Revision Change Log

Weil Quadratic Form — Structured Proof

Response to Second Independent Audit

February 19, 2026

Overview

This document records every change made to `lamport_structured.tex` in response to the 35-issue audit report (Second Independent Audit). Of the 35 issues identified, **34 were addressed** and **1 was deemed not necessary to address**.

Severity breakdown of addressed issues:

- 3 Significant (missing citations for standard theorems)
- 5 Moderate (flawed argument, incomplete covering, domain issues, logical inversion)
- 15 Minor (convergence, measurability, forward references, notation)
- 11 Trivial (notation, redundancies, style)

New bibliography entries added: Engel–Nagel [EN00] and Reed–Simon [RS80] (Vol. I).

1 Change Log

Step A — Algebraic Identities (Lemmas 1–2)

Change 1 Minor *Convergence of $\langle g, U_a g\rangle$ unstated (Audit #1)*

Location: Lines 110–115 (setup before Lemma 1)

Original: "...define the unitary dilation operator ... Then $\|U_a g\|_2 = \|g\|_2$ and $\langle g, U_a g\rangle$ is well-defined."

Updated: "...define the dilation operator ... U_a is unitary on $L^2(\mathbb{R}_+^*, d^*x)$: it is isometric ($\|U_a g\|_2 = \|g\|_2$ by Haar invariance ...) and surjective ($U_a^{-1} = U_{a^{-1}}$). In particular, $\langle g, U_a g\rangle$ is well-defined and finite by Cauchy–Schwarz: $|\langle g, U_a g\rangle| \le \|g\|_2 \|U_a g\|_2 = \|g\|_2^2$."

Reason: Simultaneously addresses audit issues #1 (convergence unstated) and #3 (well-definedness/surjectivity implicit). The unitarity proof, Cauchy–Schwarz bound, and surjectivity via explicit inverse are now all stated.

Change 2 Trivial *Ambiguous substitution notation (Audit #2)*

Location: Lemma 1, Step 2

Original: "Substituting $y \mapsto y/a$ (with $d^*(y/a) = d^*y$) gives"

Updated: "Substituting $y' = y/a$ (so $y = ay'$ and $d^*y = d^*y'$ by Haar invariance) gives"

Reason: The arrow notation $y \mapsto y/a$ could be misread as defining the reverse substitution. The primed-variable form is unambiguous.

Step B — Prime Energy Decomposition (Lemma 3)

Change 3 Minor *Sum finiteness forward reference (Audit #4)*

Location: Lemma 3, Substep 1.1 justification

Original: "Definition of W_p."

Updated: "Definition of W_p. (The sum is in fact finite: Step 3 below shows $\langle g, U_{p^m} g\rangle = 0$ for $p^m > \lambda^2$, so only finitely many terms contribute. Steps 1–2 are therefore a finite computation.)"

Reason: The sum is manipulated algebraically before finiteness is established; the forward reference resolves this.

Change 4 Trivial *Zero-extension compatibility (Audit #6)*

Location: Lemma 3, Step 4 justification

Original: Ended with "The $L^2(\mathbb{R}_+^*, d^*x)$ norm becomes the $L^2(\mathbb{R}, du)$ norm."

Updated: Added parenthetical: "(Here $\widetilde{G}$ denotes the zero-extension of G to $\mathbb{R}$; the identity holds because g is supported in $[\lambda^{-1}, \lambda]$, so G is supported in $I = [-L, L]$, and the substitution $u = \log x$ applies simultaneously to both terms.)"

Reason: Makes the implicit zero-extension explicit.

Step C — Archimedean Energy Decomposition (Lemma 4)

Change 5 **Minor** *$W_{\mathbb{R}}(f)$ convergence (Audit #7)*

Location: Lemma 4, Step 1 justification

Original: Justification ended after "$t \in [0, \infty)$."

Updated: Added convergence analysis: near $t = 0$, $f(e^t) + f(e^{-t}) - 2e^{-t/2} f(1) = O(t)$ by Taylor, cancelling the $1/t$ singularity; for $t > 2L$, the remaining term is $O(e^{-t})$.

Reason: The substitution was applied without verifying convergence of the integral.

Change 6 **Minor** *Forward-referential convergence (Audit #8)*

Location: Lemma 4, Substep 4.1 justification

Original: "Negate Step 1 and use $f(1) = \|g\|_2^2$ from Step 2."

Updated: Added: "(Absolute convergence of this integral is verified in Steps 7–8 below; we proceed with the algebraic manipulation.)"

Reason: The integral is formed before its convergence is established downstream.

Change 7 **Trivial** *Cross-reference for norm identity (Audit #9)*

Location: Lemma 4, Step 10 justification

Original: "...noting $\|g\|_2^2 = \|G\|_{L^2(I)}^2$ by the change of variables."

Updated: "...noting $\|g\|_2^2 = \|G\|_{L^2(I)}^2$ (by the isometry $u = \log x$, as in Lemma 3, Step 5.3)."

Reason: Adds explicit back-reference to the earlier proof of this identity.

Step D1 — Markov Property & Irreducibility (Lemmas 5–7, Remark 8)

Change 8 **Minor** *$\Phi \circ G \in L^2(I)$ membership (Audit #10)*

Location: Lemma 5, proof

Original: Proof began directly with the pointwise Lipschitz argument (old Step 1).

