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NOTES

ARIMA AND COINTEGRATION TESTS OF PPP UNDER FIXED AND FLEXIBLE EXCHANGE RATE REGIMES

Walter Enders*

Abstract—Real exchange rates between the United States and its major trading partners were calculated for the Bretton Woods and flexible exchange rate periods. Unit root tests indicate that Purchasing Power Parity performed poorly in both periods. Tests for cointegration reveal limited instances in which it is possible to estimate the deviations from PPP as an error correcting model. The estimated error correcting models indicate that foreign, but not U.S., prices responded to deviations from PPP.

Frenkel's (1981b) finding that Purchasing Power Parity (PPP) worked better during the 1920s than the 1970s caused considerable controversy. For example, Davutyan and Pippenger (1985) contend that the so-called "collapse" of PPP is a result of an increase in the relative importance of real versus monetary shocks. They argue that the 1970s, as opposed to the 1920s, was characterized by real supply shocks and the international coordination of monetary policies. The argument is that PPP did not fail; rather, there was an increase in the volatility of those factors giving rise to deviations from PPP. Hakkio (1984) reestimated PPP over the 1920s and 1970s; using cross-country tests (i.e., SURE estimates) to improve the efficiency of his estimates, he was able to support the hypothesis that PPP worked better in the 1970s than in the 1920s. On the other hand, papers by Adler and Lehman (1983), Dornbusch (1980), Frenkel (1981a), Junge (1985), and Krugman (1978) report findings contrary to the PPP hypothesis. Moreover, Kenen and Rodrik (1986) find that the volatility of real exchange rates has increased throughout the flexible rate period.

This paper tries to shed some light on the importance and persistence of the observed deviations from Purchasing Power Parity under alternative exchange rate systems. While it is interesting to compare PPP in the 1920s versus the 1970s, it is equally useful to compare the 1960s versus the 1970s and 1980s. If real supply shocks and lack of monetary coordination are characteristic of the latter period, PPP should perform better in the 1960s.

To illustrate the issues involved, consider the following econometric model of (Relative) Purchasing Power Parity:

$$Ex(t)P^*(t) - \alpha P(t) = d(t) \tag{1}$$

where $Ex(t)$ = U.S. dollar price of foreign exchange in period t relative to a base year; $P^*(t)$ = index of the foreign price level in t; $P(t)$ = index of the U.S. price level in t; $d(t)$ is a stochastic disturbance representing a deviation from PPP; and α = constant.

Using equation (1), long-run PPP is said to hold if $\alpha = 1$ and the $d(t)$ series is stationary. The difficulty in estimating equation (1) is that all variables are jointly determined. This paper employs two alternative tests (ARIMA and Cointegration) which can be used to estimate the Purchasing Power Parity relationship under fixed or flexible exchange rates.

I. The ARIMA Model

This section of the paper attempts to characterize the deviations from Purchasing Power Parity using an ARIMA model. Rewrite equation (1) as

$$\frac{Ex(t)P^*(t)}{P(t)} = \alpha + d1(t) \tag{2}$$

where $Ex(t)P^*(t)/P(t)$ = "real" exchange rate in t; and $d1(t)$ is a stochastic disturbance. For notational simplicity, define the real exchange rate as

$$R(t) = Ex(t)P^*(t)/P(t).$$

In this form, standard Box-Jenkens ARIMA techniques can be used to characterize the $d1(t)$ series and to provide an estimate of α. Clearly, for PPP to hold, $d1(t)$ must be integrated of order zero. If, for example, $d1(t)$ is ARIMA $(n, 0, 0)$, the real rate can be estimated by

$$R(t) = \alpha_0 + \sum_{i=1}^{n} \alpha_i R(t-i) + \epsilon(t) \tag{3}$$

where the estimate of α in equation (2) is $\alpha_0/(1 - \Sigma\alpha_i)$ and $\epsilon(t)$ is white noise.

Notice that equation (2) and/or (3) can be estimated regardless of whether the exchange rate is fixed or

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flexible. If the exchange rate is fixed (i.e., $Ex(t) = Ex(t-1) = \ldots$), PPP predicts that nations will ultimately have the same inflation rate; deviations from PPP are deviations in the domestic inflation rate from the foreign rate. Given the specification of equation (3), PPP in either a fixed or a flexible rate system requires that $\alpha_0/(1 - \Sigma\alpha_i) = 1$ and for all characteristic roots to lie within the unit circle.

