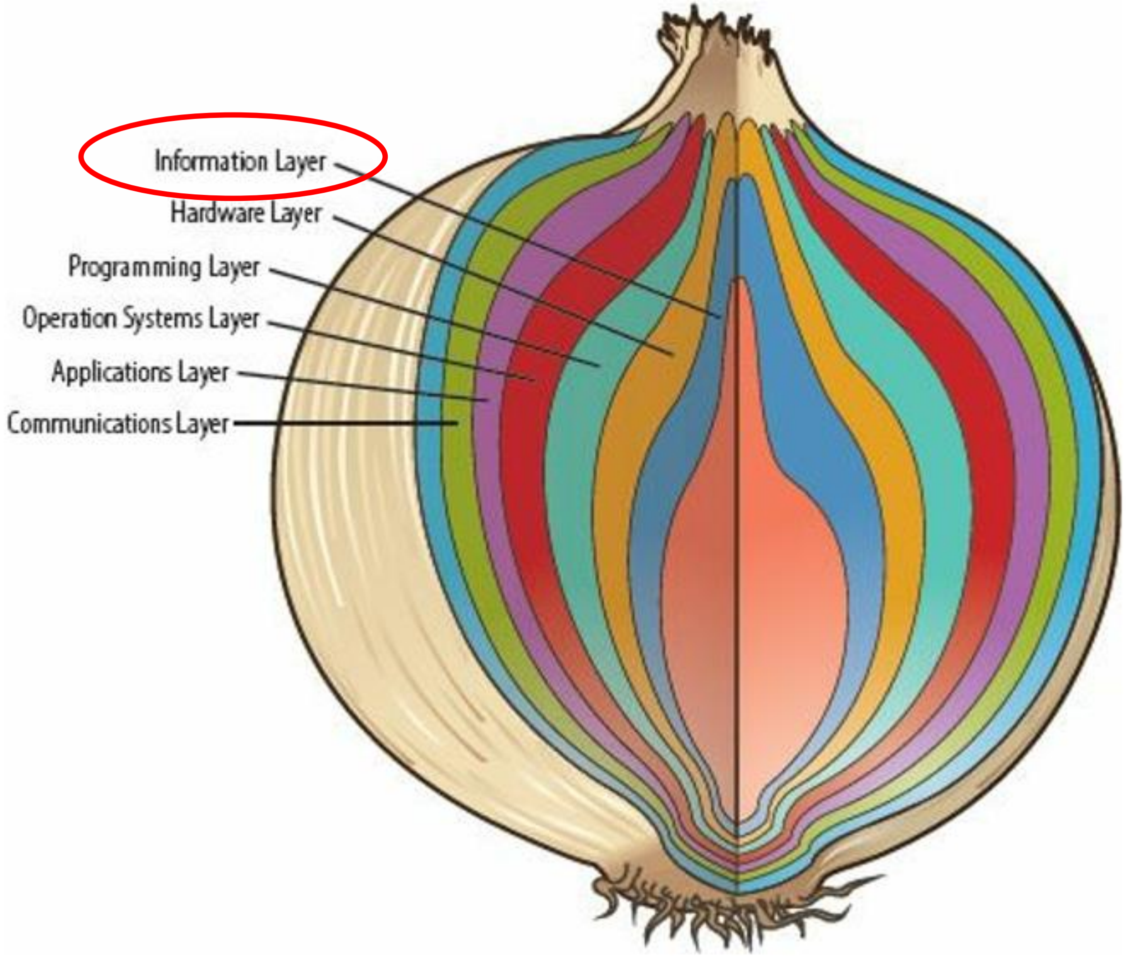
Introduction to Computing
Section 2 – The Information Layer

Binary Values and Number Systems

Data Representation

The Information Layer



PART 1

Binary Values and Number Systems

Number and Computing

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Number: crucial to computing

$\rightarrow$ All types of information: stored as number

$\rightarrow$ Explore the numbers

Natural, negative, rational, irrational numbers and many others (not to the understanding of computing)

Number: A unit of an abstract mathematical system subject to the laws of arithmetic

Numbers

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Natural Numbers

Zero and any number obtained by repeatedly adding one to it.

