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TRENDS AND RANDOM WALKS IN MACROECONMIC TIME SERIES

Some Evidence and Implications

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This paper investigates whether macroeconomic time series are better characterized as stationary fluctuations around a deterministic trend or as non-stationary processes that have no tendency to return to a deterministic path. Using long historical time series for the U.S. we are unable to reject the hypothesis that these series are non-stationary stochastic processes with no tendency to return to a trend line. Based on these findings and an unobserved components model for output that decomposes fluctuations into a secular or growth component and a cyclical component we infer that shocks to the former, which we associate with real disturbances, contribute substantially to the variation in observed output. We conclude that macroeconomic models that focus on monetary disturbances as a source of purely transitory fluctuations may never be successful in explaining a large fraction of output variation and that stochastic variation due to real factors is an essential element of any model of macroeconomic fluctuations.

1. Introduction

It is common practice in macroeconomics to decompose real variables such as output, and sometimes nominal variables, into a secular or growth component and a cyclical component. In the case of output, the secular component is viewed as being in the domain of growth theory with real factors such as capital accumulation, population growth, and technological change as the primary determinants. The cyclical component, on the other hand, is assumed to be transitory (stationary) in nature with monetary and, to a lesser extent, real factors being featured as primary causes. Since cyclical fluctuations are assumed to dissipate over time, any long-run or permanent

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movement (non-stationarity) is necessarily attributed to the secular component.

The notion that the secular component does not fluctuate much over short periods of time, such as a year or a quarter, but rather moves slowly and smoothly relative to the cyclical component has led to the practice of 'detrending' time series by regression on time (or perhaps a polynomial in time). The residuals are then interpreted as the cyclical component to be explained by business cycle theory.[1] For example, Bodkin (1969), Lucas (1973), Barro (1978), Sargent (1978), Taylor (1979), Hall (1980), and Kydland and Prescott (1980) all implicitly or explicitly regard residuals from fitted linear or quadratic time trends as the relevant data for business cycle analysis.[2]

Secular movement, however, need not be modeled by a deterministic trend. For example, the class of integrated stochastic processes exemplified by the random walk, also exhibit secular movement but do not follow a deterministic path. If the secular movement in macroeconomic time series is of a stochastic rather than deterministic nature, then models based on time trend residuals are misspecified.[3]

The types of misspecification that arise from inappropriate detrending can be illustrated by considering the properties of residuals from a regression of a random walk on time. These properties are investigated in recent papers by Chan, Hayya and Ord (1977) and Nelson and Kang (1981). The autocorrelation function of the residuals is shown to be a statistical artifact in the sense that it is determined entirely by sample size and it implies strong positive autocorrelation at low lags with pseudo-periodic behavior at long lags.[4] Empirical investigations of output fluctuations that do not consider the possible source of this autocorrelation might be led to over-estimate both the persistence and variance of the business cycle. Conversely, the importance of real factors that influence the secular component would be under-estimated.

In this paper we investigate whether macroeconomic times series are consistent with the time trend decomposition usually employed. Section 2 discusses the statistical issues involved in testing for deterministic trends and section 3 presents the results of formal and informal tests using long historical time series for the U.S. We are unable to reject the hypothesis that

[1]Or equivalently, time is included as an explanatory variable.

[2]Burns and Mitchell (1946), in their pioneering empirical investigation of business cycles, were concerned with the method of trend removal and went to great lengths to justify their procedures (see pp. 37–41, and ch. 7).

[3]Hall's (1980) use of the time trend model for real GNP is particularly puzzling since in previous work [Hall (1978)] he argues that aggregate consumption behaves like a random walk. Without some rather implausible restrictions on the other components of GNP, aggregate GNP will then include random walk characteristics and linear detrending is likely to be inappropriate.

[4]It is interesting to note that McCulloch (1975) finds evidence of periodicity in logs of real income, investment, and consumption after fitting a linear trend, but does not find periodicity in their first differences.

these series are non-stationary stochastic processes with no tendency to return to a trend line. The implications of this finding are explored in sections 4 and 5. Assuming that any stochastic fluctuations in output of a permanent variety must be associated with secular movements, and thus real factors, the evidence presented in section 3 leads us to the inference that (i) real shocks associated with the secular component contribute substantially to the variation in observed output, and (ii) either these shocks are correlated with the innovations in the cyclical component or the secular component contains transitory fluctuations (or both). We conclude that macroeconomic models that focus on monetary disturbances as a source of purely transitory (stationary) fluctuations may never be successful in explaining a very large fraction of output fluctuations and that stochastic variation due to real factors is an essential element of any model of economic fluctuations. Some recent efforts in this direction include the equilibrium stochastic growth models studied by Black (1979), Long and Plosser (1980), King and Plosser (1981) and Kydland and Prescott (1981).

2. Statistical background

The basic statistical issue is the appropriate representation of non-stationarity in economic time series. We are primarily concerned with non-stationarity in the mean of the series. Such behavior implies that the series lacks a fixed long-term mean, or put positively, has a tendency to move farther away from any given initial state as time goes on.

We consider two fundamentally different classes of non-stationary processes as alternative hypotheses. The first class of processes consists of those that can be expressed as a deterministic function of time, called a trend, plus a *stationary* stochastic process with mean zero. We refer to these as trend-stationary (TS) processes. The tendency of economic time series to exhibit variation that increases in mean and dispersion in proportion to absolute level motivates the transformation to natural logs and the assumption that trends are linear in the transformed data. We also assume that the deviations from trend have a representation as a stationary and invertible ARMA process. Denoting the natural logs of the series by z_t and the deviations from trend by c_t, the linear TS class has the form

$$\begin{aligned} z_t &= \alpha + \beta t + c_t, \\ \phi(L)c_t &= \theta(L)u_t;\ u_t \sim \text{i.i.d.}(0, \sigma_u^2), \end{aligned} \tag{1}$$

where α and β are fixed parameters, L is the lag operator, and $\phi(L)$ and $\theta(L)$ are polynomials in L that satisfy the conditions for stationarity and invertibility.

The fundamental determinism of the process is captured in the properties of long-term forecasts and uncertainty around such forecasts. While autocorrelation in c_t can be exploited in short-term forecasting, it is clear that over long horizons the only information about a future z is its mean ($\alpha+\beta t$). Therefore neither current nor past events will alter long-term expectations. Further, the long-term forecast error must be c which has finite variance. Thus uncertainty is bounded, even in the indefinitely distant future.

The second class of non-stationary process considered in this paper is that class for which first or higher order differences is a stationary and invertible ARMA process (DS processes). The counterpart of the linear TS process is the first-order DS process in natural logs written as

$$\begin{aligned} &(1-L)z_t=\beta+d_t, \\ &\delta(L)d_t=\lambda(L)u_t;\, u_t\sim \text{i.i.d.}(0,\sigma_u^2), \end{aligned} \tag{2}$$

where $(1-L)$ is the difference operator and $\delta(L)$ and $\lambda(L)$ are polynomials satisfying the stationarity and invertibility conditions. The simplest member of the class is the random walk for which the changes are serially uncorrelated, that is $d_t=u_t$.

