CS 559: Machine Learning: Fundamentals and Applications

HW 6 Due: 4/5/2024 Friday 11:59 p.m.

- The assignment must be individual work and must not be copied or shared. Any tendency to cheat/copy evidence will lead to a 0 mark for the assignment.
- Students must only use Pandas, NumPy, matplotlib, and Spacy if the coding problem does not specify libraries/packages. Use of libraries other than the specified will be penalized.
- All problems must be submitted in a single notebook file. Do not work in the lecture notebook file.

1 Karush-Kuhn-Tucker Condition Verification [45 pts]

Lagrangian multipliers play an important role in SVM in finding the support vectors. In this assignment, the objective is to verify the conditions of Karush-Kuhn-Tucker (KKT).

a. Load the following data set

```
x1 = np.array([4, 4, 1, 2.5, 4.9, 1.9, 3.5, 0.5, 2, 4.5])
x2 = np.array([2.9, 4, 2.5, 1, 4.5, 1.9, 4, 1.5, 2.1, 2.5])
y = np.array([1,1,-1,-1,1,-1,1,-1,-1,1])
```

b. [15 pts] Make a random Lagrangian multiplier array, $\mathbf{a}$ where $0 \leq a_i \leq 0.5$. Calculate the bias parameter and the weight vectors $\mathbf{w}$. Find the hyperplane equation. Calculate the margin and determine points in the margin using the obtained hyperplane equation. Find the support vectors if there are any. Using the visualization, justify if this is a good SVM classifier. Predict a class of a point $\mathbf{x}^* = [3, 3]$. Is this point in a margin?

c. [15 pts] Numerically verify that Karush-Kuhn-Tucker (KKT) conditions are satisfied for data points above using the objective equation below

$$L = \frac{\lambda}{2}\mathbf{w}^2 - \sum_{i=1}^{N} a_i \Big[y_i(\mathbf{w}\mathbf{x}_i + b) - 1 \Big]. \tag{1}$$

Use the same $\mathbf{a}$ from Question a.

d. [15 pts] Using the Lagrangian multiplier, $\mathbf{a} = [0.414, 0, 0, 0.018, 0, 0, 0.018, 0, 0.414, 0]$, confirm that KKT conditions are satisfied. Find the hyperplane equation and find the support vectors. Determine if a point ($\mathbf{x}^* = [3, 3], y^* = 1$) is in a margin. Guess a possible slack variable ξ_i for the point, $\mathbf{x}^*$ that satisfies the KKT condition.