Revitalize or Relocate:

Optimal Place-based Transfers for Local Recessions

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Many regions in the US experience depressed labor demand and high unemployment, even when the rest of the United States does not. How should the US government respond? In this paper, I characterize optimal place-based transfers in a dynamic economic geography model with nominal wage rigidity and compare them to observed government transfers. I show that transfers not only have a stimulus effect—by boosting local demand—but also a migration effect—by encouraging local residents to stay. Analytically, I provide optimal transfer formulas that capture this trade-off and show, perhaps surprisingly, that the optimal transfer to a distressed region may be a tax due to the migration effect. All else equal, transfers should be larger in the short-run and when distressed places are geographically concentrated. Quantitatively, I find that observed transfers are both too small in the short-run and too large in the medium-run, achieving about a third of the gains from the fully optimal response to idiosyncratic local shocks. I conclude by exploring how the US government could have responded to the China trade shock in the 2000s.

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1 Introduction

The Janesville Assembly Plant produced its final car for GM on December 23, 2008.[1] In the following months and years, large numbers of workers lost their jobs. Though a large factory stood empty and many people were willing to work for low wages, no new company moved in to offer lower wages and employment opportunities. Instead, the area experienced high unemployment and growing poverty for years afterwards. This is not an isolated case. Autor et al. (2013) report widespread declines in employment, often times larger than the declines in manufacturing employment, in regions of the United States that compete directly with Chinese goods. North Hickory, North Carolina, for example, saw a steady surge of Chinese imports that competed with their own furniture products for more than a decade and suffered for it.

In the case of the China shock, there is now a large literature that has shown how it affected, not only labor markets, but also mortality (Pierce and Schott, 2020), political beliefs (Autor et al., 2020; Che et al., 2022), marriage rates (Autor et al., 2019), and many other outcomes. Sophisticated trade and geography models have been developed to evaluate the average incidence of the shock as well (Galle et al., 2017; Caliendo et al., 2019). But surprisingly, little has been said about how the government should have responded. Should the government encourage people to leave, to find jobs elsewhere? Should the government provide funds to help reinvigorate the region? Do the answers differ in the short- and long-run? Does it depend on the nature of the shock? The goal of this paper is to provide a normative framework to help address these questions.

To set the stage, I first show that the government does not sit idle in response to regional increases in unemployment. Instead, national and state governments transfer money to regions after a shock through a variety of tax and transfer programs, including unemployment insurance and a progressive income tax. The first contribution of this paper is then a set of analytical results. I provide a sufficient statistic for the optimal place-based transfers and then derive qualitative results on how they change with time and the nature of the shock. The second key contribution is a set of quantitative results that compares the fully optimal transfers in my calibrated model of the US economy to the observed policies.

The starting point of my analysis is that wages may not fully adjust after demand for labor in a region goes down, leading to involuntary underemployment.[2] Since economic

[1] See Goldstein (2017) for a moving account of what happened to Janesville, Wisconsin after the factory closed.

[2] There is now a large literature documenting downward nominal wage rigidity (Grigsby et al., 2021; Hazell and Taska, 2020; Jo, 2024), and a growing literature suggesting that downward wage rigidity is important to understand how regions adjust to idiosyncratic demand shocks (Costinot et al., 2022; Rodríguez-Clare et al., 2020).

conditions may vary across regions, monetary policy is not sufficient to put everyone back to work, so place-based transfers may help. I first formalize this idea in the context of a two-period economic geography model with fully rigid wages. I model all transfers as explicitly place-based to capture the key trade-off in designing policy, but I discuss how the results apply more generally to place-biased policies as well. I set up the second-best planner's problem where workers are free to live where they would like (subject to migration frictions) and the planner can tax or subsidize certain areas. While the planner cannot directly move people, it can indirectly influence where people want to live by making certain regions more or less attractive with transfers.

In addition to their direct redistributional effects, place-based transfers have two macroeconomic effects: a stimulus effect and a migration effect.[3] The stimulus effect comes from the fact that people spend disproportionately on goods and services near them, and so giving a region money will increase demand in the local area. When wages are rigid, there will be an aggregate demand externality leading to first order welfare benefits, as emphasized by Kenen (1969) and formalized by Farhi and Werning (2017). All other things equal, transferring money from a booming area to a busting area will cool down the booming economy while heating up the area in a recession, efficiently putting people back to work.

The migration effect emerges because transfers influence where people want to live. If the government gives tax breaks to people living in an area, other people will be more likely to move there, and people already living there will be less likely to move out. When output is demand-determined because wages are sticky, this movement of people will have an important impact on underemployment. Each region produces some traded goods for the country and the amount demanded is independent of local spending and population. Consider the GM factory in Janesville. With sticky prices, it needs to build a certain number of cars to meet the demand of the outside world. It only needs a certain number of man-hours to do that. In the short run, that will not adjust so movement of people in and out of the region will change the population without affecting employment in the traded sector. This force implies that, if anything, the federal government should tax hard-hit areas to encourage people to find jobs somewhere else.

I derive three analytical results that demonstrate how the migration and stimulus effects interact to shape optimal place-based policy. First, I consider what fiscal transfers should be in a small region that just had a negative shock to the demand of its traded output, like Janesville. Starting from a point with no transfers, a transfer to Janesville improves macroeconomic stability if and only if the local multiplier is larger than per capita earnings

[3] The redistribution effect will not be the focus of my analysis here. See Gaubert et al. (2021) and Donald et al. (2023) for in depth discussions of how place-based policy can be used for redistribution.

multiplied by the semi-elasticity of population to a transfer (holding fixed labor supply); thus, the optimal transfer could be a tax. This might seem counterintuitive since, when there is no migration, transferring money to a region in a recession always helps stimulate the economy, improving welfare. One might have thought that allowing migration would simply mute that effect. In fact, the migration effect can overturn that result, making a place-based transfer counterproductive. This is because government transfers directly increase the utility of living in a location, independent of the stimulus effect, and that increase in utility leads to migration which reduces the employment rate. Therefore, the fully optimal transfer could be positive or negative, depending on the local multiplier and the migration semi-elasticity.

While the previous result provides a clear cut-off to weigh the relative strength of the migration effect versus the stimulus effect, in practice many demand shocks do not hit only one region. Instead, they are spatially correlated. My next result considers what the spatial nature of the shock implies for the optimal transfer. I find that if migrants to and from Janesville disproportionately come from and to areas that are in a recession, then the optimal transfer is larger than that suggested by the local multiplier and the migration semi-elasticity. That is due to the migration effect. If workers disproportionately leave areas in a recession to go to Janesville, that might hurt the recession in Janesville, but it will help the areas that those workers left. Therefore, considering Janesville in a vacuum misses an important effect. When demand shocks are correlated, there might be more scope for the national government to use transfers to stimulate an entire area.

My final analytical result considers the effects of dynamics on the optimal place-based transfers. In particular, I show that the transfer to Janesville in period 2 is smaller than that suggested by the local multiplier and the migration semi-elasticity. This is due to a dynamic migration effect. One might have thought that transfers in the second period would have the same trade-off between the stimulus effect and the migration effect, but because people have more time to move, the migration effect is stronger and so the optimal transfer is smaller. That is not the full story because period 2 transfers not only affect where people live in period 2, but also period 1. If the government has made it clear that it will tax households that are in Janesville in period 2, households that have the opportunity to leave in period 1 will do so. Thus, the planner can encourage out-migration in period 1 without losing stimulus.

In the quantitative portion of the paper, I develop a dynamic New Keynesian economic geography model to derive the quantitative implications for optimal transfers in response to two different demand shocks, an idiosyncratic one, like Janesville, and the China trade shock, and compare those transfers to the observed transfers in the United States. To do so, I move to a continuous time, parametric version of my theoretical model where wages are only partially rigid, due to a standard Calvo friction, and there are finite trade costs in the traded sector, in order to capture realistic geographic features of the US economy. In contrast to the leading dynamic economic geography models studying the response to the China trade shock, I calibrate the model to the 722 commuting zones in the continental United States rather the states to assess the effectiveness of transfers for fighting the local recessions that arise in each of the distinct labor markets of the US. I matched observed trade flows between states, observed migration flows between commuting zones, and economic activity at the commuting level.

I then consider what optimal fiscal transfers look like in the aftermath of an idiosyncratic demand shock. Comparing the optimal policy to observed policy, fiscal transfers should be four times larger immediately after the shock to efficiently put households back to work. However, those transfers should then more quickly scale back. I find that the government should give less transfers to households than that suggested by redistributive motives in commuting zones 10 years after the shock to encourage out-migration. Observed policy gets only 35% of the welfare gains of optimal policy over no policy at all. I also find that making unemployment insurance more generous after a commuting zone-wide shock could get much of the welfare gains. Thus, perhaps the US government should consider making the special unemployment benefits that workers have access to in times of high unemployment more generous, not just longer lasting. Alternatively, the local government could engage in its own fiscal stimulus, borrowing money to jump start the economy, and paying it back over the period 5-20 years after the shock.

I revisit how the national government could have used place-based policy to fight against the local recessions that resulted from competition with Chinese exporters. If the planner had anticipated how bad the China shock was going to be, the planner should have gradually ramped up transfers towards those region directly affected until the end of the China shock. That is because, before the China shock peaks, the planner wants to decrease the population so that they are not around when the worst of the recession happens. Balancing that against the stimulus effects leads to slowly increasing transfers. After the peak of the China shock, the optimal transfers slowly fall towards their long run redistributive levels, never falling below them, suggesting the migration effect has a smaller influence with this spatially correlated shock. Transfers to nearby regions are especially effective since they stimulate the commuting zones that were hit, while encouraging workers to leave relatively worse hit regions.

The rest of the paper is structured as follows. There is a short Related Literature section below where I mention a number of papers related to the current study. In section 2, I present descriptive facts about how government transfers in the United States respond to unexpected increases in local unemployment. I present a two-period model economic geography model with wage rigidity in section 3, before analytically characterizing the optimal policy and teasing out the implications in section 4. The continuous time version of this model used for quantification is in section 5. I show what the model implies for optimal policy in response to an idiosyncratic demand and compare it to observed policy in section 6, and then I demonstrate what policy is in response to the China trade shock in section 7. I give some concluding remarks in section 8. All proofs of propositions are in the appendix.

Related Literature

This paper most directly contributes to the literature on placed-based policy. The literature has identified two motives for place-based policy: redistribution and efficiency. Gaubert et al. (2021) and Donald et al. (2023) both discuss the redistributive reasons for policy. On the efficiency side for policy, Abdel-Rahman and Anas (2004), Wildasin (1980), Fajgelbaum and Gaubert (2020) and Kline and Moretti (2014) all study how optimal spatial policy could correct for agglomeration externalities. More closely related to this paper are those studying labor market distortions. Austin et al. (2018) shows that if the employment elasticity differs between regions, government policy should vary across the US. Kline and Moretti (2013) find optimal place-based policy when finding a job is subject to search and matching frictions, and Bilal (2023a) considers a similar setting where heterogeneous firms sort across markets. I contribute to this literature by considering what place-based policy can do in response to a different market failure: wage rigidity. I show that the implications for optimal policy are different and the timing of the transfers play an important role.

My paper also contributes to a large literature studying how regions respond to idiosyncratic shocks. Blanchard and Katz (1992) and Yagan (2019) study how states respond to shocks that are not uniform across the US. Autor et al. (2013), Topalova (2010), and Dix-Carneiro (2014) all study how regions respond to trade shocks. Costinot et al. (2022) studies the effect of the collapse of trade between Finland and the USSR on worker outcomes and rationalizes some of the results with a model of wage rigidity. A growing dynamic trade and economic geography literature tries to quantify the welfare impacts of such trade shocks. Galle et al. (2017) and Caliendo et al. (2019) are two such neoclassical examples. Lyon and Waugh (2019) consider the welfare implications when households have imperfect savings tools. Rodríguez-Clare et al. (2020) incorporate wage rigidities, and Kim et al. (2023) shows that currency pegs play a key role in explaining the large impact of the China shock. My paper differs primarily in focus. I am mostly interested in the normative question: what should the government do to fight the local recessions that arise from the shock? Thus, I differ from much of the literature by modeling more granular geography (CZs) and modeling sticky wages so that I can consider optimal policy without being subject to a Lucas critique.

To make progress on this issue, I built on the themes and ideas in the Optimal Currency Area (OCA) literature. This literature has emphasized a number of important features of successful currency unions like factor mobility (Mundell, 1961), trade openness (Mundell, 1961), fiscal integration (Kenen, 1969), and financial integration (Mundell, 1973). My paper can be viewed as formalizing the results from Kenen (1969) when there is significant factor mobility as expressed by Mundell (1961).

Within this literature, my paper is most closely related to Farhi and Werning (2014, 2017). Farhi and Werning (2017) consider what optimal fiscal policy should look like in a currency unions when people are stuck in a location. I show that some of the results are overturned when there is significant factor mobility. Like the present paper, Farhi and Werning (2014) allows for factor mobility in a currency union. However, Farhi and Werning (2014) compares equilibrium migration to the migration a planner would enact if the planner could directly control where people live, hence they have nothing to say about place-based policy. My paper takes as given that people can live where they want and then solves an optimal reallocation of funds exercise.

2 Local Recessions & Transfers: Motivating Facts

In this section, I show how wage earnings and transfers from the government respond to changes in local unemployment at the commuting zone level. In the spirit of Blanchard and Katz (1992), I interpret an innovation in unemployment as the start of a local recession. I then trace out the impulse response functions using local projection methods.[4] I briefly describe the data I use in the text, further details are in Appendix A.

I use data on unemployment and labor force counts by county for 1990-2022 from the Local Area Unemployment Statistics (LAUS) managed by the US Bureau of Labor Statistics. I then aggregate to the commuting zone level following Tolbert and Sizer (1996) and Autor and Dorn (2013).

2.1 Wage Adjustment

I start by studying how wages in a commuting zone adjust to an innovation in unemployment relative to wages in the rest of the United States. I use data on individual wage

[4]These methods were pioneered by Jordà (2005) and have become a standard tool for macroeconomists looking to describe impulse response functions. See Jordà and Taylor (2024) for a review.

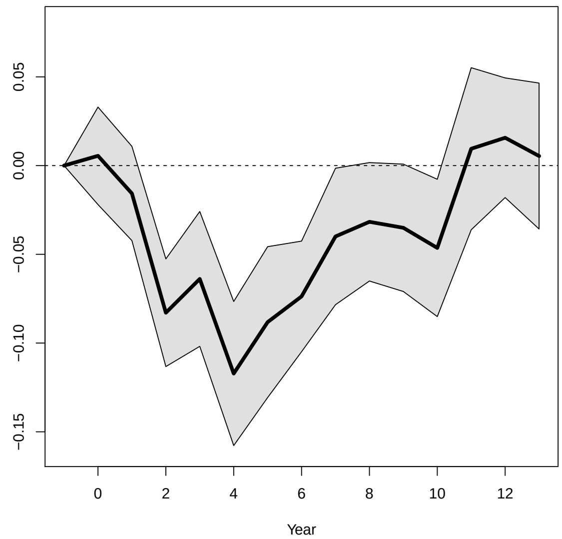


Figure 1: Log Wage Response.

Note: I plot the local Jorda projection of log wage earnings in a commuting zone on innovations in local unemployment controlling for the log number of weeks worked and detailed demographic controls described in the text. Results are normalized to correspond to a jump in unemployment of 10 percentage points. Bands indicate 95% confidence intervals clustering on commuting zone.

earnings along with county of residence, demographic information, and weeks worked from the Annual Social and Economic Supplements (ASEC) of the Current Population Survey (CPS). I then project wage earnings of individuals in commuting zone n on an innovation in unemployment h periods earlier, controlling for the number of weeks worked.

My main regression specification is:

$$\log E^w_{i,t+h} = \delta^h \log \text{weeks}_{i,t+h} + \beta_h u_{n(i)t} + \gamma^h_{n(i)} + \gamma^h_t + \sum_{L=1}^{\overline{L}} \gamma^h_{uL} u_{n(i),t-L} + \Gamma^h X_{ith} + \varepsilon^w_{ith},$$

where $E^w_{i,t}$ is the wage earnings of individual i in year t, $\text{weeks}_{i,t}$ is the number of weeks that individual worked, $u_{n(i)t}$ is the unemployment in i's commuting zone in year t, γ^h_n and γ^h_t are commuting zone and year fixed effects respectively, and X_{ith} is a vector of individual level controls including education, race, sex, industry, age, and age^2. Controlling for lagged unemployment $u_{n(i),t-L}$ controls for the expected path of unemployment, so that β_h identifies the impact of an innovation in unemployment at time t on log wages h periods after. I use $\overline{L} = 2$, though including more (or less) lags does not materially affect the results.

I plot the estimates of β_h in Figure 1, normalizing the results to correspond to a 10 percentage point increase in unemployment. I find that weekly wage earnings do not move at all the year of the increase in unemployment. Instead, weekly wage earnings in the commuting zone slowly decrease relative to earnings in the rest of the US over the following 4 years, before leveling off and recovering. This is inconsistent with a neoclassical model of economic geography with no capital, where wages would drop immediately after the increase in unemployment. Then wages would recover to some extent as the shock that caused the decline subsides or workers leave to find better opportunities elsewhere. It is also inconsistent with Dix-Carneiro and Kovak (2017)'s model where wages fall long after a shock as capital slowly degrades for two reasons. First, there is no initial drop in wages. The wage earnings only become statistically significantly different from zero 2 years after the shock. Second, wages start to recover after 4 years whereas wages would continue to fall if capital were continually depreciating.

However, this is consistent with the well documented phenomenon of downward wage rigidity. Blanchard and Katz (1992) find that wages drop for 4 years in response to similar employment innovations at the state level. In the trade literature, Rodríguez-Clare et al. (2020) show that wage rigidity can account for the employment response to the China trade shock, and Costinot et al. (2022) find that wage rigidity can account for the wage dynamics in Finland after a collapse in trade with the Soviet Union. And in the macro literature, there is now significant micro evidence that wages are sticky downwards for both continuously employed workers (Grigsby et al., 2021) and new hires (Hazell and Taska, 2020).

2.2 Government Response

Next, I turn to how the government responds to unemployment innovations in a commuting zone. Using the same projection technique, I first analyze how much money the government sends to the region through various public assistance programs in response to an innovation in local unemployment. I then turn to payments in income tax to see how much less money the government collects in taxes from the region. Throughout, I will continue to normalize the results to correspond to a 10 percentage point jump in unemployment.

I use data on government transfers to each county from the Regional Economic Accounts (REA) managed by the Bureau of Economic Analysis (BEA). They report the aggregate payments to all households in a county for which no service is provided, what they call the personal current transfer receipts. This includes social security benefits, medical benefits, veterans' benefits, and unemployment benefits. It also includes some payments from businesses for personal injury, though businesses only make up 1.7% of total current transfers in 2022.

Using τ^c_{nt} to denote personal current transfer receipts per capita, my main specification

(a) Public Assistance Programs (b) Log Income Retention Rate

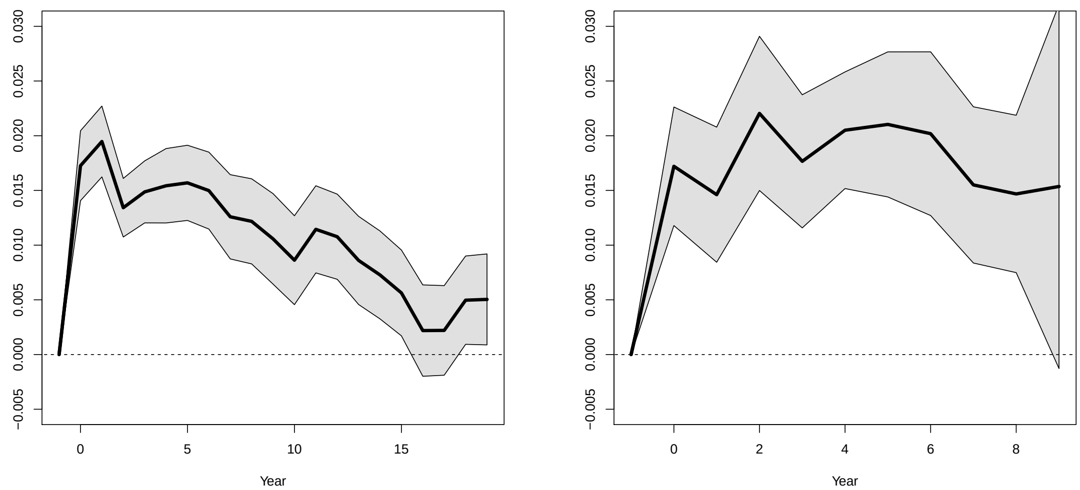


Figure 2: Government Transfer Impact on Log Income

Note: Panel a and b plot local Jorda projections of log public assistance programs and log income retention rates in a commuting zone on innovations in local unemployment, respectively. Results are normalized to correspond to a jump in unemployment of 10 percentage points. Bands indicate 95% confidence intervals clustering on commuting zone.

is:

$$\log \tau^c_{n,t+h} = \beta_h u_{nt} + \gamma^h_n + \gamma^h_{s(n)t} + \sum_{L=1}^{\overline{L}} \gamma^h_{uL} u_{n,t-L} + \gamma^h_O \log \mathrm{OldShare}_{n,t+h} + \varepsilon^c_{nth},$$

where $\mathrm{OldShare}_{nt}$ is the share of adults in the commuting zone over 65. Since retirement makes up a large component of the transfers, controlling for the share of people over 65 removes the mechanical increase in τ^c_{nt+h} that would occur as working age people leave the commuting zone to find work elsewhere and retired people stay. I plot the estimates of β_h normalized by the share of income that is personal current transfers to find the log first order impact of the transfer on total log income in the region in Figure 2a.[5] I find that on impact, these transfers spike to increase total take home pay by almost 2%. The size of the transfers then slowly decrease over the next 15 years.

Finally, I turn to the other side of the government ledger. The Internal Revenue Service (IRS) maintains the Statistics of Income (SOI), and starting in 2010, they record total income tax paid by each county along with total gross income. Thus, I can construct the

[5]Details are in Appendix A.

income retention rate by commuting zone for the years 2010-2021 to see how income tax collection responds to local recessions. I plot the Jorda projection controlling for only one lag of unemployment so that I can get a longer time horizon in Figure 2b. I find that the income retention rate jumps by around 0.015 log points immediately after the shock and remains there for all years that I have data. This is driven primarily by the fact that the United States has a progressive income tax. After the unemployment shock, earnings in the region drop, so that people end up in a lower tax bracket. Therefore, they have to pay a smaller percentage of their income in taxes.

3 A Two Period Model of Local Recessions

I propose as a starting point that local underemployment may arise from the inability of wages to adjust. To capture the economic forces in the most transparent way, I assume that wages are perfectly rigid, workers are hand-to-mouth, and goods are either freely traded with no trade costs or non-traded. In this setting, I can fully characterize the solution to a second best planner's problem choosing place-based transfers to fight the local recessions. In section 5, I will introduce a continuous time version of the same model and relax the assumptions on fully rigid wages and no trade costs. This will allow me to quantify optimal place-based transfers both in response to idiosncratic shocks (in Section 6) and the China shock (in Section 7).

For expositional purposes, I model all fiscal transfers as explicitly place-based to illustrate the key mechanism in this section, however, as shown above, most transfers to regions in a recession are facially place-neutral. They only end up place-biased because what they target correlates with local recessions. I will return to the distinction between the two in section 4.5 and in my quantitative analysis.

3.1 Environment

Consider an economy with N regions indexed by $n, m \in \mathcal{N} = \{1, \ldots, N\}$ and two periods indexed by $t \in \{1, 2\}$. Throughout, I will use subscripts to index values and superscripts to index functions. I will then use subscripts on functions to denote partial derivatives.

Households. There is a continuum of households that I index by $i \in \mathcal{I}$. I let $n_t(i)$ denote the region where i lives at time t. Each household starts in a region $n_0(i)$. Then, at the beginning of period $t \in \{1, 2\}$, each household observes preference shocks for every region, $\boldsymbol{\varepsilon}_t(i) = (\varepsilon_{1t}(i), \ldots, \varepsilon_{Nt}(i)) \in \mathbb{R}^N$. These shocks are distributed according to a continuous cumulative distribution function that may depend on household i's location at time $t-1$, $G_{n_{t-1}(i)}(\cdot)$. Thus, these preference shocks can include migration costs or idiosyncratic preferences for location. The utility that household i gets from living in region n at time 1 and region m at time 2 is given by

$$U_{n1} + \varepsilon_{n1}(i) + \beta\left(U_{m2} + \varepsilon_{m2}(i)\right),$$

where U_{nt} is the fundamental utility of region n, and $\beta \in [0, 1]$ is the discount rate.[6]

Then the population of region n at time t, ℓ_{nt}, is given by

$$\ell_{nt} = \int_{\mathcal{I}} \mathbb{1}_{n_t(i)=n} di, \tag{1}$$

where I have normalized the population to measure 1.

All of the households agree on the fundamental utility of a location. This fundamental utility in region n period t is determined by a nested set of functions

$$\begin{aligned} U_{nt} &= U^n(C_{nt}, H_{nt}), \\ C_{nt} &= C^n(C_{Tnt}, C_{NTnt}), \\ C_{Tnt} &= C^T(\{C_{Tmnt}\}), \end{aligned}$$

where C_{nt} is the sub-utility that a household in location n derives from consuming goods, H_{nt} is her per capita hours of labor supply, C_{Tnt} is the consumption of a freely traded aggregate, C_{NTnt} is the consumption of the non-traded good produced in location n, and C_{Tmnt} is the consumption of the traded good produced in location m. I assume that $U^n(C, H)$ is twice continuously differentiable, quasi-concave, strictly increasing in C, and decreasing in H. The consumption sub-utilities $C^n(C_{Tn}, C_{NTn})$ and $C^T(\{C_{Tmn}\})$ are both homogeneous of degree 1 and strictly quasi-concave.

Firms. In both the freely traded and non-traded sector, a representative firm produces using technology linear in labor. That is,

$$Y_{snt} = A_{sn} H_{snt} \ell_{nt},$$

[6]This general set up nests much of the economic geography literature that puts particular distributional restrictions on $\boldsymbol{\varepsilon}$. The assumption of additive shocks distributed according to a Gumbel distribution as used in Caliendo et al. (2019) is an explicit special case of the model. For the economic geography models that use multiplicative shocks distributed Fréchet as in Fajgelbaum and Gaubert (2020), one can simply define a new utility as log of the old utility. The set of Pareto optimal allocations will be the same in this transformed economy and it will fall under my assumptions. This setup also nests the Calvo friction to migration used by Peters (2022) as a limit case.

where Y_{snt} is the production of location n in sector $s \in \{T, NT\}$, A_{sn} is the productivity, and H_{snt} is hours per worker in sector s, region n at time t.[7]

Market Clearing. For the labor market to clear in each location, total labor supply needs to equal the labor used by the freely traded sector and the non-traded sector,

$$H_{nt}\ell_{nt} = H_{Tnt}\ell_{nt} + H_{NTnt}\ell_{nt}, \text{ for all } n, t. \tag{2}$$

The market for the non-traded good needs to clear market-by-market,

$$Y_{NTnt} = C_{NTnt}\ell_{nt}, \text{ for all } n, t. \tag{3}$$

And demand for the freely traded good produced in location i needs to equal production,

$$Y_{Tnt} = \sum_m C_{Tmnt}\ell_{nt}, \text{ for all } n, t. \tag{4}$$

Wage Rigidity. Nominal wages in each location W_n are sticky; they are therefore parameters of the model rather than equilibrium objects. The inefficiencies in the model arise because wages are either too high or too low given the realized demand for labor, given preferences and technology. When wages are too high, the quantity of labor demanded of households in a location will be below what the households would like to supply. Therefore, those households will be underemployed relative to the first best and policy can play some role in correcting that distortion.[8]

3.2 Decentralized equilibrium

Profit Maximization. Firms are perfectly competitive. They choose production to maximize profits taking as given wages and prices:

$$Y_{snt} \in \underset{Y'_s}{\text{argmax}} \quad \left\{ \left(P_{snt} - \frac{W_n}{A_{sn}} \right) Y'_s \right\}, \text{ for all } s, n, t. \tag{5}$$

[7] Note that I assume that all workers in region n work the same number of hours. Employment will be demand determined, it will therefore lead to underemployment of all workers, not unemployment of some.

[8] I write the model here as one with wage rigidities that are exogenously set. I could also consider a more standard macro model with monopolistic firms that set prices of goods (or wages) before the realization of some state of the world, but cannot change them in the ex-post stage when the state of the word is realized. I will do this in the quantitative section. For now, note that at this ex-post stage, prices (or wages) are fixed so there is no difference between my analysis and this alternative approach.

Thus, $P_{snt} = W_n / A_{sn}$ for all t. Without risk of confusion, I drop the t index on prices from now on.

Utility Maximization. I start by taking as given utility in each location and characterize the household's dynamic optimization problem. I then return to characterize the intratemporal problem.

Households are free to live wherever they would like. Thus, they move to the location that provides them the most utility, however they do not know their utility shocks for period 2 when choosing their first location. Therefore, I characterize the household migration problem using backward induction. In period 2, household i observes her utility shocks $\boldsymbol{\varepsilon}_2$ and chooses

$$n_2(i) \in \underset{m}{\operatorname{argmax}} \quad U_{m2} + \varepsilon_{m2}(i). \tag{6}$$

Denote by $\overline{U}_{n2} \equiv \mathbb{E}[\max_{n'} U_{n'2} + \varepsilon_{n'2} | n_1(i) = n]$ the expected utility in period 2 of a household who lives in location n at the end of period 1, before the idiosyncratic utility shocks $\boldsymbol{\varepsilon}_2$ are revealed. This is a function of the vector of fundamental utility levels in period 2. Then in period 1, the household chooses her location to maximize expected utility,

$$n_1(i) \in \underset{m}{\operatorname{argmax}} \quad U_{m1} + \beta \overline{U}_{m2} + \varepsilon_{m1}(i). \tag{7}$$

Conditional on living in location n at time t, households choose consumption to maximize utility subject to a single period budget constraint as they cannot save,

$$\sum_m P_{Tm} C_{Tmnt} + P_{NTn} C_{NTn} \leqslant W_n H_{nt} + T_{nt},$$

where P_{Tm} is the price of the freely traded good produced in location n, P_{NTn} is the price of the non-traded good produced in location n, W_n is the wage paid in location n, and T_{nt} is the per capita transfer from the government to people in location n at time t. That is

$$\begin{aligned} \{C_{nt}, C_{NTnt}, C_{Tnt}, \{C_{Tmnt}\}\} \in \underset{C, C_{NT}, C_T, \{C_{Tm}\}}{\operatorname{argmax}} \Bigg\{ U^n(C, H_{nt}) \Bigg| & \\ C = C^n(C_T, C_{NT}), & \\ C_T = C^T(\{C_{Tm}\}) & \\ \sum_m P_{Tm} C_{Tm} + P_{NTn} C_{NT} \leqslant W_n H_{nt} + T_{nt} \Bigg\}. & \end{aligned} \tag{8}$$

The nested nature of the preferences allows for the problem to be broken down into subcomponents. First note that $C_T(\cdot)$ is homogeneous of degree 1 and identical across locations. Then, since there are no trade costs within the traded sector, there exists a common aggregate price of the traded good $P_T = \min\{\sum_m P_{Tm} C_{Tm} | C^T(\{C_{Tm}\} \geqslant 1\}$. In turn, the price of the consumption aggregate C_{nt} in each location n is $P_n = \min\{P_{NTn} C_{NT} + P_T C_T | U^n(C_{NT}, C_T) \geqslant 1\}$.

Importantly, households do not choose their hours H_{nt}. Instead, labor is completely demand determined in each location. This creates a wedge since the marginal rate of substitution between consumption and labor may not be equal to the relative price. With flexible wages, the household would choose consumption and labor supply so that $U^n_C / P_n = -U^n_H / W_n$. The labor wedge is a measure of how far this first order condition is from being satisfied. I will denote this wedge as follows:

$$\tau_{nt} \equiv 1 + \frac{P_n}{W_n} \frac{U^n_H}{U^n_C}.$$

If an economy is in a local, then the household is working less than it would like. Therefore, $|U^n_H|$ will be low, leading to a positive labor wedge. On the other hand, the wedge will be negative if the region is going through a local boom.