To see the fundamental difference between the TS and DS classes it is useful to express z_t as the value at some reference point in the past, time zero, plus all subsequent changes,

$$z_t=z_0+\beta t+\sum_{j=1}^{t} d_t. \tag{3}$$

Eqs. (3) and (1) indicate that the two types of processes can be written as a linear function of time plus the deviation from it. The intercept in (1), however, is a fixed parameter while in (3) it is a function of historical events, and the deviations from trend in (1) are stationary while in (3) they are accumulations of stationary changes. The accumulation in (3) is not stationary but rather its variance increases without bound as t gets large. It is not difficult to see that the long-term forecast of a DS process will always be influenced by historical events and the variance of the forecast error will increase without bound.

The DS class is purely stochastic in nature while the TS class is fundamentally deterministic. When one assumes the latter class is appropriate one is implicitly bounding uncertainty and greatly restricting the relevance of the past to the future. Empirical tests may be quite sensitive to this distinction. For example, Shiller (1979) finds that the variance of holding period returns on long-term bonds is larger than would be consistent with a particular efficient markets (rational expectations) version of term structure theory if short rates are assumed to be stationary around a fixed mean. They are not too large, however, if short-term rates are assumed stationary only after differencing. The crucial factor causing the discrepancy is that under the DS assumption any movement in short rates will have some impact on the long-term expectations embodied in long rates, but will have very little impact under the TS assumption.[5]

The fundamental difference between the two classes of processes can also be expressed in terms of the roots of the AR and MA polynomials. If we first-difference the linear TS model the result is

$$\phi(L)[(1-L)z_t]=\beta\phi(L=1)+(1-L)\theta(L)u_t, \tag{4}$$

where $\phi(L=1)$ is a constant obtained by evaluating the polynomial $\phi(L)$ at $L=1$. Eq. (4) indicates that a unit root will be present in the MA part of the ARMA process describing the first differences $[(1-L)z_t]$. The simplest example would be the case of a linear trend plus random noise ($c_t=u_t$). The presence of the unit root implies that the process is not invertible; that is, it does not have a convergent autoregressive representation. Recall that the first differences of a DS process are both stationary and invertible.

Correspondingly, when we write the DS in terms of levels we obtain from (2)

$$\delta(L)(1-L)z_t=\beta\delta(L=1)+\lambda(L)u_t, \tag{5}$$

which contains a unit root in the AR polynomial. It would appear then that if a series is generated by a member of the linear TS subclass we should fail to reject the hypothesis of a unit MA root in the ARMA model for its first difference, and if it is generated by a member of the first order DS subclass we should fail to reject the hypothesis of a unit AR root in the ARMA model for its levels.[6]

Unfortunately, the standard asymptotic theory developed for stationary and invertible ARMA models is not valid for testing the hypotheses that either polynomial contains a unit root. To get some idea of the problem, consider the simplest version of (5) where the null hypothesis is that z_t is a random walk with drift,

$$z_t=\rho z_{t-1}+\mu+u_t,$$

so $\rho=1$ is the hypothesis we wish to test. The standard expression for the

[5]In a subsequent paper Shiller (1981) finds the variance of linearly detrended stock returns is similarly excessive if dividends are also assumed to be stationary around a linear trend (both variables deflated and in logs), but he does not report the impact of the TS assumption on the results.

[6]Pierce (1975) discusses a technique for distinguishing between deterministic and stochastic non-stationarity by inspection of sample autocorrelation functions.

large sample variance of the least square estimator $\hat{\rho}$ is $[(1-\rho^2)/T]$ which would be zero under the null hypothesis. The true variance of course is not zero; the problem is that the conventional asymptotic theory is inappropriate in this case. Dickey (1976) and Fuller (1976) develop the limiting distribution of $\hat{\rho}$ and the conventionally calculated least squares t-statistic, which we denote τ for the null hypothesis $\rho=1$, and tabulate the distributions. They demonstrate that if $\mu=0$ then the distribution of $\hat{\rho}$ is biased towards zero and skewed to the left, that is towards stationarity.[7]

In addition, Dickey and Fuller provide a set of results that allow us to test the DS hypothesis against the TS hypothesis as long as we are willing to assume that only AR terms are required to obtain satisfactory representations. The strategy is to embed both hypotheses in a common model. The simplest alternatives are a TS process with first order AR deviations and a random walk (DS process) with drift which are both special cases of

$$z_t=\alpha+\beta t+u_t/(1-\phi L),$$

or equivalently, after multiplication by $(1-\phi L)$, of

$$z_t=\phi z_{t-1}+[\alpha(1-\phi)+\phi\beta]+\beta(1-\phi)t+u_t. \tag{6}$$

If the TS hypothesis is correct then $|\phi|<1$. If the DS hypothesis is correct then $\phi=1$ and (6) reduces to

$$z_t=z_{t-1}+\beta+u_t.$$

It would appear then that one would want to run the regression

$$z_t=\mu+\rho z_{t-1}+\gamma t+u_t \tag{7}$$

and test the null hypothesis $\rho=1$, $\gamma=0$ which is equivalent to $\phi=1$ in (6). Under this null hypothesis the usual t-ratios are not t-distributed but Dickey and Fuller provide tabulations of the distribution of the t-ratio for ρ, again denoted τ, for testing the null hypothesis $\rho=1$.[8] Dickey and Fuller (1979)

[7]The problem of testing for unit roots in MA polynomials is more discouraging. MA processes are only identified under the restriction that the roots lie on or outside the unit circle; therefore estimates will be bounded away from the unit root. Plosser and Schwert (1977) have demonstrated in the first-order MA case that application of t-tests or likelihood ratio tests, which would be appropriate for null hypotheses within the invertibility region, lead to rejection in the vast majority of instances when the null hypothesis of a unit root is true.

[8]They do not develop a statistic for the joint test or for γ alone. However, a test on ρ alone is sufficient given that we do not consider a process with $\rho=1$ and $\gamma\neq0$ as part of the model space. A process with $\rho=1$ and $\gamma\neq0$ would be one in which differences in logs (rates of change) followed a deterministic path, implying ever increasing ($\gamma>0$) or even decreasing ($\gamma<0$) rates of change. We rule out this kind of behavior in economic time series on a priori grounds. Similarly we rule out quadratic or higher degree time polynomial trends.

state that the distributions of $\hat{\rho}$ and $\tau(\hat{\rho})$ is not affected by whether μ is zero or not, but $\tau(\hat{\rho})$ would be normal if $\gamma \neq 0$ (the case we have excluded). To illustrate these properties we have conducted a Monte Carlo experiment which is summarized in table 1. The sample length is 100 observations with 500 replications. In Case I the generating process is a random walk with a zero drift; in Case II there is non-zero drift. As indicated by Dickey and Fuller, customary testing procedures reject the null in favor of stationarity far too often in both cases. The distribution of $\hat{\rho}$ is centered around 0.9 instead of 1.0 in both cases. In addition, it is clear that the t-ratio for testing the hypothesis that $\gamma=0$ [denoted $t(\hat{\gamma})$], is biased towards indicating a trend. Thus, standard testing procedures are strongly biased towards finding stationarity around a trend; they tend to reject the hypothesis $\rho=1$ when it is true in favor of $\rho<1$ and tend to reject the hypothesis $\gamma=0$ when it is true.