Using monthly data from the International Monetary Fund's *International Financial Statistics*, real exchange rates for three major U.S. trading partners—Germany (representing Europe), Canada (representing North America), and Japan (representing Asia)—were constructed. The data series was divided into two periods: January 1960–April 1971 (representing a period of fixed exchange rates) and January 1973–November 1986 (representing a period of flexible exchange rates).[1] Each nation's Wholesale Price Index was multiplied by an index of the U.S. dollar price of the domestic currency and then divided by the U.S. Wholesale Price Index. For each period, the relative version of PPP was tested by forcing the real exchange rate to equal unity at the beginning of the period.

The Maximum Likelihood estimates of the "best" ARIMA models are reported in table 1.[2] There are several interesting points to note:

1. For each country in each period, the predicted steady-state value of the real exchange rate does not statistically differ from unity. In table 1, the first entry in the column labeled "Mean" is the point estimate of the expression $\alpha_0/(1 - \Sigma\alpha_i)$. Since all such values are within two standard deviations of unity, one of the conditions for the long-run version of Purchasing Power Parity is satisfied.

2. Point estimates of the characteristic roots indicate that real exchange rates are convergent. For the four AR(1) models, the point estimates of the slope coefficients are all less than unity. In the post-Bretton Woods period (1973–1986), the point estimates of the characteristic roots for Japan's second order process are

[1] There is some subjectivity in determining the line of demarcation between the two periods. For purposes of this paper, the fixed rate period runs through April 1971 (before the sharp devaluations of the dollar were widely anticipated). The flexible rate period (beginning January 1973) captures the second major devaluation of the dollar and the 1973 oil price shock.

[2] Details of the estimation procedure are available from the author.

TABLE 1.—MAXIMUM LIKELIHOOD ARIMA ESTIMATES
$R(t) = \alpha_0 + \alpha_1 R(t-1) + \alpha_2 R(t-2)$

	α_1	α_2	Mean	rho/DW	Chi-sq	SD/SEE	H_0: $\alpha_1 = 1$ t stat.
1973–1986							
Canada	0.97805		1.0556	0.059	0.194	5.47	1.421
	(0.01545)		(0.0344)	1.883		1.16	
Japan	1.25321	−0.29737	1.0077	−0.007	0.226	10.44	N.A.
	(0.07488)	(0.07529)	(0.0458)	2.008		2.81	
Germany	1.22218	−0.24712	1.1124	−0.014	0.858	20.68	N.A.
	(0.07625)	(0.07625)	(0.0998)	2.035		3.71	
1960–1971							
Canada	0.96947		1.0165	−0.107	0.434	0.0141	1.593
	(0.01916)		(0.0084)	2.206		0.0036	
Japan	0.97068		0.9805	0.046	0.330	0.0168	1.045
	(0.02806)		(0.0150)	1.908		0.0054	
Germany	0.98498		1.0065	0.038	0.097	0.0257	1.226
	(0.01225)		(0.0165)	1.925		0.0039	
Tests for AR(2) Processes							
			F		Prob. > F		
Japan	$\alpha_1 + \alpha_2 = 1$		3.5653		0.0609		
Germany	$\alpha_1 + \alpha_2 = 1$		2.1640		0.1433		

Notes:
1. The standard errors are in parentheses.
2. Mean is the estimated mean of the series: Mean = $\alpha_0/(1 - \alpha_1 - \alpha_2)$.
3. SD is the actual standard of the real exchange rate; SEE is the estimated standard deviation of the residuals (i.e., the standard error of the estimate); Chi-sq is the significance level of the joint test that inclusion of additional lags up to lag 12 adds additional explanatory power to the regression; DW is the Durbin-Watson statistic for first-order autocorrelation and "rho" is the estimated correlation of the residuals.
4. t-stat. is the t-statistic for the hypothesis $\alpha_1 = 1$. For the AR(2) processes, the appropriate hypothesis test is $\alpha_1 + \alpha_2 = 1$; the calculated F statistic and the significance level of that value of F, are reported separately.