Government Policy. The government serves two roles. First, it transfers money between regions. The budget constraint at period t for the national government is

$$\sum_n \ell_{nt} T_{nt} = 0, \text{ for all } t. \tag{9}$$

The government may set the level of aggregate demand through monetary policy. In this simplified setup, I assume that the government can choose nominal GDP directly

$$E_t = \sum_n P_n C_{nt} \ell_{nt}, \text{ for all } t. \tag{10}$$

In a richer, dynamic model, the government would do this by setting the interest rate.

Definition 1. *Given nominal GDP in each period E_t and per capita transfers T_{nt}, an* equilibrium *is a set of location choices $n_t(i)$, utility levels U_{nt}, regional population ℓ_{nt}, prices for freely trade goods P_{Tn}, prices for non-traded goods P_{NTn}, consumption levels C_{Tmnt}, C_{NTnt}, labor supplies H_{nt}, and output Y_{NTnt}, Y_{Tnt}, such that:*

- *Households choose consumption and their location to maximize utility,* (6), (7), (8)*;*
- *Population is consistent with location choices,* (1)*;*

- *Firms maximize profits taking prices as given,* (5)*;*
- *The government's budget constraints hold,* (9)*;*
- *The total value of consumption is equal to nominal GDP* (10)*; and*
- *Markets clear,* (2)*,* (3)*,* (4)*.*

3.3 The Planner's Problem

The planner chooses monetary policy E_t, place-based transfers T_{nt}, and associated expected utilities $U(i) \equiv \max_n U_{n1} + \varepsilon_{n1}(i) + \beta \overline{U}_{n2}$ to maximize social welfare. I assume that social welfare is a weighted sum of utility with weight $\lambda(i)$ on household i. Formally, the planner's problem (PP) is,

$$\max_{E_t, \{T_{nt}\}, \{U(i)\} \in \mathcal{E}} \mathcal{W}, \tag{PP}$$

where $\mathcal{W} \equiv \int_{\mathcal{I}} \lambda(i) U(i) di$ and $\mathcal{E}$ is the set of utility profiles attainable in a competitive equilibrium, as described in Definition 1.

4 Optimal Place-based Transfers

In this section, I derive the implications for optimal place-based transfers. Before I do that, I characterize the economy of a region n at time t as a function of monetary policy, the population ℓ_{nt}, and the transfer from the government T_{nt}. This will provide intuition for how government policies can affect regions in a recession, and also simplify the planner's problem. In setting this up, it will be easier to think of monetary policy as choosing the national spending on the traded sector, E_{Tt} where $E_{Tt} \equiv \sum_m P_T C_{Tmt} \ell_{mt}$, rather than total spending. I show these are equivalent, and provide all of the proofs for this section, in appendix B.

For all the analytical results in this section, I focus on the limit as the discount factor $\beta \to 0$. This allows me to focus on the static implications for policy in the first period without worrying about the second period. Then, in the second period, I illustrate the dynamic implications of policy while ignoring feedback effects of the first period back on the second period. I will dispense with this limit assumption as well in my quantitative analysis.

4.1 Preliminary: Characterizing Hours & Utility in Equilibria

In this section, I characterize hours and utility in a location m period t as a function of monetary policy E_{Tt}, the transfer T_{mt}, and population ℓ_{mt}. This will allow me to simplify the planner's problem. The characterization proceeds in two steps. I start by solving the consumption decision of households in each location summarized in equation (8). I then find what hours worked is consistent with those consumption choices and government policy.

Since prices are fixed and the consumption aggregator over the traded output of each location is homothetic, the consumption decision (8) implies that households spend a fixed proportion ϕ_m of their traded expenditures on the output of location m, i.e.

$$P_{Tm}C_{Tmnt} = \phi_m P_T C_{Tnt}.$$

Multiplying by the population in location n, ℓ_{nt}, and summing across all locations we find, from the market clearing condition for traded production, satisfies (4), that total spending on the traded output of location m is a fixed share of traded output,

$$P_{Tm}Y_{mt} = \phi_m E_{Tt}.$$

Total labor earnings in location m, $W_j H_{mt}\ell_{mt}$, is then that spending on traded output plus spending on the non-traded good. Again from the consumption decision (8), spending on the non-traded good is simply a fixed share of total income α_m, and total income is labor earnings $W_m H_{mt}\ell_{mt}$, plus the transfer from the government $T_{mt}\ell_{mt}$, therefore, by the market clearing for non-traded goods, (3),

$$P_{NTm}Y_{NTmt} = \alpha_m \left(W_m H_{mt}\ell_{mt} + T_{mt}\ell_{mt}\right).$$

Then using the market clearing condition for labor in region m, (2),

$$W_m H_{mt}\ell_{mt} = \phi_m E_{Tt} + \alpha_m \left(W_m H_{mt}\ell_{mt} + T_{mt}\ell_{mt}\right).$$

This defines hours worked as a function of monetary policy E_{Tt}, population ℓ_{mt}, and the transfer from the government T_{mt}. In what follows I define this function as,

$$H^m(E_T, \ell, T) \equiv \frac{1}{W_m}\left(\frac{\phi_m E_T}{1-\alpha_m}\frac{1}{\ell} + \frac{\alpha_m}{1-\alpha_m}T\right). \tag{11}$$

I also define an indirect utility function for households in location m only as a function of the transfer T_{mt} and hours worked H_{mt}. Substituting in that real consumption is total earnings $W_m H$ plus the transfer T divided by the price level P_m, I find that

$$V^m\left(H, T\right) \equiv U^m\left(\frac{W_m}{P_m}H + \frac{T}{P_m}, H\right). \tag{12}$$

The derivatives of the two previous functions, H^m and U^m, will play a crucial role in my characterization of optimal place-based transfers. I formally describe them in the lemma below.

Lemma 1. *The derivatives of the hours worked function are*

$$\frac{\partial H^n}{\partial \log E_T} = \frac{1}{W_n}\frac{\phi_n E_T}{1-\alpha_n}\frac{1}{\ell}; \quad \frac{\partial H^n}{\partial \log \ell} = -\frac{1}{W_n}\frac{\phi_n E_T}{1-\alpha_n}\frac{1}{\ell}; \quad \frac{\partial H^n}{\partial T} = \frac{1}{W_n}\frac{\alpha_n}{1-\alpha_n}. \tag{13}$$

The derivatives of the indirect utility function are

$$\frac{\partial V^n}{\partial H} = W_n \frac{U_C^n}{P_n}\tau_{nt}; \quad \frac{\partial V^n}{\partial T} = \frac{U_C^n}{P_n}. \tag{14}$$

First, consider how E_T shapes the hours worked H^m, as described in equation (13). When the central government heats up the entire economy by increasing spending in the freely traded sector, the people in each location will work more in the freely traded sector, $\frac{\partial H^m}{\partial \log E_T} > 0$. However, at the same time, they will get more money, and they will want to spend that money on traded and non-traded goods. This will increase demand for the local non-traded good, increasing the labor supplied to that sector leading to a feedback loop. The size of that feedback loop is summarized by the proportion of spending on the non-traded good, α_n. What this means for the utility V^n of households in region n depends on whether the location is in a boom or bust. If it is in a bust ($\tau_{nt} > 0$), then the households there value the opportunity to work more and earn more money, $\frac{\partial V^n}{\partial H} > 0$, as shown in (14). On the other hand, if the labor market is already hot, household utility will decrease from having to work even harder.

In this model, migration ends up having a similar effect on hours worked and utility as does an increase in the level of expenditures on freely traded goods, as can also be seen from equations (13) and (14). Suppose that more people move to location n. The demand for the traded output of the location remains the same, which means they cannot start producing more. Instead, every households needs to reduce the number of hours they are working so that the total number of hours worked at a location remains the same when including the extra workers. Then the feedback loop leads to reduced hours per household in the non-traded sector as well, $\frac{\partial H^m}{\partial \log \ell} < 0$. The effect on utility then depends on the labor wedge of (14). If the area is in a recession, workers leaving will increase the utility of those left behind because those left behind can work and earn more, since $\frac{\partial V^n}{\partial H} > 0$.

Direct monetary transfers from the government behave very differently. In particular, they provide a direct utility benefit by increasing consumption of the traded goods (14) on top of the stimulus effect (13). Whether the increase in hours increases utility depends again on the state of the economy. If the economy is in a recession ($\tau_{nt} > 0$), then the social value of an extra dollar is higher than the marginal utility of income, $\frac{V^n}{\partial T} + \frac{\partial V^n}{\partial H}\frac{\partial H^n}{\partial T} > \frac{U_C^n}{P_n}$, and there are positive externalities from spending more. If the economy is already booming ($\tau_{nt} < 0$), then working more will hurt the residents and the total benefit from a transfer is smaller than the private internalized benefit.

4.2 The Simplified Planner's Problem

Having characterized hours and utility as a function of monetary policy E_{Tt}, transfers T_{nt}, and population ℓ_{nt}, I now restate the planner's problem in a simplified form that only includes the government's policy (E_{Tt} and T_{nt}), and the fundamental utilities and population in each region (U_{nt} and ℓ_{nt}). Define the expected utility in period 2 of living in location n in period 1 as $\overline{U}^{n2}(\{U_{m2}\}) = \mathbb{E}[\max_m U_{m2} + \varepsilon_{m2} | n_1(i) = n]$. To make the formula slightly more compact, I introduce the notation $\overline{U}_{n1}(i) \equiv U_{n1} + \varepsilon_{n1}(i) + \beta \overline{U}^{n2}(\{U_{m2}\}$. Then the simplified planner's problem is as follows:

$$\max_{\{\overline{U}_{n1}(i)\}, E_{Tt}, \{T_{nt}\}, \{U_{nt}\}, \{\ell_{nt}\}} \int_{\mathcal{I}} \lambda(i) \sum_n \mathbb{1}_{n \in \arg\max_m \overline{U}_{m1}(i)} \left[\overline{U}_{n1}(i)\right] di, \tag{SPP}$$

such that utility is given by the indirect utility functions derived in section 4.1,

$$U_{mt} = V^m \left(T_{mt}, H^m \left(E_{Tt}, \ell_{mt}, T_{mt}\right)\right) \text{ for all } m, t; \tag{15}$$

population in period 1 is consistent with free mobility, (1), (7),

$$\ell_{n1} = \ell^{n1}\left(\{\overline{U}_{m1}\}\right) \text{ for all } n, \tag{16}$$

where $\ell^{n1}(\{\overline{U}_{m1}\}) \equiv \mathbb{P}[n \in \arg\max_m \overline{U}_{m1}]$; population in period 2 is consistent with location choice in period 1 and free mobility (1), (6),

$$\ell_{m2} = \sum_n \ell_{n1} \mu^{nm}\left(\{U_{k2}\}\right) \text{ for all } m, \tag{17}$$

where $\mu^{nm}\left(\{U_{k2}\}\right) \equiv \mathbb{P}[m \in \text{argmax}_k U_{k2} + \varepsilon_{k2} | n_1(i) = n]$; and the government budget constraints hold, (9).

4.3 Optimal Short-Run Transfers

To characterize the optimal short-run transfers, I focus on the first order necessary conditions of (SPP). I start by summarizing how monetary policy adjusts in the background to ensure that the average labor wedge across locations is zero.

Lemma 2. *In any interior solution to* (SPP)*,*

$$\sum_n \frac{W_n H_{Tn1}}{1-\alpha_n} \ell_{n1} \frac{\tau_{n1}}{1+\frac{\alpha_n}{1-\alpha_n}\tau_{n1}} = 0.$$

By increasing the overall spending in the entire economy, the planner can stimulate all regions. Thus, the planner sets the average labor wedge to zero, properly weighting each region according to its economic importance. Before I show the relevant first order condition for place-based transfers, I introduce a variable ζ_{n1} to denote the social marginal utility of income in region n period 1. It is defined as $\zeta_{n1} \equiv \frac{\overline{\lambda}_{n1} U_C^n}{P_n}$, where $\overline{\lambda}_{nt} = \mathbb{E}[\lambda(i) | n_t(i) = n]$ is the average Pareto weight on households in location n at time t. This measures how much social welfare increases if the income of the average household in location n increases slightly, holding all else fixed. The household's utility increase depends on the price index in the location P_n and her marginal utility of consumption U_C^n. What that means for social welfare then depends on the average weight the planner puts on those in the location, $\overline{\lambda}_{nt}$.

The first order condition for a transfer to location n implies the next lemma.

Lemma 3. *In any interior solution to* (SPP)*, first period transfers must satisfy*

$$\underbrace{\sum_m \ell_{m1} T_{m1} \nu_{n1}^{m1}}_{\textit{fiscal externality}} = \ell_{n1} \Bigg[\underbrace{\frac{\zeta_{n1}}{\lambda_{G1}} - 1}_{\textit{redistribution}} + \underbrace{\frac{\zeta_{n1}}{\lambda_{G1}} \frac{\alpha_n}{1-\alpha_n} \tau_{n1}}_{\textit{stimulus effect}} \Bigg] - \underbrace{\sum_m \frac{W_m H_{Tm1}}{1-\alpha_m} \ell_{m1} \frac{\tau_{m1}}{1+\frac{\alpha_m}{1-\alpha_m}\tau_{m1}} \nu_{n1}^{m1}}_{\textit{migration effect}},$$

where $\nu_{n1}^{m1} \equiv \frac{\partial \log \ell^{m1}}{\partial U_{n1}} \left(\frac{\partial V^n}{\partial T_{n1}} + \frac{\partial V^n}{\partial H_{n1}} \frac{\partial H^{n1}}{\partial T_{n1}} \right)$ *is the migration semi-elasticity of population in location m to a transfer in location n holding fixed utility in locations other than n, and $\lambda_{G1} > 0$ is the social value of the government having another dollar in period 1.*

Increasing the transfer to location n has four effects, each labeled in Lemma 3. The first effect is a fiscal externality. By increasing the transfer to location n, households move away from other locations and into location n. The extent to which the planner values this movement depends on how much people were being taxed in their old location versus their tax in their new location. If households were being taxed in their previous location m but gaining a transfer in their new location n, this will hurt the government's ability to raise money.

The next effect is a direct redistributive effect.[9] Ignoring any effect on labor demand, giving a transfer to households in location n increases utility. The amount that that improves social welfare depends on the social marginal utility of consumption divided by the value of an extra dollar to the government β_{n1}/λ_{G1}.

The final two effects are the macroeconomic effects that are the focus of this paper. First, there is the stimulus effect. When the government increases transfers to a location n, utility increases over and above the direct utility benefit when n is in a recession (i.e. $\tau_{n1} > 0$) because total work hours demanded increases by a factor of $\frac{\alpha_n}{1-\alpha_n}$ as discussed in Lemma 1. Whether or not the government values that stimulus depends on the labor wedge τ_{n1}. Second, there is the migration effect. Providing a transfer to location n will increase the population in location n and decrease the population in every other location m. If the regions households leave are in a recession, the out-migration improves social welfare, while if those regions are in a boom, that will be harmful as discussed in Lemma 1. The total migration effect of a transfer then depends on the distribution of recessions τ_{m1} and the matrix of migration semi-elasticities ν_{n1}^{m}.

Localized Shock. Specializing these equations to the case of Janesville, where there is one small region in a recession within the large US, I find the following.

Proposition 1. *Suppose that there are two locations, j (Janesville) and u (Rest of the US), location j is arbitrarily small, $\ell_{jt} \to 0$, and there are no redistributive reasons for policy, i.e. $\zeta_{nt} = 1$. Then in any interior solution to* (SPP)*, the optimal period 1 transfer to location j must satisfy*

$$T_{j1} = \frac{1}{\nu_{j1}^{j1}} \left(\frac{\alpha_j}{1-\alpha_j} - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}} \right) \tau_{j1},$$

where $\frac{\partial \log \ell^{j1}}{\partial T_{j1}} \equiv \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \frac{\partial V^j}{\partial T}$ is the semi-elasticity of location 1 population to a transfer, holding fixed hours worked, and $\nu_{j1}^{j1} \equiv \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \left(\frac{\partial V^j}{\partial T_{j1}} + \frac{\partial V^j}{\partial H_{j1}} \frac{\partial H^{j1}}{\partial T_{j1}} \right)$ is the semi-elasticity of location 1 population to a transfer, allowing hours to vary.[10]

Proposition 1 shows that the optimal transfer depends on five statistics: the labor wedge τ_{j1}, the local multiplier $\frac{\alpha_j}{1-\alpha_j}$, the micro migration semi-elasticity $\frac{\partial \log \ell^{j1}}{\partial T_{j1}}$, The per capita wage earnings in the traded sector divided by the traded share of consumption $\frac{W_j H_{Tj1}}{1-\alpha_j}$, and the macro migration semi-elasticity ν_{j1}^{j1}. I will take each of these in turn.

[9] This can also be thought of as an insurance effect from the perspective of a household before her utility draws are revealed. Mongey and Waugh (2024) discuss this perspective in the context of a discrete choice model similar to mine.

[10] In the limit where households do not move across locations the planner will use transfers to set the labor wedge in Janesville to 0 since the planner has no redistributive reasons for policy. In Farhi and Werning (2017), the optimal stimulus transfers are weighed against the redistributive reasons for policy.

The first statistic is the labor wedge τ_{j1}. This determines if the region is in a recession or not and so whether the planner wants to stimulate the economy or cool it down. In the following discussion, I assume that Janesville is in a recession, so that $\tau_{j1} > 0$.

The sign of the optimal transfer to Janesville then depends on the relative size of the local multiplier $\frac{\alpha_j}{1-\alpha_j}$ and the micro migration semi-elasticity with the traded sector adjustment $\frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}}$. In particular, the optimal transfer could actually be a tax on Janesville if households are sufficiently mobile. Why? Because a transfer to Janesville has a direct effect on both the demand and supply for total labor.

To demonstrate this, suppose that, starting from an equilibrium with no transfers, the national government gives a small transfer to Janesville, $dT_{j1} > 0$, paid for with a small tax on the rest of the US, $dT_{u1} = -\frac{\ell_{j1}}{\ell_{u1}} dT_{j1}$ in an attempt to stimulate Janesville since households in Janesville are working less than they would like. I assume that monetary policy sets the labor wedge in u to 0. Then the total effect on social welfare, when there are no redistributive reasons for policy ($\beta_{j1} = \beta_{u1} = 1$), is given by

$$\begin{aligned}
d\mathcal{W} &= \overline{\lambda}_{j1} \ell_{j1} dU_{j1} + \overline{\lambda}_{u1} \ell_{u1} dU_{u1} \\
&= \overline{\lambda}_{j1} \ell_{j1} \left(\frac{U_C^j}{P_j} dT_{j1} + W_j \frac{U_C^j}{P_j} \tau_{j1} dH_{j1} \right) + \overline{\lambda}_{u1} \ell_{u1} \frac{U_C^u}{P_u} dT_{u1} \\
&= \ell_{j1} dT_{j1} + \ell_{j1} W_j \tau_{j1} dH_{j1} - \ell_{u1} \frac{\ell_{j1}}{\ell_{u1}} dT_{j1} \\
&= \ell_{j1} W_j \tau_{j1} dH_{j1},
\end{aligned}$$

using the indirect utility function derivatives from Lemma 1. Therefore, since Janesville is in a recession, $\tau_{j1} > 0$, the transfer increases social welfare if and only if it increases per capita hours worked in Janesville, $dH_{j1} > 0$. The direct effect of the transfer will increase the number of hours worked per capita because households spend some of their money on non-traded goods. However, the transfer will also have an indirect effect because it will affect how many people would like to live there which will also affect hours.

I graph the equilibrium in Figure 3a in order to illustrate the comparative static. For notational convenience, I omit the dependence on monetary policy E_{Tt} and variables in the rest of the United States u since Janesville is infinitesimal and so has no effect on those aggregates. The top panel plots the optimal number of hours the households would like to supply, holding fixed the transfer from the government and total population. Distinct from the usual supply and demand framework, wages are rigid at W_j and so do not clear the market. I have therefore left off the labor demand curve. $W_j \tau_{j1}$ then measures how far households in location j are from their ideal labor supply.

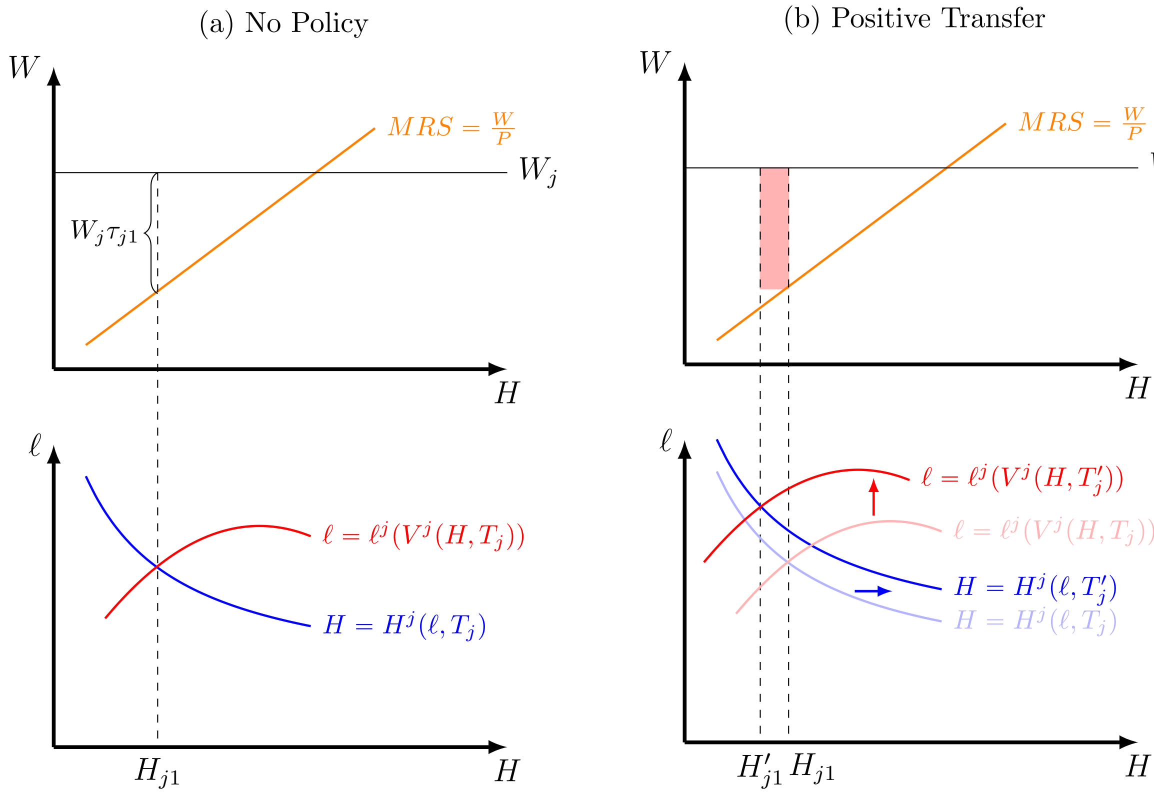


Figure 3: Illustration of Stimulus and Migration Effect of a Transfer

Notes: The top panel of (a) plots the per capita hours of work demanded and the first best level of hours supplied in Janesville holding fixed population with no government transfer. Population is endogenously determined by the population supply and the hours demanded curves in the bottom panel which both take as given the transfer. (b) plots the comparative static with respect to a small increase in the transfer to Janesville. The top panel shades the welfare loss due to the decrease in hours worked per capita.

To complete the description of equilibrium, I endogenize ℓ_{j1} and H_{j1} in the bottom panel of Figure 3a. I plot the population supply curve in red. This curve shows how many households would like to live in location j as a function of the hours worked per capita. It is increasing for most H_{j1} because the region is in a recession and fundamental utility increases in hours. I also plot the hours demanded curve as a function of population. Where they cross determines the equilibrium population and hours.

I plot how the equilibrium changes when the national government gives a small transfer to Janesville in Figure 3b. The stimulus effect leads to the hours demanded curve in the bottom panel shifting to the right by $\frac{1}{W_j}\frac{\alpha_j}{1-\alpha}$ as shown in Lemma 1. That is, for a given population, if those households have extra income, there will be more demand for their labor because there is home bias in consumption. If this were the only direct effect of a transfer, then the transfer might affect total population, but the inflow of population would only come from a shift along the population supply curve and could not decrease hours demanded.

However, that is not the case here because transfers directly increase utility independent of the stimulus effect. Therefore, the population supply curve also shifts up by $\frac{\partial \log \ell^j}{\partial U_{j1}} \frac{\partial V^j}{\partial T_j}$. It is this shift that determines whether or not the migration effect can dominate the stimulus effect, which is why the migration semi-elasticity that matters for the migration effect is this micro semi-elasticity, holding fixed hours worked, rather than the macro semi-elasticity which would take into account moves along the supply curve.

The curve that shifts up the most dominates. That is, if the hours demanded curve shifts up more, then hours worked will increase and welfare will improve from a transfer. If the population supply curve shifts more, then hours will decrease since too many people move in. I note that the slope of the hours demanded curve is $-\frac{W_j}{\frac{W_j H_{Tj1}}{1-\alpha_j}}$ so that a shift to the right of $\frac{1}{W_j} \frac{\alpha_j}{1-\alpha}$ corresponds to a shift up of $\frac{1}{\frac{W_j H_{Tj1}}{1-\alpha_j}} \frac{\alpha_j}{1-\alpha}$. Thus, the stimulus effect dominates, and the optimal policy features a positive transfer to Janesville, if and only if

$$\frac{\alpha_j}{1-\alpha_j} > \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^j}{\partial U_{j1}} \frac{\partial V^j}{\partial T_j}.$$

Not drawn is the fiscal externality that comes from increasing or decreasing a transfer to Janesville. The ultimate size of the transfer balances the direct stimulus and migration effects on the labor wedge against that force. Therefore, the formula is divided by the migration semi-elasticity ν_{j1}^{j1}. Importantly, this elasticity takes into account the effect on hours worked of a transfer since that determines the total utility effect of the transfer, and therefore the total increase in the population. That is why it is a macro migration semi-elasticity that matters for the fiscal externality rather than the micro migration semi-elasticity of the migration effect.

Spatially Correlated Shock. In practice, many labor demand shocks do not hit only one small region. Instead, they hit whole industries, as is the case with the China trade shock. In that case, the migration effect of a transfer can have more complicated effects. If giving a transfer to a region in a recession causes households to leave a region that is in a worse recession, the migration effect will be a net positive. The next proposition makes precise how the spatial distribution of shocks interacts with migration patterns to shape optimal spatial policy.

Proposition 2. *Suppose that there are two large locations, s (southern US) and n (northern US), and one small location, j (Janesville). Then, if there are no redistributive reasons for* *transfers* $\zeta_{nt} = \zeta_{st} = \zeta_{jt} = 1$*, in any interior solution to* (SPP)*,*

$$T_{j1} > \frac{1}{\nu_{j1}^{j1}} \left(\left(\frac{1}{\lambda_{G1}} - 1 \right) + \frac{1}{\lambda_{G1}} \frac{\alpha_j}{1-\alpha_j} - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}} \right) \tau_{j1},$$

if and only if migrants to j *disproportionately come from the region in a recession, i.e.* $\mathrm{Cov}(|\nu_{j1}^{k1}|, \tau_{k1}) > 0$.

Proposition 2 says that if migrants to location j disproportionately come from parts of the US which are in a recession, the national government should give more money to location j than that suggested by the local multiplier and migration semi-elasticity, taking into account the social marginal value of a government dollar λ_{G1}. Importantly, the formula in Proposition 2 looks slightly different than that in 1 because, in Proposition 1, the social value of another government dollar is 1. When there are two locations, and one is in a recession, that is no longer the case because the value of another dollar is not simply the marginal value of consumption (which we have assumed is 1 because there are no redistributive reasons for transfers). That extra dollar can now also be used to stimulate the region in a recession.

Above and beyond that difference, the transfer to Janesville is larger than that suggested by the local trade-off if and only if migrants to Janesville disproportionately come from the region in a recession. To see why that is, suppose that migrants to Janesville came proportionately from the north and the south, i.e., $\nu_{j1}^{n1} = \nu_{j1}^{s1}$. In that case, increasing the transfer to Janesville will have the effects on Janesville discussed in Proposition 1, but it will also have an impact on the fiscal externality and the migration effect in the north and the south. In particular, households will leave the south and the north proportionately to their population. However, the average labor wedge and the average transfer across the two locations are zero due to monetary policy and budget balance, respectively. Therefore, the net effect on the fiscal externality and the net migration effect are both zero.

By contrast, if migrants to Janesville disproportionately come from the region in a recession, increasing the transfer to Janesville will have a net migration effect and an effect on the fiscal externality that depends on the social value of having slightly fewer households in the north relative to the south. Importantly, the fact that the planner has the optimal transfer on the north and south already tells us something about the combined fiscal externality and migration effect. The combined fiscal externality and migration effect that come from migration into the region in a recession in response to the transfer are balanced against the stimulus effect of the transfer. However, because the region is in a recession, the stimulus effect must be positive, and therefore the combined fiscal externality and migration effect must be negative. Thus, the planner values encouraging households to disproportionately leave the region in a recession by giving extra money to those in Janesville.

This implies that the nature of the demand shock matters for the optimal policy. If the shock is very correlated, then regions that are in recessions will be near other regions in recessions. Therefore, a transfer to one of those regions will not have a large net migration effect since all migrants in response to the transfer will come from other areas also in a recession. The China trade shock might call for more aggressive transfers from the national government than an idiosyncratic shock like the closure of the Janesville Assembly Plant. I will return to this quantitatively in sections 6 and 7.

4.4 Optimal Long-Run Transfers

Having shown that fiscal transfers to a region in the immediate aftermath of a factory closure have competing stimulus and migration effects, I next turn to the effects of a transfer in the long run. One might think that the same basic trade-off between the migration effect and the stimulus effect apply in the second period as it did in period 1. The only difference is that people have more time to move so that the migration effect will likely be stronger. But that intuition turns out to be incomplete, as I now discuss.

I start by stating the first order necessary condition for a transfer to location n in period 2. I define the social marginal utility of income in region n period 2, $\zeta_{n2} \equiv \frac{\beta \bar{\lambda}_{n2} U_C^n}{P_n}$.

Lemma 4. *In any interior solution to* (SPP)*, second period transfers must satisfy*

$$\sum_t \frac{\lambda_{Gt}}{\lambda_{G2}} \sum_m \ell_{mt} T_{mt} \nu_{n2}^{mt} = \ell_{n2} \left[\frac{\beta_{n2}}{\lambda_{G2}} - 1 + \frac{\beta_{n2}}{\lambda_{G2}} \frac{\alpha_n}{1-\alpha_n} \tau_{n2} \right] - \sum_t \frac{\lambda_{Gt}}{\lambda_{G2}} \sum_m \frac{W_m H_{Tmt}}{1-\alpha_m} \ell_{mt} \frac{\tau_{mt}}{1+\frac{\alpha_m}{1-\alpha_m}\tau_{mt}} \nu_{n2}^{mt},$$

where λ_{G2} is the social value of the government having another dollar in period 2, and ν_{n2}^{mt} is the elasticity of population in location m at time t to a transfer to location i at time 2.

Lemma 4 shows the same four effects of a transfer from the period 1 first order condition shown in Lemma 3: fiscal externality, redistribution, stimulus, and migration. The redistribution and stimulus effects remain the same as before. Transferring an extra dollar to households in location n improves social welfare by β_{n2} directly through consumption. The planner weights that use of the money against the marginal value of a dollar in period 2, λ_{G2}. Similarly, the transfer leads to a stimulus of $\frac{\alpha_n}{1-\alpha_n}$. The only difference is that real consumption and the labor wedge might be different in period 2 as compared to period 1.

Both the fiscal externality and the migration effect now have dynamic components. That is because a promise to tax certain locations in period 2 will affect where households decide to live at time 1. Therefore, the planner has to take into account how that movement in the first period will affect the fiscal externality and recessions in the first period. Under the limit $\beta \to 0$, this effect is infinitesimal. However, λ_2^G is also infinitesimal, so the effect still shapes the optimal policy.

In the next proposition, I consider what this implies for optimal policy in Janesville in period 2.

Proposition 3. *Suppose that there are two locations, j (Janesville) and u (Rest of the US), location j is arbitrarily small, $\ell_{jt} \to 0$, there are no redistributive reasons for policy, $\beta_{nt} = 1$, and j is in a recession, $\tau_{jt} > 0$. Then in any interior solution to* (SPP)*, the optimal period 2 transfer to location j satisfies*

$$T_{j2} < \frac{1}{\nu_{j2}^{j2}} \left(\frac{\alpha_j}{1-\alpha_j} - \frac{W_j N_{Tj2}}{1-\alpha_j} \frac{\partial \log \ell^{j2}}{\partial T_{j2}} \right) \tau_{j2},$$

when the share of workers in location j in period 1 who stay in location j in period 2 is greater than zero.