Results of Fuller (1976) allow us to use the distributions of $\hat{\rho}$ and $\tau(\hat{\rho})$ in higher order cases. In general we want to distinguish between a TS process with AR component of order k and a DS process with AR representation of order $(k-1)$. In levels we write the DS model as an AR of order k with one

Table 1

Sampling distributions for the estimators in regression model (7),[a]

$$z_t = \hat{\mu} + \hat{\rho} z_{t-1} + \hat{\gamma} t + \hat{u}_t,$$

Case I: $\mu=0, \quad \rho=1.0, \quad \gamma=0,$

Case II: $\mu=1.0, \quad \rho=1.0, \quad \gamma=0.$

	Mean	Standard deviation	Skewness	Excess Kurtosis	Studentized range	Percent rejections
Case I						
$\hat{\mu}$	−0.007	0.525	0.490	4.12	10.73	
$t(\hat{\mu})$	−0.049	1.84	0.079	−0.615	5.97	32.8
$\hat{\rho}$	0.895	0.064	−1.11	1.53	6.15	
$\tau(\hat{\rho})$	−2.26	0.863	−0.237	0.655	7.41	65.2
$\hat{\gamma}$	−0.001	0.013	0.212	2.27	7.91	
$t(\hat{\gamma})$	−0.093	1.84	0.124	−0.807	5.08	35.6
Case II						
$\hat{\mu}$	0.878	0.503	−0.390	0.732	6.70	
$t(\hat{\mu})$	4.00	2.18	−0.524	−0.569	4.94	79.0
$\hat{\rho}$	0.900	0.034	−0.757	0.493	5.67	
$\tau(\hat{\rho})$	−2.22	0.769	0.196	0.284	6.35	66.0
$\hat{\gamma}$	0.099	0.054	0.698	0.303	5.49	
$t(\hat{\gamma})$	2.22	0.777	−0.202	0.454	6.94	65.0

[a]The sampling distributions are based on 500 replications of a random walk of sample size 100, with (Case II) and without (Case I) drift. $t(\hat{\mu})$ and $t(\hat{\gamma})$ are the ratios of $\hat{\mu}$ and $\hat{\gamma}$ to their respective standard errors and $\tau(\hat{\rho})$ is the ratio of $(\hat{\rho}-1)$ to its standard error. The percent rejections are computed based on the frequency that $|t(\hat{\mu})|$, $|\tau(\hat{\rho})|$, and $|t(\hat{\gamma})|$ are greater than 1.96.

root on the unit circle. The alternatives are imbedded in the model

$$z_t = \phi_1 z_{t-1} + \cdots + \phi_k z_{t-k} + \mu + \gamma t + u_t, \tag{8}$$

where $\sum \phi_i = 1$ and $\gamma = 0$ if the DS hypothesis is true. The terms in lagged z's can be rearranged in the format

$$z_t = \left(\sum_{i=1}^{k} \phi_i\right) z_{t-1} + \left(-\sum_{i=2}^{k} \phi_i\right)(z_{t-1} - z_{t-2})$$

$$+ \cdots + (-\phi_k)(z_{t-k+1} - z_{t-k}) + \mu + \gamma t + \mu_t. \tag{9}$$

Fuller (1976, p. 374) shows that if the coefficient of z_{t-1} is unity in (9), as it would be under the DS hypothesis, then the least squares estimator of that coefficient has the same large sample distribution as $\hat{\rho}$ in model (7), and similarly for its τ-ratio.[9]

We note that in the Dickey–Fuller procedure the *null* hypothesis is the DS specification while the alternative is the TS specification. As usual, acceptance of the null hypothesis is not disproof of the alternative hypothesis. It is important therefore to have a check on the power of the test. To provide this check we include in our data set a series that on *a priori* grounds is likely to be a member of the TS class (albeit with zero slope) rather than the DS class, namely the unemployment rate.

3. Analysis of U.S. historical data

We turn now to the analysis of the U.S. historical time series listed in table 2 which include measures of output, spending, money, prices, and interest rates.[10] The data are annual, generally averages for the year, with starting dates from 1860 to 1909 and ending in 1970 in all cases. All series except the bond yield are transformed to natural logs.

Sample autocorrelations of the levels are tabulated in table 2 and typically start at around 0.96 at lag one and decay slowly with increasing lag. This is consistent with the behavior of sample autocorrelations from a random walk as indicated by the values calculated from a formula due to Wichern (1973). One exception to this characterization is the unemployment rate which exhibits more rapid decay as would be expected of a stationary

[9] Dickey and Fuller (1981) have recently extended their analysis to likelihood ratio statistics.

[10] Data cources are as follows. GNP series, industrial production, employment 1929–1970, unemployment rate, consumer prices, and stock prices: *Long Term Economic Growth*, (1973). Wages, money stock, and bond yield: *Historical Statistics of the U.S., Colonial Times to 1970*, (1975). Velocity: Friedman and Schwartz (1963) with revisions kindly provided by Anna Schwartz. Employment 1890–1928: Lebergott (1964). Data files available from the authors upon request.

Table 2

Sample autocorrelations of the natural logs of annual data.[a]

Series	Period	Sample autocorrelations						
		T	r_1	r_2	r_3	r_4	r_5	r_6
Random walk[b]		100	0.95	0.90	0.85	0.81	0.76	0.70
Time aggregated[b] random walk		100	0.96	0.91	0.86	0.82	0.77	0.73
Real GNP	1909–1970	62	0.95	0.90	0.84	0.79	0.74	0.69
Nominal GNP	1909–1970	62	0.95	0.89	0.83	0.77	0.72	0.67
Real per capita GNP	1909–1970	62	0.95	0.88	0.81	0.75	0.70	0.65
Industrial production	1860–1970	111	0.97	0.94	0.90	0.87	0.84	0.81
Employment	1890–1970	81	0.96	0.91	0.86	0.81	0.76	0.71
Unemployment rate	1890–1970	81	0.75	0.47	0.32	0.17	0.04	−0.01
GNP deflator	1889–1970	82	0.96	0.93	0.89	0.84	0.80	0.76
Consumer prices	1860–1970	111	0.96	0.92	0.87	0.84	0.81	0.77
Wages	1900–1970	71	0.96	0.91	0.86	0.82	0.77	0.73
Real wages	1900–1970	71	0.96	0.92	0.88	0.84	0.80	0.75
Money stock	1889–1970	82	0.96	0.92	0.89	0.85	0.81	0.77
Velocity	1869–1970	102	0.96	0.92	0.88	0.85	0.81	0.79
Bond yield	1900–1970	71	0.84	0.72	0.60	0.52	0.46	0.40
Common stock prices	1871–1970	100	0.96	0.90	0.85	0.79	0.75	0.71

[a]The natural logs of all the data are used except for the bond yield. T is the sample size and r_i is the ith order autocorrelation coefficient. The large sample standard error under the null hypothesis of no autocorrelation is $T^{-\frac{1}{2}}$ or roughly 0.11 for series of the length considered here.

