As you know, Friday, May 28 at 14:00, you will have face a Lab test which counts 10% from the final mark. Also, please remember two compulsory conditions to pass the final exam:

- to have attended at least 6 Labs (Lab 7 is counted too). If this condition is not fulfilled you can not even participate at the final exam this year;
- to obtain at least 4.5 points (from the 10) at the Lab test.

The Lab test is of type open-book. Take care to not loose all the time "reading books", instead of solving the problems.

During the Lab test you are NOT allowed to communicate with any person.

Please find in the sequel some models of Lab tests. I hope this will help you to prepare for the test.

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Laboratory test

1. Find the general solution to the following system (here $x = x(t)$ and $y = y(t)$):
$x' = x + y + t - 1\,, \quad y' = -2x + 4y + e^t.$

2. We consider the nonlinear planar system (here $x = x(t)$ and $y = y(t)$) $\dot{x} = y + x^2, \quad \dot{y} = -x + xy.$
a) Find its equilibria.

b) Find the matrix of the linearized system around the equilibrium point $(0, 0)$ and its eigenvalues. Is $(0, 0)$ a hyperbolic equilibrium point?

c) Remind that the cartesian differential equation of the orbits is: $\frac{dy}{dx} = \frac{-x+xy}{y+x^2}$ (here $y = y(x)$). Find its general solution.

d) Note that the general solution found at c) can be written in the form $H(x, y) = c$. Find the expression $H(x, y)$. Check whether H is a first integral.

e) Plot the level curves of H in a small neighborhood of $(0, 0)$ (like, for example, the box $[-0.1, 0.1] \times [-0.1, 0.1]$).

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Laboratory test

1. a) Find the general solution of $x'' + 3x' + x = 1$ (here $x = x(t)$).

   b) Decide if the following statement is true or false:
   All the solutions of $x'' + 3x' + x = 1$ satisfies $\lim_{t\to\infty} x(t) = 1$.

2. a) Find the general solution of the differential equation $x^2y'' - 2xy' + y = 0$.

   b) Find the solution of the IVP $x^2y'' - 2xy' + y = 0, \quad y(1) = 2, \quad y'(1) = 3$.

   c) Plot the solution obtained at item b).

3. We consider the planar nonlinear system (here $x = x(t)$ and $y = y(t)$)

$$\dot{x} = 2x - x^2 - xy, \quad \dot{y} = -y + xy\,.$$

   a) Find its equilibria, find the matrix of the linearized system around each equilibrium point together with its corresponding eigenvalues.

   b) Plot few orbits that starts near each equilibrium point.

4. For the IVP $y' = y^2 + x^2, \quad y(0) = 0$, apply the Euler's method and the improved Euler's method in the interval $[0, 1]$ with step size $h = 0.1$. Write here the approximate values in $x = 1$ obtained in each case.