Comparing Proposition 3 to Proposition 1 reveals that in period 2, the optimal transfer to a region in a recession is always lower than that implied by the simple static trade-off between the stimulus effect and the migration effect.

A transfer in the second period has the same stimulus, migration, and fiscal externality effects on period 2 as first period transfers did in period 1. However, giving a transfer to people in Janesville in period 2 also increases the expected utility of living in Janesville in period 1 if people who live in Janesville in period 1 are likely to live there in period 2 (due to moving costs). Therefore, if the planner promises to give a transfer to people who are in Janesville in period 2, people who would have left in period 1 because they had a job opportunity somewhere else will be less likely to leave. So the period 2 transfer will increase population in period 1 Janesville, impacting the first period fiscal externality and migration effect.

What is the net effect on social welfare? To answer that, we need to know the signs and relative strength of those two forces. The key is to note that, just as in Proposition 2, period 1 transfers already reveal something about their combined effect. Period 1 transfers optimally trade off those exact forces that come from an increase in population against the positive stimulus effect of giving a little extra money to people in location 1. Therefore, the net effect of increasing population in period 1 Janesville must be negative, and a transfer in period 2 makes that worse. Therefore, transfers in period 2 should be smaller than what would be suggested by the static trade-off since taxes in period 2 allow the planner to encourage out migration without decreasing stimulus in the first period.

The actual size of the transfer in period 2 could be larger or smaller than that in period 1. For most models, the migration semi-elasticity in period 2 will be larger than the semi-elasticity in period 1, suggesting the transfer should be lower. However the labor wedge in period 2, τ_{j2} will often be closer to 0 than the labor wedge in period 1 τ_{j1}, shrinking the transfer towards 0. I will demonstrate how this plays out quantitatively in sections 6 and 7.

4.5 Extensions and Robustness

The model so far has been stylized in order to shed light on the key forces shaping optimal fiscal policy in the most transparent way possible. Here I summarize how the results change when I include other real world features. Formal derivations can found in Appendix C.

Downward Wage Rigidity and Costly Price Adjustments. This model features perfect wage rigidity, but empirical evidence suggests that wages are more rigid going downwards. In appendix C.1, I consider a variant of this model with 2 locations, downward wage rigidity, and costly upward price adjustments. In that case, I derive a new version of Proposition 1. The formula is largely unchanged because I consider a small region in a recession, where wages are rigid in both the downwardly rigid case and the completely rigid case.

Place-biased Policy. In appendix C.2, I consider an extension of the model with multiple types and transfers that can be partially targeted towards those types and locations. I show that starting from an equilibrium with no taxes, whether or not a place-biased transfer helps with macroeconomic stability still depends on the same sufficient conditions: the local multiplier and the migration semi-elasticity. Importantly, the stimulus effect depends on the observed place-biased nature of the transfer while the migration effect is determined by how place-biased the transfer is within a type.

One way to think of this extension is having to do with automatic stabilizers. The extension then solves for the conditions under which making a particular place-biased program more generous helps macroeconomic stability. Consider the income tax. The income tax will have stimulus effects if income decreases in a recession. But also, because the tax rate is progressive, higher paying jobs are less attractive. Therefore, households have less incentive to take a higher paying job in a region with higher demand. Similarly, unemployment benefits will stimulate the region, but it will reduce the incentive to find a job. Assuming that it is easier to find a job in a low unemployment area, this reduces the attractiveness of other regions not in a recession.

Another interpretation of this extension is as transfers that can be targeted based on starting location. The type is then starting location. In that case, this extension says that the planner would target money towards types who tend to be in recessionary regions, that is, those who were there before. However, the migration effect still operates within the group, so that the planner might want to offer households more money to go somewhere else if the migration semi-elasticity is high enough.

Households Affect Demand. In appendix C.3, I consider an extension of the model to have multiple household types who can affect demand for a particular region. These could represent entrepreneurs, for example. When they move into a region, they open up new businesses that export new products to the rest of the country. I find an adjusted version of Proposition 1. The migration effect then also has an effect on demand that depends on the covariance between the household type's effect on demand and their migration semi-elasticity to the transfer. In practice, this covariance is likely small since entrepreneurs likely move to areas with good economic conditions, regardless of the government transfers, though this force could suggest other place-based policies to fight local recessions.

Wage Stickiness Only in Traded Goods. While Autor et al. (2013) found that earnings decreased significantly, they found no evidence that average wages decreased in the manufacturing sector. Instead, all of the wage movement was in services. In appendix C.4, I consider an extension of the model where labor is imperfectly substitutable across the traded and non-traded sector, and wages are not sticky in the non-traded sector. In that case, there is no stimulus effect of a transfer because there is no wedge on the non-traded labor. Instead, there is only a migration effect, so I show that in an adjusted version of Proposition 1, the optimal transfer to Janesville is always negative.

Monetary Policy. I also consider the implications for monetary policy in Appendix B.1. I show that in the baseline model, there is a contractionary bias to monetary policy. By underheating the economy, the regions in a recession become less attractive since unemployment is higher. Conversely, regions that are booming become more attractive because they are not working too much. Thus, households are encouraged to move out of recessionary regions into regions doing well.[11]

[11] This force is closely related to the contractionary bias in times of industrial reallocation of Guerrieri et al. (2021) when wages are perfectly sticky and workers can reallocate across sectors.

5 Dynamic New Keynesian Economic Geography Model

The two period model with freely traded and non-traded goods in section 3 reveals the key forces in a transparent manner, but it is too stylized to bring to the data to quantify how large place-based transfers should be. On the trade side, I have abstracted from any geographic considerations that may create non-zero trade costs on traded goods. On the macro side, I have abstracted from any wage adjustment by assuming completely rigid wages.

In this section, I present a continuous time model of New Keynesian economic geography where I allow for non-zero trade costs and partially rigid wages. I also impose parametric restrictions on preferences and migrations costs that allow me to capture the key features of the data while remaining tractable. I then briefly describe how I approximate the model and compute the fully optimal time-varying spatial policy in response to time-varying demand shocks like the China trade shock. In contrast to the leading dynamic models assessing the impact of the China shock,[12] I will be able to consider the effects at the commuting zone level rather than the state level. Finally, I describe how I calibrate the model to the 722 commuting zones in the contiguous United States.

5.1 Environment

There are N regions indexed by $n, m \in \mathcal{N} = \{1, \ldots, N\}$, one non-traded sector and one traded sector, and continuous time indexed by $t \in [0, \infty)$.

Households. There is a continuum of households that I index by $i \in \mathcal{I}$. I will start by describing the dynamic welfare taking as given flow utility before returning to describe the flow utility.

I denote the location of agent i at time t by $n(i,t)$. Then each household starts in some location $n(i,0)$ and it gets the opportunity to move at a Poisson rate $\delta_\ell > 0$.[13] At that point, the household observes additive utility shocks of moving to every location m, $\varepsilon_m(i,t)$. The utility shocks are distributed Gumbel with shape parameter ν. The household can then move subject to an additive migration cost of moving to a location m, $\tau_{\ell nm}$.

Denoting the set of all times where household i moves from location n to m by $\mathcal{M}_{nm}(i) \subset$

[12] See Caliendo et al. (2019) and Rodríguez-Clare et al. (2020).

[13] Bilal (2023b) uses the same migration restriction in a continuous time model and Peters (2022) also uses this migration restriction in a discrete time model. In a continuous time model, this is necessary to transform population in a region into a state variable. To first order, making the arrival rate lower is similar to raising moving costs to all other locations.

$[0, \infty)$. Then realized utility of household i is

$$\int_0^\infty e^{-\rho t}\left[U_{n(i,t)}(t) + \sum_{n,m} \delta_{t\in\mathcal{M}_{nm}(i)}[-\tau_{\ell nm} + \varepsilon_m(i,t)]\right] dt,$$

where $U_n(t)$ is the flow utility of living in location n, $\rho > 0$ is household's discount rate, and $\delta_{t\in\mathcal{M}_{nm}(i)}$ is the dirac delta function.

The immediate flow utility of a household in location n at time t, $U_n(t)$ is a function of consumption and labor supply,

$$U_n(t) = \frac{C_n(t)^{1-\theta}}{1-\theta} - \frac{H_n(t)^{1+\eta}}{1+\eta},$$

where $C_n(t)$ is the consumption aggregate, θ is the elasticity of intertemporal substitution, $H_n(t)$ is hours supplied, and η is the Frisch labor elasticity. The consumption aggregate is a Cobb-Douglas aggregation of consumption of the traded good and the non-traded good,

$$C_n(t) = C_{NTn}(t)^\alpha C_{Tn}(t)^{1-\alpha},$$

where $C_{sn}(t)$ is consumption of the sector s good and $\alpha \in (0,1)$ is the share of spending on non-traded goods. The traded good is an aggregation of the varieties produced in each location,

$$C_{Tn}(t) = \left(\sum_{m\in\mathcal{N}} \phi_m^{\frac{1}{\sigma}} C_{Tmn}(t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where ϕ_m is the consumption weight on the variety produced by location m, which I normalize so that $\sum_m \phi_m = 1$, $C_{Tmn}(t)$ is consumption of the traded good produced in location m by the consumer in n, and σ is the elasticity of substitution between varieties produced by the locations.

Firms. In each location n, there is a continuum of intermediate producers $\omega \in [0,1]$ who produce an intermediate using labor. Firm ω produces

$$Y_n(\omega,t) = H_n(\omega,t)\ell_n(t),$$

where $Y_n(\omega,t)$ is production and $H_n(\omega,t)$ is the amount of per capita labor supplied to intermediate ω.

A final producer then combines those intermediates according to a CES aggregator

$$Y_n(t) = A_n \left[\int_0^1 Y_n(\omega, t)^{\frac{\epsilon-1}{\epsilon}} d\omega \right]^{\frac{\epsilon}{\epsilon-1}},$$

where $Y_n(t)$ is the aggregate production of location n and $\epsilon > 1$ is the elasticity of substitution across intermediates. This final good can then be consumed as a non-traded or traded good.

Market Clearing. For the labor market to clear, labor supplied equals the sum of labor demand by each intermediate producer,

$$H_n(t) = \int_0^1 H_n(\omega, t) d\omega, \text{ for all } n, t. \tag{18}$$

Aggregate production of location n is consumed as a traded good and non-traded good. The non-traded good is only consumed by the local households. Trade is subject to iceberg trade costs. Therefore, goods market clearing requires production in location n is equal to consumption of non-traded goods in the location plus consumption of its produce as a traded good across all locations,

$$Y_n(t) = C_{NTn}(t)\ell_n(t) + \sum_m \tau_{nm} C_{Tnm}(t)\ell_m(t), \text{ for all } n, t, \tag{19}$$

where $\tau_{nm} \geqslant 1$ is the iceberg trade costs of delivering a good from location n to location m.

5.2 Decentralized equilibrium

5.2.1 Utility Maximization

I start by characterizing the household's migration decision taking as given flow utility in location n at time t, $U_n(t)$. I then turn to the consumption decision. Just as before, workers do not choose labor, and instead supply the labor demanded.

Migration Decision. The Bellman equation for a household in location n is

$$\rho v_n(t) - \dot{v}_n(t) = U_n(t) + \delta_\ell \left(V_n(t) - v_n(t) \right), \tag{20}$$

where $v_n(t)$ is the expected lifetime utility of a household in location n at time t and $V_n(t)$ is the expected utility if that households gets the opportunity to move. Because the utility shocks are distributed Gumbel,

$$V_n(t) = \frac{1}{\nu} \log \left(\sum_m \exp(\nu(v_m(t) - \tau_{\ell nm})) \right). \tag{21}$$

This implies that a $\exp\left(\nu(v_m(t) - \tau_{\ell nm} - V_n(t)\right)$ share of households in location n who have the chance to move will move to location m. The population in location m changes according to

$$\dot{\ell}_m(t) = \delta_\ell \left[\sum_n \exp\left(\nu(v_m(t) - \tau_{\ell nm} - V_n(t)\right) \ell_n(t) - \ell_m(t) \right]. \tag{22}$$

Intratemporal Consumption Decision. Given expenditures $E_n(t)$, households in location n at time t choose consumption to maximize utility taking prices as given. In particular,

$$\begin{aligned} \{C_{NTn}(t), C_{Tn}(t), \{C_{Tmn}(t)\}\} \in \underset{C_{NT}, C_T\{C_{Tm}\}}{\operatorname{argmax}} \Bigg\{ (C_{NT})^\alpha (C_T)^{1-\alpha} \Bigg| \\ C_T = \left(\sum_m \phi_m^{\frac{1}{\sigma}} (C_{Tm})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \\ \sum_m p_{Tmn}(t) C_{Tm} + p_{NTn} C_{NT} \leqslant E_n(t) \Bigg\}. \end{aligned} \tag{23}$$

This problem is standard so the characterization is left for the appendix D. I denote by $P_n(t)$ the perfect price index so that $E_n(t) = P_n(t) C_n(t)$.

Households are hand-to-mouth so they spend all of their income in each period. Income comes from two different sources: labor earnings and government transfers. That is,

$$E_n(t) = \left(\int_0^1 W_n(\omega, t) H_n(\omega, t) d\omega \right) + T_n(t), \tag{24}$$

where $W_n(\omega, t)$ is the wage offered by intermediate producer ω in location n and $T_n(t)$ is the transfer to households in location n.

5.2.2 Production

Profit Maximization. A competitive, representative firm for each intermediate ω in location n maximizes profits taking prices and wages set by the union as given using a linear technology. Therefore, the price of the intermediate is simply the wage $p_n(\omega, t) = W_n(\omega, t)$.

The final producer is competitive and so maximizes profits taking as given the price of the final good $p_n(t)$ and intermediates $W_n(\omega, t)$. That is,

$$
\begin{aligned}
Y_n(t), \{Y_n(\omega,t)\} \in \underset{Y, Y(\omega)}{\operatorname{argmax}} \Bigg\{ p_n(t) Y - \int_0^1 W_n(\omega,t) Y(\omega) d\omega \Bigg| \\
Y = A_n \left[\int_0^1 Y(\omega)^{\frac{\epsilon-1}{\epsilon}} d\omega \right]^{\frac{\epsilon}{\epsilon-1}} \Bigg\}.
\end{aligned} \tag{25}
$$

Trade is also competitive so that $p_{Tnm}(t) = \tau_{nm} p_n(t)$ and $p_{NTn}(t) = p_n(t)$.

Labor Unions. For each intermediate ω in location n, there is a union that can unilaterally set the wage it demands. Wages are sticky, and the union only gets the chance to change the wage demanded at a poisson rate δ_w.

Given wages, the union supplies the labor necessary to meet demand for intermediate ω. I assume that there is efficient rationing. When a union gets the chance to change its wage, it sets the wage to maximize utility of the average household in its location. As is standard in this literature, I assume the local government has a wage subsidy κ to undo the monopoly distortion, funded by a tax on the residents. That is, the unions who can change their wage at time t choose a new wage $\tilde{W}_n(t)$ that solves

$$
\tilde{W}_n(t) \in \underset{W'}{\operatorname{argmax}} \int_t^\infty e^{-(\rho+\delta_w)(t'-t)} \left[\kappa \frac{C_n(t')^{-\theta}}{P_n(t')} (W')^{1-\epsilon} - H_n(t')^{\eta} (W')^{-\epsilon} \right] A_n^{\epsilon-1} P_n(t')^{\epsilon} Y_n(t') dt'. \tag{26}
$$

Appendix D describes further details.

5.2.3 Government

The government sets aggregate spending $E(t)$, such that

$$
E(t) = \sum_n E_n(t) \ell_n(t), \text{ for all } t, \tag{27}
$$

and also chooses the place specific transfers between locations. The government budget constraint then must hold in each period,

$$
\sum_n \ell_n(t) T_n(t) = 0, \text{ for all } t. \tag{28}
$$

Definition 2. *Given monetary policy $E(t)$ and per capita transfers $T_n(t)$, an* equilibrium *is a set of location choices $n(i,t)$, utility levels $U_n(t)$, regional population $\ell_n(t)$, prices $P_n(t)$, wages $W_n(\omega,t)$, consumption levels $C_{Tmn}(t)$, $C_{NT}(t)$, labor supplies $H_n(t), H_n(\omega,t)$, and output $Y_n(t)$, such that:*

- *Households choose consumption and their location to maximize utility* (20)*,* (21)*,* (22)*,* (23)*,* (24)*;*
- *Firms maximize profits taking prices as given,* (25)*;*
- *Unions set wages to maximize expected utility of the local households,* (26)*;*
- *The government's budget constraints hold,* (28)*;*
- *Total spending is equal to nominal GDP* (27)*; and*
- *Markets clear* (18)*,* (19)*.*

5.3 The Planner's Problem

The government chooses monetary policy $E(t)$, place-based transfers $T_n(t)$ and associated flow utilities

$$U(i,t) \equiv U_{n(i,t)} + \sum_{n,m} \delta_{t\in\mathcal{M}_{nm}(i)}[-\tau_{\ell nm} + \varepsilon_m(i,t)],$$

to maximize social welfare. Following Dávila and Schaab (2022), I allow the government to have a time varying pareto weight $\lambda(i,t)$ on households. That is, the planner could care about the consumption of a household more at some time t than another time t'. I will adjust these time varying Pareto weights to justify no government policy in the steady state. However, in contrast to section 4, I will include redistributive reasons for policy. Formally, the planner faces the problem

$$\max_{E(t),\{T_n(t)\},\{U(i,t)\}\in\mathcal{E}} \int_{\mathcal{I}} \int_0^\infty e^{-\rho t}\lambda(i,t)U(i,t)dtdi, \tag{29}$$

where $\mathcal{E}$ is the set of utility profiles attainable in equilibrium, as described in Definition 2.

5.4 Computation

This is a non-linear model with state variables utility $v_n(t)$, population $\ell_n(t)$, along with wages $W_n(\omega,t)$ for each intermediate. Solving the optimal planner's problem with the 722 commuting zones of the United States would be infeasible. Therefore, I follow the macro literature in doing a log-quadratic approximation to the social welfare function and a log-linear approximation to all of the constraints around a no-inflation, no-fiscal transfer steady state, where Pareto weights $\lambda(i,t)$ are such that it is optimal before any shocks. Details of how I derive the loss function including distortions in migration, trade, inflation, and output along with the final linearized constraints are in appendix E. I use $\hat{x}$ to denote log deviations from that steady state, and I consider idiosyncratic demand shocks to the traded output of specific regions ϕ_m.

Table 1: Calibration Summary

Parameter	Value	Description	Source
Panel A. Stimulus effects			
Parameter	Value	Description	Source
$\frac{\alpha}{1-\alpha}$	1.6	Local multiplier	Moretti (2010)
σ	4.5	Trade EoS	Head and Mayer (2014)
τ_{nm}		Trade costs	CFS state trade flows
Panel B. Migration effects			
Parameter	Value	Description	Source
$\frac{\nu}{\rho+\delta_\ell}$	2.9	long-run migration elasticity	Hornbeck and Moretti (2024)
δ_ℓ	0.157	Migration calvo friction	ACS migration flows
$\tau_{\ell nm}$		Migration costs	
Panel C. Other Parameters			
Parameter	Value	Description	Source
ρ	0.06	Patience	Farhi and Werning (2017)
ϵ	11	Intermediate EoS	Farhi and Werning (2017)
η	2	Frisch labor supply elasticity	Peterman (2016)
δ_w	0.3	Wage calvo friction	Figure 1
θ	1	Intertemporal EoS	log preferences
A_n		Productivity	CBP labor earnings

The final linearized model features four state variables for each commuting zone: population $\hat{\ell}_n(t)$, utility $\hat{v}_n(t)$, wage $\hat{w}_n(t)$, and inflation $\hat{\pi}_n(t)$, for a total of 2,888. When solving the planner's problem, I also need to keep track of the 2,888 co-state variables. I give details of how I compute the optimal policy for time-varying shocks in appendix G.

5.5 Calibration

In this section, I provide an overview of how I calibrate the model to match the United States in 2000. Additional details can be found Appendix F. I interpret a local labor market in the model as a commuting zone (CZ), as developed by Tolbert and Sizer (1996). My analysis will focus on the 722 commuting zones of the contiguous United States, as in Autor et al. (2013). I discuss the key parameters for the stimulus effect and migration effect in detail before turning to the more standard parameters from the macro literature. A summary of how I calibrate the parameters is in Table 1.

Stimulus Effects. As I show in Appendix H, for a small open region, the stimulus effect of a transfer depends on the local multiplier $\frac{\alpha}{1-\alpha}$ when wages are perfectly rigid. While it does not estimate the local multiplier in response to a government transfer, Moretti (2010) measures the next best thing: how many jobs in the non-traded sectors are created in response to the creation of a new manufacturing job, 1.6. I set α to rationalize what he finds.

With a finite number of regions, the stimulus effect also depends on trade flows between commuting zones. I set the elasticity of substitution across varieties produced by different commuting zones to be 4.5, taking from Head and Mayer (2014). I do not have data on trade across commuting zones in the United States, so I infer those costs by looking at trade between states. In particular, I assume the iceberg trade costs between two distinct commuting zones n and m are

$$\log \tau_{nm} = \delta_D \log \text{distance}_{nm} + \delta_H,$$

where distance_{nm} is the bilateral distance between the population centroids of CZs n and m. I then guess δ_D and δ_H and find the implied productivity of each commuting zone to match observed employment and earnings. I can then back out the implied expenditure flows between states. I search over δ_D and δ_H to minimize the square distance between the implied share of state's earnings spent on another state and the observed shares from the 2002 Commodity Flow Survey.[14]

Migration Effects. As I show in Appendix H, the migration effect depends on the long-run migration elasticity $\frac{\nu}{\rho+\delta_\ell}$ and the speed of transition δ_ℓ when wages are perfectly rigid. I set ν to match the average long-run migration elasticity of Metropolitan Statistical Area (MSA) population to earnings found in Hornbeck and Moretti (2024), 2.9.[15] This is not ideal as it is the elasticity in response to earnings rather than a transfer, but under the envelope theorem, the elasticities are the same at the point $T_n = 0$, which determines the sign of the transfer.

The speed of transition is then jointly determined by δ_ℓ and the matrix of migration costs $\tau_{\ell nm}$. I calibrate these parameters using migration reported in the American Community Survey (ACS). In particular, I construct yearly CZ-to-CZ commuting flows from where people report being in the previous year and their current location. This matrix has many zeros so

[14] This procedure is very similar to that used in Allen and Arkolakis (2014) but using great arc distance rather than using the observed transportation network, and stopping at the CZ level rather than going down to counties.

[15] As opposed to CZs, MSAs do not cover all of the United States, leaving off rural areas. However, they are similar-sized: some CZs fully encompass an MSA and some MSAs encompass a CZ. Bryan and Morten (2019) find a value of 2.7 for the US and 3.2 in Indonesia, and Hsieh and Moretti (2019) find a value of 3.3. Other papers studying the effect of the China trade shock like Artuç et al. (2010), Caliendo et al. (2019), and Rodríguez-Clare et al. (2020) consider the elasticity across sectors and/or states.

I assume that migration costs have the gravity structure

$$\tau_{\ell nm} = \delta_{\ell D} \log \text{distance}_{nm} + \delta_{\ell H}.$$

I then jointly calibrate $\delta_{\ell D}$, $\delta_{\ell H}$ and δ_{ℓ} to match the elasticity of migration to distance and the share of workers who do not move in any given year. I find that $\delta_{\ell} = 0.1575$ which is double the value of 0.07 that Peters (2022) finds in Germany in the post-war years. I also find that, conditional on getting the opportunity to leave, a household will almost always leave. This is consistent with the evidence of Yagan (2019) and Monras (2018) that while population of a region responds to economic shocks, the likelihood of an individual household leaving does not.

Wage Rigidity. The wage rigidity that matters for my mechanism is the relative wage across commuting zones. There is reason to believe that that relative wage rigidity is higher than absolute wages since many firms set national wages (Hazell et al., 2022). Therefore, I set wage rigidity $\delta_w = 0.3$ to match the fact that, for an average commuting zone, the half-life for wage adjustment is just above two years in Figure 1. These are very sticky wages and I will consider how robust the results are to this parameter.

Other Parameters. For patience, ρ, I take the standard value of the literature used by Farhi and Werning (2017). The elasticity of substitution across intermediates ϵ determines the loss from inflation. I similarly set this according to the literature. I take a value of 2 for the Frisch labor supply elasticity η to be closer to the macro estimates of Peterman (2016). And finally, I set $\theta = 1$ implying log preferences.

6 Optimal Policy After an Idiosyncratic Shock

In section 5, I presented a New Keynesian economic geography model and calibrated it to the continental US. In this section, I use this model to compute the optimal policy in the average commuting zone after an idiosyncratic demand shock for its traded output. This will allow me to demonstrate how the migration and stimulus effect from Proposition 1 and Proposition 3 interact. I can also assess how effective imperfect policies like unemployment insurance and income tax can be when place-based policy is not feasible. I then show how the optimal changes when there is an aggregate shock like the China trade shock in section 7.

6.1 Impulse Response with no Policy

I consider a commuting zone with the average amount of home bias in consumption and in migration. Larger locations will have stronger stimulus effects and weaker migration effects on average while smaller locations will have weaker stimulus effects and stronger migration effects. I then simulate a local recession by considering a drop in demand for traded output that matches the first year drop in employment of the local projections in section 2, and assuming that every other location in the United States is unaffected. The model is log linear, so all results can be scaled up or down to consider a different sized recession.

I plot the impulse response functions in Figure 4 when the central government implements a smoothed-out version of the observed policies in Figure 2 in black. Details of how I construct the observed policy are in Appendix F. I compare that to what would have happened in the absence of policy (blue dotted line), and the optimal policy (magenta dashed line) which I will discuss more below. Every variable is in log differences from its steady state value except for fiscal transfers which are relative to the size of original income.

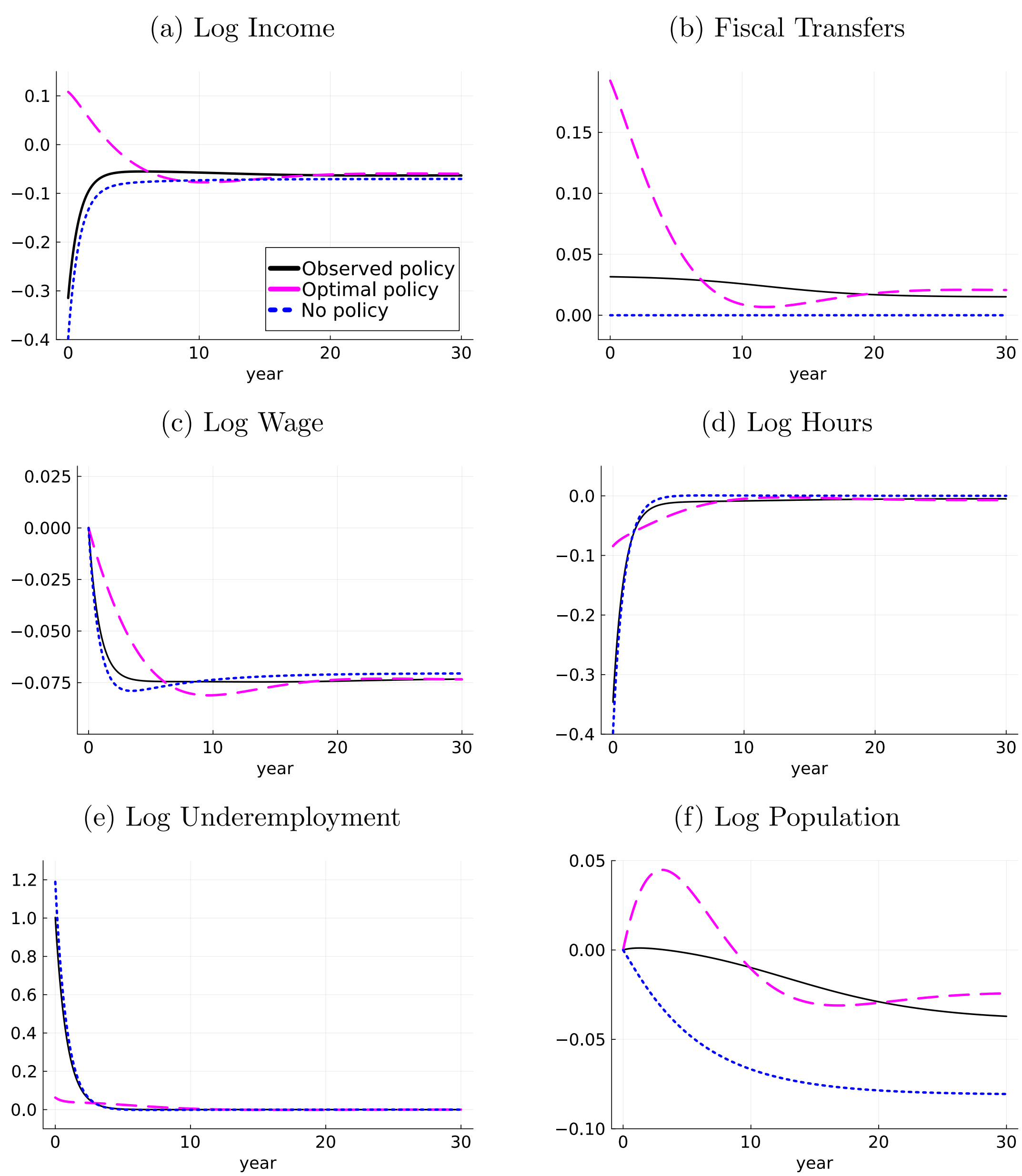
Figure 4b plots the fiscal transfers to the region. These transfers start around 3.5% of original income, but they slowly fade out over the next 20 years. I assume that the retention rate remains at 1.5% of pre-shock income so that the government continues to provide payments long after the recession has receded.

I plot the time path of log wages in Figure 4c. Consistent with the patterns observed in Figure 1, wages fall for the first 4 years following the shock. Only after that do wages recover slightly as people leave the commuting zone for employment somewhere else. Without the observed policy, the wages would have fallen still further since there is no increased local demand from the policy. Wages would then have recovered more as more people left the region.

While wages did not fall on the impact of the shock, earnings did. I plot the log per capita income in Figure 4a. Immediately on impact, earnings drop by more than 30%, completely driven by a decline in hours. As wages decline, demand for the traded output recovers, and log hours increase back toward their steady state value in 4d. Income recovers to around 5% less than its original amount 5 years after the shock. In the absence of policy, income would have dropped further.

There is no unemployment in this model, but there is a labor wedge. To give an idea of what that might mean for unemployment in response to a demand shock, I plot the log difference between how much the representative worker would like to work relative to how much he does work in Figure 4e. The impulse response suggests that the gap jumps by an entire log point. Unemployment then slowly drops over the next 5 years. In the absence of policy this looks very similar, though the jump is larger.

Figure 4: Impulse Response



Note: This figure shows the impulse response in an average commuting zone to a demand shock when under various policies. This is calculated by feeding a demand shock for the average CZ's tradable output into the model described in Section 5 assuming the rest of the country remains unchanged. All values are in log differences from the steady state except transfers which is relative to original income.

Finally, population also slowly adjusts to the new economic situation. Households are very slow to move in this model because they rarely get the opportunity. But when they do, they often decide to live somewhere else. In the meantime, very few people outside of the region want to move in. Thus, the population slowly drops more than 4% in the 30 years following the demand shock as shown in Figure 4f. By contrast, in the absence of policy, population would drop much quicker and further.

6.2 Optimal Policy Response

I plot how the national government should respond and what that implies for local economic variables in a magenta dashed line on the same Figure 4. Figure 4b plots the time path of the optimal transfers relative to earnings in the steady state. There are three distinct stages to the optimal transfer that roughly correspond to each of the three roles transfers can play: stimulus, migration, and finally, redistribution.

Stage one lasts for about six and a half years. In this time period, the stimulus effects of the transfer dominate. Immediately after the demand shock, there is a large amount of unemployment, but people do not have time to move in response to government policy, so the government can get free stimulus by giving people a check immediately upon being laid off. Thus, optimal transfers jump to around 20% of the original commuting zone income. In fact, the transfers are so large, one can see in Figure 4a that total income of the region actually increases. That is because, immediately after the shock migration cannot respond. Therefore, transfers only have two effects: redistribution and stimulus. Redistribution would suggest that the planner should exactly make up for the lost income so that the marginal utility of consumption remains the same. However, at that level of spending, the household is still working less than he would like, so the planner would like to give extra money for the added stimulus. Because of that, log underemployment in Figure 4e peaks at around 6% of initial labor supply rather than more than 100% increase seen with observed policy. Optimal transfers then taper in size as the migration effect of the transfer becomes more important.