[b]Computed by the authors from the approximation due to Wichern (1973).

series. Sample autocorrelations of first differences are presented in table 3 and in each instance are positive and significant at lag one, but in many cases are not significant at longer lags.

One explanation of positive autocorrelation at lag one only is that the annual series are constructed by averaging shorter interval observations which themselves are generated by a DS process. Working (1960) demonstrates this effect of time aggregation on a random walk and shows that positive autocorrelation at lag one approaches +0.25 as the number of underlying observations being aggregated becomes large. Tiao (1972) shows that the Working result generalizes to the temporal aggregation of any DS process as long as the span of serial dependence in the underlying process is shorter than the interval of aggregation, in this case one year.

The autocorrelation structures of real GNP, nominal GNP, real per capita GNP, employment, nominal and real wages, and common stock prices display positive autocorrelation at lag one only which is characteristic of first-order MA processes. This representation of the data is inconsistent with the TS model. The only TS process that gives rise to autocorrelation only at lag one is the case of serially random deviations around the trend. The value of the lag one autocorrelation, however, would be −0.50. To salvage a TS

Table 3

Sample autocorrelations of the first difference of the natural logs of annual data.[a]

Series	Period	Sample autocorrelations							
		T	r_1	r_2	r_3	r_4	r_5	r_6	$s(r)$
Time aggregated random walk[b]			0.25	0.00	0.00	0.00	0.00	0.00	
Real GNP	1909–1970	62	0.34	0.04	−0.18	−0.23	−0.19	0.01	0.13
Nominal GNP	1909–1970	62	0.44	0.08	−0.12	−0.24	−0.07	0.15	0.13
Real per capita GNP	1909–1970	62	0.33	0.04	−0.17	−0.21	−0.18	0.02	0.13
Industrial production	1860–1970	111	0.03	−0.11	−0.00	−0.11	−0.28	0.05	0.09
Employment	1890–1970	81	0.32	−0.05	−0.08	−0.17	−0.20	0.01	0.11
Unemployment rate	1890–1970	81	0.09	−0.29	0.03	−0.03	−0.19	0.01	0.11
GNP deflator	1889–1970	82	0.43	0.20	0.07	−0.06	0.03	0.02	0.11
Consumer prices	1860–1970	111	0.58	0.16	0.02	−0.00	0.05	0.03	0.09
Wages	1900–1970	71	0.46	0.10	−0.03	−0.09	−0.09	0.08	0.12
Real wages	1900–1970	71	0.19	−0.03	−0.07	−0.11	−0.18	−0.15	0.12
Money stock	1889–1970	82	0.62	0.30	0.13	−0.01	−0.07	−0.04	0.11
Velocity	1869–1970	102	0.11	−0.04	−0.16	−0.15	−0.11	0.11	0.10
Bond yield	1900–1970	71	0.18	0.31	0.15	0.04	0.06	0.05	0.12
Common stock prices	1871–1970	100	0.22	−0.13	−0.08	−0.18	−0.23	0.02	0.10

[a]The first differences of the natural logs of all the data are used except for the bond yield. T is the sample size and r_i is the estimated ith order autocorrelation coefficient. The large sample standard error for r is given by $s(r)$ under the null hypothesis of no autocorrelation.

[b]Theoretical autocorrelations as the number of aggregated observations becomes large; result due to Working (1960).

representation for these series we would need to hypothesize the presence of an autoregressive component in the deviations from trend that has a root close enough to unity to obscure the effect of differencing on the autocorrelation structure. For example, suppose the deviations from trend were generated by the ARMA process

$$(1-\phi L)c_t=(1-\theta L)u_t \tag{10}$$

so that the first difference of z_t have the representation

$$(1-L)z_t=\beta+\frac{(1-L)(1-\theta L)}{(1-\phi L)}u_t. \tag{11}$$

The ratio $(1-L)/(1-\phi L)$ has the expansion $[1-(1-\phi)L-\phi(1-\phi)L^2-\ldots]$, which may be difficult to distinguish empirically from unity if ϕ is close to one, leaving the appearance of a first order MA process $(1-\theta L)$ for $\{(1-L)z_t\}$.

The GNP deflator, consumer prices, the money stock, and the bond yield, exhibit more persistent autocorrelation in first differences. None, however, shows evidence of being generated from a process containing MA terms with a unit root or AR terms arising from inversion of such an MA term as one would expect to find in a TS process that has been differenced. The presence of strong positive autocorrelation in deviations from trend (or from a fixed mean) may again be the explanation. The conclusion we are pointed toward is that if these series do belong to the TS class, then the deviations from trend must be sufficiently autocorrelated to make it difficult to distinguish them from the DS class on the basis of sample autocorrelations.

The evidence against the TS representation from levels and differences is reinforced by the sample autocorrelations of the deviations from fitted trend lines presented in table 4. The pattern is strikingly similar across series (except for the unemployment rate) starting at about 0.9 at lag one and declining roughly exponentially. The first two lines in table 4 give the expected sample autocorrelations for deviations of random walks of 61 and 101 observations from a fitted trend line [Nelson and Kang (1981)] and again suggest the consistency of the data with a simple form of the DS hypothesis.[11] Nelson and Kang also show that these results are rather insensitive to moderate autocorrelation in first differences, such as would be present in a time aggregated DS process.

[11]The approximate expected sample autocorrelations are based on the ratios of expected sample autocovariances. Simulation experiments by Nelson and Kang (1981) for 100 observations suggest that the exact expected sample autocorrelations are smaller. At lag one the mean sample autocorrelation was 0.88 instead of 0.91 and at lag 6 it was 0.43 instead of 0.51.