0.93525 and 0.31796; for Germany the roots are 0.96649 and 0.25569. However, for all countries, the (dominant) root is not statistically different from unity at conventional significance levels; the random walk hypothesis cannot be rejected. For the four AR(1) models, the slope coefficients are never more than 1.6 standard deviations from unity. To test for convergence of an AR(2) model, consider the linear restriction $\alpha_1 + \alpha_2 = 1$; if this restriction holds, one of the roots will equal unity and the other will equal $(\alpha_1 - 1)$. If it is not possible to reject this restriction of the coefficients, it is not possible to reject the hypothesis that the real exchange rate follows a random walk. As shown at the bottom of table 1, a standard F-test indicates that it is not possible to reject (i.e., it is possible to accept) this hypothesis for Germany at the 1%, 5% or 10% levels. A similar test for Japan seemingly reveals that it is possible to reject this hypothesis for Japan at the 10% significance level. However, under the null hypothesis of non-stationarity, confidence intervals for hypothesis testing are greatly expanded. Using the expanded confidence intervals of Dickey and Fuller (1981), it is not possible to reject the hypothesis that, for Japan, $\alpha_1 + \alpha_2 = 1$ at even the 10% level.

Even if the random walk hypothesis could be rejected, the point estimates indicate that there is a great amount of persistence in any deviation from Purchasing Power Parity. For example, the fastest speed of adjustment is exhibited by Japan's real rate during the 1973–1986 period. Yet, even for this case, the half-life of any deviation from Purchasing Power Parity is over 15 months.[3]

3. As measured by the standard deviation (SD), real exchange rates were far more volatile in the 1973–1986 period than in the 1960–1971 period. Moreover, real exchange rate volatility is associated with real rate unpredictability (as measured by the standard error of the estimate). Table 1 shows that for each country, the standard error of the estimate (SEE) during the flexible rate period exceeds that of the fixed rate period by several hundred-fold. Within the flexible rate period, ordering nations according to the standard deviation of their real rate yields the same ordering as does the standard error of the estimate.

4. Overall, the results find mixed support for the claim that Purchasing Power Parity collapsed during the 1970s. It is true that the average deviation was smaller and more predictable during the 1960s. However, the point estimates of the characteristic roots indicate faster convergence in the aftermath of Bretton Woods. The most important point is that it is not possible to reject the hypothesis of a unit root for any of the estimated series; in this sense, it is hard to claim that Purchasing Power Parity held in either period.

[3] For each period, the three equations were jointly estimated using SURE as suggested by Hakkio (1984). The SURE point estimates and the qualitative results concerning non-stationarity are almost identical to those reported in table 1.

II. Cointegration and the Error Correcting Model

The ambiguity concerning the convergence of the real exchange rate might possibly be resolved by testing for cointegration between the series $P(t)$ and $Ex(t)P^*(t)$. Following Engle and Granger (1987), the following steps were used to estimate α and to test for stationarity:

Step 1

Regress $Ex(t)P^*(t)$ on $P(t)$ to get the "equilibrium" regression or long-run PPP relationship. It can be proven that the estimated slope coefficient of the equilibrium regression is a consistent estimator of α if $d(t)$ is stationary while $Ex(t)P^*(t)$ and $P(t)$ are non-stationary.

The estimates of the "equilibrium" regressions are presented in table 2. The results conflict with the ARIMA tests in that five of the six estimated values of α are significantly below unity; the other is significantly above unity. Notice that Germany, in the flexible rate period, yields the poorest fit; the estimated value of α is furthest from unity and the R-square is remarkably low for a time series regression.

Step 2

Check the residuals of the "equilibrium" regression for stationarity using the Dickey-Fuller (1981) test for unit roots. Specifically, let $\hat{d}(t)$ be the estimated residual of the equilibrium regression. If $d(t)$ is stationary, the coefficient for π_0 in the following regression should be statistically different from zero:

$$(1 - L)\hat{d}(t) = -\pi_0\hat{d}(t-1) + \sum_{i=1}^{n}(1 - L)\pi_i\hat{d}(t-i) + v(t) \tag{4}$$

where $v(t)$ is an i.i.d. disturbance with zero mean and L is the lag operator.