In stage two, the migration effect of the transfer dominates, consistent with Proposition 3. This lasts from year 7 to around year 20 and features transfers that are lower than the long run redistributive transfers. After the demand shock, the planner commits to an entire time path of fiscal transfers. The planner promises very generous transfers in the immediate aftermath of the shock, but he also includes a promise to tax people who stay in the commuting zone in the medium run (around 10 years after the shock). Because of that promise, workers who get the opportunity to move to a different location (because of a new job opportunity, etc.) take it. Thus, the planner can have her cake and eat it to. She can get the immediate stimulus with the front loaded transfers while still encouraging workers to find work elsewhere through the promise of less generous transfers in the medium run. Therefore, the bump in population in Figure 4f is relatively small, even though the transfers immediately after the demand shock are quite large.

The third stage is the long run, more than 20 years after the shock. At this point, wages have completely adjusted and population has started to stabilize. There is no longer any reason for policy to affect macroeconomic stabilization. This transfer optimally trades off redistribution to people who are now poorer because of the shock against misallocation that comes from worker migration response as explored in Gaubert et al. (2021). I show that the basic shape of the optimal polic is robust to varying key parameters in Appendix I.1. As speed of migration increases, the migration effect becomes more prominent leading to taxes 10 years after the shock. On the other hand, if wages adjust much quicker the migration effect becomes less important as any migration occurs too slowly to affect the recession.

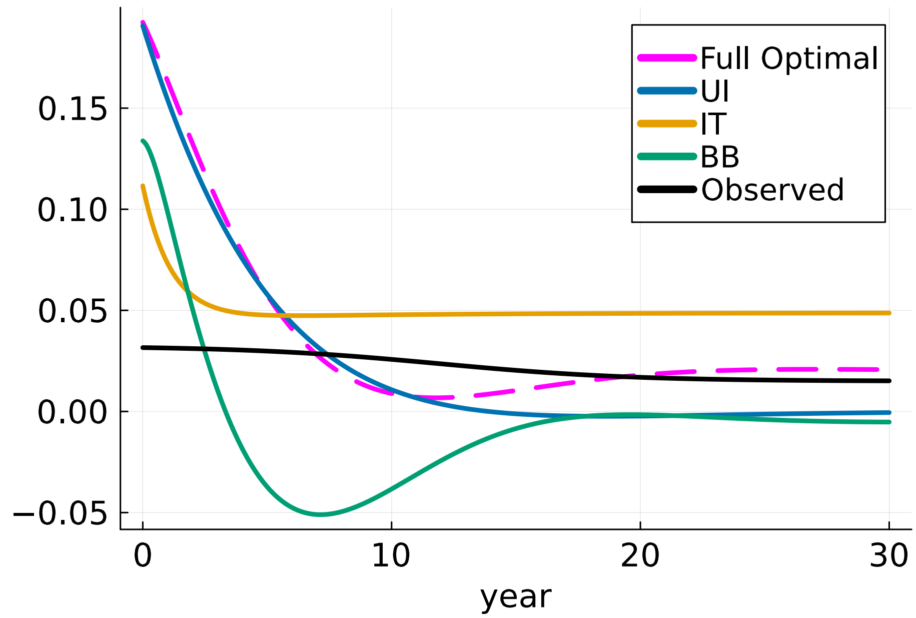
The observed policy falls short in two main ways. First of all, it is not nearly generous enough immediately after the demand shock, so that unemployment rises inefficiently high. It then also does not fade away quickly enough, encouraging workers to remain in the area for too long. In particular, transfers driven by the retirement and disability programs seem to hurt macroeconomic stability. However the observed policy does a decent job at matching the long run optimal redistribution. In the end, the observed policy achieves 35.4% of the welfare gains offered by the fully optimal policy.

6.3 Alternate Policy Instruments

While the United States might never have access to fully optimal place-based taxes, it could make adjustments to its current programs of automatic stabilizers so that they do a better job of ensuring macroeconomic stability for cities going through recessions. For example, currently the federal government offers an extra 13 weeks of unemployment insurance for households who are in a state with elevated unemployment levels. The government could make further adjustments to respond to local conditions.

In this section, I assess how well these automatic stabilizers could work to fight local recessions when we account for the stimulus and migration effects of policy. In particular, I will consider 3 types of policies: unemployment insurance, income tax, and local budget balance. I model unemployment insurance as a transfer to the region that must be proportional to the labor wedge. With the income tax, the transfer must be proportional to lost income. Local budget balance is different. I assess how effective policy can be when it is constrained to have a present discounted value of 0, taking as given the taxes and transfers

Figure 5: Imperfect Policies



Note: This figure plots optimal policy against various imperfect policy instruments

currently offered by the national government. I optimize over the possible policies within each class and assess how well they can compare to the full optimal policy in response to the idiosyncratic demand shock to a commuting zone.

I plot the time path of transfers for the best policy within each class in Figure 5. The fully optimal policy and the observed policy are both reprinted for easy comparison. The optimal unemployment insurance does a very good job of matching the general shape of the fully optimal policy. It allows for extremely generous transfers on impact that decay over the next ten years as wages adjust. Compared to the fully optimal policy, it only fails to recover and offer the efficient long run redistribution. Yet, despite that, it still manages to achieve 94.7% of the welfare gains of the fully optimal transfer. This unemployment insurance policy is much more generous than any reasonable unemployment insurance system. It suggests that each unemployed person should get a transfer equivalent to 5 times their original income. While that does not make sense as an individual transfer, it does suggest that the central government could transfer money to commuting zones that have high unemployment rate shocks. It also suggests that the federal government should consider making the special benefits authorized for periods of high unemployment more generous, rather than extending the period for which you are eligible.

The income tax has a small bump in transfers on impact, but it then falls close to its long run level after 5 years. This high long run transfer implies that it continues to distort migration too much, both in the medium run and long run. The income tax only manages to get 65.6% of the welfare gains of the fully optimal policy even while it makes up 50% of the lost income in the commuting zone.

Turning to the budget balanced policy, I find that a local government can fight a local recession by borrowing to fund a large stimulus program after the demand shock. The stimulus is not enough to increase income on impact, but it does put many of the households back to work. The local government then pays for that policy with taxes in the medium and long run. By taxing heavily around year 8, the local government can encourage people to leave and find good employment somewhere else at the same time it funds its stimulus payments. The government then settles in with a moderate long run tax to make up the rest of the shortfall. With this policy, and no change in the central government's tax and transfer program, a local government can get 74.7% of the welfare gains from the fully optimal policy, much better than the 35.4% implied by the current policies.

7 Optimal Policy After the China Trade Shock

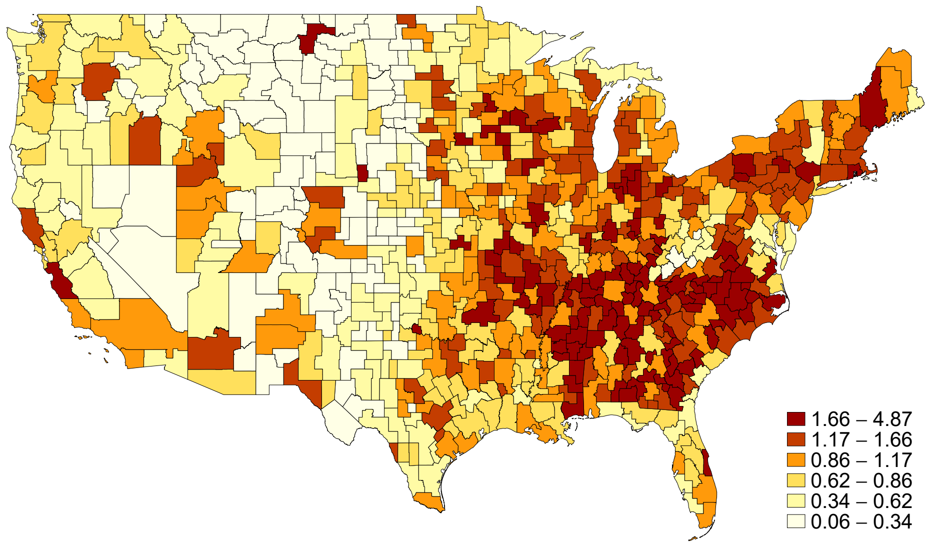
In section 6, I analyzed what this model suggests for place-based policy in response to an idiosyncratic demand shock to a single region. In this section, I consider what this model suggests for fighting the regional recessions caused by the China trade shock. With the full model and a spatially correlated shock, I can assess how the migration and stimulus effect change as suggested by Proposition 2.

7.1 The Trade Shock

I model the trade shock as a uniformly increasing demand shock for traded production of commuting zones starting in the year 2000 and ending at the beginning of year 2010 as Autor et al. (2021) showed that imports from China plateaued at that point. I further assume that starting in the year 2000, the planner fully anticipates the size of the entire trade shock. I follow Autor et al. (2021) in constructing the China Trade shock to each commuting zone. In particular, I use the notion of average change in import penetration across industries, weighted by industry shares in initial CZ employment:

$$\Delta IP_n = \sum_s \frac{\ell_{n,s,2000}}{\ell_{n,2000}} \Delta IP_s^{US},$$

Figure 6: 100× China Shock to Trade Demand



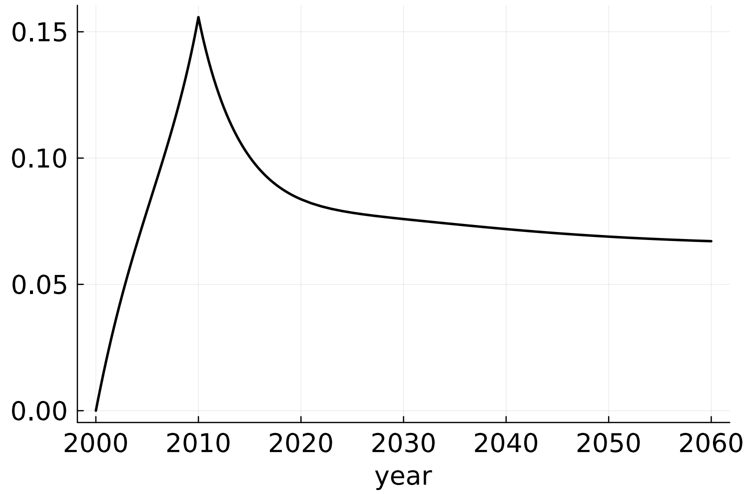
Note: This figure plots the incidence of the China Trade shock across the 722 commuting zones of the contiguous United States. The shock is constructed instrumenting for the increase in Chinese import penetration in 4 digit industries to the United States from 2000 to 2012 with the increase in export penetration to a group of other developed countries, as in Autor et al. (2021). The impact on each commuting zone is determined by share of commuting zone employment in the sector in year 2000.

where $\Delta IP_s^{US} = \Delta M_{china,US,s}/(Y_{US,s,2000} + M_{US,s,2000} - X_{US,s,2000})$ is the growth of Chinese import penetration for U.S. manufacturing industry s over the period 2000 to 2012,[16] $\frac{\ell_{n,s,2000}}{\ell_{n,2000}}$ is the share of industry s in CZ n's total employment in the year 2000, and $Y_{US,s,2000} + M_{US,s,2000} - X_{US,s,2000}$ is total US absorption of industry s production in the year 2000. I then instrument for that import penetration using import penetration of China to eight other developed countries[17] and the share of employment in industry s commuting zone n in the year 1990,

$$\Delta IP_n^{IV} = \sum_s \frac{\ell_{n,s,1990}}{\ell_{n,1990}} \Delta IP_s^{oc},$$

where $\Delta IP_s^{oc} = \Delta M_{china,oc,s}/(Y_{US,s,1997} + M_{US,s,1997} - X_{US,s,1997})$ following Autor et al. (2021). Then I interpret the predicted exposure as the negative demand shock to the traded output of CZ n, $-\hat{\phi}_n$. I plot the distribution of shocks in Figure 6.

Figure 7: Optimal Policy Response to China Trade Shock



Note: This figure plots the coefficients of a regression of optimal transfers relative to original income on the size of the China shock for each time t, weighting by pre-shock earnings.

7.2 Average Optimal Policy

I start by plotting the average optimal policy directed at commuting zones affected by the China trade shock. I regress the optimal transfer, as a share of initial earnings, to each CZ on the size of the shock it received weighted by total labor earnings before the shock. I plot the results in Figure 7.

The time path of the transfers is significantly different from that found in Section 6 because starting in the year 2000, the planner expects future shocks. Therefore, the planner does not want to encourage too many households from entering the CZs hit by the China shock before the worst of the shock hits. Therefore, the optimal transfer starts small and slowly builds until 2010 when the China shock stops intensifying. At that point, the optimal transfers start to fall. I include a plot of the optimal policy responses against a China trade shock that happens all at some future date τ in Appendix I.2. It shows that for expected shocks, the central government should actually tax locations that are about to be hit, but then greatly increase transfers once the region experiences the shock. Thus, depending on the planner's expectations of how much the shock will intensify, the government might have wanted to tax in the early 2000s or provide more generous transfers.

Importantly, the optimal transfer to regions hit by the China trade shock never falls below

[16] The authors use that time frame as 2000 is the year before China enters the WTO and 2012 is sufficiently after the 2008 financial crisis that the volatility in global trade has subsided.

[17] The eight other countries are Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland.

the long run redistribution level suggesting that the migration effect is not as important for fighting this spatially correlated shock. Instead, the optimal policy features significantly elevated payments even in the year 2020, a full 10 years after the China shock has stopped.

7.3 The Geography of Optimal Policy

The average policy hides a significant amount of spatial heterogeneity. I plot some of that heterogeneity in Figure 8.

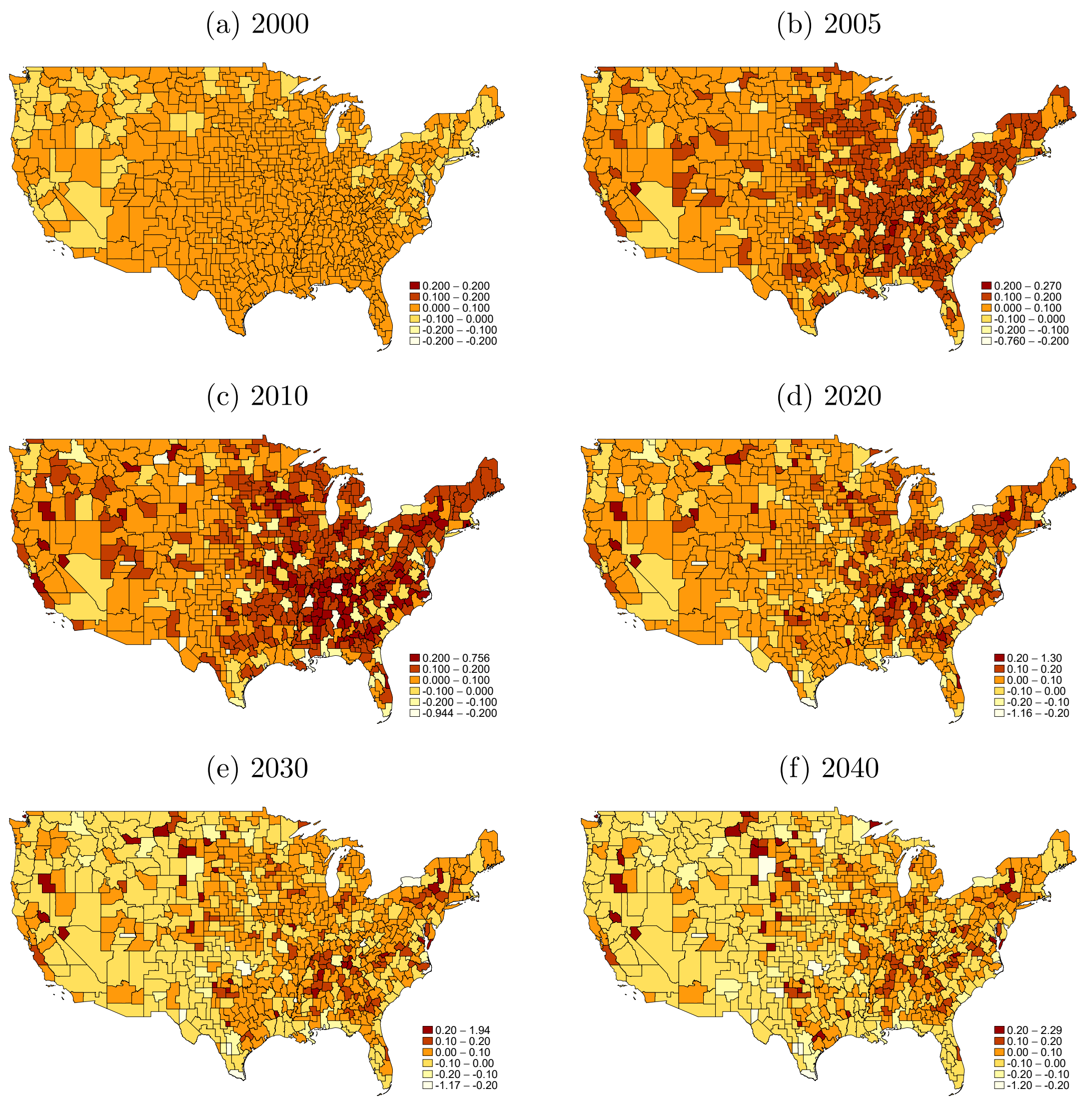
In the year 2000, Figure 8a, transfers are small but roughly targeted toward those CZs that will receive the China trade shock. That is because there is no migration effect from these unexpected transfers. Households cannot respond to transfers immediately announced and enacted. Thus, the planner targets the transfers towards CZs where he can get a small stimulus effect.

By 2005, as one can see in Figure 8b, the transfers are targeted toward those CZs that are directly impact by the China shock and those CZs nearby. One can see this most clearly with the Appalachian mountains, which were not hit by the China shock since they were not particularly industrialized, but they do receive generous transfers from the government. The other notable feature in 2005 is that there a few isolated CZs in Georgia and Atlanta that did not see strong effects from the China shock. Those CZs tend to see mild taxes. That is because the optimal transfers feature such large transfers to the surrounding areas that those regions end up overstimulated.

By 2010, the same basic transfers from 2005 are there but they are intensified. In particular, there are generous transfers directed towards much of the eastern half of the United States that saw much worse trade shocks. The transfers are also targeted towards the regions in the west directly impacted by the shock, or those commuting zones exactly adjacent to those. After the peak of the shock, the optimal transfers slowly scale back. Transfers in 2020 are only large in those areas badly hit by the shock. By 2030, the transfers shrink even more. And in 2040, the optimal transfers feature modest redistributive payments to those CZs most impacted by the China trade shock, both directly, and indirectly through migration and demand feedback effects emphasized by Adao et al. (2019).

I summarize the lifetime implications of these transfers in Figure 9. In Figure 9a I plot the lifetime value of transfers relative to original income in every CZ. These transfers are overwhelmingly targeted towards regions directly impacted by the shock. Figure 9b shows the total lifetime value of transfers minus the lifetime value of the redistribution transfers. Here, the Appalachian CZs stand out as regions that received a significant amount of transitional transfers from the central government. These are regions not directly impacted by the shock

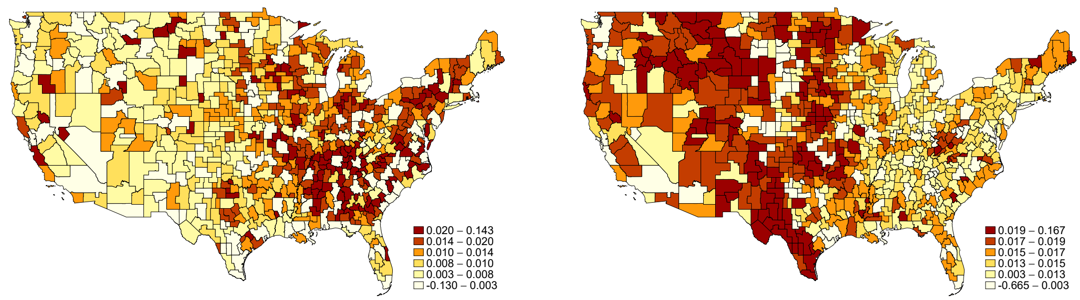
Figure 8: 100× Optimal Transfer Relative to Original Income



Note: This figure shows the geography of optimal transfers in response to the China trade shock for commuting zones at various years.

Figure 9: Summary of Optimal Transfers

(a) Total Value of Transfers (b) Value of Transitional Transfers



Note: This figure shows the total value of transfers and the value of transfers relative to the long run redistribution levels of transfers.

but are close by and so can provide stimulus while also encouraging households to move out. Similarly, regions just to the west of those regions hit by the shock saw significant transitional transfers for similar reasons.

8 Concluding Remarks

Regions are subject to idiosyncratic shocks. Changes in trade policies can lead to large shifts in demand. Economic structural change can make the product one location produces less enticing. And idiosyncratic shocks to individual firms can end up greatly hurting a town. Central governments cannot use monetary policy to fight the resulting local recessions, but it can use other policies.

In this paper, I focused on one key market failure that shapes how regions respond to these shocks: wage rigidity. In such a case, I have shown that fiscal policy can be used to fight the resulting local recession. The resulting optimal transfers should be aggressive, but short lived. For idiosyncratic shocks, more generous unemployment insurance could provide the necessary stimulus without distorting location choice greatly. More aggregate shocks likely call for a more coordinated response.

My analysis leaves many questions unanswered. Are there other tools available to a central government for fighting local recessions? What if households lose skills from not working? Can retraining programs work to stimulate the local economy without distorting migration decisions? When can a commuting zone reinvent itself and rebuild demand for its traded output? I hope to address these topics in future research.

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A Empirical Details

A.1 Data Details

Local Area Unemployment Statistics (LAUS). The Local Area Unemployment Statistics are maintained by the Bureau of Labor Statistics (BLS) and provide counts of the labor force, the number of employed workers, and the number of unemployed workers by county in the United States for the years 1990-2023.

For Los Angeles County, New York City, Chicago-Naperville-Arlington Heights, Cleveland-Elyria, Detroit-Warren-Dearborn, Miami-Miami Beach-Kendall, and Seattle-Bellevue-Everett, the BLS constructs the counts by smoothing out the responses from the Current Population Survey (CPS). They assume that in any given month, the reported unemployment in the CPS has some measurement error. They then model how the true values move around with some autocorrelation and back out an estimate.

For every other county, the BLS use an approach known as the Handbook method. The total employment estimate comes from the Current Employment Statistics (CES) survey and the Quarterly Census of Employment and Wages (QCEW) which are designed to find non-farm employment. For the remaining employment, they use CPS estimates combined with ACS estimates. The count of unemployment primarily comes from the Unemployment Insurance system. Those covered by the UI system are counted. The BLS then includes estimates of how many who are still unemployed but no longer qualify for benefits. For those who are never covered, the BLS uses the CPS. All series are then adjusted so that they sum up to be consistent with the state-level data.

Details can be found at https://www.bls.gov/opub/hom/lau/calculation.htm. The data can be downloaded from https://www.bls.gov/lau/data.htm.

Regional Economic Accounts (REA). The Regional Economic Accounts (REA) are maintained by the Bureau of Economic Analysis (BEA). I use it to get information on population and transfers from outside of the county. They get estimates of the total population by county from the Census Bureau. The personal current transfer receipts include benefits received by people for which no service was provided. It is broken up into retirement and other benefits, income maintenance benefits, and unemployment insurance compensation. I will describe what programs each of these categories include in turn.

The unemployment insurance compensation category includes a wide variety of unemployment benefits. It counts state unemployment insurance compensation, the unemployment compensation for Federal employees, unemployment compensation for railroad employees, unemployment compensation for Ex-servicemembers, and other unemployment compensation including trade adjustment assistance, Redwood park benefits, public service employment benefits, and transitional benefits. Most of these transfers are kept track of by the US Department of Labor's Employment and Training Administration (ETA) or state officials.

Income maintenance benefits include supplemental security income (SSI) benefits, earned income tax credit (EITC), supplemental nutritional assistance, family assistance, general assistance benefits, foster care and adoption assistance, additional child tax credit (ACTC), energy assistance, and special supplemental nutrition for women, infants, and children (WIC) benefits. These payments are tracked by the relevant US agency.

The retirement category includes all current transfer receipts that are not in unemployment insurance or income maintenance benefits. This includes Social Security benefits, Railroad retirement and disability benefits, Workers' compensation, temporary disability benefits, black lung benefits, Pension Benefit Guaranty benefits, Medicare benefits, Medicaid benefits, other medical care benefits, military medical insurance benefits, Veterans' pension and disability benefits, Veteran's readjustment benefits, Veterans' life insurance benefits, other assistance to veterans, Federal fellowship benefits, Federal educational exchange benefits, Interest on guaranteed student loans, Higher education student assistance, Job Corps benefits, State educational assistance, Compensation to survivors of public safety officers, Compensations to victims of crime, Alaska Permanent Fund benefits, Disaster relief benefits, Radiation exposure compensation, Japanese interns redress benefits, Anti-terrorism judgment receipts, Compensation to victims of September 11, Bureau of Indian Affairs benefits, TV converter box coupons, American Recovery and Reinvestment Act of 2009 compensation, American opportunity tax credit, Home Affordable Modification Program, Temporary High-Risk Health Insurance premium reduction, World Trade Center health benefits, Economic Stimulus Act of 2008 rebates, Alternative Minimum Tax credit, Adoption tax credit, Health Coverage Tax Credit, Health insurance premium assistance tax credit, Cost-sharing reduction subsidy, Health benefits for retired United Mine Workers of America members, Economic Impact Payments, and Lost wages supplemental payments. These payments are tracked by the relevant US agency.

See https://www.bea.gov/resources/methodologies/local-area-personal-income-employment for details.

Current Population Survey (CPS). I use data on wage earnings from the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS) which I download from IPUMS. I use the variable INCWAGE where respondents are asked their pre-tax wage and salary income in the previous year. I use the variable WKSWORK1 where respondents are asked how many weeks they worked in the previous year.

Internal Revenue Service (IRS) Statistics of Income (SOI). The IRS creates the Statistics of Income (SOI) by county based on the addresses reported on the individual income tax returns field. Data on income is available for the years 1989-2021. Data on on total income tax paid by county begins in 2010. I use the variable A06500 for total income tax, which corresponds to line 5 on Form 1040. I then also use the variable A00100 for Adjusted gross income to calculate the retention rate. The data can be easily downloaded from https://www.irs.gov/statistics/soi-tax-stats-county-data.

US County Population Data. I download data on US population by county broken up by age from National Cancer Institute Surveillance, Epidemiology, and End Results Program. It is available at https://seer.cancer.gov/popdata/.

A.2 Aggregating up to Commuting Zones

All data is aggregated up to the 1990 commuting zone level following Tolbert and Sizer (1996) and Autor et al. (2013). The crosswalks I use are available at https://www.ddorn.net/data.htm.

A.3 Regional Income

The net income of a household in commuting zone n at time t is:

$$E_{nt} = R_{nt}\left(W_{nt}H_{nt} + \tau_{nt}\right),$$

where τ_{nt} is the transfer to the region, W_{nt} is the wage, H_{nt} is per capita hours worked, and R_{nt} is the retention rate of commuting zone n. Then taking a log linearization around the point of average earnings implies that,

$$\hat{E}_{nt} = \hat{R}_{nt} + \frac{\overline{WH}}{\overline{WH} + \overline{\tau}}\left(\hat{W}_{nt} + \hat{H}_{nt}\right) + \frac{\overline{\tau}}{\overline{WH} + \overline{\tau}}\hat{\tau}_{nt}.$$

B Proofs for Section 4

Throughout, I make a few technical assumptions to ensure that the limits I take are well-defined. First, I assume that the labor wedge is bounded away from infinity and away from being too negative.

Assumption 1. *There exists a $\overline{\varepsilon} > 0$ and $\overline{B} > 0$ such that, in any interior solution to* (SPP)*,*

$$\tau_{nt} < \overline{B}$$

and

$$1 + \frac{\alpha_n}{1-\alpha_n}\tau_{nt} > \overline{\varepsilon}.$$

This restricts my analysis to equilibria where regions are not booming or busting too much. The assumption that $1 + \frac{\alpha_n}{1-\alpha_n}\tau_{nt}$ is bounded away from 0 also implies that utility in each location is increasing in a transfer as $\frac{\partial V^n}{\partial T} + \frac{\partial V^n}{\partial H}\frac{\partial H^n}{\partial T} = \frac{U^n_C}{P_n}\left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{nt}\right)$.

The next assumption is that the migration semi-elasticities are all bounded away from infinity.

Assumption 2. *There exists a $\overline{C} > 0$ such that*

$$\left|\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\right| \leqslant \overline{C}; \quad \left|\frac{\partial \log \mu^{mn}}{\partial U_{n1}}\right| \leqslant \overline{C}.$$

I also assume that H_{Tnt} is bounded above and away from 0.

Assumption 3. *There exists a $\overline{A} > 0$ and $\varepsilon_H > 0$ such that, in any interior solution to* (SPP)*,*

$$\varepsilon_H < H_{Tnt} < \overline{A}.$$

I assume that the optimal transfers are also bounded.

Assumption 4. *There exists a* $\overline{D} > 0$ *such that*

$$|T_{nt}| \leqslant \overline{D},$$

in any interior solution to (SPP)*.*

And I also assume that the marginal utility of expenditures is bounded away from 0 and infinity.

Assumption 5. *There exists* $\varepsilon_C > 0$ *and* $\overline{D}_C$ *such that*

$$\varepsilon_C \leqslant \frac{U_C^n}{P_n} \leqslant \overline{D}_C,$$

in all interior solution to (SPP)*.*

I start by proving Lemma 1.

Lemma 1. *The derivatives of the indirect utility function are*

$$\frac{\partial V^n}{\partial T} = \frac{U_C^n}{P_n}; \qquad \frac{\partial V^n}{\partial H} = W_n \frac{U_C^n}{P_n} \tau_{nt}.$$

The derivatives of the hours worked function are

$$\frac{\partial H^n}{\partial T} = \frac{1}{W_n} \frac{\alpha_n}{1-\alpha_n}; \quad \frac{\partial H^n}{\partial \log E_T} = \frac{1}{W_n} \frac{\phi_n E_T}{1-\alpha_n} \frac{1}{\ell}; \quad \frac{\partial H^n}{\partial \log \ell} = -\frac{1}{W_n} \frac{\phi_n E_T}{1-\alpha_n} \frac{1}{\ell}.$$

Proof. The derivatives of the hours are trivial. Recall that

$$H^m(E_T, \ell, T) = \frac{1}{W_m} \left(\frac{\phi_m E_T}{1-\alpha_m} \frac{1}{\ell} + \frac{\alpha_m}{1-\alpha_m} \right).$$

Then

$$\begin{aligned}
\frac{\partial H^m}{\partial T} &= \frac{1}{W_m} \frac{\alpha_m}{1-\alpha_m} \\
\frac{\partial H^m}{\partial \ell} &= -\frac{1}{W_m} \frac{\phi_m E_T}{1-\alpha_m} \frac{1}{\ell^2} \\
\frac{\partial H^m}{\partial E_T} &= \frac{1}{W_m} \frac{\phi_m}{1-\alpha_m} \frac{1}{\ell}.
\end{aligned}$$

These can then be rewritten to get the derivatives.

The derivatives for the indirect utility function are

$$\begin{aligned}\frac{\partial V^n}{\partial T} &= \frac{d}{dT}\left[U^n\left(\frac{W_n}{P_n}H + \frac{T}{P_n}, H\right)\right] \\ &= \frac{U_C^n}{P_n} \\ \frac{\partial V^n}{\partial H} &= \frac{d}{dH}\left[U^n\left(\frac{W_n}{P_n}H + \frac{T}{P_n}, H\right)\right] \\ &= \frac{W_n}{P_n}U_C^n + U_H^n \\ &= W_n\frac{U_C^n}{P_n}\left(1 + \frac{P_n}{W_n}\frac{U_H^n}{U_C^n}\right) \\ &= W_n\frac{U_C^n}{P_n}\tau_{nt}.\end{aligned}$$

□

Next I prove the lemmas associated with the first order conditions.