Table 4

Sample autocorrelations of the deviations from the time trend.[a]

		Sample autocorrelations						
Series	Period	T	r_1	r_2	r_3	r_4	r_5	r_6
Detrended random walk[b]		61	0.85	0.71	0.58	0.47	0.36	0.27
		101	0.91	0.82	0.74	0.66	0.58	0.51
Real GNP	1909–1970	62	0.87	0.66	0.46	0.26	0.19	0.07
Nominal GNP	1909–1970	62	0.93	0.79	0.65	0.52	0.43	0.05
Real per capita GNP	1909–1970	62	0.87	0.65	0.43	0.24	0.11	0.04
Industrial production	1860–1970	111	0.84	0.67	0.53	0.40	0.30	0.28
Employment	1890–1970	81	0.89	0.71	0.55	0.39	0.25	0.17
Unemployment rate	1890–1970	81	0.75	0.46	0.30	0.15	0.03	−0.01
GNP deflator	1889–1970	82	0.92	0.81	0.67	0.54	0.42	0.30
Consumer prices	1860–1970	111	0.97	0.91	0.84	0.78	0.71	0.63
Wages	1900–1970	71	0.93	0.81	0.67	0.54	0.42	0.31
Real wages	1900–1970	71	0.87	0.69	0.52	0.38	0.26	0.19
Money stock	1889–1970	82	0.95	0.83	0.69	0.53	0.37	0.21
Velocity	1869–1970	102	0.91	0.81	0.72	0.65	0.59	0.56
Bond yield	1900–1970	71	0.85	0.73	0.62	0.55	0.49	0.43
Common stock prices	1871–1970	100	0.90	0.76	0.64	0.53	0.46	0.43

[a]The data are residuals from linear least squares regression of the logs of the series (except the bond yield) on time. See footnote for table 3.

[b]Approximate expected sample autocorrelations based on Nelson and Kang (1981).

To carry out the formal tests of Dickey and Fuller we must estimate regressions of the form of eq. (9) which may be rewritten as

$$z_t = \mu + \gamma t + \rho_1 z_{t-1} + \sum_{j=2}^{k} \rho_j (z_{t+1} - z_{t-j}) + u_t. \tag{12}$$

To specify the maximum lag k we consider both the values that would be suggested by the autocorrelations of first differences and by the partial autocorrelations of the deviations from trend. In cases where MA models for first differences seem appropriate we fit AR approximations. In general the latter procedure indicates higher order autoregressions and our rule is to utilize the higher order models on the grounds that leaving out relevant terms might bias our results but inclusion of irrelevant ones would only reduce efficiency.[12]

The results of these regressions are reported in table 5. Recall that we are interested in testing whether ρ_1 differs from unity. The values of $\hat{\rho}$ range

[12]We do not report the sample partial autocorrelations of the deviations from trend. However, the pattern is very similar across almost all of the series; a sharp cut-off after lag two where there is negative and generally significant partial autocorrelation. These characteristics suggest second-order AR representations with complex roots and therefore pseudoperiodic behavior in the trend deviations, again a property of detrended random walks [Nelson and Kang (1981)].

Table 5

Tests for autoregressive unit roots[a]

$$z_t = \hat{\mu} + \hat{\gamma}t + \hat{\rho}_1 z_{t-1} + \hat{\rho}_2(z_{t-1} - z_{t-2}) + \cdots + \hat{\rho}_k(z_{t-k+1} - z_{t-k}) + \hat{\mu}_t.$$

Series	T	k	$\hat{\mu}$	$t(\hat{\mu})$	$\hat{\gamma}$	$t(\hat{\gamma})$	$\hat{\rho}_1$	$\tau(\hat{\rho}_1)$	$s(\hat{u})$	r_1
Real GNP	62	2	0.819	3.03	0.006	3.03	0.825	−2.99	0.058	−0.02
Nominal GNP	62	2	1.06	2.37	0.006	2.34	0.899	−2.32	0.087	0.03
Real per capita GNP	62	2	1.28	3.05	0.004	3.01	0.818	−3.04	0.059	−0.02
Industrial production	111	6	0.103	4.32	0.007	2.44	0.835	−2.53	0.097	0.03
Employment	81	3	1.42	2.68	0.002	2.54	0.861	−2.66	0.035	0.10
Unemployment rate	81	4	0.513	2.81	−0.000	−0.23	0.706	−3.55*	0.407	0.02
GNP deflator	82	2	0.260	2.55	0.002	2.65	0.915	−2.52	0.046	−0.03
Consumer prices	111	4	0.090	1.76	0.001	2.84	0.986	−1.97	0.042	−0.06
Wages	71	3	0.566	2.30	0.004	2.30	0.910	−2.09	0.060	0.00
Real wages	71	2	0.487	3.10	0.004	3.14	0.831	−3.04	0.034	−0.01
Money stock	82	2	0.133	3.52	0.005	3.03	0.916	−3.08	0.047	0.03
Velocity	102	1	0.052	0.99	−0.000	−0.65	0.941	−1.66	0.067	0.11
Interest rate	71	3	−0.186	−0.95	0.003	1.75	1.03	0.686	0.283	−0.02
Common stock prices	100	3	0.481	2.02	0.003	2.37	0.913	−2.05	0.158	0.20

[a] z_t represents the natural logs of annual data except for the bond yield. $t(\hat{\mu})$ and $t(\hat{\gamma})$ are the ratios of the OLS estimates of μ and γ to their respective standard errors. $\tau(\hat{\rho}_1)$ is the ratio of $\hat{\rho}_1 - 1$ to its standard error. $s(\hat{u})$ is the standard error of the regression and r_1 is the first-order autocorrelation coefficient of the residuals. The values of $\tau(\hat{\rho}_1)$ denoted by an (*) are smaller than the 0.05 one tail critical value of the distribution of $\tau(\hat{\rho}_1)$ and similarly for $\hat{\rho}_1$. It should also be noted that $t(\hat{\mu})$ and $t(\hat{\gamma})$ are not distributed as normal random variables.

from a low of 0.706 for the unemployment rate to a high of 1.03 for the bond yield. The majority of the estimates fall in the range 0.85 to 0.93 which is quite consistent with the mean of 0.900 and standard deviation of 0.054 reported in table 1 for realizations of a random walk. Also of interest is that all but two of the t-statistics for the hypothesis $\rho_1=1$ [i.e., $\tau(\hat{\rho})$] are significant by conventional standards. However, we know from the sampling experiments in table 1 that $\tau(\hat{\rho}_1)$ has a mean of about -2.22 under the null hypothesis that ρ_1 is unity. Using the distributions tabulated by Fuller (1976), only the unemployment rate exhibits a value of $\tau(\hat{\rho})$ below the 0.05 critical value of -3.45 for samples sizes of 100. In this case, $\hat{\rho}$ is also smaller than the 0.05 critical value given by Fuller. Moreover, there is no evidence from this regression that the slope is non-zero and we conclude that the series is well described as a stationary process.

To sum up, the evidence we have presented is consistent with the DS representation of non-stationaruty in economic time series.[13] We recognize that none of the tests presented, formal and informal, can have power against a TS alternative with an AR root arbitrarily close to unity. However, if we are observing stationary deviations from linear trends in these series then the tendency to return to the trend line must be so weak as to avoid detection even in samples as long as sixty years to over a century.