Examples: 100, 0, 45645, 32

Negative Numbers

A value less than 0, with a – sign

Examples: -24, -1, -45645, -32

Numbers

Integers

A natural number, a negative number, zero

Examples: 249, 0, - 45645, - 32

Rational Numbers

An integer or the quotient of two integers

Examples: -249, -1, 0, 3/7, -2/5

Natural Numbers

How many ones are there in 642?

600 + 40 + 2 ?

Or is it

384 + 32 + 2 ?

Or maybe…

1536 + 64 + 2 ?

Natural Numbers

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642 is 600 + 40 + 2 in **BASE 10**

The **base** of a number determines the number of digits used in the system and the value of digit positions.
The digits always begin with 0 and continue through one less than the base.
Numbers are written using positional notation.

Positional Notation

Numbers are written using **positional notation**

*The rightmost digit its value * the base to the zeroth power*

→*The digit to the left of that one: its value*the base to 1^{st} power*

→*The next digit: its value*the base to 2^{nd} power and so on*

Positional notation A system of expressing numbers in which the digits are arranged in succession, the position of each digit has a place value, and the number is equal to the sum of the products of each digit by its place value.

Positional Notation

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Continuing with our example…
642 in base 10 *positional notation* is:

$$6 \times 10^2 = 6 \times 100 = 600$$
$$+\ 4 \times 10^1 = 4 \times 10 = 40$$
$$+\ 2 \times 10^0 = 2 \times 1 = 2 \qquad = 642 \text{ in base 10}$$

This number is in base 10

The power indicates the position of the number

Positional Notation

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R is the base of the number

As a formula:

$$d_n * R^{n-1} + d_{n-1} * R^{n-2} + ... + d_2 * R + d_1$$

n is the number of digits in the number

d is the digit in the i^{th} position in the number

$$642 \text{ is } 6_3 * 10^2 + 4_2 * 10 + 2_1$$

Positional Notation

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What if 642 has the base of 13?

$$
\begin{aligned}
+ 6 \times 13^2 &= 6 \times 169 &= 1014 \\
+ 4 \times 13^1 &= 4 \times 13 &= 52 \\
+ 2 \times 13^0 &= 2 \times 1 &= 2 \\
& &= 1068 \text{ in base 10}
\end{aligned}
$$

642 in base 13 is equivalent to 1068 in base 10

Binary

Decimal is base 10 and has 10 digits:

0,1,2,3,4,5,6,7,8,9

Binary is base 2 and has 2 digits:

0,1

For a number to exist in a given number system, the number system must include those digits. For example, the number 284 only exists in base 9 and higher.

Bases Higher than 10

How are digits in bases higher than 10 represented?

With distinct symbols for 10 and above.

Base 16 has 16 digits:
0,1,2,3,4,5,6,7,8,9,A,B,C,D,E, and F

Converting Binary to Decimal

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What is the decimal equivalent of the binary number 1101110?

$1 \times 2^6 = 1 \times 64 = 64$
$+ 1 \times 2^5 = 1 \times 32 = 32$
$+ 0 \times 2^4 = 0 \times 16 = 0$
$+ 1 \times 2^3 = 1 \times 8 = 8$
$+ 1 \times 2^2 = 1 \times 4 = 4$
$+ 1 \times 2^1 = 1 \times 2 = 2$
$+ 0 \times 2^0 = 0 \times 1 = 0$
$= 110$ in base 10

Converting Octal to Decimal

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What is the decimal equivalent of the octal number 642?

$$
\begin{aligned}
6 \times 8^2 &= 6 \times 64 = 384 \\
+\ 4 \times 8^1 &= 4 \times 8 = 32 \\
+\ 2 \times 8^0 &= 2 \times 1 = 2 \\
&= 418 \text{ in base 10}
\end{aligned}
$$

Converting Hexadecimal to Decimal

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What is the decimal equivalent of the hexadecimal number DEF?