TABLE 2.—THE EQUILIBRIUM REGRESSIONS

	Germany	Japan	Canada
1973–1986			
Estimated α	0.5734	0.8938	0.7749
	(0.0415)	(0.0316)	(0.0077)
R-square	0.5454	0.8341	0.9847
1960–1971			
Estimated α	0.6660	0.7361	1.0809
	(0.0262)	(0.0154)	(0.0200)
R-square	0.8283	0.9448	0.9544

TABLE 3.—DICKEY-FULLER TESTS OF RESIDUALS

	Germany	Japan	Canada
1973–1986			
$n = 0$: Estimated π_0	−0.0225	−0.0151	−0.1001
(standard error)	(0.0169)	(0.0236)	(0.0360)
t for $\pi_0 = 0$	−1.331	−0.6398	−2.781
$n = 4$: Estimated π_0	−0.0316	−0.0522	−0.0983
(standard error)	(0.0170)	(0.0236)	(0.0388)
t for $\pi_0 = 0$	−1.859	−2.212	−2.533
1960–1971			
$n = 0$: Estimated π_0	−0.0189	−0.1137	−0.0528
(standard error)	(0.0196)	(0.0449)	(0.0286)
t for $\pi_0 = 0$	−0.966	−2.535	−1.846
$n = 4$: Estimated π_0	−0.0294	−0.1821	−0.0509
(standard error)	(0.0198)	(0.0530)	(0.0306)
t for $\pi_0 = 0$	−1.468	−3.437	−1.663

If π_0 is not statistically different from zero, it is not possible to reject the null hypothesis of no cointegration (i.e., if $\pi_0 = 0$, it is possible to conclude that the $\hat{d}(t)$ series is not stationary). Table 3 shows the estimates of π_0 for 2 different lag lengths ($n = 0$ and $n = 4$). Japan in the fixed rate period shows the greatest likelihood of cointegration. With four lags of $\hat{d}(t-i)$ the critical value for t at the 5% significance level is 3.17.[4] The next best candidate is Canada during the flexible rate period; it is possible to reject the null hypothesis of no cointegration (i.e., conclude that the residuals are stationary) at slightly more than the 10% significance level.

Step 3

If the null of no cointegration is rejected, the residuals of the "equilibrium" regression can be used as instruments to estimate an error-correcting model. Consider the following two equations for Canada during the 1973–1986 period:

$$(1-L)\,Ex(t)\,P^*(t) = \underset{(0.00173)}{0.00688} - \underset{(0.03773)}{0.109558}\,\hat{d}(t-1) \tag{5}$$

$$(1-L)\,P(t) = \underset{(0.00103)}{0.00819} - \underset{(0.02249)}{0.01228}\,\hat{d}(t-1) \tag{6}$$

where $Ex(t)$ is the Canadian dollar rate; $P^*(t)$ is the Canadian WPI; $P(t)$ is the U.S. WPI; and $\hat{d}(t-1)$ is the residual of the Canadian equilibrium regression.

Equation (5) is quite well behaved—the Canadian price level and/or the foreign exchange value of the Canadian dollar fall in response to a positive deviation from PPP. Simply put, it is estimated that approximately 11% of any deviation in PPP is corrected within a month by a fall in the U.S. dollar value of Canadian prices.

Notice that the coefficient of the error-correcting term in equation (6) is not significantly different from zero. To the extent that Canada is a small country relative to the United States, we would not expect the U.S. price level to respond to any deviation from its PPP level with Canada. For equations (5) and (6), tests for longer lag lengths did not reveal that additional lags added significant explanatory power.

For Japan during the fixed rate period, the estimated error-correcting model is

$$(1-L)\,Ex(t)\,P^*(t) = \underset{(0.00044)}{0.00119} - \underset{(0.04184)}{0.10548}\,\hat{d}(t-1) \tag{7}$$

$$(1-L)\,P(t) = \underset{(0.00033)}{0.00156} + \underset{(0.03175)}{0.01114}\,\hat{d}(t-1) \tag{8}$$

where all $Ex(t) = 1$; $P^*(t)$ is the Japanese WPI; $P(t)$ is the U.S. WPI; and $\hat{d}(t-1)$ is the residual of the U.S./Japanese "equilibrium" regression.