4. Stochastic representations of the secular component

Our tests in section 3 suggest that economic time series do not contain deterministic time trends but contain stochastic trends characteristic of the DS class of processes. To investigate the implications of this finding it is useful to focus on the behavior of output. Pursuing the decomposition discussed earlier we assume that actual output (presumably logged) can be viewed as the sum of a secular or growth component, $\bar{y}_t$, and a cyclical component, c_t. If the cyclical component is assumed to be transitory (stationary), then any underlying non-stationarity in output must be attributable to the secular component. Thus, if actual output can be viewed

[13]The contrasting implications of the TS and DS models for long-run uncertainty can be illustrated by real per capita GNP. Under the TS hypothesis, uncertainty about future values is founded by the marginal standard deviation of fluctuations around a linear trend which is estimated to be 0.133 (in natural logs) over the sample period. According to the DS model for this series, however, the standard deviation of forecast errors is given by $SD[e_t(k)]=0.062[1+(k-1)1.711]^{\frac{1}{2}}$, where $e_t(k)$ denotes the forecast error for k years in the future. This standard deviation exceeds 0.133 for any more than four years in the future and obviously grows without bound but at a decreasing rate. Taking into account an estimated mean growth rate of 0.016, the lower point of a 95% confidence interval reaches its minimum when the forecast horizon is 24 years. At a horizon of 24 years the standard deviation is 0.39 compared with accumulated mean growth of 0.38. Thus the possibility that actual real per capital would not only show no increase 24 years hence but decline by about 33% from its current level is not excluded in 95% interval.

as belonging to the DS class, then so must the secular component. This decomposition of output can be expressed as

$$\begin{aligned} y_t &= \bar{y}_t + c_t \\ &= (1-L)^{-1}\theta(L)v_t + \psi(L)u_t, \end{aligned} \tag{13}$$

where $\bar{y}_t = (1-L)^{-1}\theta(L)v_t$ or $(1-L)\bar{y}_t = \theta(L)v_t$, and $c_t = \psi(L)u_t$. The (possibly infinite order) polynomials $\theta(L)$ and $\psi(L)$ are assumed to satisfy the conditions for stationarity and invertibility and v_t and u_t are mean zero serially uncorrelated random variables. Eq. (13) assigns the non-stationarity of y to $\bar{y}$ through the factor $(1-L)^{-1}$. (Also note that we are ignoring any drift in $\bar{y}$ for convenience.) Separation of the secular component from observed data may be thought of as a problem in signal extraction when only information in the observed series itself is used, or it may be cast as a regression problem when determinants of the growth process are regarded as known and observable.

4.1. Regression strategies

Perhaps the ideal method of dealing with non-stationarity in output (i.e., growth or secular movements) is to include the variables in a regression that would account for such behavior. For example, Perloff and Wachter (1979), among many others, fit real GNP to labor, capital, and energy as inputs in a translog production function and represent technological change as a time trend. The first-order autocorrelation coefficient of the residuals, however, is reported to be 0.881, roughly the value expected from regression of a random walk on time given the number of observations in question. It would seem then that measured input and time trend variables may not adequately account for the growth component in real GNP.[14] Another regression strategy is to work in per capita values under the assumption that population is the primary source of non-stationarity. We can reject this strategy based on our results in section 3 that indicate per capita real GNP also belongs to the DS class. Thus, using observable variables to account for growth components seems unsatisfactory since neither factor inputs nor population seem to suffice and direct measures of technology are not readily available.[15]

[14]Plosser and Schwert (1979) discuss this issue and others that must be considered when intepreting regressions such as those estimated by Perloff and Wachter (1979).

[15]It is interesting that the DS nature of technological change is evident in the early empirical estimates of the implied stock of technology calculated by Solow (1957) for the period 1909–1949 as corrected by Hogan (1958). The log of Solow's $A(t)$ variable exhibits little autocorrelation in first differences ($Q(12) = 8.8$) and $\hat{\rho}$ and its τ-ratio for regression (12) are both close to their expected values under the Fuller/Dickey distribution, suggesting that the stock of technology is well characterized as a random walk.

4.2. Signal extraction and an unobservable components model of output

Signal extraction procedures imply, or are implied by, some model of the underlying component structure of the series. Therefore, it seems that prior to adoption of an unobserved components model, it should be investigated for consistency with the data, or, perhaps preferably, an attempt should be made to identify a class of models from the observed sample autocorrelations.

The classic example of signal extraction in economics is the permanent income model of Friedman (1957).[16] One version measures permanent income as an exponentially weighted average of past observed incomes. Muth (1960) shows that an optimal estimate of permanent income has that form if permanent income follows a random walk and transitory income is serially random and independent of changes in permanent income. The Friedman/Muth permanent income model may be written as a special case of (13) with $\theta(L)=\psi(L)=1$

$$y_t=\bar{y}_t+u_t, \qquad \bar{y}_t=\bar{y}_{t-1}+v_t, \tag{14}$$

where $\bar{y}_t$ is now the permanent component, generated by a random walk with innovations v_t, and u_t is the transitory component, a purely random series that is independent of v_t. The first differences are the stationary process

$$(1-L)y_t=v_t+u_t-u_{t-1}, \tag{15}$$

illustrating the general fact that differencing does not 'remove the trend' since the innovation in the permanent component, v_t, is part of the first difference. The first differences are autocorrelated at lag one only with coefficient

$$\rho_1=-\sigma_u^2/(\sigma_v^2+2\sigma_u^2), \tag{16}$$

which is confined to the range $-0.05\leqq\rho_1\leqq 0$ and depends on the relative variances of u and v. Apparently, this model cannot account for the positive autocorrelation at lag one only observed in the first difference of the historical series studied in section 3.

In general, if an unobserved components version of (13) is restricted *a priori* by assuming that (i) $\bar{y}_t$ is a random walk [i.e., $\theta(L)=1$] and (ii) v_t and u_t are independent, then the parameters of the unobserved components model will be identified. This is clearly the case for the permanent income model

[16]For a general discussion of signal extraction in economic time series see Pierce (1978) and Nerlove, Grether, and Carvalho (1979). Some recent examples of signal extraction techniques applied to unobserved components models are Beveridge and Nelson (1981) and Hodrick and Prescott (1980).

since σ_u^2 is computable from the autocovariance of the first differences at lag one [the numerator of (16)] and σ_v^2 from the variance of the first differences [the denominator of (16)] and the computed value of σ_u^2. If the cyclical or stationary component of (13) has the MA representation $\psi(L)u_t$ and is of order q, then the first difference will be

$$(1-L)y_t = v_t + (1-L)\psi(L)u_t \tag{17}$$

with non-zero autocovariances through lag $(q+1)$. The value of $(q+1)$ can in principle be inferred from a realization of y. There are then $(q+2)$ parameters to be solved for from the $(q+2)$ autocovariance relations implied by (17), using values for the autocovariances computed from the data.[17]

It is clear from our discussion, however, that a decomposition satisfying both the above restrictions is not always feasible. The simplest example is a process with positive autocorrelation in first differences at lag one only. Eq. (17) implies that the Friedman/Muth model is the only linear model that satisfies both restrictions and leads to non-zero autocorrelation at lag one only. However, it is unable to account for positive autocorrelation at lag one only. To do so we must relax either the assumption that $\bar{y}$ is a random walk (i.e., containing no transitory, only permanent movements) or the assumption that v and u are independent. In general, if either of these assumptions is relaxed the parameters of the unobserved components model are not identified.