Lemma 2. *In any interior solution to* (SPP),

$$\sum_n \frac{W_n H_{Tn1}}{1-\alpha_n}\ell_{n1}\frac{\tau_{n1}}{1+\frac{\alpha_n}{1-\alpha_n}\tau_{n1}} = 0.$$

Lemma 3. *In any interior solution to* (SPP), *first period transfers must satisfy*

$$\underbrace{\sum_m \ell_{m1}T_{m1}\nu_{n1}^{m1}}_{\textit{fiscal externality}} = \ell_{n1}\Bigg[\underbrace{\frac{\zeta_{n1}}{\lambda_{G1}}}_{\textit{redistribution}}\underbrace{\left(1+\frac{\alpha_n}{1-\alpha_n}\tau_{n1}\right)}_{\textit{stimulus effect}}-1\Bigg] - \underbrace{\sum_m \frac{W_m H_{Tm1}}{1-\alpha_m}\ell_{m1}\frac{\tau_{m1}}{1+\frac{\alpha_m}{1-\alpha_m}\tau_{m1}}\nu_{n1}^{m1}}_{\textit{migration effect}},$$

where $\nu_{n1}^{m1} \equiv \frac{\partial \log \ell^{m1}}{\partial U_{n1}}\left(\frac{\partial V^n}{\partial T_{n1}} + \frac{\partial V^n}{\partial H_{n1}}\frac{\partial H^{n1}}{\partial T_{n1}}\right)$ *is the migration semi-elasticity of population in location m to a transfer in location n holding fixed utility in locations other than n, and $\lambda_1^G > 0$ is the social value of the government having another dollar.*

Lemma 4. *In any interior solution to* (SPP), *second period transfers must satisfy*

$$\begin{aligned}\sum_t \frac{\lambda_{Gt}}{\lambda_{G2}}\sum_m \ell_{mt}T_{mt}\nu_{n2}^{mt} = \ell_{n2}&\left[\frac{\zeta_{n2}}{\lambda_{G2}}\left(1+\frac{\alpha_n}{1-\alpha_n}\tau_{n2}\right)-1\right] \\ &- \sum_t \frac{\lambda_{Gt}}{\lambda_{G2}}\sum_m \frac{W_m H_{Tmt}}{1-\alpha_m}\ell_{mt}\frac{\tau_{mt}}{1+\frac{\alpha_m}{1-\alpha_m}\tau_{mt}}\nu_{n2}^{mt},\end{aligned}$$

where λ_{G2} is the social value value of the government having another dollar in period 2, and ν_{n2}^{mt} is the elasticity of population in location m at time t to a transfer to location i at time 2.

Lemma 5. *In any interior solution to* (SPP),

$$\sum_n \frac{W_n H_{Tn2}}{1-\alpha_n} \ell_{n2} \frac{\tau_{n2}}{1+\frac{\alpha_n}{1-\alpha_n}\tau_{n2}} = 0.$$

Proof. The planners problem is

$$\max_{E_{Tt},\{T_{nt}\},\{U_{nt}\},\{\ell_{nt}\}} \int_{\mathcal{I}} \lambda(i) \sum_n \mathbb{1}_{n \in \arg\max U_{n'1}+\varepsilon_{n'1}(i)+\beta \overline{U}_{n'2}} \left[U_{n1} + \varepsilon_{n1}(i) + \beta \overline{U}_{n2}\right] di$$

subject to the constraints

$$\begin{aligned}
\ell_{n1} &= \ell^{n1}\left(\left\{U_{n1} + \beta \overline{U}_{n2}\right\}\right), \\
\ell_{n2} &= \sum_m \ell_{m1} \mu^{mn}\left(\{U_{k2}\}\right), \\
U_{nt} &= \tilde{V}^n\left(T_{nt}, E_{Tt}, \ell_{nt}\right), \\
\sum_n \ell_{nt} T_{nt} &= 0,
\end{aligned}$$

where $\tilde{V}^n(T, E_T, \ell) = V^n(T, H(T, \ell, E_T))$. I can then take the first order conditions. This gives

$$\begin{aligned}
U_{n1} &: 0 = \overline{\lambda}_{n1}\ell_{n1} + \sum_m \lambda^{\ell}_{m2} \frac{\partial \ell^{m1}}{\partial U_{n1}} - \lambda^V_{n1} \\
U_{n2} &: 0 = \beta \overline{\lambda}_{n2}\ell_{n2} + \sum_m \lambda^{\ell}_{m1} \beta \frac{\partial \ell^{m1}}{\partial U_{n1}} \mu_{mn} + \sum_m \sum_k \lambda^{\ell}_{m2} \ell_{k1} \frac{\partial u^{kj}}{\partial U_{n2}} - \lambda^V_{n2} \\
\ell_{n1} &: 0 = -\lambda^{\ell}_{n1} + \sum_m \lambda^{\ell}_{m2} \mu_{nm} + \lambda^V_{n1} \frac{\partial \tilde{V}^n}{\partial \ell_{n1}} - \lambda_{G1} T_{n1} \\
\ell_{n2} &: 0 = -\lambda^{\ell}_{n2} + \lambda^V_{n2} \frac{\partial \tilde{V}^n}{\partial \ell_{n2}} - \lambda_{G2} T_{n2} \\
T_{n1} &: 0 = \lambda^V_{n1} \frac{\partial \tilde{V}^n}{\partial T_{n1}} - \lambda^G_1 \ell_{n1} \\
T_{n2} &: 0 = \lambda^V_{n2} \frac{\partial \tilde{V}^n}{\partial T_{n2}} - \lambda^G_2 \ell_{n2},
\end{aligned}$$

where λ^V_{nt} is the Lagrange multiplier on the utility constraint, λ^{ℓ}_{nt} is the Lagrange multiplier on the population constraint, and λ_{Gt} is the Lagrange multiplier on the government budget constraint, and $\mu_{nm} \equiv \mu^{mn}\left(\{U_{k2}\}\right)$. Then in the first order conditions for the transfers, we can solve for λ_{nt}^V and then substitute in for it in the other equations. That gives

$$\begin{aligned}
U_{n1} &: 0 = \overline{\lambda}_{n1}\ell_{n1} + \sum_m \lambda_{m1}^{\ell}\frac{\partial \ell^{m1}}{\partial U_{n1}} - \frac{\lambda_{G1}\ell_{n1}}{\frac{\partial \tilde{V}^n}{\partial T_{n1}}} \\
U_{n2} &: 0 = \beta\overline{\lambda}_{n2}\ell_{n2} + \sum_m \lambda_{m1}^{\ell}\beta\frac{\partial \ell^{m1}}{\partial U_{n1}}\mu_{mn} + \sum_m\sum_k \lambda_{m2}^{\ell}\ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}} - \frac{\lambda_{G2}\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}} \\
\ell_{n1} &: 0 = -\lambda_{n1}^{\ell} + \sum_m \lambda_{m2}^{\ell}\mu_{nm} + \frac{\lambda_1^G\ell_{n1}}{\frac{\partial \tilde{V}^n}{\partial T_{n1}}}\frac{\partial \tilde{V}^n}{\partial \ell_{n1}} - \lambda_{G1}T_{n1} \\
\ell_{n2} &: 0 = -\lambda_{n2}^{\ell} + \frac{\lambda_{G2}\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}\frac{\partial \tilde{V}^n}{\partial \ell_{n2}} - \lambda_2^G T_{n2}.
\end{aligned}$$

Then we look to take the limit $\beta \to 0$. Turning to the first order conditions for the second period, note that the first order condition for ℓ_{n2} is,

$$0 = -\lambda_{n2}^{\ell} + \frac{\lambda_2^G\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}\frac{\partial \tilde{V}^n}{\partial \ell_{n2}} - \lambda_2^G T_{n2}$$

and the first order condition for U_{n2},

$$0 = o(\beta) + \sum_m\sum_k \lambda_{m2}^{\ell}\ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}} - \frac{\lambda_2^G\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}},$$

since the migration semi-elasticity is bounded by assumption 2. Then substituting out λ_{n2}^{ℓ}, we find that

$$0 = o(\beta) + \sum_m\sum_k \left[\frac{\lambda_{G2}\ell_{m2}}{\frac{\partial \tilde{V}^m}{\partial T_{m2}}}\frac{\partial \tilde{V}^m}{\partial \ell_{m2}} - \lambda_2^G T_{m2}\right]\ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}} - \frac{\lambda_{G2}\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}.$$

Summing across all n implies that

$$\begin{aligned}
0 &= o(\beta) + \sum_n\left\{\sum_m\sum_k \left[\frac{\lambda_{G2}\ell_{m2}}{\frac{\partial \tilde{V}^m}{\partial T_{m2}}}\frac{\partial \tilde{V}^m}{\partial \ell_{m2}} - \lambda_{G2}T_{m2}\right]\ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}} - \lambda_{G2}\ell_{n2}\frac{1}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}\right\} \\
&= o(\beta) + \sum_m \left[\frac{\lambda_{G2}\ell_{m2}}{\frac{\partial \tilde{V}^m}{\partial T_{m2}}}\frac{\partial \tilde{V}^m}{\partial \ell_{m2}} - \lambda_2^G T_{m2}\right]\left(\sum_n\sum_k \ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}}\right) - \lambda_{G2}\sum_n \ell_{n2}\frac{1}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}.
\end{aligned}$$

Then note that because idiosyncratic utility shocks are additive, a uniform increase in utility across all locations does not change population, $\left(\sum_n\sum_k \ell_{k1}\frac{\partial \mu^{km}}{\partial U_{n2}}\right) = 0$. Therefore,

$$0 = o(\beta) - \lambda_{G2}\sum_n \ell_{n2}\frac{1}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}.$$

And since $\frac{\partial \tilde{V}^n}{\partial T_{n2}} = \frac{U_C^n}{P_n}\left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1}\right)$ is bound away from infinity and zero by assumptions 1 and5, $\lambda_2^G \in o(\beta)$. And therefore, $\lambda_{n2}^\ell = \frac{\lambda_{G2}\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}\frac{\partial \tilde{V}^n}{\partial \ell_{n2}} - \lambda_{G2}T_{n2} \in o(\beta)$.

Therefore, focusing on the first period we can solve for the Lagrange multiplier on population in the first period

$$\lambda_{n1}^\ell = \frac{\lambda_1^G \ell_{n1}}{\frac{\partial \tilde{V}^n}{\partial T_{n1}}}\frac{\partial \tilde{V}^n}{\partial \ell_{n1}} - \lambda_1^G T_{n1} + o(\beta).$$

Plugging this into the first order condition for utility gives

$$\frac{\lambda_1^G \ell_{n1}}{\frac{\partial \tilde{V}^n}{\partial T_{n1}}} = \overline{\lambda}_{n1}\ell_{n1} + \sum_m \left[\frac{\lambda_1^G \ell_{m1}}{\frac{\partial \tilde{V}^m}{\partial T_{m1}}}\frac{\partial \tilde{V}^m}{\partial \ell_{m1}} - \lambda_1^G T_{m1} + o(\beta)\right]\frac{\partial \ell^{m1}}{\partial U_{n1}}.$$

Rewriting slightly

$$\sum_m \ell_{m1}T_{m1}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\frac{\partial \tilde{V}^n}{\partial T_{n1}} = \ell_{n1}\left(\frac{\overline{\lambda}_{n1}}{\lambda_1^G}\frac{\partial \tilde{V}^n}{\partial T_{n1}} - 1\right) + \sum_m \ell_{m1}\frac{\frac{\partial \tilde{V}^m}{\partial \log \ell_{m1}}}{\frac{\partial \tilde{V}^m}{\partial T_{m1}}}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\frac{\partial \tilde{V}^n}{\partial T_{n1}} + o(\beta).$$

Substituting in,

$$\frac{\partial \tilde{V}^n}{\partial T} = \frac{U_C^n}{P_n}\left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{nt}\right)$$
$$\frac{\partial \tilde{V}^n}{\partial \log \ell} = -\frac{\phi_n E_T}{1-\alpha_n}\frac{1}{\ell_n}\frac{U_C^n}{P_n}\tau_{nt},$$

and $\nu_{n1}^{m1} \equiv \frac{\partial \log \ell^{m1}}{\partial U_{n1}}\frac{\partial \tilde{V}^n}{\partial T_{n1}}$ and taking the limit as $\beta \to 0$ then completes the proof of the formula in Lemma 3. Next, I note that if the government were to increase transfers to location n by $\frac{\overline{T}}{\frac{\partial \tilde{V}^n}{\partial T}} > 0$, then utility in every location increases by $\frac{\partial \tilde{V}^n}{\partial T}\frac{\overline{T}}{\frac{\partial \tilde{V}^n}{\partial T}} = \overline{T}$. Then since utility shocks are additive, no one changes where they live and utility increases in every location. Therefore, the Lagrange multiplier on the budget constraint must be positive, i.e. $\lambda_{G1} > 0$.

Next I take the first order condition with respect to E_{T1}. That gives

$$E_{T1} : 0 = \sum_n \lambda_{n1}^V \frac{\partial \tilde{V}^n}{\partial E_T}.$$

Substituting in for λ_{n1}^V then implies

$$\sum_n \lambda_1^G \ell_{n1}\frac{\frac{\partial \tilde{V}^n}{\partial E_T}}{\frac{\partial \tilde{V}^n}{\partial T_{n1}}} = 0.$$

Plugging in the derivatives then completes the proof of Lemma 2.

Next we return to the second period first order conditions. Dividing those equations by

β gives

$$0=\overline{\lambda}_{n2}\ell_{n2}+\sum_m \lambda^{\ell}_{m1}\ell_{m1}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\mu_{mn}+\sum_m\sum_k \frac{\lambda^{\ell}_{m2}}{\beta}\ell_{k1}\mu_{km}\frac{\partial \log \mu^{km}}{\partial U_{n2}}-\frac{\lambda^G_2}{\beta}\frac{\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}$$

$$\frac{\lambda^{\ell}_{n2}}{\beta}=\frac{\lambda^G_2}{\beta}\frac{\frac{\partial \tilde{V}^n}{\partial \log \ell_{n2}}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}-\frac{\lambda^G_2}{\beta}T_{n2}.$$

We can then plug in for $\frac{\lambda^{\ell}_{n2}}{\beta}$ and λ^{ℓ}_{m1}. Therefore

$$\begin{aligned}\frac{\lambda^G_2}{\beta}\frac{\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}=\overline{\lambda}_{n2}\ell_{n2}&+\sum_m\left[\frac{\lambda_{G1}}{\frac{\partial V^m}{\partial T_{m1}}}\frac{\partial V^m}{\partial \log \ell_{m1}}-\lambda_{G1}T_{m1}+o(\beta)\right]\ell_{m1}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\mu_{mn}\\&+\sum_m\sum_k\left[\frac{\lambda^G_2}{\beta}\frac{\frac{\partial \tilde{V}^m}{\partial \log \ell_{m2}}}{\frac{\partial \tilde{V}^m}{\partial T_{m2}}}-\frac{\lambda^G_2}{\beta}T_{m2}\right]\ell_{k1}\mu_{km}\frac{\partial \log \mu^{km}}{\partial U_{n2}}.\end{aligned}$$

Then rewriting slightly

$$\begin{aligned}&\sum_m \frac{\lambda^G_1}{\lambda^G_2/\beta}T_{m1}\ell_{m1}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\mu_{mn}\frac{\partial \tilde{V}_n}{\partial T_{n2}}\\&+\sum_m T_{m2}\ell_{m2}\left(\sum_k \frac{\ell_{k1}\mu_{km}}{\ell_{m1}}\frac{\partial \log \mu^{km}}{\partial U_{n2}}\frac{\partial \tilde{V}^n}{\partial T_{n2}}\right)=\ell_{n2}\left(\frac{\overline{\lambda}_{n2}}{\lambda^G_2/\beta}\frac{\partial \tilde{V}^n}{\partial T_{n2}}-1\right)+o(\beta)\\&\qquad+\sum_m \frac{\lambda^G_1}{\lambda^G_2/\beta}\frac{\frac{\partial V^m}{\partial \log \ell_{m1}}}{\frac{\partial V^m}{\partial T_{m1}}}\ell_{m1}\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\mu_{mn}\frac{\partial \tilde{V}^n}{\partial T_{n2}}\\&\qquad+\sum_m \frac{\frac{\partial V^m}{\partial \log \ell_{m2}}}{\frac{\partial V^m}{\partial T_{m2}}}\ell_{m2}\left(\sum_k \frac{\ell_{k1}\mu_{km}}{\ell_{m1}}\frac{\partial \log \mu^{km}}{\partial U_{n2}}\frac{\partial \tilde{V}^n}{\partial T_{n2}}\right).\end{aligned}$$

Then to complete the proof, we plug in the derivative values and note that

$$\nu^{m1}_{n2}\equiv \beta\frac{\partial \log \ell^{m1}}{\partial U_{n1}}\mu_{mn}\frac{\partial \tilde{V}^n}{\partial T_{n2}},$$

and

$$\nu^{m2}_{n2}=\sum_k \frac{\ell_{k1}\mu_{km}}{\ell_{m1}}\frac{\partial \log \mu^{km}}{\partial U_{n2}}\frac{\partial \tilde{V}^n}{\partial T_{n2}}.$$

Just as before, I can consider a deviation where the planner increases the transfers to every location by $\frac{\overline{T}}{\frac{\partial \tilde{V}^n}{\partial T}}$ in period 2 to conclude that $\lambda^G_2/\beta>0$.

Finally we turn to the first order condition for the monetary policy in period 2. We have

$$E_{T2}:0=\sum_n \lambda^V_{n2}\frac{\partial \tilde{V}^n}{\partial E_{T2}}.$$

Then we can plug in for the λ^V_{n2},

$$0 = \sum_n \ell_{n2} \frac{\frac{\partial \tilde{V}^n}{\partial E_{T2}}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}.$$

Plugging in the values for the derivatives proves the result. □

Next I turn to proving the propositions.

Proposition 1. *Suppose that there are two locations, j (Janesville) and u (Rest of the US), location j is arbitrarily small compared to location u, $\ell_{jt} \to 0$, and there are no redistributive reasons for policy, $\zeta_{nt} = 1$. Then in any interior solution to* (SPP)*, the optimal period 1 transfer to location j must satisfy*

$$T_{j1} = \frac{1}{\nu^{j1}_{j1}} \left(\frac{\alpha_j}{1-\alpha_j} - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}} \right) \tau_{j1},$$

where $\frac{\partial \log \ell^{j1}}{\partial T_{j1}} \equiv \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \frac{\partial V^j}{\partial T}$ is the semi-elasticity of location 1 population to a transfer, holding fixed hours worked, and $\nu^{j1}_{j1} \equiv \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \left(\frac{\partial V^j}{\partial T_{j1}} + \frac{\partial V^j}{\partial H_{j1}} \frac{\partial H^{j1}}{\partial T_{j1}} \right)$ is the semi-elasticity of location 1 population to a transfer, allowing hours to vary.

Proof. With no redistributive reasons for policy, $\frac{\overline{\lambda}_{n1} U^n_C}{P_n} = 1$. We then have that the budget constraint is

$$T_{u1}\ell_{u1} + T_{j1}\ell_{j1} = 0,$$

while the first order condition for monetary policy is

$$\frac{W_j H_{Tj1}}{1-\alpha_j} \ell_{j1} \frac{\tau_{j1}}{1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1}} + \frac{W_u H_{Tu1}}{1-\alpha_u} \ell_{u1} \frac{\tau_{u1}}{1 + \frac{\alpha_u}{1-\alpha_u}\tau_{u1}} = 0.$$

And then, with two locations, the number of people in Janesville is only a function of the difference in utility $\ell_{j1} = \ell^{j1}(U_{j1} - U_{j2})$. Then there is a ν so that

$$\frac{\partial \ell^{j1}}{\partial U_{j1}} = -\frac{\partial \ell^{j1}}{\partial U_{u1}} = \frac{\partial \ell^{u1}}{\partial U_{u1}} = -\frac{\partial \ell^{u1}}{\partial U_{j1}} = \nu.$$

And the first order condition for the transfer to Janesville is

$$\begin{aligned} -T_{u1}\nu \frac{\partial \tilde{V}^{j1}}{\partial T_{j1}} + T_{j1}\nu \frac{\partial \tilde{V}^{j1}}{\partial T_{j1}} = \ell_{j1} &\left[\frac{1}{\lambda^G_1} + \frac{1}{\lambda^G_1} \frac{\alpha_j}{1-\alpha_j} \tau_{j1} - 1 \right] \\ &- \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\tau_{j1}}{1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1}} \nu \frac{\partial \tilde{V}^{j1}}{\partial T_{j1}} \\ &+ \frac{W_u H_{Tu1}}{1-\alpha_u} \frac{\tau_{u1}}{1 + \frac{\alpha_u}{1-\alpha_u}\tau_{u1}} \nu \frac{\partial \tilde{V}^{j1}}{\partial T_{j1}}. \end{aligned}$$

Then from the budget constraint,

$$T_{u1} = -\frac{\ell_{j1}}{\ell_{u1}} T_{j1}.$$

From the monetary policy,

$$\frac{W_u H_{Tu1}}{1-\alpha_u}\frac{\tau_{u1}}{1+\frac{\alpha_u}{1-\alpha_u}\tau_{u1}} = -\frac{\ell_{j1}}{\ell_{u1}}\frac{W_j H_{Tj1}}{1-\alpha_j}\frac{\tau_{j1}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}}.$$

Plugging in we find that the transfer to Janesville is

$$T_{j1} = \frac{\ell_{j1}\ell_{u1}}{\nu\bar{\ell}}\frac{1}{\frac{\partial\tilde{V}^{j1}}{\partial T_{j1}}}\left[\frac{1}{\lambda_{G1}}\left(1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}\right)-1\right] - \frac{W_j H_{Tj1}}{1-\alpha_j}\frac{\tau_{j1}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}}.$$

We can do similar algebra for the rest of the U.S.,

$$T_{u1} = \frac{\ell_{j1}\ell_{u1}}{\nu\bar{\ell}}\frac{1}{\frac{\partial\tilde{V}^{u1}}{\partial T_{u1}}}\left[\frac{1}{\lambda_{G1}}\left(1+\frac{\alpha_u}{1-\alpha_u}\tau_{u1}\right)-1\right] - \frac{W_u H_{Tu1}}{1-\alpha_u}\frac{\tau_{u1}}{1+\frac{\alpha_u}{1-\alpha_u}\tau_{u1}}.$$

I will then rewrite this with migration semi-elasticities to j, absorbing ν,

$$T_{j1} = \frac{\ell_{u1}}{\bar{\ell}}\frac{1}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{j1}}{\partial T_{j1}}}\left[\frac{1}{\lambda_{G1}}\left(1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}\right)-1\right] - \frac{W_j H_{Tj1}}{1-\alpha_j}\frac{\tau_{j1}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}}.$$

and

$$T_{u1} = \frac{\ell_{u1}}{\bar{\ell}}\frac{1}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{u1}}{\partial T_{u1}}}\left[\frac{1}{\lambda_{G1}}\left(1+\frac{\alpha_u}{1-\alpha_u}\tau_{u1}\right)-1\right] - \frac{W_u H_{Tu1}}{1-\alpha_u}\frac{\tau_{u1}}{1+\frac{\alpha_u}{1-\alpha_u}\tau_{u1}}.$$

Budget balance then implies

$$0 = \frac{\ell_{j1}}{\bar{\ell}}\frac{1}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{j1}}{\partial T_{j1}}}\left[\frac{1}{\lambda_{G1}}\left(1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}\right)-1\right] + \frac{\ell_{u1}}{\bar{\ell}}\frac{1}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{u1}}{\partial T_{u1}}}\left[\frac{1}{\lambda_{G1}}\left(1+\frac{\alpha_u}{1-\alpha_u}\tau_{u1}\right)-1\right],$$

using the fact that monetary policy makes the average labor wedge 0. Solving for λ_{G1} then gives

$$\begin{aligned}\lambda_{G1} &= \frac{\frac{\ell_{j1}}{\bar{\ell}}\frac{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{j1}}{\partial T_{j1}}} + \frac{\ell_{u1}}{\bar{\ell}}\frac{1+\frac{\alpha_u}{1-\alpha_u}\tau_{u1}}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{u1}}{\partial T_{u1}}}}{\frac{\ell_{j1}}{\bar{\ell}}\frac{1}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{j1}}{\partial T_{j1}}} + \frac{\ell_{u1}}{\bar{\ell}}\frac{1}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{u1}}{\partial T_{u1}}}} \\ &= 1 + \frac{\ell_{j1}}{\bar{\ell}}\frac{\frac{\frac{\alpha_j}{1-\alpha_j}\tau_{j1}}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{j1}}{\partial T_{j1}}}}{\frac{\ell_{j1}}{\bar{\ell}}\frac{1}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{j1}}{\partial T_{j1}}} + \frac{\ell_{u1}}{\bar{\ell}}\frac{1}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{u1}}{\partial T_{u1}}}} + \frac{\ell_{u1}}{\bar{\ell}}\frac{\frac{\frac{\alpha_u}{1-\alpha_u}\tau_{u1}}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{u1}}{\partial T_{u1}}}}{\frac{\ell_{j1}}{\bar{\ell}}\frac{1}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{j1}}{\partial T_{j1}}} + \frac{\ell_{u1}}{\bar{\ell}}\frac{1}{\frac{\partial\log\ell_{j1}}{\partial U_{j1}}\frac{\partial\tilde{V}^{u1}}{\partial T_{u1}}}}\end{aligned}$$

Then since hours and the labor wedge are bounded, assumptions 3 and 1, monetary policy implies

$$0 = \frac{W_u H_{Tu1}}{1-\alpha_u}\ell_{u1}\frac{\tau_{u1}}{1+\frac{\alpha_u}{1-\alpha_u}\tau_{u1}} + o(\ell_{j1}),$$

and therefore, $\tau_{u1} \in o(\ell_{j1})$ as H_{Tu1} is bounded away from 0 by assumption 3 and $1 + \frac{\alpha_u}{1-\alpha_u}\tau_{u1}$ is bounded by assumption 1. Therefore, $\lambda_{G1} = 1 + o(\ell_{j1})$.

Then returning to the tax in Janesville,

$$T_{j1} = \frac{\overline{\ell} - \ell_{j1}}{\overline{\ell}} \frac{1}{\frac{\partial \log \ell^{j1}}{\partial u_{j1}} \frac{\partial \tilde{V}^{j1}}{\partial T_{j1}}} \left[\frac{1}{1 + o(\ell_{j1})} \left(1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1}\right) - 1 - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{1}{1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1}} \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \frac{\partial \tilde{V}^{j1}}{\partial T_{j1}} \tau_{j1} \right].$$

Finally, I note that, taking the limit as $\ell_{j1} \to 0$, and plugging in

$$\nu_{j1}^{j1} = \frac{\partial \log \ell^{j1}}{\partial u_{j1}} \frac{\partial \tilde{V}^{j1}}{\partial T_{j1}}$$

and

$$\frac{\partial \log \ell^{j1}}{\partial U_{j1}} \frac{\partial V^{j1}}{\partial T_{j1}} = \frac{1}{1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1}} \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \frac{\partial \tilde{V}^{j1}}{\partial T_{j1}},$$

completes the proof.

□

Next I move on to proposition 2.

Proposition 2. *Suppose that there are two large locations, s (southern US) and n (northern US), and one small location, j (Janesville). Then, if there are no redistributive reasons for transfers $\zeta_{nt} = \zeta_{st} = \zeta_{jt} = 1$, in any interior solution to* (SPP),

$$T_{j1} > \frac{1}{\nu_{j1}^{j1}} \left(\left(\frac{1}{\lambda_{G1}} - 1 \right) + \frac{1}{\lambda_{G1}} \frac{\alpha_j}{1-\alpha_j} - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}} \right) \tau_{j1},$$

if and only if migrants to j disproportionately come from the region in a recession, i.e. $\mathrm{Cov}(|\nu_{j1}^{k1}|, \tau_{k1}) > 0$.

Proof. Without loss of generality, I am going to assume that the north is in a recession. The budget constraint is

$$o(\ell_{j1}) + T_{n1}\ell_{n1} + T_{s1}\ell_{s1} = 0,$$

since, by assumption 4, transfers to Janesville are bound. The first order condition for monetary policy is

$$o(\ell_{j1}) + \frac{W_s H_{Ts1}}{1-\alpha_s} \ell_{s1} \frac{\tau_{s1}}{1 + \frac{\alpha_s}{1-\alpha_s}\tau_{s1}} + \frac{W_n H_{Tn1}}{1-\alpha_n} \ell_{n1} \frac{\tau_{n1}}{1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1}} = 0,$$

where I use the fact that hours and the labor wedges are bounded by assumptions 1 and 3. Then the first order condition for transfers is

$$\sum_m \ell_{m1} T_{m1} \nu_{k1}^{m1} = \ell_{k1} \left[\frac{\overline{\lambda}_{k1} U_C^k}{\lambda_{G1} P_k} \left(1 + \frac{\alpha_k}{1-\alpha_k}\tau_{k1}\right) - 1 \right] - \sum_m \frac{W_m H_{Tm1}}{1-\alpha_m} \ell_{m1} \frac{\tau_{m1}}{1 + \frac{\alpha_m}{1-\alpha_m}\tau_{m1}} \nu_{k1}^{m1}.$$

Then solving for the transfers in the north and south gives

$$T_{n1} = \frac{\ell_{n1}\ell_{s1}}{\nu\bar{\ell}} \frac{1}{\frac{\partial \tilde{V}^{n1}}{\partial T_{n1}}} \left[\frac{1}{\lambda_1^G} \left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1} \right) - 1 \right] - \frac{W_n H_{Tn1}}{1-\alpha_n} \frac{\tau_{n1}}{1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1}} + o(\ell_{j1}),$$

and

$$T_{s1} = \frac{\ell_{s1}\ell_{n1}}{\nu\bar{\ell}} \frac{1}{\frac{\partial \tilde{V}^{s1}}{\partial T_{s1}}} \left[\frac{1}{\lambda_1^G} \left(1 + \frac{\alpha_s}{1-\alpha_s}\tau_{s1} \right) - 1 \right] - \frac{W_s H_{Ts1}}{1-\alpha_s} \frac{\tau_{s1}}{1 + \frac{\alpha_s}{1-\alpha_s}\tau_{s1}} + o(\ell_{j1}),$$

where $\nu = \frac{\partial \ell^{s1}}{\partial U_{s1}}$. Then the transfer to Janesville must satisfy

$$\begin{aligned} T_{j1}\nu_{j1}^{j1} = & \frac{1}{\lambda_1^G} \left(1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1} \right) - 1 - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\tau_{j1}}{1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1}} \nu_{j1}^{j1} \\ & - \frac{W_n H_{Tn1}}{1-\alpha_n} \ell_{n1} \frac{\tau_{n1}}{1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1}} \frac{\nu_{j1}^{n1}}{\ell_{j1}} - \frac{W_s H_{Ts1}}{1-\alpha_s} \ell_{s1} \frac{\tau_{s1}}{1 + \frac{\alpha_s}{1-\alpha_s}\tau_{s1}} \frac{\nu_{j1}^{s1}}{\ell_{j1}} \\ & - T_{s1}\ell_{s1}\frac{\nu_{j1}^{s1}}{\ell_{j1}} - T_{n1}\ell_{n1}\frac{\nu_{j1}^{n1}}{\ell_{j1}}. \end{aligned}$$

Then note that

$$T_{j1} = \frac{1}{\nu_{j1}^{j1}} \left[\frac{1}{\lambda_1^G} \left(1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1} \right) - 1 - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\tau_{j1}}{1 + \frac{\alpha_j}{1-\alpha_j}\tau_{j1}} \nu_{j1}^{j} \right] + \frac{1}{\nu_{j1}^{j1}} X + o(\ell_{j1})$$

where X is the weighted average of $\ell_{n1}\frac{W_n H_{Tn1}}{1-\alpha_n}\frac{\tau_{n1}}{1+\frac{\alpha_n}{1-\alpha_n}\tau_{n1}} + \ell_{n1}T_{n1}$ and $\ell_{s1}\frac{W_s H_{Ts1}}{1-\alpha_s}\frac{\tau_{s1}}{1+\frac{\alpha_s}{1-\alpha_s}\tau_{s1}} + \ell_{s1}T_{s1}$ with weights $-\frac{\nu_{j1}^{n1}}{\ell_{j1}}$ and $-\frac{\nu_{j1}^{s1}}{\ell_{j1}}$ respectively. Note from the expressions for T_{n1} and T_{s1}, we have

$$\ell_{n1}T_{n1} + \ell_{n1}\frac{W_n H_{Tn1}}{1-\alpha_n} \frac{\tau_{n1}}{1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1}} = \ell_{n1}\frac{\ell_{n1}\ell_{s1}}{\nu\bar{\ell}} \frac{1}{\frac{\partial \tilde{V}^{n1}}{\partial T_{n1}}} \left[\frac{1}{\lambda_1^G} \left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1} \right) - 1 \right],$$

and

$$\ell_{s1}T_{s1} + \ell_{s1}\frac{W_s H_{Ts1}}{1-\alpha_s} \frac{\tau_{s1}}{1 + \frac{\alpha_s}{1-\alpha_s}\tau_{s1}} = \ell_{s1}\frac{\ell_{s1}\ell_{n1}}{\nu\bar{\ell}} \frac{1}{\frac{\partial \tilde{V}^{s1}}{\partial T_{s1}}} \left[\frac{1}{\lambda_1^G} \left(1 + \frac{\alpha_s}{1-\alpha_s}\tau_{s1} \right) - 1 \right].$$

Furthermore, adding together equals zero. Since n is in a recession so that $\tau_{n1} > 0$ and s is in a boom $\tau_{s1} < 0$, it must be the case that

$$\frac{\ell_{n1}\ell_{s1}}{\nu\bar{\ell}} \frac{1}{\frac{\partial \tilde{V}^{n1}}{\partial T_{n1}}} \left[\frac{1}{\lambda_1^G} \left(1 + \frac{\alpha_n}{1-\alpha_n}\tau_{n1} \right) - 1 \right] > 0 > \frac{\ell_{s1}\ell_{n1}}{\nu\bar{\ell}} \frac{1}{\frac{\partial \tilde{V}^{s1}}{\partial T_{s1}}} \left[\frac{1}{\lambda_1^G} \left(1 + \frac{\alpha_s}{1-\alpha_s}\tau_{s1} \right) - 1 \right].$$

Therefore, if $|\nu_{j1}^n| > |\nu_{j1}^s|$, $X > 0$ proving the result when I take the limit as $\ell_{j1} \to 0$. □

Proposition 3. *Suppose that there are two locations, j (Janesville) and u (Rest of the US), location j is arbitrarily small, $\ell_{jt} \to 0$, there are no redistributive reasons for policy, $\beta_{nt} = 1$, and j is in a recession, $\tau_{jt} > 0$. Then in any interior solution to* (SPP)*, the optimal period*

2 transfer to location j satisfies

$$T_{j2} < \frac{1}{\nu_{j2}^{j2}} \left(\frac{\alpha_j}{1-\alpha_j} - \frac{W_j N_{Tj2}}{1-\alpha_j} \frac{\partial \log \ell^{j2}}{\partial T_{j2}} \right) \tau_{j2},$$

when the share of workers in location j in period 1 who stay in location j in period 2 is greater than zero.

