0. A crash course on quantum information

0.1 Classical vs. Quantum bits

$\boxed{\text{Lecture 1: The qubit}}$

classical bits are just 0 or 1

$\longrightarrow$ let's associate those bits w/ a vector

$$0 \longrightarrow |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

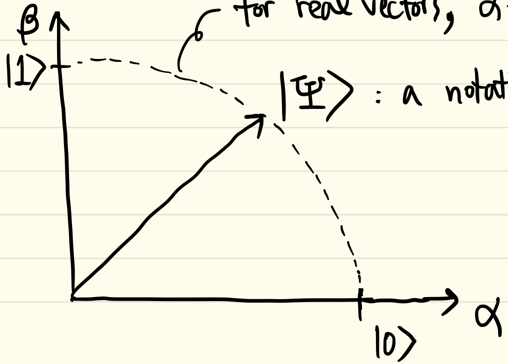
$$1 \longrightarrow |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

these two vectors are orthogonal

$\Downarrow$

we could assume two axis

qubits



for real vectors, $\alpha^2 + \beta^2 = 1$

$|\Psi\rangle$: a notation of vector in quantum information

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

"a qubit has a little of the zero and a little of one bit at the same time"

$\Rightarrow$ Qubit can be a superposition of 0 & 1

what do qubits look like? what does superposition really means?

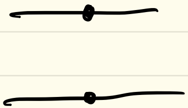
Quantumly, we could send a particle to left & right. at the same time.

i.e. the particle would be in a superposition of being on the left & right.

qubits could be in the ground and the excited state at the same time.

$|1\rangle$

$|0\rangle$



standard basis

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

to construct the qubits, we started from the classical bits as vectors.

"standard basis"

$$|\Psi\rangle = \underbrace{\alpha |0\rangle + \beta}_{\text{amplitude}} |1\rangle \in \mathbb{C}^2$$ (amplitude can be complex numbers too)

Kets and Bras

Ket : $|\Psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

conjugate

Bra : $\langle\Psi| = \left(|\Psi\rangle^{*}\right)^{T}$ transpose $= \begin{pmatrix} \alpha^{*} \\ \beta^{*} \end{pmatrix}^{T} = \begin{pmatrix} \alpha^{*} & \beta^{*} \end{pmatrix}$

Inner product : $\langle\Psi||\Psi\rangle = \langle\Psi|\Psi\rangle$

$$= \begin{pmatrix} \alpha^{*} & \beta^{*} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha^{*}\alpha + \beta^{*}\beta = |\alpha|^2 + |\beta|^2 = 1$$