Nevertheless, the assumption that the cyclical component is stationary combined with the observation that autocorrelations in the first differences of output are positive at lag one and zero elsewhere are sufficient to imply that the variation in actual output changes is dominated by changes in secular component $\bar{y}_t$ rather than the cyclical component c_t.

The above proposition can be demonstrated by considering first differences of (13)

$$(1-L)y_t = \theta(L)v_t + (1-L)\psi(L)u_t. \tag{18}$$

The presence of first-order autocorrelation only in $(1-L)y_t$ implies (barring fortuitous cancelations) that $\theta(L)$ is first-order and $\psi(L)$ is zero order so that we can write

$$(1-L)y_t = v_t + \theta v_{t-1} + u_t - u_{t-1}, \tag{19}$$

[17]The argument is easily extended to the case where the stationary component includes AR terms since they may be inferred from the autocovariances of the first differences for lags greater than $(q+1)$ using Yule–Walker equations. We note, however, that we do not have a formal proof that the non-linear autocovariance equations will always have a solution or unique solution in terms of invertible values of the ψ's.

with $|\theta|<1$ being required by invertibility. While u_t and v_t may be contemporaneously correlated, lagged cross-correlations would imply higher than first-order autocorrelation in $(1-L)y_t$ and therefore are ruled out. The autocovariance of output changes at lag one is therefore

$$\gamma_1=\theta\sigma_v^2-(1-\theta)\sigma_{uv}-\sigma_u^2, \tag{20}$$

where σ_{uv} is the contemporaneous covariance between u and v. Note that γ_1 consists of the autocovariance of the change in the secular component, $\bar{y}$, at lag one, $\theta\sigma_v^2$, the sum of the cross-covariances at lag one, $-(1-\theta)\sigma_{uv}$, and the autocovariance of the change is the cyclical component, c, at lag one, $-\sigma_u^2$, which is necessarily negative. The factors that would account for the positive value of γ_1 we observe are therefore (1) a positive value of θ (positive autocorrelation in first differences of the secular component) combined with a sufficiently large value of σ_v^2, and/or (2) a sufficiently large negative value of the covariance σ_{uv} which also puts a lower bound on σ_v^2 due to the familiar inequality $\sigma_u\sigma_v\geqq|\sigma_{uv}|$. We now prove that if $\gamma_1>0$ then $\sigma_v^2>\sigma_u^2$.

Since the value of σ_{uv} is unknown, consider first the case $\sigma_{uv}\geqq 0$. For $\gamma_1>0$ and $\sigma_{uv}\geqq 0$, eq. (20) implies that $\theta>0$, i.e., the secular component must be positively autocorrelated. Given this, eq. (20) also implies

$$\sigma_v^2>\theta^{-1}\sigma_u^2+(\theta^{-1}-1)\sigma_{uv}>\sigma_u^2,$$

using the fact that $0<\theta<1$. The other possible case is $\sigma_{uv}<0$. Using the fact that $\sigma_u\sigma_v\geqq|\sigma_{uv}|$, we have

$$\theta\sigma_v^2+(1-\theta)\sigma_u\sigma_v-\sigma_u^2\geqq\theta\sigma_v^2-(1-\theta)\sigma_{uv}-\sigma_u^2>0.$$

Factoring the first expression yields

$$(\theta\sigma_v+\sigma_u)(\sigma_v-\sigma_u)>0$$

and hence both factors must be positive or negative. If they are both positive then the second factor gives us $\sigma_v>\sigma_u$. Note that there is nothing in this case to prevent θ from being negative since if the first factor is positive we have only that $\theta>-(\sigma_u/\sigma_v)$. If both factors are negative then the first factor would imply $\theta<0$ but also that $\sigma_v>-\theta^{-1}\sigma_u>\sigma_u$ (again using the fact that $0<\theta<1$), however, the second factor would imply $\sigma_v<\sigma_u$ thus leading to a contradiction that rules out negative factors. We conclude therefore that the standard deviation of innovations in the secular or growth component is larger than the standard deviation of innovations in the cyclical component.

We can now use these results to obtain a plausible range of values of σ_v/σ_u under alternative assumptions. Consider first the case $\sigma_{uv}=0$, so that the growth and cyclical components are uncorrelated. From (19) and (20) it is easy to show that

$$\rho_1 = \frac{\theta\sigma_v^2 - \sigma_u^2}{(1+\theta^2)\sigma_v^2 + 2\sigma_u^2}, \quad \text{or}$$

$$\sigma_v/\sigma_u = [-(1+2\rho_1)/\rho_1(1+\theta^2-\rho_1^{-1}\theta)]^{\frac{1}{2}}. \tag{21}$$

Our empirical results give us a relevant range of values for ρ_1 and we know that $0<\theta<1$ when $\sigma_{uv}=0$. Computed values of the ratio of the standard deviations are given in the following table:

Values of σ_v/σ_u for various values of ρ_1 and θ when $\sigma_{uv}=0$.

	θ			
ρ_1	0.3	0.5	0.6	0.8
0.1	2.5	1.8	1.6	1.4
0.3	—	3.6	2.9	2.3
0.4	—	∞	5.7	3.5

The blanks in the table are due to the fact that ρ_1 cannot be larger than $\theta/(1+\theta^2)$ regardless of how large we make σ_v^2/σ_u^2. The values in the table suggest that the standard deviation of innovations in the non-stationary component may be several times larger than the standard deviation of innovations in the cyclical component.

Now consider the case where the secular or non-stationary component is a strict random walk, so that $\theta=0$. The value of ρ_1 is then given by

$$\rho_1 = \frac{-\rho_{uv} - (\sigma_v/\sigma_u)^{-1}}{(\sigma_v/\sigma_u) + 2(\sigma_v/\sigma_u)^{-1} + 2\rho_{uv}},$$

where ρ_{uv} is the contemporaneous correlation between u and v. To account for positive values of ρ_1, ρ_{uv} must be negative, in fact $\rho_{uv} < -(\sigma_v/\sigma_u)^{-1} < 0$. Thus, imposing the random walk assumption on $\bar{y}$ implies either strong negative correlation between u and v, or a large variance ratio, or both. This is borne out by the values of ρ_{uv} and (σ_v/σ_u) consistent with observed values of ρ_1 presented in the following table:

Values of σ_v/σ_u for various values of ρ_1 and ρ_{uv} when $\theta=0$.

	ρ_{uv}		
ρ_1	−0.2	−0.6	−0.9
0.0	5.0	1.7	1.1
0.1	—	4.5	1.2
0.3	—	—	1.8

The blanks indicate values of ρ_1 and ρ_{uv} that are inconsistent with any σ_v/σ_u. It is interesting to note that the magnitude of σ_v/σ_u implied by assuming $\sigma_{uv}=0$ or $\theta=0$ are similar.