Proof. In period 1, we have that

$$\begin{aligned}
T_{j1} &= \frac{1}{\nu_{j1}^{j1}} \left(\frac{\alpha_j}{1-\alpha_j} - \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}} \right) \tau_{j1} + o(\ell_{j1}) \\
T_{u1} &= o(\ell_{j1}) \\
\lambda_{G1} &= 1 + o(\ell_{j1}).
\end{aligned}$$

Recall that

$$0 = \beta \overline{\lambda}_{n2} \ell_{n2} + \sum_m \lambda_{m1}^{\ell} \beta \frac{\partial \ell^{m1}}{\partial U_{n1}} \mu_{mn} + \sum_m \sum_k \lambda_{m2}^{\ell} \ell_{k1} \frac{\partial \mu^{km}}{\partial U_{n2}} - \frac{\lambda_{G2} \ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}}.$$

Then summing across all n implies that

$$\lambda_{G2} \sum_n \frac{\ell_{n2}}{\frac{\partial \tilde{V}^n}{\partial T_{n2}}} = \beta \sum_n \overline{\lambda}_{n2} \ell_{n2},$$

since no one moves from uniform increases in utility. Therefore,

$$\frac{\lambda_{G2}}{\beta} = \overline{\lambda}_{u2} \frac{\partial \tilde{V}^u}{\partial T_{u2}} + o(\ell_{j2}).$$

Meanwhile, the monetary policy is

$$0 = \frac{W_u H_{Tu1}}{1-\alpha_u} \ell_{u1} \frac{\tau_{u1}}{1 + \frac{\alpha_u}{1-\alpha_u} \tau_{u1}} + o(\ell_{j2}).$$

Therefore, $\tau_{u1} \in o(\ell_{j2})$ and $\overline{\lambda}_{u2} \frac{\partial \tilde{V}^u}{\partial T_{u2}} = \overline{\lambda}_{u2} \frac{U_C^u}{P_u} \left(1 + \frac{\alpha_u}{1-\alpha_u} o(\ell_{j2})\right) = 1 + o(\ell_{j2})$. Then we can turn to the first order conditions in Janesville in the second period. They are,

$$\begin{aligned}
\sum_t \frac{\lambda_{Gt}}{\lambda_{G2}} \sum_m \ell_{mt} T_{mt} \nu_{n2}^{mt} = \ell_{n2} &\left[\frac{\beta \overline{\lambda}_{n2} U_C^n}{\lambda_{G2} P_n} \left(1 + \frac{\alpha_n}{1-\alpha_n} \tau_{n2}\right) - 1 \right] \\
&- \sum_t \frac{\lambda_{Gt}}{\lambda_{G2}} \sum_m \frac{W_m H_{Tmt}}{1-\alpha_m} \ell_{mt} \frac{\tau_{mt}}{1 + \frac{\alpha_m}{1-\alpha_m} \tau_{mt}} \nu_{n2}^{mt}.
\end{aligned}$$

Note that,

$$\begin{aligned}
\nu_{j2}^{j1} &= \beta \frac{\partial \log \ell^{j1}}{\partial U_{j1}} \mu_{jj} \frac{\partial \tilde{V}^j}{\partial T_{j2}} \\
&= \beta \mu_{jj} \nu_{j1}^{j1} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}}.
\end{aligned}$$

And we have $\frac{\lambda_1^G}{\lambda_2^G} = \frac{1+o(\ell_{j1})}{\beta(1+o(\ell_{j2}))}$. Transfers to the rest of the United States are $T_{u2} \in o(\ell_{j2})$ as transfers are bounded. Then the transfers to Janesville satisfy

$$\begin{aligned}
& o(\ell_{j1}) + o(\ell_{j2}) + \\
& \ell_{j1} T_{j1} \mu_{jj} \nu_{j1}^{j1} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}} \frac{1+o(\ell_{j1})}{1+o(\ell_{j2})} + \ell_{j2} T_{j2} \nu_{j2}^{j2} = \ell_{j2} \left[\frac{1}{1+o(\ell_{j2})} - 1 + \frac{1}{1+o(\ell_{j2})} \frac{\alpha_j}{1-\alpha_j} \tau_{j2} \right] \\
&\qquad - \frac{W_j H_{Tj2}}{1-\alpha_j} \ell_{j2} \frac{\tau_{j2}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j2}} \nu_{j2}^{j2} \\
&\qquad - \frac{W_j H_{Tj1}}{1-\alpha_j} \ell_{j1} \frac{\tau_{j1}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}} \frac{\nu_{j2}^{j1}}{\beta} \\
\ell_{j2} T_{j2} \nu_{j2}^{j2} &= \ell_{j2} \left[\frac{1}{1+o(\ell_{j2})} - 1 + \frac{1}{1+o(\ell_{j2})} \frac{\alpha_j}{1-\alpha_j} \tau_{j2} \right] \\
&\qquad - \frac{W_j H_{Tj2}}{1-\alpha_j} \ell_{j2} \frac{\tau_{j2}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j2}} \nu_{j2}^{j2} \\
&\qquad - \frac{W_j H_{Tj1}}{1-\alpha_j} \ell_{j1} \frac{\tau_{j1}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j1}} \mu_{jj} \nu_{j1}^{j} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}} \\
&\qquad - \ell_{j1} \mu_{jj} \frac{\alpha_j}{1-\alpha_j} \tau_{j1} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}} \frac{1+o(\ell_{j1})}{1+o(\ell_{j2})} \\
&\qquad + \ell_{j1} \mu_{jj} \frac{W_j H_{Tj1}}{1-\alpha_j} \frac{\partial \log \ell^{j1}}{\partial T_{j1}} \tau_{j1} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}} \frac{1+o(\ell_{j1})}{1+o(\ell_{j2})} \\
&\qquad - o(\ell_{j1}) - o(\ell_{j2}) \\
T_{j2} \nu_{j2}^{j2} &= \frac{\alpha_j}{1-\alpha_j} \tau_{j2} - \frac{W_j H_{Tj2}}{1-\alpha_j} \frac{\tau_{j2}}{1+\frac{\alpha_j}{1-\alpha_j}\tau_{j2}} \nu_{j2}^{j2} \\
&\qquad - \frac{\mu_{jj} \ell_{j1}}{\ell_{j2}} \frac{\alpha_j}{1-\alpha_j} \tau_{j1} \frac{\frac{\partial \tilde{V}^j}{\partial T_{j2}}}{\frac{\partial \tilde{V}^j}{\partial T_{j1}}},
\end{aligned}$$

taking the limit as $\ell_{j1} \to 0$ and $\ell_{j2} \to 0$. Noting that $\tau_{j1} > 0$ completes the proof. $\square$

B.1 Monetary Policy

Proposition A1. *Define the natural monetary policy weights* $\omega_{k1} = \frac{\overline{\lambda}_{k1} U_C^k}{P_k} \frac{W_k H_{Tk1}}{1-\alpha_k} \ell_{k1}$*. Then in any interior solution to the planner's problem, with no redistributive reasons for policy,*

$$\sum_k \omega_{k1} \tau_{k1} \geqslant 0.$$

And the inequality is strict if migration elasticities are positive and the first best is not achievable.

Proof. From the first order conditions, monetary policy satisfies

$$\begin{aligned} 0 &= \sum_k \frac{W_k H_{Tk1}}{1-\alpha_k} \ell_{k1} \frac{\tau_{k1}}{1+\frac{\alpha_k}{1-\alpha_k}\tau_{k1}} \\ &= \sum_k \omega_{k1} \tau_{k1} \frac{1}{\frac{\overline{\lambda}_{k1} U_C^k}{P_k}\left(1+\frac{\alpha_k}{1-\alpha_k}\tau_{k1}\right)}. \end{aligned}$$

Then with no insurance reasons for policy, $\frac{\overline{\lambda}_{k1} U_C^k}{P_k} = 1$. Clearly, if the first best is achievable $\sum_k \omega_{k1}\tau_{k1} = 0$. If the first best is not achievable then for all $\tau_{k1} > 0$,

$$\omega_{k1}\tau_{k1} \frac{1}{1+\frac{\alpha_k}{1-\alpha_k}\tau_{k1}} < \omega_{k1}\tau_{k1}.$$

Similarly, if $\tau_{k1} < 0$,

$$|\omega_{k1}\tau_{k1}| \frac{1}{1+\frac{\alpha_k}{1-\alpha_k}\tau_{k1}} > |\omega_{k1}\tau_{k1}|$$

so that $\omega_{k1}\tau_{k1}\frac{1}{1+\frac{\alpha_k}{1-\alpha_k}\tau_{k1}} < \omega_{k1}\tau_{k1}$. Therefore,

$$0 = \sum_k \omega_{k1}\tau_{k1} \frac{1}{1+\frac{\alpha_k}{1-\alpha_k}\tau_{k1}} < \sum_k \omega_{k1}\tau_{k1}.$$

□

C Extensions of the Simple Model

Next I consider how the model results change under a variety of assumptions. In all of the extensions, I only focus on period 1, so I drop the t subscript for simplicity.

C.1 Downward wage rigidity and costly price adjustments

C.1.1 Environment

There are $\overline{\ell}$ total people, and two locations $n \in \{j, u\}$.

Households. The migration set-up is exactly the same as in the main text. But now, each household is endowed with 1 unit of labor supply that the household supplies in elastically with no utility cost. Therefore $H_n \leqslant 1$. Utility of a household living in location n is

$$\begin{aligned} U_n &= U^n(C_n) \\ C_n &= (C_{NTn})^{\alpha}(C_{Tn})^{1-\alpha} \end{aligned}$$

where α is the weight put on the non-traded sector. The households then choose non-traded consumption and traded consumption to maximize utility subject to the budget constraint taking prices as given,

$$p_n C_{NTn} + P_T C_{Tn} \leqslant W_n H_n + T_n + \Pi,$$

where p_n is the price of the local good, P_T is the price of the traded good, and Π are the profits.

Production. In each location, there is a continuum of firms that choose prices to maximize profits. A representative firm competitively produces a final good with the varieties produced by the firms, with a CES aggregator with elasticity of substitution ϵ.

The firms compete monopolistically. Changing the price requires a Rotemberg real cost in the freely traded good. That is, the firm that produces variety ω solves the problem

$$\max_{p_n(\omega), y_n(\omega), H_n(\omega)} \tau p_n(\omega) y_n(\omega) - W_n H_n(\omega) \ell_n - \psi \left(\frac{p_n(\omega) - p_{n0}}{p_{n0}} \right)^2 P_{Tt} Y_{nt}$$

where Y_n is total production of the region, and p_{n0} is the previous price, subject to the the technology constraint

$$Y_n(\omega) = A_n H_n(\omega) \ell_n$$

and demand

$$p_n(\omega) y_n(\omega) = \left(\frac{p_n(\omega)}{p_n} \right)^{1-\epsilon} p_n Y_n.$$

I further include τ which is a subsidy on prices to undo the monopoly distortion. I assume that this is paid for by the local workers. Assuming that all the firms are symmetric, we get that prices solve

$$\frac{p_n - p_{n0}}{p_{n0}} \frac{P_T}{p_{n0}} = \frac{\epsilon}{\psi} \left(\frac{W_n}{p_n A_n} \right).$$

Then the profit losses of the firm are

$$\Pi = -\sum_n \psi \left(\frac{p_n - p_{n0}}{p_{n0}} \right) P_T Y_n$$

Wages are downwardly rigid so

$$W_n \geqslant W_{n0}, H_n \leqslant 1,$$

where W_{n0} is the previously set wage.

A single firm aggregates up the goods produced by each location

$$Y_T = \left[\sum_n \phi_n^{\frac{1}{\sigma}} (Y_{Tn})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

This firm is competitive so therefore

$$P_T = \left[\sum_n p_n^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

Market clearing. Market clearing implies that

$$Y_n = A_n H_n \ell_n.$$

Demand for the good produced in the location is

$$Y_n = C_{NTn} \ell_n + Y_{Tn}.$$

Finally, demand for the traded good comes from consumption and the goods required for Rotemberg price adjustment

$$Y_T = \sum_n C_{Tn} \ell_n + \sum_n \psi \left(\frac{p_n - p_{n0}}{p_{n0}} \right) Y_n.$$

C.1.2 Adjusted Proposition

Proposition A2. *Suppose that location j is arbitrarily small compared to location u, location j is in a recession, there are no redistributive reasons for policy $\frac{\overline{\lambda}_n U_C^n}{P_n} = 1$, and monetary policy is such that there is no inflation in u. Then in any interior equilibrium, the optimal period 1 transfer to location j must satisfy*

$$T_j = \frac{1}{\frac{d \log \ell_j}{d T_j}} \frac{\frac{\alpha}{1-\alpha} - \frac{\phi_j E_T}{1-\alpha} \frac{1}{\ell_j} \frac{\partial \log \ell^j}{\partial T_j}}{1 + \frac{\phi_j E_T}{1-\alpha} \frac{1}{\ell_j} \frac{\partial \log \ell^j}{\partial T_j}}.$$

Proof. The monetary policy ensures that there is full employment in u. Therefore, wages in j are downwardly rigid, no prices change, and $\Pi = 0$. Defining $\tilde{\phi}_j = \phi_j \left(\frac{p_j}{P_T} \right)^{1-\sigma}$, hours worked in j is given by

$$W_j H_j = \frac{\alpha}{1-\alpha} T_j + \frac{\tilde{\phi}_j E_T}{1-\alpha} \frac{1}{\ell_j}.$$

The utility in location j is

$$U_j = U^j \left(\frac{W_j H_j + T_j}{P_j} \right)$$

while utility in location u is

$$U_u = U^u \left(\frac{W_u + T_u}{P_u} \right).$$

Furthermore, population remains the same so that

$$\ell_j = \ell^j \left(U_j - U_u \right).$$

Then I will consider a change in taxes

$$\ell_j T_j d \log \ell_j + \ell_j dT_j + \ell_u T_u d \log \ell_u + \ell_u dT_u = 0.$$

Therefore,

$$dT_u = -\frac{1}{\ell_u} \left(\ell_j T_j d \log \ell_j + \ell_j dT_j + \ell_u T_u d \log \ell_u \right).$$

Therefore, the total change in welfare is

$$\begin{aligned}
d\mathcal{W} &= \overline{\lambda}_j \ell_j dU_j + \overline{\lambda}_u \ell_u dU_u \\
&= \overline{\lambda}_j \ell_j U_C^j \left(\frac{W_j}{P_j} dH_j + \frac{1}{P_j} dT_j \right) + \overline{\lambda}_u \ell_u U_C^u \frac{1}{P_u} dT_u \\
&= \overline{\lambda}_j \ell_j \frac{U_C^j}{P_j} W_j dH_j + \overline{\lambda}_j \ell_j \frac{U_C^j}{P_j} dT_j - \overline{\lambda}_u \frac{U_C^u}{P_u} \left(\ell_j T_j d \log \ell_j + \ell_j dT_j + \ell_u T_u d \log \ell_u \right) \\
&= \ell_j W_j dH_j - T_j \ell_j d \log \ell_j - T_u \ell_u d \log \ell_u,
\end{aligned}$$

where we use the fact that there is no insurance reason for policy,

$$\overline{\lambda}_j \frac{U_C^j}{P_j} = \overline{\lambda}_u \frac{U_C^u}{P_u} = 1.$$

Then population changes according to

$$\begin{aligned}
d \log \ell_j &= \frac{\partial \log \ell^j}{\partial U} \left(dU_j - dU_u \right) \\
&= \frac{\partial \log \ell^j}{\partial U} \left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j - \frac{U_C^u}{P_u} dT_u \right)
\end{aligned}$$

Meanwhile, hours change according to

$$W_j dH_j = \frac{\alpha}{1-\alpha} dT_j - \frac{\phi_j E_T}{1-\alpha} \frac{1}{\ell_j} d \log \ell_j.$$

Next, I note that $d\ell_j = -d\ell_u$ so that $\frac{\ell_j}{\ell_u} d \log \ell_j = d \log \ell_u$. Therefore, the change in total welfare, normalized by the population in Janesville, is given by

$$\frac{d\mathcal{W}}{\ell_j} = W_j dH_j - T_j d \log \ell_j - T_u d \log \ell_j.$$

Then taking the limit as $\ell_j \to 0$ holding fixed the $\frac{\partial \log \ell^j}{\partial U}$,

$$\frac{d\mathcal{W}}{\ell_j} = W_j dH_j - T_j d\log \ell_j,$$

since $T_u \to 0$. In the limit,

$$\begin{aligned}
d\log \ell_j &= \frac{\partial \log \ell^j}{\partial U}\left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j - \frac{U_C^u}{P_u} dT_u\right) \\
&= \frac{\partial \log \ell^j}{\partial U}\left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j + \frac{U_C^u}{P_u}\left(\ell_j T_j d\log \ell_j + \ell_j dT_j + \ell_u T_u d\log ell_u\right)\right) \\
&= \frac{\partial \log \ell^j}{\partial U}\left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j\right).
\end{aligned}$$

I then turn to solving for the change in hours. This is

$$\begin{aligned}
W_j dH_j &= \frac{\alpha}{1-\alpha} dT_j - \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j} d\log \ell_j \\
&= \frac{\alpha}{1-\alpha} dT_j - \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j\right) \\
W_j\left(1 + \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\frac{U_C^j}{P_j}\right) dH_j &= \left(\frac{\alpha}{1-\alpha} - \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\frac{U_C^j}{P_j}\right) dT_j
\end{aligned}$$

Then plugging into the welfare equation,

$$\frac{d\mathcal{W}}{\ell_j} = \frac{\frac{\alpha}{1-\alpha} - \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\frac{U_C^j}{P_j}}{1 + \frac{\phi_j E_T}{1-\alpha}\frac{1}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\frac{U_C^j}{P_j}} - T_j d\log \ell_j.$$

In the optimum, this must be zero. Rearranging gives the expression. □

C.2 Place-biased policy

C.2.1 Environment

The environment is the same as in the text, but now there are types $\theta \in \Theta$ and I assume that there are only two location j and u. These types can represent a wide variety of categories. If a policy can target households based on where they started, then θ can denote starting location. It also could denote people over 65.

I will use a superscript θ for functions and subscript θ for values just as in the main text. I assume that utility is given by

$$U^{\theta n}(C, H) = C - v(H).$$

This ensures that even if agents of different types earn different amounts in the same location, they still supply the same number of hours worked when hours are efficiently rationed. Also, each type θ might have a different distribution of idiosyncratic utilities $G_{\theta n}(\cdot)$.

I also suppose that there is some policy parametrized by a parameter κ. Agents of type θ get a transfer of $T_{\theta n}\kappa$ for living in location n when there is κ amount of the policy. This captures the place-biased nature of the policy. It can be place-biased for two reasons. One is that within a type θ, it is biased toward a single location. Alternatively, the policy is biased toward a type θ that is disproportionately in one particular location. Then earnings of type θ in location n is

$$E_{\theta n} = W_n H_n + T_{\theta n}\kappa + T,$$

where T is the lump sum transfer from the government. The government budget constraint is

$$\sum_\theta \sum_n T_{\theta n}\ell_{\theta n}\kappa + \bar{\ell}T = 0.$$

C.2.2 Adjusted Proposition

Proposition A3. *Suppose that location j is arbitrarily small compared to location u, location j is in a recession, there are no redistributive reasons for policy $\frac{\bar{\lambda}_{nt}U_C^n}{P_{nt}} = 1$, and monetary policy is such that there is no labor wedge in u. Then starting from an equilibrium with no transfers, using a place-biased policy paid for with a lump-sum transfer increases welfare if and only if*

$$\frac{\alpha_j}{1-\alpha_j}\left(\sum_\theta \frac{T_{\theta j}\ell_{\theta j}}{\ell_j} - \frac{\sum_\theta \sum_n \ell_{\theta n}T_{\theta n}}{\bar{\ell}}\right) > \frac{\phi_j E_T/\ell_j}{1-\alpha_j}\frac{d\log \ell_j}{d\kappa}.$$

Proof. I will start by characterizing the equilibrium. Spending on the traded output in j is given by

$$P_{Tj}Y_{Tj} = \phi_j E_T.$$

Spending on the non-traded goods is

$$P_{NTj}Y_{NTj} = \alpha_n \left[\sum_\theta \left(W_j H_j + T_{\theta j}\kappa + T\right)\ell_{\theta j}\right].$$

Summing together and solving for hours worked I get

$$H_j = \frac{1}{W_j}\left[\frac{\phi_j E_T/\ell_j}{1-\alpha_j} + \frac{\alpha_j}{1-\alpha_j}\left(\sum_\theta \frac{T_{\theta j}\ell_{\theta j}\kappa}{\ell_n} + T\right)\right],$$

where $\ell_n = \sum_\theta \ell_{\theta n}$. Therefore, the utility of agents of type θ for living in Janesville is

$$V^{\theta j}(\kappa, T, H) = U^{\theta j}\left(\frac{W_j H + T_{\theta j}\kappa + T}{P_j}, H\right).$$

With that, I can then turn to find what happens with a small increase in the place-biased policy to a region, and the lump sum transfer changes to maintain budget balance. Budget balance implies

$$\sum_\theta T_{\theta j}\ell_{\theta j}d\kappa + \sum_\theta T_{\theta u}\ell_{\theta u}d\kappa + \bar{\ell}dT = 0.$$

Then the change in total welfare is

$$\begin{aligned}
d\mathcal{W} &= \sum_\theta \bar{\lambda}_{\theta j}\ell_{\theta j}dU_{\theta j} + \sum_\theta \bar{\lambda}_{\theta u}\ell_{\theta u}dU_{\theta u} \\
&= \sum_\theta \bar{\lambda}_{\theta j}\ell_{\theta j}\left(U_C^{\theta j}\frac{T_{\theta j}}{P_j}d\kappa + \frac{U_C^{\theta j}}{P_j}dT + W_j\frac{U_C^{\theta j}}{P_j}\tau_j dH_j\right) \\
&\quad + \sum_\theta \bar{\lambda}_{\theta u}\ell_{\theta u}\left(U_C^{\theta u}\frac{T_{\theta u}}{P_u}d\kappa + \frac{U_C^{\theta u}}{P_u}dT\right) \\
&= \ell_j W_j \tau_j dH_j.
\end{aligned}$$

Taking the derivative of the hours function, I find that

$$\begin{aligned}
W_j dH_j = &-\frac{\phi_j E_T/\ell_j}{1-\alpha_j}\left(\sum_\theta \frac{\ell_{\theta j}}{\ell_j}d\log\ell_{\theta j}\right) + \frac{\alpha_j}{1-\alpha_j}\left(\sum_\theta \frac{T_{\theta j}\ell_{\theta t}}{\ell_j}\right)d\kappa \\
&+ \frac{\alpha_j}{1-\alpha_j}dT.
\end{aligned}$$

Then plugging in for transfers and noting that $d\log\ell_j = \sum_\theta \frac{\ell_{\theta j}}{\ell_j}d\log\ell_{\theta j}$ we get that utility increases from an increase in the policy if and only if

$$\frac{\alpha_j}{1-\alpha_j}\left(\sum_\theta \frac{T_{\theta j}\ell_{\theta j}}{\ell_j} - \frac{\sum_\theta\sum_n \ell_{\theta n}T_{\theta n}}{\bar{\ell}}\right)d\kappa > \frac{\phi_j E_T/\ell_j}{1-\alpha_j}d\log\ell_j.$$

$\square$

Importantly, the stimulus effect depends on the observed place-bias. The migration effect depends the place-bias within a single type. To see that, note that

$$\begin{aligned}
d\log\ell_j &= \sum_\theta \frac{\ell_{\theta j}}{\ell_j}d\log\ell_{\theta j} \\
&= \sum_\theta \frac{\ell_{\theta j}}{\ell_j}\frac{\partial\log\ell^{\theta j}}{\partial U_j}\left(\frac{U_C^{\theta j}}{P_j}\left[T_{\theta j} - \frac{\sum_\theta\sum_n \ell_{\theta n}T_{\theta n}}{\bar{\ell}}\right]d\kappa\right. \\
&\qquad - \frac{U^{\theta u}}{P_u}\left[T_{\theta u} - \frac{\sum_\theta\sum_n \ell_{\theta n}T_{\theta n}}{\bar{\ell}}\right]d\kappa \\
&\qquad \left. + W_j\frac{U_C^j}{P_j}\tau_j dH_j\right).
\end{aligned}$$

C.3 Households affect demand

C.3.1 Environment

Next I suppose that households affect demand for a particular region. In particular, assume that there are types $\theta \in \Theta$ and that the share of spending on the traded good is a function of the people living in the location. That is,

$$\phi_n = \phi^n\left(\{\ell_{\theta n}\}\right).$$

I will also continue focusing on the two location version of the model. These types have the same fundamental utility across all locations but might have different distributions of idiosyncratic shocks.

C.3.2 Adjusted Proposition

Proposition A4. *Suppose that location j is arbitrarily small compared to location u, location j is in a recession, there are no redistributive reasons for policy $\frac{\overline{\lambda}_n U_C^n}{P_n} = 1$, and monetary policy is such that there is no inflation in u. Then in any interior equilibrium, the optimal period 1 transfer to location j must satisfy*

$$T_j = \frac{1}{\frac{d\log\ell_j}{dT_j}} \frac{\frac{\alpha_j}{1-\alpha_j} - \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[\frac{\ell_{\theta j}}{\ell_j} - \frac{\partial\log\phi^j}{\partial\log\ell_{\theta j}}\right]\frac{\partial\log\ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}}{1+\Omega}\tau_j,$$

where

$$\Omega \equiv \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[-\frac{\ell_{\theta j}}{\ell_j} + \frac{\partial\log\phi^j}{\partial\log\ell_{\theta j}}\right]\frac{\partial\log\ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}.$$

Proof. In this slightly adjusted setting, hours worked is now given by

$$H^n(E_T,\{\ell_\theta\},T) = \frac{1}{W_n}\left(\frac{\phi_n(\{\ell_\theta\})E_T}{1-\alpha_n}\frac{1}{\sum_\theta\ell_\theta} + \frac{\alpha_n}{1-\alpha_n}T\right).$$

Then I will consider a change in the transfer to j paid for with a small tax on the rest on u. Budget balance implies that

$$\ell_j dT_j + T_j\ell_j d\log\ell_j + \ell_u dT_u + T_u\ell_u d\log\ell_u = 0.$$

The change in total welfare is

$$\begin{aligned}
d\mathcal{W} &= \overline{\lambda}_j\ell_j dU_j + \overline{\lambda}_u\ell_u dU_u \\
&= \overline{\lambda}_j\ell_j U_C^j\left(\frac{W_j}{P_j}\tau_j dH_j + \frac{1}{P_j}dT_j\right) + \overline{\lambda}_u\ell_u U_C^u\frac{1}{P_u}dT_u \\
&= \ell_j W_j\tau_j dH_j - T_j\ell_j d\log\ell_j - T_u\ell_u d\log\ell_u,
\end{aligned}$$

where we use the fact that there is no insurance reason for policy. Then population of each type changes according to

$$\begin{aligned} d\log\ell_{\theta j} &= \frac{\partial \log \ell^{\theta j}}{\partial U}\left(dU_j - dU_u\right) \\ &= \frac{\partial \log \ell^{\theta j}}{\partial U}\left(U_C^j \frac{W_j}{P_j} dH_j + \frac{U_C^j}{P_j} dT_j - \frac{U_C^u}{P_u} dT_u\right). \end{aligned}$$

Meanwhile, hours change according to

$$W_j dH_j = \frac{\alpha_j}{1-\alpha_j} dT_j - \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j} d\log\ell_j + \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}} d\log\ell_{\theta j}.$$

I will then take the limit as $\ell_{\theta j} \to 0$ holding fixed the migration semi-elasticities to j. Then the change in welfare, normalized by the population is

$$\frac{d\mathcal{W}}{\ell_j} = W_j \tau_j dH_j - T_1 d\log\ell_j$$

since $T_u \to 0$. In the limit,

$$d\log\ell_{\theta j} = \frac{\partial \log \ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}\left(W_j dH_j + dT_j\right).$$

I then turn to solving for the change in hours. This is

$$\begin{aligned} W_j dH_j &= \frac{\alpha_j}{1-\alpha_j} dT_j - \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j} d\log\ell_j + \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}} d\log\ell_{\theta j} \\ &= \frac{\alpha_j}{1-\alpha_j} dT_j + \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[-\frac{\ell_{\theta j}}{\ell_j} + \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right] d\log\ell_{\theta j} \\ &= \frac{\alpha_j}{1-\alpha_j} dT_j + \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[-\frac{\ell_{\theta j}}{\ell_j} + \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right]\frac{\partial \log \ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}\left(W_j dH_j + dT_j\right) \end{aligned}$$

Which we can rearrange

$$W_j\left(1+\Omega\right) = \left[\frac{\alpha_j}{1-\alpha_j} + \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[-\frac{\ell_{\theta j}}{\ell_j} + \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right]\frac{\partial \log \ell^{j\theta}}{\partial U}\frac{U_C^j}{P_j}\right] dT_j$$

where

$$\Omega \equiv \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[-\frac{\ell_{\theta j}}{\ell_j} + \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right]\frac{\partial \log \ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}.$$

Therefore, plugging this into the expression for welfare change, the optimal transfer needs to satisfy

$$T_j = \frac{1}{\frac{d\log\ell_j}{dT_j}}\frac{\frac{\alpha_j}{1-\alpha_j} + \sum_\theta \frac{\phi_j E_T}{1-\alpha_j}\frac{1}{\ell_j}\left[-\frac{\ell_{\theta j}}{\ell_j} + \frac{\partial \log \phi^j}{\partial \log \ell_{\theta j}}\right]\frac{\partial \log \ell^{\theta j}}{\partial U}\frac{U_C^j}{P_j}}{1+\Omega}\tau_j.$$

$\square$

C.4 Wage stickiness only in traded goods

C.4.1 Environment

The final extension I consider is when wages are only sticky in the traded sector. Suppose that supplying labor to the non-traded sector and the traded are not perfect substitutes so that the fundamental utility of living in location n is

$$U_n = U^n(C_n, H_{Tn}, H_{NTn}).$$

I then assume that wages in the traded sector are completely sticky, just as before, and that wages in the non-traded sector are free to adjust. I will further assume Cobb-Douglas preferences so that

$$C_n = (C_{Tn})^{1-\alpha}(C_{NTn})^{\alpha}.$$

C.4.2 Adjusted Proposition

I will take this in two steps. First, I will define an indirect utility function

$$v^n\left(T_n, H_{Tn}, W_{NTn}\right) = U^n\left(\frac{W_{Tn}H_{Tn} + \frac{\alpha}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right) + T_n}{P_n\left(W_{NTn}\right)}, H_{Tn}, \frac{\frac{\alpha}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right)}{W_{NTn}}\right)$$

where $P_n(W_{NTn})$ is the local price index as a function of the non-traded wages, and I have already substituted in for earnings in the non-traded sector, using the equation for non-traded demand,

$$W_{NT}H_{NTn}\ell_n = \alpha\left(W_{Tn}H_{Tn}\ell_n + W_{NTn}H_{NTn}\ell_n + T_n\ell_n\right).$$

I then define another indirect utility function

$$V^n(T, H_T) = \max_{W_{NT}} v^n\left(T_n, H_T, W_{NT}\right).$$

The derivatives are then

$$\begin{aligned}
\frac{\partial v^n}{\partial T_n} &= \frac{1}{1-\alpha}\frac{U_C^n}{P_n} + \frac{1}{W_{NTn}}U^n_{HNT}\frac{\alpha}{1-\alpha} \\
&= \frac{U_C^n}{P_n}\left(1 + \frac{\alpha}{1-\alpha}\tau_{NTn}\right) \\
\frac{\partial v^n}{\partial H_{Tn}} &= W_{Tn}\frac{U_C^n}{P_n}\tau_{Tn}.
\end{aligned}$$

The derivative with respect to the non-tradable wage is slightly more complicated

$$\begin{aligned}
\frac{\partial v^n}{\partial W_{NTn}} = &-\frac{U_C^n}{P_n}\frac{W_{TnH_{Tn}} + \frac{\alpha}{1-\alpha}(W_{Tn}H_{Tn} + T_n) + T_n}{P_n}\frac{\partial P_n}{\partial W_{NTn}} \\
&- U^n_{HNT}\frac{\alpha}{1-\alpha}\frac{W_{Tn}H_{Tn} + T_n}{W_{NTn}^2}
\end{aligned}$$

By the envelope theorem,

$$\frac{\partial \log P_n}{\partial \log W_{NTn}} = \alpha.$$

Therefore, we can write

$$\begin{aligned}
\frac{\partial v^n}{\partial W_{NTn}} &= -\frac{U_C^n}{P_n}\frac{W_{Tn}H_{Tn} + \frac{\alpha}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right) + T_n}{P_n}\frac{\partial P_n}{\partial W_{NTn}} \\
&\quad - U_{HNT}^n \frac{\frac{\alpha}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right)}{W_{NTn}^2} \\
&= -\frac{U_C^n}{P_n}\frac{\frac{1}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right)}{W_{NTn}}\frac{\partial \log P_n}{\partial \log W_{NTn}} \\
&\quad - U_{HNT}^n \frac{\frac{\alpha}{1-\alpha}\left(W_{Tn}H_{Tn} + T_n\right)}{W_{NTn}^2} \\
&= -\frac{\alpha}{1-\alpha}\frac{U_C^n}{P_n}\frac{W_{Tn}H_{Tn} + T_n}{W_{NTn}}\left[1 + \frac{P_n}{W_{NTnt}}\frac{U_{HNT}^n}{U_C^n}\right] \\
&= -\frac{\alpha}{1-\alpha}\frac{U_C^n}{P_n}\frac{W_{Tn}H_{Tn} + T_n}{W_{NTn}}\tau_{NTnt}.
\end{aligned}$$

That means that under V^n and the market allocation $\tau_{NTnt} = 0$. Meanwhile, by the envelope theorem,

$$\begin{aligned}
\frac{\partial V^n}{\partial T} &= \frac{U_C^n}{P_n} \\
\frac{\partial V^n}{\partial H_{Tn}} &= W_n \frac{U_C^n}{P_n}\tau_{Tn}.
\end{aligned}$$

Finally I note that traded hours are given by

$$H_{Tn} = \frac{\phi_n E_T}{W_{Tn}}\frac{1}{\ell_n}.$$

Then I can state the adjusted proposition.

