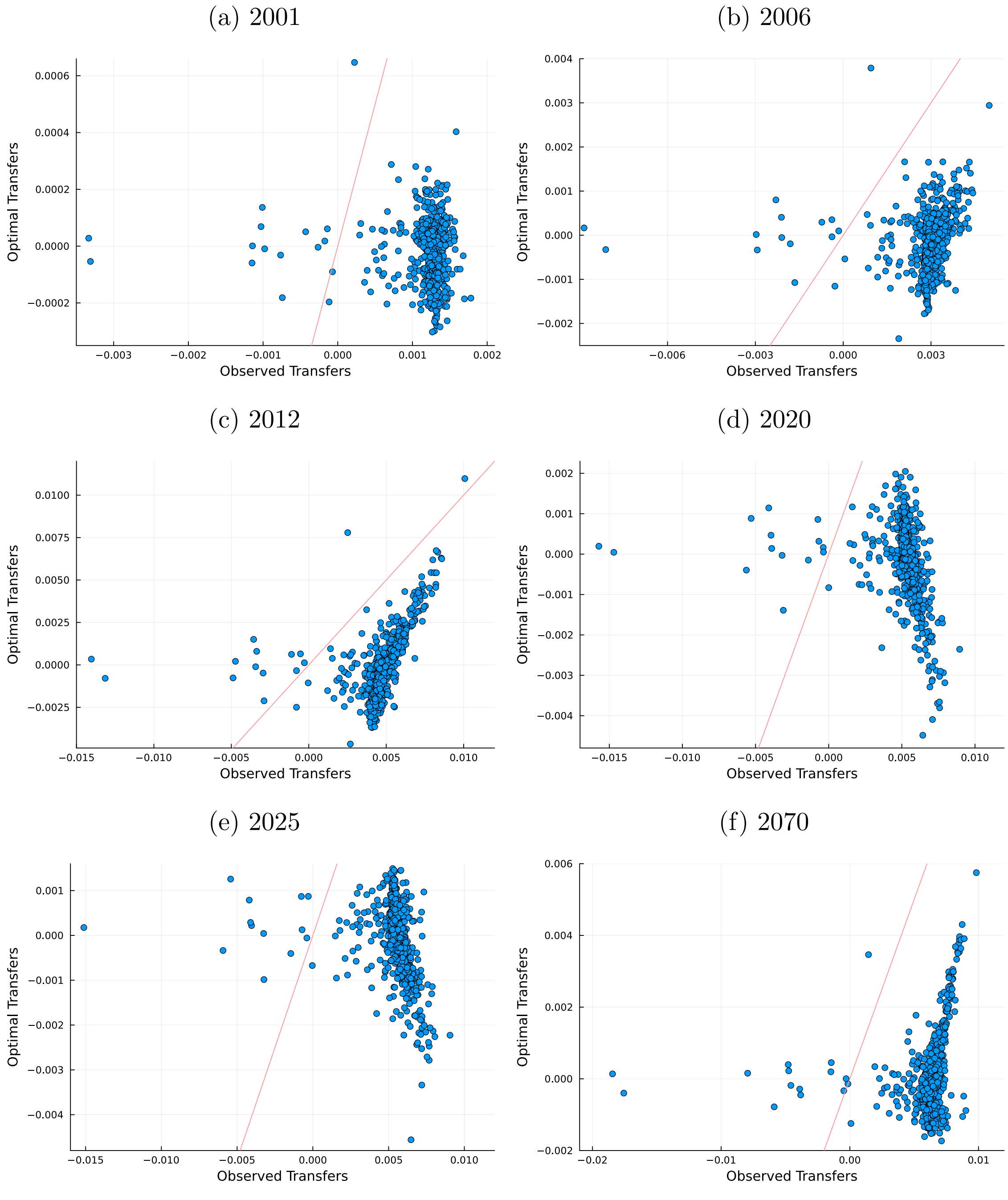
government will want to tax a recessionary city more to encourage people to move out.

I.2 Further China Trade Shock Analysis

In this appendix, I do a more in-depth comparison of the optimal policy against other policies. In Figure A9, I again plot the coefficient of a regression of the optimal transfer relative to the initial earnings on the size of the trade shock. I also include the coefficient of the same regression if the policy followed the optimal policy rule implied by the one-time, one-location shock, $\gamma^w = -0.05$ and $\gamma^H = -1.7$. I find that the one location optimal policy rule matches the maximum stimulus transfer in the year 2012. The transfer then does not fall as quickly as that suggested by the optimal policy.

In Figure A10, I look at the scatter plots comparing the observed transfers against the optimal transfers for various years. The observed transfers do the best job of replicating optimal transfers in 2012, when the stimulus effect dominates, and in 2070, where the transfers are providing insurance.

Figure A10: Comparing the Observered Transfers to Optimal Transfers



Note: This figure plots the observed transfers against the fully optimal transfers across commuting zones for a variety of years. The 45 degree line is plotted in red.