The above results are dependent on the stochastic structure of output being a first-order MA process with positive autocorrelation at lag one. As mentioned in section 3, the positive autocorrelation could be attributed entirely to temporal aggregation. If this is the case then our inferences about σ_v/σ_u are distorted since it is well-known that time aggregation amplifies low frequency (i.e., long-run) movements relative to high frequency (i.e., short-run) movements. However, we are somewhat reluctant to accept this interpretation of the results since it implies that the short-run or cyclical variability we are reducing through aggregation is variation that is dissipated within the aggregation interval of a year [see Tiao (1972)]. Another way of making this point is to say that by looking at annual data, we can make no inference regarding the variance of components whose memory (or life) is less than a year. We do not believe, however, this is a significant disadvantage of the annual time interval since most economists probably identify business cycles (transitory components) with periods that are longer than a year.[18]

It is instructive to contrast our analysis to the signal extraction strategy proposed by Hodrick and Prescott (1980). Hodrick and Prescott decompose observed variables into growth and cyclical components under the maintained hypothesis that the growth component moves smoothly through time. The standard deviation of innovations in the growth component is assumed to be very small relative to the standard deviation of innovations in the cyclical component (specifically 1/40th). Optimal estimates are chosen through a criterion function that penalizes variance in the second differences of the growth component as well as variance in the cycle. A linear time trend emerges as a limiting case.

The Hodrick and Prescott strategy implicitly imposes a components model on the data without investigating what restrictions are implied (a difficult task in their model) and whether those restrictions are consistent with the data. Our strategy, on the other hand, is to use the data as an aid in identifying certain characteristics of an appropriate components model. Our results suggest that the ratio of the standard deviations of growth to cyclical innovations has a minimum in the neighborhood of one with likely values up to five or six rather than the value of 1/40th assumed by Hodrick and Prescott.

[18]Although we have not carried out an analysis using quarterly data, our experience with such data suggests that our conclusions are not likely to be sensitive to the interval of observation. In other words, the autocorrelation structure of the quarterly data are not much different from that observed in the annual data.

5. Some implications for business cycle theorizing

The analysis of unobserved components models leads us to the inference that if (a) output is the sum of a non-stationary component of the DS class and a stationary (transitory) component, and (b) we observe non-negative autocorrelation at lag one only in the first differences of output then (i) the variance of the innovations in the non-stationary component must be as large or larger than that of the purely stationary or transitory component, and (ii) either the non-stationary component contains significant transitory components (i.e., it is not a random walk) or, if the non-stationary component is assumed to be a random walk, the innovations in the random walk are correlated with the transitory component.

These inferences have potentially important implications for business cycle research. For example, most of the recent developments in business cycle theory stress the importance of monetary disturbances as a source of output fluctuations.[19] However, the disturbances are generally assumed to have only transitory impact (i.e., monetary disturbances have no permanent real effects).[20] Therefore, the inference that the innovations in the non-stationary component have a larger variance than the innovations in a transitory component implies that real (non-monetary) disturbances are likely to be a much more important source of output fluctuations than monetary disturbances.[21] This conclusion is further strengthened if monetary disturbances are viewed as only one of several sources of cyclical disturbances. In addition, while we have focused on real GNP, we believe the fact that other real variables such as real per capita GNP, employment, and real wages have similar characteristics provides some corroborating evidence. In fact, by investigating in detail several series jointly one might be able to get a more complete picture of the relative sizes of various shocks.[22] Several additional points are worth noting. First there is nothing in theory or in our empirical results that implies that the unobserved components model of (13) is economically meaningful. For example, we cannot reject the hypothesis that actual output contains only one non-stationary component (i.e., $\sigma_u^2=0$) and thus observed autocorrelation simply reflects autocorrelation in movements in a stochastic growth component. Indeed, a stochastic growth process that contains both permanent and

[19]For example, see the models of Lucas (1975) and Barro (1976).

[20]We are ignoring in this discussion the potential permanent effects of inflation in the models described by Tobin (1965), Stockman (1982), and others.

[21]As noted near the end of section 3, given the observed behavior of output, this result holds even if monetary disturbances and real (non-stationary) disturbances are correlated (perhaps through policy response).

[22]For example one might be able to use a known decomposition of output to in turn decompose the unemployment rate into 'natural' and 'cyclical' movements in the unemployment rate. Such an effort, however, would probably require more structure to the problem that we have used here.

transitory characteristics can arise in the models developed by Long and Plosser (1980) and Kydland and Prescott (1981). In these models, dynamic competitive equilibrium is capable of generating fluctuations in a 'natural rate of output' that, in many ways, mimics the behavior of observed output.

Second, we also cannot prove empirically that cyclical fluctuations are stationary. The stationarity of this component is also an assumption, but one we believe most economists would accept. Nevertheless, the hypothesis that the business cycle is a stochastic process of the DS class is not refutable from the empirical evidence. The general point is that some unobserved components representations are rejected by the data, but the data by itself cannot reveal the true structure.

6. Summary and conclusions

In this paper we try to distinguish between two alternative hypotheses concerning the nature of non-stationarity in macroeconomic time series, one is the widely held view that such series represent stationary fluctuations around a deterministic trend and the other is that non-stationarity arises from the accumulation over time of stationary and invertible first differences. Our test results are consistent with the latter hypothesis and would be consistent with the former only if the fluctuations around a deterministic trend are so highly autocorrelated as to be indistinguishable from non-stationary series themselves in realizations as long as one hundred years.

The distinction between the two classes of processes is fundamental and acceptance of the purely stochastic view of non-stationarity has broad implications for our understanding of the nature of economic phenomena. For example, if aggregate output is thought of as consisting of a non-stationary growth component plus a stationary cyclical component, then the growth component must itself be a non-stationary stochastic process rather than a deterministic trend as has been generally assumed in empirical work. Instead of attributing all variation in output changes to the cyclical component, the stochastic model allows for contributions from variations in both components. Therefore, empirical analyses of business cycles based on residuals from fitted trends lines are likely to confound the two sources of variation, greatly overstating the magnitude and duration of the cyclical component and understating the importance of the growth component. Moreover, to impose the trend specification is to assume away long-run uncertainty in these variables and to remove much of their variation *a priori*.

We also remind the reader that first differencing does not remove a stochastic growth component although it may render the series stationary. The first differences of the observed series will consist of the sum of the first differences of both the secular and cyclical components. While first differences do not exhibit the spurious periodicity of trend residuals neither do they discard variation in the secular component; the problem of inferring the behavior of each unobserved component from the sum remains.

Finally, the empirical observation that changes on real output (as well as employment and real wages) displays non-negative autocorrelation at lag one and zero elsewhere suggests that shocks to the secular or non-stationary component account for a substantial portion of the variation observed. Assigning a major portion of variance in output to innovations in this non-stationary component gives an important role to real factors in output fluctuations and places limits on the importance of monetary theories of the business cycle.

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