Proposition A5. *Suppose that location j is arbitrarily small compared to location u, location j is in a recession, there are no redistributive reasons for policy $\frac{\bar{\lambda}_n U_C^n}{P_n} = 1$, and monetary policy is such that there is no inflation in u. Then in any interior equilibrium, the optimal period 1 transfer to location j must satisfy*

$$T_j = -\frac{1}{\frac{d\log \ell_j}{dT_j}}\frac{\phi_j E_T/\ell_j \frac{\partial \log \ell^j}{\partial T_j}}{1 + \frac{\phi_j E_T}{\ell_j}\frac{\partial \log \ell^j}{\partial U}\frac{U_C^j}{P_j}\tau_{Tj}}\tau_{Tj}.$$

Proof. Just as before, I will consider a change in taxes. By budget balance, we have

$$\ell_j T_j d\log \ell_j + \ell_j dT_j + \ell_u T_u d\log \ell_u + \ell_u dT_u = 0.$$

Therefore,

$$dT_u = -\frac{1}{\ell_u}\left(\ell_j T_j d\log\ell_j + \ell_j dT_j + \ell_u T_u d\log\ell_u\right).$$

The total change in welfare is then

$$\begin{aligned}
d\mathcal{W} &= \overline{\lambda}_j \ell_j dU_j + \overline{\lambda}_u \ell_u dU_u \\
&= \overline{\lambda}_j \ell_j U_C^j \left(\frac{W_{Tj}}{P_j}\tau_{Tj} dH_{Tj} + \frac{1}{P_j} dT_j\right) + \overline{\lambda}_u \ell_u U_C^u \frac{1}{P_u} dT_u \\
&= \ell_j W_{Tj}\tau_{Tj} dH_{Tj} - T_j \ell_j d\log\ell_j - T_u \ell_u d\log\ell_u,
\end{aligned}$$

where we use the fact that there is no insurance reason for policy. Then population changes according to

$$\begin{aligned}
d\log\ell_j &= \frac{\partial\log\ell^j}{\partial U}(dU_j - dU_u) \\
&= \frac{\partial\log\ell^j}{\partial U}\left(U_C^j \frac{W_{Tj}}{P_j} dH_{Tj} + \frac{U_C^j}{P_j} dT_j - \frac{U_C^u}{P_u} dT_u\right).
\end{aligned}$$

Meanwhile, traded hours change according to

$$W_{Tj} dH_{Tj} = -\frac{\phi_j E_T}{\ell_j} d\log\ell_j.$$

Next I note that $d\ell_j = -d\ell_u$ so that $\frac{\ell_j}{\ell_u} d\log\ell_j = d\log\ell_u$. Therefore, the change in total welfare, normalized by the population in Janesville is given by

$$\frac{d\mathcal{W}}{\ell_j} = W_{Tj}\tau_{Tj} dH_{Tj} - T_j d\log\ell_j - T_u d\log\ell_j.$$

Then taking the limit as $\ell_j \to 0$ holding fixed the $\frac{\partial\log\ell^j}{\partial U}$,

$$\frac{d\mathcal{W}}{\ell_j} = W_{Tj}\tau_{Tj} dH_{Tj} - T_j d\log\ell_j,$$

since $T_u \to 0$. In the limit,

$$d\log\ell_j = \frac{\partial\log\ell^j}{\partial U}\left(U_C^j \frac{W_j}{P_j}\tau_{Tj} dH_{Tj} + \frac{U_C^j}{P_j} dT_j\right).$$

I then turn to solving for the change in hours. This is

$$\begin{aligned} W_{Tj} dH_{Tj} &= -\frac{\phi_j E_T}{\ell_j} d\log \ell_j \\ &= -\frac{\phi_j E_T}{\ell_j} \frac{\partial \log \ell^j}{\partial U} \left(U_C^j \frac{W_j}{P_j} \tau_{Tj} dH_{Tj} + \frac{U_C^j}{P_j} dT_j \right) \\ W_{Tj} \left(1 + \frac{\phi_j E_T}{\ell_j} \frac{\partial \log \ell^j}{\partial U} \frac{U_C^j}{P_j} \tau_{Tj} \right) dH_{Tj} &= -\frac{\phi_j E_T}{\ell_j} \frac{\partial \log \ell^j}{\partial U} \frac{U_C^j}{P_j} dT_j. \end{aligned}$$

Putting it together, the optimal transfer needs to satisfy

$$T_j = -\frac{1}{\frac{d\log \ell_j}{dT_j}} \frac{\phi_j E_T / \ell_j \frac{\partial \log \ell^j}{\partial T_j}}{1 + \frac{\phi_j E_T}{\ell_j} \frac{\partial \log \ell^j}{\partial U} \frac{U_C^j}{P_j} \tau_{Tj}} \tau_{Tj}.$$

□

D Characterizing the Dynamic Model

In this section, I provide details on characterizing the consumer and union problems of the quantitative model presented in section 5.

D.1 Intratemporal Consumption Decision

Given expenditures $E_n(t)$, households in location n at time t choose consumption to maximize utility taking prices as given. In particular,

$$\begin{aligned} \{C_{NTn}(t), C_{Tn}(t), \{C_{Tmn}(t)\}\} \in \underset{C_{NT}, C_T\{C_{Tm}\}}{\operatorname{argmax}} \Bigg\{ (C_{NT})^{\alpha} (C_T)^{1-\alpha} \Bigg| \\ C_T = \left(\sum_m \phi_m^{\frac{1}{\sigma}} (C_{Tm})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \\ \sum_m p_{Tmn}(t) C_{Tm} + p_{NTn} C_{NT} \leqslant E_n(t) \Bigg\}. \end{aligned}$$

I further break this problem down into a traded consumption problem and then an aggregated consumption problem. Suppose that the household is spending E_T on trade goods. Then the household solves the problem

$$\max_{C_{tm}} \left(\sum_m \phi_m^{\frac{1}{\sigma}} (C_{Tm})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

such that

$$\sum_m p_{Tmn}(t) C_{Tm} \leqslant E_T.$$

Raising the maximand to $\frac{\sigma-1}{\sigma}$ and taking the first order condition with respect to C_{Tm} gives

$$\lambda p_{Tmn}(t) = \frac{\sigma-1}{\sigma} \phi_m^{\frac{1}{\sigma}} (C_{Tm})^{-\frac{1}{\sigma}}.$$

Defining $\mu = (\lambda \frac{\sigma}{\sigma-1})^{-\sigma}$,$C_{Tm} = \mu \phi_m p_{Tmn}(t)^{-\sigma}$. Then the budget constraint is

$$\begin{aligned}
E_T &= \sum_m p_{Tmn}(t) C_{Tm} \\
&= \sum_m \mu p_{Tmn}(t)^{1-\sigma} \phi_m \\
\mu &= \frac{E_T}{\sum_m \phi_m p_{Tmn}(t)^{1-\sigma}}
\end{aligned}$$

Therefore, traded consumption is

$$\begin{aligned}
C_T &= \left(\sum_m \phi_m^{\frac{1}{\sigma}} (C_{Tm})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\sum_m \phi_m^{\frac{1}{\sigma}} (\mu \phi_m p_{Tmn}(t)^{-\sigma})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\
&= \mu \left(\sum_m \phi_m p_{Tmn}(t)^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}} \\
&= E_T \left(\sum_m \phi_m p_{Tmn}(t)^{1-\sigma} \right)^{\frac{1}{\sigma-1}}.
\end{aligned}$$

Defining $p_{Tn}(t) \equiv (\sum_m \phi_m p_{Tmn}(t)^{1-\sigma})^{\frac{1}{1-\sigma}}$, $C_T = \frac{E_T}{p_{Tn}}$. Furthermore,

$$p_{Tmn}(t) C_{Tmn}(t) = \phi_m \left(\frac{p_{Tmn}(t)}{p_{Tn}} \right)^{1-\sigma} p_{Tn}(t) C_{Tn}(t). \tag{A1}$$

Then choosing between traded and non-traded, the household solves the problem

$$\max_{C_{NT}, C_T} (C_{NT})^{\alpha} (C_T)^{1-\alpha}$$

such that

$$p_{Tn} C_T + p_{NTn} C_{NT} \leqslant E_n(t).$$

Taking the first order conditions,

$$p_{Tn}(t)C_{Tn}(t) = (1-\alpha)E_n(t) \tag{A2}$$

and

$$p_{NTn}(t)C_{NTn}(t) = \alpha E_n(t). \tag{A3}$$

D.2 Labor Unions

In this subsection, I derive the key equations describing how unions operate in this model. I start by taking as given wages of each union and characterizing labor supply and production. I then turn to the maximization problem of the unions and derive the equations describing how wages move.

D.2.1 Labor Demand

The final, competitive producer looks to maximize profits taking as given wages of each of the unions. That is

$$Y_n(t), \{Y_n(\omega,t)\} \in \underset{Y,Y(\omega)}{\operatorname{argmax}} \left\{ p_n(t)Y - \int_0^1 W_n(\omega,t)Y(\omega)d\omega \right| \\ Y = A_n\left[\int_0^1 Y(\omega)^{\frac{\epsilon-1}{\epsilon}}d\omega\right]^{\frac{\epsilon}{\epsilon-1}} \left.\right\}.$$

The usual CES algebra, reviewed above in the consumer maximization problem, implies that,

$$p_n(t) = \frac{1}{A_n}\left[\int_0^1 W_n(\omega,t)^{1-\epsilon}d\omega\right]^{\frac{1}{1-\epsilon}}.$$

Furthermore, demand for the labor of union ω is

$$Y_n(\omega,t) = \frac{1}{A_n^{1-\epsilon}}\left(\frac{W_n(\omega,t)}{p_n(t)}\right)^{-\epsilon} Y_n(t).$$

Production is $Y_n(\omega,t) = H_n(\omega,t)\ell_n(t)$. Therefore, the total amount of labor demanded is given by

$$\begin{aligned}
H_n(t) &= \int_0^1 H_n(\omega,t)d\omega \\
&= \int_0^1 \frac{Y_n(\omega,t)}{\ell_n(t)}d\omega \\
&= \frac{1}{\ell_n(t)A_n(t)^{1-\epsilon}}\int_0^1 \left(\frac{W_n(\omega,t)}{p_n(t)}\right)^{-\epsilon} Y_n(t)d\omega \\
&= \frac{Y_n(t)}{\ell_n(t)A_n(t)^{1-\epsilon}}p_n(t)^{\epsilon}\int_0^1 W_n(\omega,t)^{-\epsilon}d\omega.
\end{aligned}$$

Then I solve for wage earnings. That is

$$\begin{aligned}
\int_0^1 W_n(\omega,t)H_n(\omega,t)d\omega &= \int_0^1 W_n(\omega,t)\frac{Y_n(\omega,t)}{\ell_n(t)}d\omega \\
&= \frac{1}{\ell_n(t)A_n(t)^{1-\epsilon}}\int_0^1 W_n(\omega,t)\left(\frac{W_n(\omega,t)}{p_n(t)}\right)^{-\epsilon} Y_n(t)d\omega \\
&= \frac{Y_n(t)p_n(t)^{\epsilon}}{\ell_n(t)A_n(t)^{1-\epsilon}}\int_0^1 W_n(\omega,t)^{1-\epsilon}d\omega \\
&= \frac{Y_n(t)p_n(t)^{\epsilon}}{\ell_n(t)A_n(t)^{1-\epsilon}}(A_n p_n(t))^{1-\epsilon} \\
&= \frac{p_n(t)Y_n(t)}{\ell_n(t)}.
\end{aligned}$$

Then defining

$$\begin{aligned}
W_n(t) &\equiv \frac{\int_0^1 W_n(\omega,t)H_n(\omega,t)d\omega}{H_n(t)} \\
&= \frac{p_n(t)Y_n(t)/\ell_n(t)}{\frac{Y_n(t)}{\ell_n(t)A_n(t)^{1-\epsilon}}p_n(t)^{\epsilon}\int_0^1 W_n(\omega,t)^{-\epsilon}d\omega} \\
&= \frac{A_n^{1-\epsilon}p_n(t)^{1-\epsilon}}{\int_0^1 W_n(\omega,t)^{-\epsilon}d\omega} \\
&= \frac{A_n p_n(t)}{\int_0^1 \left(\frac{W_n(\omega,t)}{A_n p_n(t)}\right)^{-\epsilon}d\omega},
\end{aligned}$$

we get an expression for wages as a function of $v_n^p(t) \equiv \int_0^1 \left(\frac{W_n(\omega,t)}{A_n p_n(t)}\right)^{-\epsilon} d\omega$. Then we can also write hours

$$H_n(t) = \frac{\int_0^1 W_n(\omega,t)H_n(\omega,t)d\omega}{W_n(t)} = v_n^p(t)\frac{Y_n(t)}{A_n(t)}. \tag{A4}$$

I can further solve for prices. $p_n(t) = \frac{1}{A_n}v_n^p(t)W_n(t)$.

D.2.2 The Union Problem

Next I characterize the labor union's problem. A union that gets an opportunity to choose wages at time t looks to maximize welfare of the workers there. The utility that the households get from more earnings at time s is $\frac{C_n(s)^{-\theta}}{P_n(s)}$. Meanwhile, the utility loss from working more is $-H_n(s)^{\eta}$.

Setting a wage of $\tilde{W}$ leads to hours demanded H at time s given by

$$\begin{aligned} H(\tilde{W}) &= \frac{Y_n(\tilde{W})}{\ell_n(t)} \\ &= \frac{1}{A_n^{1-\epsilon}} \left(\frac{\tilde{W}}{p_n(t)}\right)^{-\epsilon} Y_n(t) \frac{1}{\ell_n(t)} \\ &= \tilde{W}^{-\epsilon} A_n^{\epsilon-1} p_n(t)^{\epsilon} Y_n(t) \frac{1}{\ell_n(t)}. \end{aligned}$$

Therefore, the flow utility at time s is

$$\tau \frac{C_n(s)^{-\theta}}{P_n(s)} \tilde{W}^{1-\epsilon} A_n^{\epsilon-1} p_n(s)^{\epsilon} Y_n(s) - H_n(s)^{\eta} \tilde{W}^{-\epsilon} A_n^{\epsilon-1} p_n(s)^{\epsilon} Y_n(s),$$

when there is a subsidy of κ on wage earnings. The union then chooses $\tilde{W}$ to maximize

$$\tilde{W}_n(t) = \underset{\tilde{W}}{\operatorname{argmax}} \int_t^{\infty} e^{-(\rho+\delta_w)(s-t)} \left[\kappa \frac{C_n(s)^{-\theta}}{P_n(s)} \tilde{W}^{1-\epsilon} - H_n(s)^{\eta} \tilde{W}^{-\epsilon}\right] A_n^{\epsilon-1} p_n(s)^{\epsilon} Y_n(s) ds.$$

To undo the monopoly distortion, $\kappa = \frac{\epsilon}{\epsilon-1}$. Taking the first order condition with respect to $\tilde{W}$ and rearranging I get

$$\tilde{W}_n(t) = \frac{\int_t^{\infty} e^{-(\rho+\delta_w)(s-t)} N_n(s)^{\eta} A_n(s)^{\epsilon-1} p_n(s)^{\epsilon} Y_n(s) ds}{\int_t^{\infty} e^{-(\rho+\delta_w)(s-t)} \frac{C_n(s)^{-\theta}}{P_n(s)} A_n(s)^{\epsilon-1} P_n(s0 Y_n(s) ds}.$$

I then define a variable $X_{1n}(t)$ as the numerator and $X_{2n}(t)$ as $W_n(t)$ times the denominator. Then these variables change according to

$$\dot{X}_{1n}(t) = -H_n(t)^{\eta} A_n(t)^{\epsilon-1} P_n(t)^{\epsilon} Y_n(t) + (\rho + \delta_w) X_{1n}(t), \tag{A5}$$

and

$$\dot{X}_{2n}(t) = -W_n(t) \frac{C_n(t)^{-\theta}}{P_n(t)} A_n(t)^{\epsilon-1} P_n(t)^{\epsilon} Y_n(t) + (\rho + \delta_w + \pi_n^w(t)) X_{2n}(t), \tag{A6}$$

where $\pi_n^w(t) \equiv \frac{\dot{W}_n(t)}{W_n(t)}$. Then to describe how wages change, note that

$$W_n(t)^{1-\epsilon} = \int_{-\infty}^{t} \delta_w e^{-\delta_w(t-\tau)} \tilde{W}_n(\tau)^{1-\epsilon} d\tau.$$

Taking the derivative with respect to time I find that

$$\pi_n^w(t) = \frac{\delta_w}{1-\epsilon} \left[\left(\frac{\tilde{W}_n(t)}{W_n(t)}\right)^{1-\epsilon} - 1\right],$$

or

$$\pi_n^w(t) = \frac{\delta_w}{1-\epsilon}\left[\left(\frac{X_{1n}(t)}{X_{2n}(t)}\right)^{1-\epsilon} - 1\right]. \tag{A7}$$

Next we need to describe how the misallocation term changes. rewriting in terms of when wages were set,

$$v_n^p(t) = \int_{-\infty}^{\tau} \delta_w e^{-\delta(t-\tau)} \left(\frac{\tilde{W}_n(\tau)}{W_n(t)}\right)^{-\epsilon} d\tau.$$

Taking the derivative with respect to time I find that

$$\dot{v}_n^p(t) = \delta_w \left(\frac{X_{1n}(t)}{X_{2n}(t)}\right)^{-\epsilon} + (\epsilon\pi_n^w(t) - \delta_w)v_n^p(t). \tag{A8}$$

E Linear-Quadratic Approximation

E.1 Summarizing Equations

Summarizing the equations describing equilibrium, I have the following. For total welfare,

$$\mathcal{W} = \sum_{\gamma}\int_0^{\infty} e^{-\rho t}\left[\sum_n \lambda_n(t)U_n(t)\ell_n(t) - \delta_\ell \sum_n\sum_m \tilde{\lambda}_{nm}(t)\ell_n(t)\exp\left(\nu(v_m(t) - \tau_{\ell nm} - V_n(t))\right)(v_m(t) - V_n(t))\right]dt,$$

where $\lambda_n(t)$ is the planner's average weight on households living in location n, and $\tilde{\lambda}_{nm}(t)$ is the planner's average weight on households moving from location n to location m at time t. The constraint can be written as follows:

$$\dot{v}_n(t) = -U_n(t) - \delta_\ell V_n(t) + (\rho + \delta_\ell)v_n(t)$$

$$\exp\left(\nu V_n(t)\right) = \sum_m \exp\left(\nu\left(v_m(t) - \tau_{\ell nm}\right)\right)$$

$$\dot{\ell}_n(t) = \delta_\ell\left(\sum_m \exp\left(\nu(v_n(t) - \tau_{\ell mn} - V_m(t))\right)\ell_m(t) - \ell_n(t)\right)$$

$$U_n(t) = \log E_n(t) - \log P_n(t) - \frac{H_n(t)^{1+\eta}}{1+\eta}$$

$$P_{Tn}(t) = \left(\sum_{m\in\mathcal{N}} \phi_m\left(\tau_{mn}p_m(t)\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

$$P_n(t) = p_n(t)^{\alpha}P_{Tn}(t)^{1-\alpha}$$

$$E_n(t) = W_n(t)H_n(t) + T_n(t)$$

$$\frac{\dot{W}_n(t)}{W_n(t)} = \pi_n(t)$$

$$y_n(t) = \frac{A_n}{v_n^p(t)} H_n(t)\ell_n(t)$$

$$p_n(t)y_n(t) = \alpha E_n(t)\ell_n(t) + p_n(t)y_{Tn}(t)$$

$$p_m(t)y_{Tm}(t) = \sum_n \phi_m \left(\frac{\tau_{mn}p_m(t)}{P_{Tn}(t)}\right)^{1-\sigma} (1-\alpha)E_n(t)\ell_n(t)$$

$$\dot{X}_{1n}(t) = -H_n(t)^\eta A_n^{\epsilon-1} p_n(t)^\epsilon Y_n(t) + (\rho + \delta_w)X_{1n}(t)$$

$$\dot{X}_{2n}(t) = -W_n(t)\frac{C_n(s)^{-\theta}}{P_n(s)} A_n^{\epsilon-1} p_n(t)^\epsilon Y_n(t) + (\pi_n^w(t) + \rho + \delta_w)X_{2n}(t)$$

$$\pi_n(t) = \frac{\delta_w}{1-\epsilon}\left[\left(\frac{X_{1n}(t)}{X_{2n}(t)}\right)^{1-\epsilon} - 1\right]$$

$$\dot{v}_n^p(t) = \delta_w \left(\frac{X_{1n}(t)}{X_{2n}(t)}\right)^{-\varepsilon} + \left(\epsilon\pi_n^w(t) - \delta_w\right) v_n^p(t)$$

$$\sum_n \ell_n(t)T_n(t) = 0.$$

E.2 Loss Function

I do a second order approximation to the welfare function. I then do a second order approximation to the constraints and make substitutions until the welfare function is only second order. Derivations are available upon request.

In doing this, I need to make two assumptions so that the observed equilibrium is efficient. First, I assume that $\theta = 1$ and $\overline{\lambda}_n = E_n$. This guarantees that the planner has no redistributive reasons to transfer money across locations in the steady state. The other assumption I make is related. I need that there is a ψ_1 such that

$$\psi_1 = \overline{\lambda}_n U_n - \delta_\ell \sum_m \tilde{\lambda}_{nm}\pi_{nm}(v_m - V_n).$$

I further assume that

$$\tilde{\lambda}_{nm}[1 + \nu(v_m - V_n)] = \psi_{n2},$$

for some ψ_{n2}. I set

$$\psi_1 = \sum_n \frac{\ell_n}{\overline{\ell}}\lambda_n\overline{U}_n + \frac{\delta_\ell}{\nu}\log N\left(\sum_n \frac{\ell_n}{\ell}\lambda_n\right)$$

and back out ψ_{n2} from the equation,

$$\psi_{n2} = \frac{\overline{\lambda}_n\overline{U}_n - \psi_1}{\delta_\ell\left[\sum_m \overline{\pi}_{nm}\frac{\overline{v}_m - \overline{V}_n}{1+\nu\left(\overline{v}_m - \overline{V}_n\right)}\right]}.$$

Then the loss function is,

$$
\begin{aligned}
\tilde{\mathcal{W}}(t) = &-\frac{1}{2}\sum_n\sum_m \nu\delta_\ell\tilde{\lambda}_{nm}\overline{\ell}_n\mu_{nm}\left(\hat{v}_m(t)-\hat{V}_n(t)\right)^2 \\
&+\sum_n \overline{\lambda}_n\overline{\ell}_n\hat{U}_n(t)\hat{\ell}_n(t) \\
&-\frac{1+\eta}{2}\sum_n \overline{\lambda}_n\overline{\ell}_n\hat{H}_n(t)^2 \\
&+\frac{1}{2}\sum_n \overline{\lambda}_n\overline{\ell}_n\left(\hat{\ell}_n(t)+\hat{H}_n(t)\right)^2-\frac{\alpha}{2}\sum_n \overline{\lambda}_n\overline{\ell}_n\left(\hat{E}_n(t)+\hat{\ell}_n(t)-\hat{w}_n(t)\right)^2-\frac{1-\alpha}{2}\sum_n \overline{\lambda}_n\overline{\ell}_n\hat{y}_{Tn}(t)^2 \\
&-\frac{1-\alpha}{2}\sum_m \overline{\lambda}_m\overline{\ell}_m\Bigg[\sum_n s_{mn}\left(\hat{\phi}_m+\hat{E}_n(t)+\hat{\ell}_n(t)+(1-\sigma)\hat{w}_m(t)-(1-\sigma)\hat{P}_{Tn}(t)\right)^2 \\
&\qquad -\left(\hat{w}_m(t)+\hat{y}_{Tm}(t)\right)^2\Bigg] \\
&-\sigma\frac{1-\alpha}{2}\sum_n \overline{\lambda}_n\overline{\ell}_n\left[\sum_m \phi_{mn}\frac{1}{1-\sigma}\left(\hat{\phi}_m+(1-\sigma)\hat{w}_m(t)\right)^2-(1-\sigma)\hat{P}_{Tn}(t)^2\right] \\
&-\frac{1}{2}\frac{\epsilon}{\delta_w(\rho+\delta_w)}\sum_n \overline{\lambda}_n\overline{\ell}_n\hat{\pi}_n(t)^2 \\
&+\text{other terms independent of policy,}
\end{aligned}
$$

where

$$s_{mn} \equiv \frac{p_{Tmn}C_{Tmn}\ell_n}{p_m y_{Tm}},$$

is the share of spending on tradable goods from location m that people in location n account for and

$$\phi_{mn} \equiv \frac{p_{Tmn}C_{Tmn}}{\sum_k p_{Tkn}C_{Tkn}},$$

is the share of tradable spending on location m for a household in n.

E.3 Linearized Constraints

The constraints can then be linearized. Derivations are available upon request.

$$\hat{U}_n(t) = \hat{E}_n(t) - \hat{P}_n(t) - \hat{H}_n(t)$$

$$\hat{P}_n(t) = \alpha\hat{w}_n(t) + (1-\alpha)\hat{P}_{Tn}(t)$$

$$\hat{P}_{Tn}(t) = \sum_m \phi_{mn}\left[\frac{1}{1-\sigma}\hat{\phi}_m + \hat{w}_m(t)\right]$$

$$\hat{V}_m(t) = \sum_n \frac{\ell_{mn}}{\ell_m}\hat{v}_n(t)$$

$$\dot{\hat{w}}_n = \hat{\pi}_n(t)$$
$$\hat{E}_n(t) = \hat{w}_n(t) + \hat{H}_n(t) + \hat{T}_n(t)$$

$$\sigma\hat{w}_m(t) + \hat{y}_{Tm}(t) = \hat{\phi}_m + \sum_n s_{mn}\left[(\sigma-1)\hat{P}_{Tn}(t) + \hat{E}_n(t) + \hat{\ell}_n(t)\right]$$
$$\hat{H}_n(t) + \hat{\ell}_n(t) = \alpha\left(\hat{E}_n(t) + \hat{\ell}_n(t) - \hat{w}_n(t)\right) + (1-\alpha)\hat{y}_{Tn}(t)$$
$$\dot{\hat{\pi}}_n(t) = \delta_w(\rho+\delta_w)\left[\hat{w}_n(t) - \hat{E}_n(t) - \eta\hat{H}_n(t)\right] + \rho\hat{\pi}_n(t)$$
$$\dot{\hat{\ell}}_n(t) = \delta_\ell\left[\sum_m \frac{\ell_{mn}}{\ell_n}\left[\nu\left(\hat{v}_n(t) - \hat{V}_m(t)\right) + \hat{\ell}_m(t)\right] - \hat{\ell}_n(t)\right]$$
$$\dot{\hat{v}}_n(t) = -\hat{U}_n(t) - \delta_\ell\hat{V}_n(t) + (\delta_\ell+\rho)\hat{v}_n(t)$$

F Calibration Details

In this appendix, I go through the details of how I calibrate the trade flows, the migration flows, and the observed policy response.

F.1 Trade flows

As described in the main text, I get state spending on other states from the 2002 Commodity Flow Survey. I then construct a matrix of the share of each state's traded spending on every other state. I assume that the trade costs between two distinct commuting zones n and m are

$$\log\tau_{nm} = \delta_D \log \text{distance}_{nm} + \delta_H$$

where distance_{nm} is the bilateral distance between the population centroids of CZs n and m.

I then guess values for δ_D and δ_H. I then solve for $\frac{W_m}{A_m}$ and P_{Tn} in

$$W_m H_m \ell_m = \sum_n \frac{(\tau_{mn})^{1-\sigma}\left(\frac{W_m}{A_m}\right)^{1-\sigma}}{(P_{Tn})^{1-\sigma}}(1-\alpha)E_n$$

where

$$(P_{Tn})^{1-\sigma} = \sum_m \tau_{mn}^{1-\sigma}\left(\frac{W_m}{A_m}\right)^{1-\sigma}$$

and $W_m H_m \ell_m$ and E_n are observed in the County Business Patterns data. Having calculated that, I then get spending flows between CZs,

$$X_{mn} = \frac{\tau_{mn}^{1-\sigma}\left(\frac{W_m}{A_m}\right)^{1-\sigma}}{P_{Tn}^{1-\sigma}}(1-\alpha)E_n.$$

I then aggregate up to the state level and calculate the square loss of the implied consumptions shares in the model and the implied consumption shares from the data.