$$
\begin{aligned}
&D \times 16^2 = 13 \times 256 = 3328 \\
&\quad + E \times 16^1 = 14 \times 16 = 224 \\
&\quad + F \times 16^0 = 15 \times 1 = 15 \\
&\quad\quad = 3567 \text{ in base } 10
\end{aligned}
$$

Remember, the digits in base 16 are 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Converting Binary to Decimal

SCSE

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$1 \times 2^6 = 1 \times 64 = 64$

$+ 1 \times 2^5 = 1 \times 32 = 32$

$+ 0 \times 2^4 = 0 \times 16 = 0$

$+ 1 \times 2^3 = 1 \times 8 = 8$

$+ 1 \times 2^2 = 1 \times 4 = 4$

$+ 1 \times 2^1 = 1 \times 2 = 2$

$+ 0 \times 2^0 = 0 \times 1 = 0$

$= 110$ in base 10

Arithmetic in Binary

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Remember that there are only 2 digits in binary, 0 and 1

Position is key, carry values are used:

```
  1 1 1 1 1
    1 0 1 1 1 0
+     1 1 0 1 1
  1 0 0 1 0 0 1
```

Carry Values

1100110011+1110111101=?

1 1 0 1 1 1 1 0 0 0 0

Subtracting Binary Numbers

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When you borrow one from the next left digit of the number you are subtracting, you borrow one power of the base.

Every time you borrow in a binary subtraction, you borrow 2.

EX:

```
      -1 2
        -1 2
   1 1 1 0 0 1
-        1 1 0
   -----------
   1 1 0 0 1 1
```

```
    -1 2
      -1 2
 1 1 1 1 0 1
-      1 1 0
 -----------
 1 1 0 1 1 1
```

1010111-111011=?

0 0 1 1 1 0 0

Subtracting Octal Numbers

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Every time you borrow in a Octal subtraction, you borrow 8.

EX:

```
   -1 8
     -1 8
    1 0 6 6
-       7 7 6
   ________
    0 0 7 0
```

```
   -1 8
     -1 8
       -1 8
    1 2 3 0
-       7 6 5
   ________
    0 2 4 3
```

5544-766=?

4556

Practice

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1. Match the solution with the problem.

Problem	Solution
1/ 1110011+11001	A. 10001100
2/ 1010101+10101	B. 10011110
3/ 1111111+11111	C. 1101010
4/ 1110011-111	D. 1011110
5/ 1100101-111	E. 1010001
6/ 1010110-101	F. 1101100

2. Perform the following Octal subtractions

a. 23656-763

b. 2346-527

c. 3736-777

3. Perform the following Hexadecimal subtractions

a. ABC-DF

b. 9988-AB

c. C9F8-1F92

Power of 2 Number System

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Binary	Octal	Decimal
000	0	0
001	1	1
010	2	2
011	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	10	8
1001	11	9
1010	12	10

345

3:011

4:100

5:101

345: 011100101

Converting Binary to Octal

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- Groups of Three (from right)
- Convert each group

10101011   **<u>10</u> <u>101</u> <u>011</u>**

2 5 3

10101011 is 253 in base 8

Converting Binary to Hexadecimal

- Groups of Four (from right)
- Convert each group

10101011   **1010** **1011**

A **B**

10101011 is AB in base 16

Converting Decimal to Other Bases

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Algorithm for converting base 10 to other bases

While the quotient is ***not*** zero:

* Divide the decimal number by the new base

* Make the remainder the next digit to the left in the answer

* Replace the original dividend with the quotient

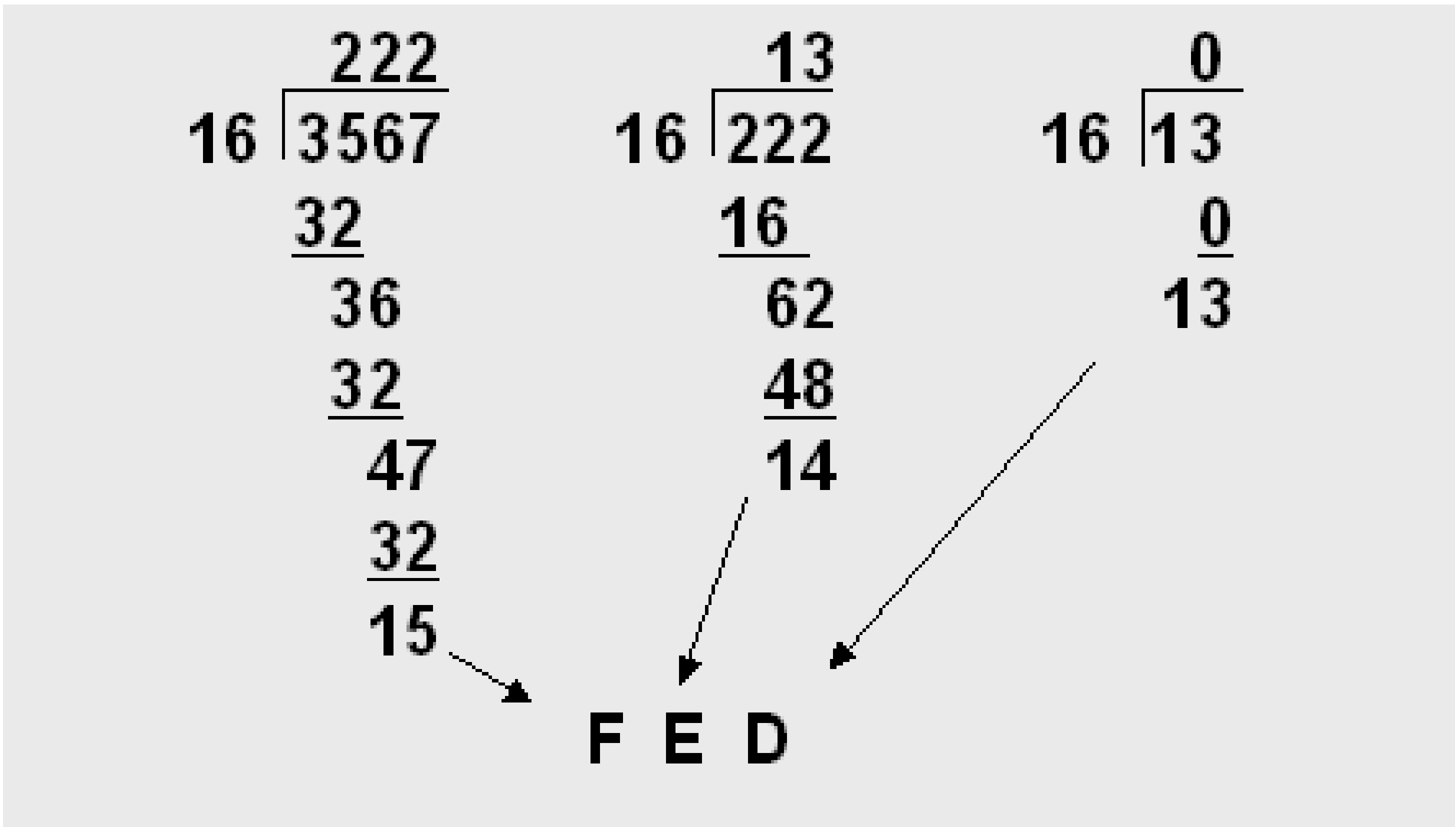
Converting Decimal to Hexadecimal

SCSE

Try a Conversion

The base 10 number 3567 is what number in base 16?

Converting Decimal to Hexadecimal



Binary and Computers

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Binary computers have storage units called binary digits or bits

Low Voltage = 0
High Voltage = 1 all bits have 0 or 1

Binary and Computers

SCSE

Byte 8 bits

The number of bits in a word determines the word length of the computer, but it is usually a multiple of 8

- 32-bit machines
- 64-bit machines etc.

Practice

4. Convert the following numbers from the base shown to base 10.

a. 111 (base 2)
b. 777 (base 8)
c. FEC (base 16)
d. 777 (base 16)
e. 111 (base 8)

5. Convert the following binary numbers to octal and hexadecimal

a. 111110110
b. 1000001
c. 10000010
d. 1100010