The error-correcting model for Japan during the 1960–1971 period shows a strong tendency to return to PPP; over 10% of any positive (negative) differential between the United States and Japanese Wholesale Price Indices was eliminated each month by a rise (fall) in the Japanese WPI. The U.S. WPI, however, did not respond to a divergence between the U.S. and Japanese price indices. This result is consistent with the idea that the United States was a large country relative to Japan—movements in the U.S. price level were exogenous to Japanese variables. On the other hand, the Japanese WPI rose as the U.S. price level rose.

III. Conclusion

The results of the ARIMA tests provide mixed support for the PPP hypothesis. In the Bretton Woods and flexible exchange rate periods, the point estimates of the long-run real exchange rate for Canada, Japan, and Germany did not significantly differ from unity. Point estimates for all countries indicated that real exchange rates are convergent. However, all confidence intervals are sufficiently large that the null hypothesis that the real rate follows a random walk cannot be rejected.

[4] Under the null hypothesis of non-stationarity, it is not possible to use the standard confidence levels for t-tests. Engle and Granger (1987) find the critical values for the t-statistic on π_0 to be 4.07, 3.37, and 3.03 at the 1%, 5%, and 10% levels, respectively. With four lags of the residual, the critical values are 3.77, 3.17, and 2.84 at the 1%, 5%, and 10% levels, respectively.

Tests for cointegration provide mixed evidence of PPP. Point estimates of long-run real rates (α) are far from unity. However, there is strong support for cointegration of the U.S. and Japanese price levels during the Bretton Woods period and weak support for cointegration of the U.S. and Canadian price levels after 1973. The implied error correcting models are in accord with the intuitive idea that the United States was a large country relative to its trading partners. Foreign, but not U.S., prices adjusted to any deviation from PPP. Other than an increase in the variability and a decline in the predictability of real rates, there is little evidence to support the claim that PPP collapsed during the 1970s. The point estimates and tests for stationarity indicate that PPP performs equally well, or equally poorly, in both time periods.

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COINTEGRATION AND TESTS OF PURCHASING POWER PARITY

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Abstract—Nonstationarity in the levels of spot exchange rates and domestic and foreign price indices makes the use of conventional tests of the absolute version of purchasing power parity (PPP) inappropriate. If PPP is true, inter-country commodity arbitrage ensures that deviations from a linear combination of spot exchange rates and domestic and foreign price levels should be stationary. Under these conditions, exchange rates and price levels should form a cointegrated system. We find the null hypothesis of no cointegration cannot be rejected for all five countries, thus violating the "long-run" absolute version of PPP.

I. Introduction

A plethora of theoretical and empirical models of exchange rate behavior has been built around purchasing power parity (PPP).[1] Conventional tests of PPP, which primarily use two-stage least squares and then test coefficient restrictions, find little recent evidence in favor of the empirical validity of the absolute version of PPP.[2] However, these tests neglect the fact that the levels of spot exchange rates and domestic and foreign price indices are typically nonstationary, which makes the use of standard critical values inappropriate.[3]

In contrast, this paper draws on the theory of cointegrated processes to test whether PPP holds as a "long-run" equilibrium relation.[4] The equilibrium relationship captured in the absolute version of PPP (as an aggregate interpretation of the law of one price) assumes that perfect commodity arbitrage acts as an error correction mechanism to force the dollar price of a consumption bundle of U.S. goods in line with the dollar price of a common bundle of foreign goods. If PPP is true, inter-country commodity arbitrage ensures that deviations from a linear combination of spot exchange rates and domestic and foreign price levels should be stationary. Since a cointegrated system allows individual time series to be integrated of order one, but requires a linear

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[1] Most versions of the monetary approach to exchange rate determination assume a strict adherence to short-run or continuous PPP (see Dornbush (1980), p. 145).

[2] See, for example, Frenkel (1981a, b).

[3] The effect of nonstationarity on standard testing procedures is examined in papers by Phillips (1987) and Park and Phillips (1987).

[4] Interpreting cointegration as a long-run equilibrium relation was proposed by Engle and Granger (1987), and will be discussed in detail in section III of this paper.