F.2 Migration flows

I construct CZ-to-CZ migration flows over one year from the American Community Survey (ACS) for the years 2006-2022 leaving out 2020 due to the pandemic. Households report their current Public Use microdata area (PUMA) in the current year and an adjusted Public Use microdata area called the MIGPUMA in the previous year. The MIGPUMAs occassionally include multiple PUMAs. I start by assuming that if a household did not move, they are in the same commuting zone now as they were last year.

For those household who did report moving, I use crosswalks from the census to map the MIGPUMA onto the PUMA, weighted by the relative population. I then use the PUMA commuting zone crosswalk from https://www.ddorn.net/data.htm to map that into commuting zone to commuting zone migration flows. I then run a regression of log migration flows on log distance with starting and ending location fixed effects.

Then setting δ_D to be equal to the value of that regression, I search over δ_ℓ and δ_H to match the observed share of people who still live in the same commuting zone one year later.

F.3 Observed Policy Response

I assume that the decrease in the income tax rate happens immediately after the shock and is constant and permanent. I then choose a value so as to minimize the distance between the chosen value and observed estimates in Figure 2 weighted by the standard errors.

I model the payments from public assistance programs as

$$\tau_n^c(t) = \delta_h \frac{\exp(\delta_t \cdot (t + \delta_x))}{1 + \exp(\delta_t \cdot (t + \delta_x))}.$$

I then search over δ_h, δ_x, and δ_t to minimize the square error normalized by the standard error.

G Computational Algorithm

In this section, I describe the computational algorithm. I stack all of the variables into vectors. I start by describing the state variables $x(t)$. These are stacked so that

$$\begin{aligned} x(t)[n] &= \hat{v}_n(t) \\ x(t)[N+n] &= \hat{\ell}_n(t) \\ x(t)[2N+n] &= \hat{\pi}_n(t) \\ x(t)[3N+n] &= \hat{w}_n(t). \end{aligned}$$

I consider shocks

$$u(t)[n] = \hat{\phi}_n(t).$$

I have the planner directly chooses expenditure at each time t, which is equivalent to choosing transfers,

$$y(t)[n] = \hat{E}_n(t).$$

Then I also include a vector of intermediates variables

$$\begin{aligned} z(t)[n] &= \hat{V}_n(t) \\ z(t)[N+n] &= \hat{U}_n(t) \\ z(t)[2N+n] &= \hat{H}_n(t) \\ z(t)[3N+n] &= \hat{P}_{Tn}(t) \\ z(t)[4N+n] &= \hat{y}_{Tn}(t). \end{aligned}$$

I then put the linearized system into matrix form. I write the loss function as

$$\mathcal{W}(t) = \left(\tilde{A}_x x(t) + \tilde{A}_y y(t) + \tilde{A}_u u(t) + \tilde{A}_{2z} z(t)\right)^\intercal \tilde{A}_1 \left(\tilde{A}_x x(t) + \tilde{A}_y y(t) + \tilde{A}_u u(t) + \tilde{A}_z z(t)\right)$$

where $\tilde{A}_1$ is a diagonal matrix and each entry corresponds to one summand in the expression of the loss function.

The intermediate variables obey equations that I summarize in matrix form,

$$\Omega_z z(t) = \Omega_x x(t) + \Omega_u u(t) + \Omega_y y(t).$$

And the state variables evolve according to

$$\dot{x}(t) = \tilde{B}_x x(t) + \tilde{B}_u u(t) + \tilde{B}_y y(t) + \tilde{B}_z z(t).$$

I then solve for $z(t)$ as a function of the other variables

$$z(t) = (\Omega_z)^{-1}\Omega_x x(t) + (\Omega_z)^{-1}\Omega_u u(t) + (\Omega_z)^{-1}\Omega_y y(t).$$

I then plug this into the welfare function and how the state variables evolve to get a simplified system

$$\mathcal{W}(t) = (A_x x(t) + A_y y(t) + A_u u(t))^\intercal \tilde{A}_1 (A_x x(t) + A_y y(t) + A_u u(t)),$$

with the matrices

$$\begin{aligned} A_x &= \tilde{A}_x + \tilde{A}_z (\Omega_z)^{-1}\Omega_x \\ A_y &= \tilde{A}_y + \tilde{A}_z (\Omega_z)^{-1}\Omega_y \\ A_u &= \tilde{A}_u + \tilde{A}_z (\Omega_z)^{-1}\Omega_u. \end{aligned}$$

I then multiply the matrices out to get

$$\begin{aligned} \mathcal{W} = {} & x^\intercal(t) A_{xx} x(t) + u^\intercal(t) A_{ux} x(t) + y^\intercal(t) A_{yx} x(t) + u^\intercal A_{uu} u(t) \\ & + y^\intercal(t) A_{yu} u(t) + y^\intercal(t) A_{yy} y(t), \end{aligned}$$

where

$$
\begin{aligned}
A_{xx} &= A_x^\intercal \tilde{A}_1 A_x \\
A_{ux} &= A_u^\intercal \tilde{A}_1 A_x \\
A_{yx} &= A_y^\intercal \tilde{A}_1 A_x \\
A_{uu} &= A_u^\intercal \tilde{A}_1 A_u \\
A_{yu} &= A_y^\intercal \tilde{A}_1 A_u \\
A_{yy} &= A_y^\intercal \tilde{A}_1 A_y.
\end{aligned}
$$

Computationally, constructing these matrices is not feasible for memory reasons. Instead, I break up the loss function into sub-problems and follow this exact procedure for each sub-problem. I then add all of the results together to get the final welfare loss function.

The state variables change according to

$$
\dot{x}(t) = B_x x(t) + B_u u(t) + B_y y(t),
$$

where,

$$
\begin{aligned}
B_x &= \tilde{B}_x + \tilde{B}_z(\Omega_z)^{-1}\Omega_x \\
B_y &= \tilde{B}_y + \tilde{B}_z(\Omega_z)^{-1}\Omega_y \\
B_u &= \tilde{B}_u + \tilde{B}_z(\Omega_z)^{-1}\Omega_u.
\end{aligned}
$$

G.1 Solving for Equilibrium

I start by describing how to solve for equilibrium taking as given how $y(t)$ and $u(t)$ are changing. Suppose that shocks and transfers take the form

$$
y(t) = \begin{cases} 0 & t < \tau \\ \overline{y} & t \geqslant \tau \end{cases}
$$

and

$$
u(t) = \begin{cases} 0 & t < \tau \\ \overline{u} & t \geqslant \tau. \end{cases}
$$

That is, there is a permanent shock starting at time τ. Below, I describe how to solve the model for this type of shock. Then I can approximate many time varying shocks using these results. Since the the model is linear, to get the full equilibrium response to a time varying shock, one simply needs to add together the equilibrium response to the shock at every point.

The computations then proceed in a few steps.

G.1.1 Steady State

I start by finding the steady state. To do that, I first find the eigenvectors of B_x. In this model, there will be $2N-1$ negative eigenvalues if the system is stable, and one 0 eigenvalue associated with the overall price level. I will denote by $\zeta_1, \ldots, \zeta_{4N}$ the eigenvalues and $v_1, \ldots v_{4N}$ the eigenvectors. Denote by C the change of basis so that if $\tilde{x}$ is in eigenvectors, $x = C\tilde{x}$ is in the usual basis. Then the steady state must solve

$$\begin{aligned}
-\left(B_u\overline{u} + B_y\overline{y}\right) &= B_x x^{SS} \\
-\left(B_u\overline{u} + B_y\overline{y}\right) &= C\tilde{B}_x C^{-1} x^{SS} \\
-C^{-1}\left(B_u\overline{u} + B_y\overline{y}\right) &= \tilde{B}_x C^{-1} x^{SS}_{temp}
\end{aligned}$$

where $\tilde{B}_x$ is diagonal. I then find some vector x^{SS}_{temp} that solves this equation. Then

$$x^{SS} = x^{SS}_{temp} + \alpha_{2N} v_{2N},$$

as $B_x v_{2N} = 0$.

G.1.2 After $\overline{t}$

I can rewrite the system after $t > \overline{t}$ as

$$\dot{x}(t) = B_x(x(t) - x^{SS}).$$

Therefore, in order to converge to the steady state, starting at time $\overline{t}$ there exist values $\alpha_1, \ldots \alpha_{2N-1}$ with eigenvectors $v_1, \ldots v_{2N-1}$ such that

$$x(t) - x^{SS} = \sum_{i=1}^{2N-1} \alpha_i v_i e^{\zeta_i t}.$$

G.1.3 Before $\overline{t}$

Before $\overline{t}$, we know that

$$\dot{x}(t) = B_x x(t).$$

We also need that $\hat{\ell}_n(0) = \hat{w}_n(0) = 0$. Then the system evolves according to

$$x(t) = \sum_{i=1}^{4N} \beta_i v_i e^{\zeta_i t}$$

G.1.4 Putting it Together

The equilibrium then follows

$$x(t) = \begin{cases} \sum_{i=1}^{4N} \beta_i v_i e^{\zeta_i t} & t < \overline{t} \\ \sum_{i=1}^{2N-1} \alpha_i v_i e^{\zeta_i t} + \alpha_{2N} v_{2N} + x^{SS}_{temp} & t \geqslant \overline{t}. \end{cases}$$

I then solve for α_i and β_i so that

$$\sum_{i=1}^{4N} \beta_i v_i e^{\zeta_i \bar{t}} = \sum_{i=1}^{2N-1} \alpha_i v_i e^{\zeta_i \bar{t}} + \alpha_{2N} v_{2N} + x_{temp}^{SS},$$

and $\hat{\ell}_n(0) = \hat{w}_n(0) = 0$.

G.2 Optimal Policy

Next I describe how to solve for optimal policy. The planner faces the problem of

$$\max_{y(t),x(t)} \int_0^\infty e^{-\rho t} \Big[x^\intercal(t) A_{xx} x(t) + u^\intercal(t) A_{ux} x(t) + y^\intercal(t) A_{yx} x(t) \\ + u^\intercal A_{uu} u(t) + y^\intercal(t) A_{yu} u(t) + y^\intercal(t) A_{yy} y(t) \Big] dt$$

such that

$$\dot{x}(t) = B_x x(t) + B_u u(t) + B_y y(t).$$

I set up the current value Hamiltonian and then take the first order necessary conditions

$$\begin{aligned} 0 &= 2A_{yy} y(t) + A_{yx} x(t) + A_{yu} u(t) + (\mu^\intercal(t) B_y)^\intercal \\ \rho\mu(t) - \dot{\mu}(t) &= 2A_{xx} x(t) + (u^\intercal(t) A_{ux})^\intercal + (y^\intercal(t) A_{yx})^\intercal + (\mu^\intercal(t) B_x)^\intercal . \end{aligned}$$

I rearrange to get

$$\begin{aligned} 2A_{yy} y(t) + A_{yx} x(t) + A_{yu} u(t) + B_y^\intercal \mu(t) &= 0 \\ -2A_{xx} x(t) - A_{ux}^\intercal u(t) - A_{yx}^\intercal y(t) + \left(\rho\mathbb{I} - B_x^\intercal\right) \mu(t) &= \dot{\mu}(t) \\ B_x x(t) + B_u u(t) + B_y y(t) &= \dot{x}(t). \end{aligned}$$

Solving for $y(t)$,

$$y(t) = -\frac{1}{2} A_{yy}^{-1} \left[\begin{bmatrix} A_{yx} & B_y^\intercal \end{bmatrix} \begin{bmatrix} x(t) \\ \mu(t) \end{bmatrix} + A_{yu} u(t) \right].$$

I can then set up a matrix that describes how the state and co-state variables develop. This is

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{\mu}(t) \end{bmatrix} &= \begin{bmatrix} B_x & 0 \\ -2A_{xx} & \rho\mathbb{I} - B_x^\intercal \end{bmatrix} \begin{bmatrix} x(t) \\ \mu(t) \end{bmatrix} + \begin{bmatrix} B_y \\ -A_{yx}^\intercal \end{bmatrix} y(t) + \begin{bmatrix} B_u \\ -A_{ux}^\intercal \end{bmatrix} u(t) \\ &= \left(\begin{bmatrix} B_x & 0 \\ -2A_{xx} & \rho\mathbb{I} - B_x^\intercal \end{bmatrix} - \frac{1}{2} \begin{bmatrix} B_y \\ -A_{yx}^\intercal \end{bmatrix} A_{yy}^{-1} \begin{bmatrix} A_{yx} & B_y^\intercal \end{bmatrix} \right) \begin{bmatrix} x(t) \\ \mu(t) \end{bmatrix} \\ &\quad + \left(\begin{bmatrix} B_u \\ -A_{ux}^\intercal \end{bmatrix} - \frac{1}{2} \begin{bmatrix} B_y \\ -A_{yx}^\intercal \end{bmatrix} A_{yy}^{-1} A_{yu} \right) u(t). \end{aligned}$$

I then define the matrix,

$$\Psi \equiv \begin{bmatrix} B_x & 0 \\ -2A_{xx} & \rho\mathbb{I} - B_x^\intercal \end{bmatrix} - \frac{1}{2}\begin{bmatrix} B_y \\ -A_{yx}^\intercal \end{bmatrix} A_{yy}^{-1} \begin{bmatrix} A_{yx} & B_y^\intercal \end{bmatrix},$$

with shocks

$$\psi = \begin{bmatrix} B_u \\ -A_{ux}^\intercal \end{bmatrix} - \frac{1}{2}\begin{bmatrix} B_y \\ -A_{yx}^\intercal \end{bmatrix} A_{yy}^{-1} A_{yu}.$$

The solution to the planner's problem can then be described by

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\mu}(t) \end{bmatrix} = \Psi \begin{bmatrix} x(t) \\ \mu(t) \end{bmatrix} + \psi u(t).$$

I then solve this the same way I solve the equilibrium problem above for a shock that starts at some time τ and is constant afterwards. The only difference is that now there are $8N$ eigenvectors, $4N-1$ eigenvalues are negative, and one is 0. I then include initial conditions $\hat{\ell}_n(0) = \hat{w}_n(0) = 0$. v^n and π_n are jump variables, so their associated co-state variables must start at 0, $\mu_n^v(0) = \mu_n^\pi(0) = 0$.

H Per Capita Labor Demand Effects in Dynamic Model

Proposition A5. *Suppose that there is a continuum of locations with no migration costs and wages are perfectly rigid ($\delta_w = 0$). Then, after a small demand shock $\hat{\phi}_n$, the total effect on per capita labor demand of a small transfer at time t' is*

$$\int_0^\infty e^{-\rho t}\hat{H}_n(t)\frac{d\hat{H}_n(t)}{d\hat{T}_n(t')}dt = \left(\frac{\alpha}{1-\alpha} - \frac{\nu}{\rho+\delta_\ell}\left(1 - e^{-\delta_\ell t'}\right)\right)e^{-\rho t'}\hat{\phi}_n.$$

Proof. In the limit with an infinite number of locations and fully rigid wages ($\delta_w = 0$) the system is described by

$$\dot{\hat{v}}_n(t) = -\hat{U}_n(t) + (\rho+\delta_\ell)\hat{v}_n(t)$$
$$\dot{\hat{\ell}}_n(t) = \delta_\ell(\nu\hat{v}_n(t) - \hat{\ell}_n(t))$$
$$\hat{U}_n(t) = \hat{E}_n(t) - \hat{H}_n(t)$$
$$\hat{E}_n(t) = \hat{H}_n(t) + \hat{T}_n(t)$$
$$\hat{H}_n(t) = (1-\alpha)\hat{\phi}_n - (1-\alpha)\hat{\ell}_n(t) + \alpha\hat{E}_n(t).$$

Combining, the two differential equations are

$$\dot{\hat{v}}_n(t) = -\hat{T}_n(t) + (\rho+\delta_\ell)\hat{v}_n(t)$$

$$\dot{\hat{\ell}}_n(t) = \delta_\ell(\nu\hat{v}_n(t) - \hat{\ell}_n(t)).$$

Integrating up the utility equation

$$\hat{v}_n(t) = \int_t^{\infty} e^{-(\rho+\delta_\ell)(s-t)} \hat{T}_n(s) ds,$$

Integrating up the labor equation, using $\hat{\ell}_n(0) = 0$,

$$\hat{\ell}_n(t) = \delta_\ell \nu \int_0^t e^{\delta_\ell(s-t)} \hat{v}_n(s) ds.$$

Then solving for $\hat{\ell}_n(t)$ in terms of transfers

$$\begin{aligned}
\hat{\ell}_n(t) &= \delta_\ell \nu \int_0^t e^{\delta_\ell(s-t)} \hat{v}_n(s) ds \\
&= \delta_\ell \nu \int_0^t e^{\delta_\ell(s-t)} \int_s^{\infty} e^{-(\rho+\delta_\ell)(r-s)} \hat{T}_n(r) dr ds \\
&= \delta_\ell \nu e^{-\delta_\ell t} \int_0^t \int_s^{\infty} e^{(\rho+2\delta_\ell)s} e^{-(\rho+\delta_\ell)r} \hat{T}_n(r) dr ds \\
&= \delta_\ell \nu e^{-\delta_\ell t} \int_0^t \int_0^r e^{(\rho+2\delta_\ell)s} e^{-(\rho+\delta_\ell)r} \hat{T}_n(r) ds dr \\
&\quad + \delta_\ell \nu e^{-\delta_\ell t} \int_t^{\infty} \int_0^t e^{(\rho+2\delta_\ell)s} e^{-(\rho+\delta_\ell)r} \hat{T}_n(r) ds dr \\
&= \delta_\ell \nu e^{-\delta_\ell t} \int_0^t \frac{e^{(\rho+2\delta_\ell)r} - 1}{\rho + 2\delta_\ell} e^{-(\rho+\delta_\ell)r} \hat{T}_n(r) dr \\
&\quad + \delta_\ell \nu e^{-\delta_\ell t} \int_t^{\infty} \frac{e^{(\rho+2\delta_\ell)t} - 1}{\rho + 2\delta_\ell} e^{-(\rho+\delta_\ell)r} \hat{T}_n(r) dr \\
&= -\frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{-\delta_\ell t} \int_0^{\infty} e^{-(\rho+\delta_\ell)r} \hat{T}_n(r) dr \\
&\quad + \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} \int_0^t e^{\delta_\ell(r-t)} \hat{T}_n(r) dr + \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} \int_t^{\infty} e^{-(\rho+\delta_\ell)(r-t)} \hat{T}_n(r) dr
\end{aligned}$$

Then taking the derivative with respect to $\hat{T}_n(r)$ for $r < t$ is

$$\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)} = -\frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{-\delta_\ell t} e^{-(\rho+\delta_\ell)r} + \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{\delta_\ell(r-t)} = e^{-\delta_\ell t} \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} \left[e^{\delta_\ell r} - e^{-(\rho+\delta_\ell)r} \right].$$

For $r > t$

$$\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)} = -\frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{-\delta_\ell t} e^{-(\rho+\delta_\ell)r} + \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{-(\rho+\delta_\ell)(r-t)} = \frac{\delta_\ell \nu}{\rho + 2\delta_\ell} e^{-(\rho+\delta_\ell)r} \left[e^{(\rho+\delta_\ell)t} - e^{-\delta_\ell t} \right].$$

Then I look to find the effect on hours. Hours are given by

$$\hat{H}_n(t) = \hat{\phi}_n - \hat{\ell}_n(t) + \frac{\alpha}{1-\alpha} \hat{T}_n(t).$$

Therefore,

$$\frac{d\hat{H}_n(t)}{d\hat{T}_n(r)} = -\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)} + \frac{\alpha}{1-\alpha}\mathbb{I}_{r=t}.$$

Meanwhile, starting with no transfers, hours are given by $\hat{H}_n(t) = \hat{\phi}_n$. Therefore, defining $X_n = \int_0^\infty e^{-\rho t}\hat{H}_n(t)^2 dt$, I have

$$\begin{aligned}
\frac{dX_n}{d\hat{T}_n(r)} &= \int_0^\infty e^{-\rho t}\hat{H}_n(t)\frac{d\hat{H}_n(t)}{d\hat{T}_n(r)}dt \\
&= \int_0^\infty e^{-\rho t}\hat{\phi}_n(t)\left[-\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)} + \frac{\alpha}{1-\alpha}\mathbb{I}_{r=t}\right]dt \\
&= -\int_0^\infty e^{-\rho t}\hat{\phi}_n(t)\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)}dt + e^{-\rho r}\hat{\phi}_n(r)\frac{\alpha}{1-\alpha}
\end{aligned}$$

Plugging in for how population changes,

$$\begin{aligned}
\frac{dX_n}{d\hat{T}_n(r)} &= -\int_0^\infty e^{-\rho t}\hat{\phi}_n(t)\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)}dt + e^{-\rho r}\hat{\phi}_n(r)\frac{\alpha}{1-\alpha} \\
&= \int_0^r e^{-\rho t}\hat{\phi}_n(t)\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)}dt - e^{-\rho r}\hat{\phi}_n(r)\frac{\alpha}{1-\alpha} + \int_r^\infty e^{-\rho t}\hat{\phi}_n(t)\frac{d\hat{\ell}_n(t)}{d\hat{T}_n(r)}dt \\
&= \int_0^r e^{-\rho t}\hat{\phi}_n(t)\frac{\delta_\ell \nu}{\rho+2\delta_\ell}e^{-(\rho+\delta_\ell)r}\left[e^{(\rho+\delta_\ell)t} - e^{-\delta_\ell t}\right]dt - e^{-\rho r}\hat{\phi}_n(r)\frac{\alpha}{1-\alpha} \\
&\quad + \int_r^\infty e^{-\rho t}\hat{\phi}_n(t)e^{-\delta_\ell t}\frac{\delta_\ell \nu}{\rho+2\delta_\ell}\left[e^{\delta_\ell r} - e^{-(\rho+\delta_\ell)r}\right]dt
\end{aligned}$$

I then integrate

$$\begin{aligned}
\frac{dX_n}{d\hat{T}_n(r)} &= \frac{\delta_\ell \nu}{\rho+2\delta_\ell} e^{-(\rho+\delta_\ell)r} \int_0^r e^{-\rho t}\hat{\phi}_n \left[e^{(\rho+\delta_\ell)t} - e^{-\delta_\ell t}\right] dt - e^{-\rho r}\hat{\phi}_n \frac{\alpha}{1-\alpha} \\
&\quad + \frac{\delta_\ell \nu}{\rho+2\delta_\ell}\left[e^{\delta_\ell r} - e^{-(\rho+\delta_\ell)r}\right] \int_r^\infty e^{-\rho t}\hat{\phi}_n e^{-\delta_\ell t} dt \\
&= \frac{\delta_\ell \nu}{\rho+2\delta_\ell} e^{-(\rho+\delta_\ell)r}\hat{\phi}_n \int_0^r e^{\delta_\ell t} dt - \frac{\delta_\ell \nu}{\rho+2\delta_\ell} e^{-(\rho+\delta_\ell)r}\hat{\phi}_n \int_0^r e^{-(\rho+\delta_\ell)t} dt \\
&\quad - e^{-\rho r}\hat{\phi}_n \frac{\alpha}{1-\alpha} \\
&\quad + \frac{\delta_\ell \nu}{\rho+2\delta_\ell} e^{\delta_\ell r}\hat{\phi}_n \int_r^\infty e^{-(\rho+\delta_\ell)t} dt - \frac{\delta_\ell \nu}{\rho+2\delta_\ell}\hat{\phi}_n e^{-(\rho+\delta_\ell)r} \int_r^\infty e^{-(\rho+\delta_\ell)t} dt \\
&= \frac{\delta_\ell \nu}{\rho+2\delta_\ell} e^{-(\rho+\delta_\ell)r}\hat{\phi}_n \frac{e^{\delta_\ell r}-1}{\delta_\ell} - \frac{\delta_\ell \nu}{\rho+2\delta_\ell} e^{-(\rho+\delta_\ell)r}\hat{\phi}_n \frac{e^{-(\rho+\delta_\ell)\infty}-1}{-(\rho+\delta_\ell)} \\
&\quad - e^{-\rho r}\hat{\phi}_n \frac{\alpha}{1-\alpha} \\
&\quad + \frac{\delta_\ell \nu}{\rho+2\delta_\ell} e^{\delta_\ell r}\hat{\mu}_1 \frac{e^{-(\rho+\delta_\ell)\infty} - e^{-(\rho+\delta_\ell)r}}{-(\rho+\delta_\ell)} \\
&= \hat{\phi}_n \frac{\delta_\ell \nu}{\rho+2\delta_\ell}\left[\frac{1}{\delta_\ell} + \frac{1}{\rho+\delta_\ell}\right]\left(e^{-\rho r} - e^{-(\rho+\delta_\ell)r}\right) - e^{-\rho r}\hat{\mu}_1 \frac{\alpha}{1-\alpha}
\end{aligned}$$

We can then rewrite to get the result.

□

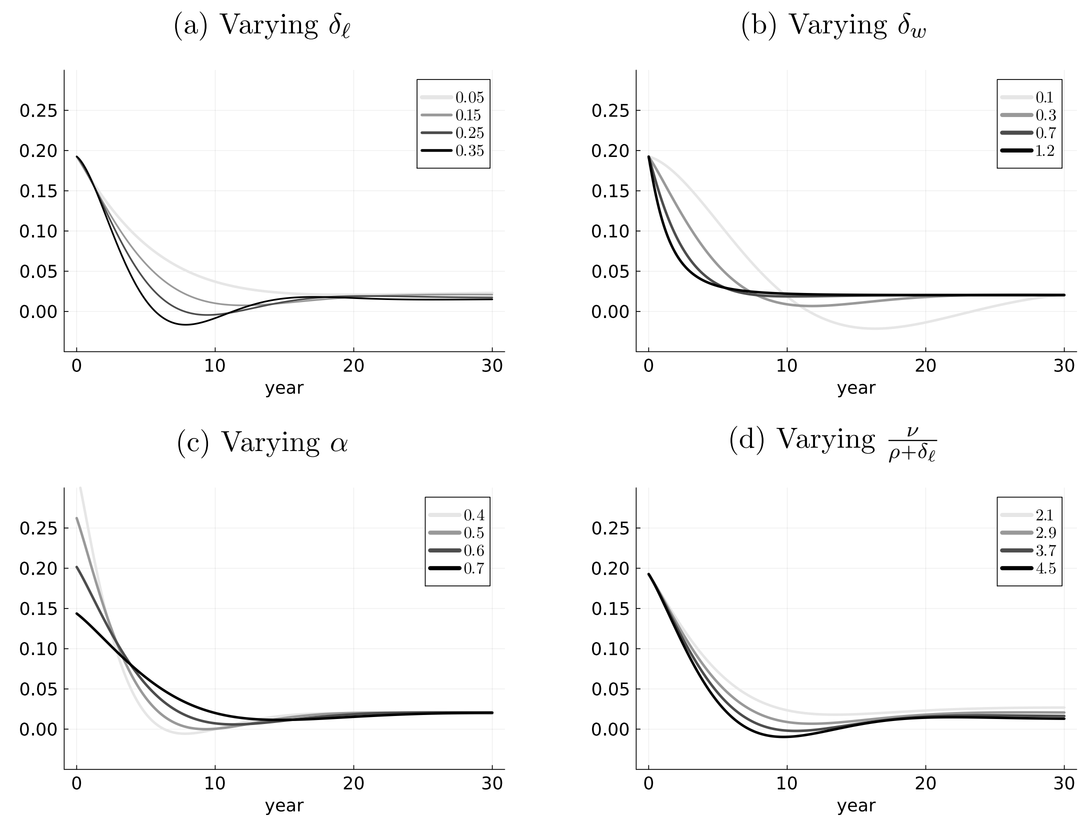
I Computational Robustness

I.1 Idiosyncratic Shock Robustness

In this section, I show how robust the optimal policy time path is to varying key model parameters. In particular, I plot the time path of optimal transfers in response to a demand shock while varying key parameters determining the relative strength of the migration and stimulus effect in Figure A1. In Figure A1a, I vary the speed of migration δ_ℓ, holding fixed the long run migration elasticity. Figure A1b varies the degree of wage rigidity. Figure A1c shows how the policy changes with the local multiplier, and Figure A1d shows how sensitive the policy is to the long run migration elasticity.

I start by discussing how the speed of population change affects the optimal policy in A1a. When population adjusts very slowly (i.e. δ_ℓ is close to 0), the optimal transfer never falls below the long run insurance level. That is because the planner cannot affect population on the time scale necessary to affect the recession. People might be very mobile in the long run, but if they will only move out 10 years after a policy change, there is no macroeconomic benefit because wages will have already adjusted by that point. When people are very quick to move, as suggested by the impulse response in Figure 1b, the migration effect becomes more important because people's migration decision is very responsive to planned taxes. Therefore, when $\delta_\ell = 0.35$, the optimal transfer becomes negative not even 7 years after the

Figure A1: Optimal Policy Robustness



Note: This figure shows how the optimal policy changes with various parameters.

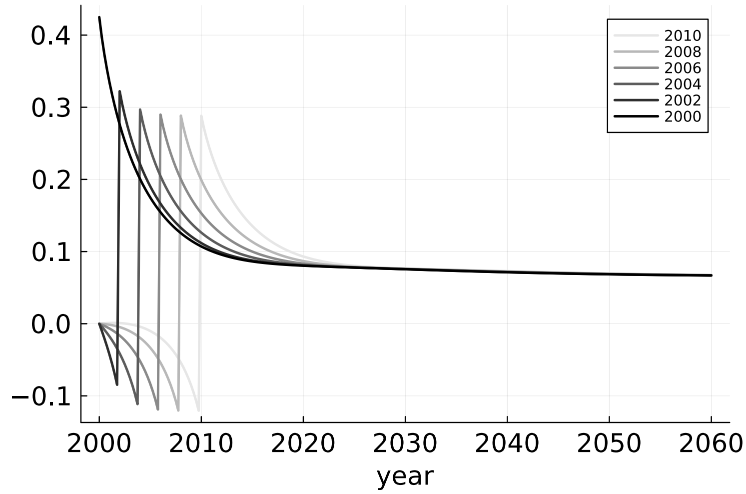
demand shock. It then rises back to the same level of long run insurance transfers.

Next I vary the speed with which wages adjust in A1b. Similar to δ_ℓ, varying δ_w plays a large role in how important the migration effect is. The main difference is that while increasing δ_ℓ speeds up the movement of households so they can respond while the recession is happening, decreasing δ_w slows down the wages so that the recession is still happening while population slowly adjusts. Thus, as wages become perfectly rigid (i.e δ_w becomes very small), the optimal transfer becomes negative for a large number of years following the demand shock. As wages adjust more quickly, migration cannot react in time so that the transfers never drop below their long run insurance levels. However, the basic structure of generous transfers that quickly fade out remains robust.

Varying the home bias in consumption α has very different impacts on the optimal transfers as seen in A1c. Increasing α makes stimulus payments much more effective. Therefore, as $\alpha \to 1$, the stimulus effect always dominates the migration effect so that there is no large dip in the optimal transfer around year 10. However, when transfers are very effective at stimulating the local economy, the government does not need to transfer as much money to a region in a recession to stimulate it. Therefore, at time 0, the optimal transfer is actually decreasing in the degree of home bias.

Finally, I show how the optimal transfer changes with the long run migration elasticity in Figure A1d. Increasing that elasticity changes the insurance effect because it increases the misallocation caused by giving a small transfer to the region. Therefore, the optimal long

Figure A2: Optimal Policy Response for China Shock with Alternate Timing



Note: This figure plots the coefficients of a regression of optimal transfers relative to original income on the size of the China shock for each time t, weighting by pre-shock earnings. It is done for a China Shock that happens all at once for different years.

run transfer decreases in the migration elasticity. This comparative static also changes the migration effect. When households' location choices are more responsive to transfers, the government will want to tax a recessionary city more to encourage people to get out. Thus, the optimal transfer becomes negative around year 10 if the long run migration elasticity is 3.7.

I.2 China Shock Timing and Expectations

Here I consider how the optimal responds to the China shock with alternate timing assumptions. In particular, I plot what the average optimal policy would look like if it all happened on one year. A2 shows the results of regressing the optimal transfer, as a share of initial earnings, to each CZ on the size of the shock it received weighted by total labor earnings before the shock for different timing assumptions. When the shock happens exactly on the year 2000, the planner immediately provides generous transfers that slowly fade out. When the shock hits in a later year, the planner starts by taxing people who are in the location so as to encourage them to move out before the China shock hits. Then after the China trade shock hits, the planner provides generous stimulus transfers that then slowly fade out.