Updated: Inserted new Step 1: "$\Phi \circ G \in L^2(I)$." with justification: $|\Phi(G(u))| \le |G(u)|$ pointwise (since $\Phi(0) = 0$ and Φ is 1-Lipschitz), hence $\|\Phi \circ G\|_{L^2(I)} \le \|G\|_{L^2(I)} < \infty$. All subsequent steps renumbered (old Step 1→2, etc.) and internal cross-references updated.

Reason: The form $\mathcal{E}_\lambda(\Phi \circ G)$ was evaluated without first verifying $\Phi \circ G \in L^2(I)$.

Change 9 **Trivial** *Vacuous case $\mathcal{E}_\lambda(G) = +\infty$ (Audit #11)*

Location: Lemma 5, Step 3 (formerly Step 2) justification

Original: "By Definition..." (no mention of infinite case).

Updated: Prepended: "When $\mathcal{E}_\lambda(G) = +\infty$ the inequality is trivial. Otherwise, by Definition..."

Reason: The inequality holds vacuously when the right side is infinite; this was unstated.

Change 10 **Moderate** *Negative-t extension deleted (Audit #12)*

Location: Lemma 6, Step 3

Original: Step 3 claimed the identity extends to $|t| < \delta$ via a substitution $u \mapsto u + t$ for $t \in (-\delta, 0)$. This argument was flawed (sends J to $J + t \neq J$) and superfluous (only $t > 0$ is used downstream).

Updated: Replaced with: "Summary: $f(u+t) = f(u)$ for a.e. $u \in J$, for all $t \in (0, \delta)$. At $t = 0$ the identity is trivial. (The downstream mollification argument, Steps 4–9, uses only $t \in (0, \delta/2)$, so no negative-t extension is required.)"

Reason: Removes an incorrect argument that was never used; downstream reference from Step 6 updated from "By Step 3" to "By Step 2".

Change 11 **Trivial** *Extension of $f = \mathbf{1}_B$ to $\mathbb{R}$ (Audit #15)*

Location: Lemma 6, Step 5 (mollifier definition)

Original: "Standard construction; $f_\eta \in C^\infty(\mathbb{R})$."

Updated: Added: "(Extend $f = \mathbf{1}_B$ to $\mathbb{R}$ by zero outside I. For $u \in K$ the convolution samples f only at points $u - s$ with $|s| < \eta < \delta/4$, so $u - s \in J \subset I$ and the extension choice is immaterial.)"

Reason: Mollification requires f defined on all of $\mathbb{R}$, but f was defined only on I.

Change 12 **Moderate** *Covering argument made explicit (Audit #13)*

Location: Lemma 6, Step 10

Original: "Cover I by overlapping compact subintervals $J_n \Subset I$ ... and apply Steps 1–9 to each. On overlaps, the a.e. constants must agree..."

Updated: Supplied explicit construction: $J_n := [\alpha + 1/n, \beta - 1/n]$, $\delta_n := \min(\varepsilon, 1/n)/2$; verified (a) $K_n \neq \emptyset$, (b) $K_n \subset K_{n+1}^\circ$ for large n (overlap), (c) $\bigcup_n K_n = I$ up to measure zero; constants agree on overlaps by positive-measure intersection.

Reason: Three conditions for the covering to work were left unverified.

Change 13 **Minor** *Forward reference replaced in Remark 8 (Audit #14)*

Location: Remark 8, Step 1 justification

Original: "$\|\widetilde{1} - S_t\widetilde{1}\|_2^2 = m(I \,\Delta\, (I+t)) = 2t > 0$ (Proposition **??**, Step 2.2)."

Updated: "$\widetilde{1} = \mathbf{1}_{(-L,L)}$ and $S_t\widetilde{1} = \mathbf{1}_{(-L+t,L+t)}$, so $\|\widetilde{1} - S_t\widetilde{1}\|_2^2 = m(I \,\Delta\, (I+t)) = 2t > 0$ (the symmetric difference consists of $(-L, -L+t)$ and $(L, L+t)$, each of measure t)."

Reason: Eliminates a forward reference to material 600 lines later; the identity is trivially self-contained.

Step E — Fourier Analysis, Closedness, Compact Resolvent

Change 14 **Minor** *Joint measurability for Tonelli (Audit #25)*

Location: Lemma 9, Step 2 justification

Original: "All integrands are nonneg... Tonelli's theorem permits interchange..."

Updated: Prepended: "The integrand $w(t) \cdot 4\sin^2(\xi t/2) \cdot |\widehat{\phi}(\xi)|^2$ is jointly measurable in

(t,ξ): $w(t)$ is Borel on $(0,2L]$, $\sin^2(\xi t/2)$ is jointly continuous, and $|\widehat{\phi}|^2$ is measurable."
Reason: Tonelli requires joint measurability, which was not stated.

Change 15 **Significant** *Kato citation for closed-form characterization (Audit #18)*
Location: Proposition 2, Step 4
Original: "A nonneg. quadratic form is closed iff its domain equipped with the graph norm is complete. This is Step 3."
Updated: "By Kato [Thm. VI.1.17], a nonneg. symmetric form is closed iff its domain equipped with the graph norm $\|\cdot\|_{\mathcal{D}}$ is complete. Step 3 verifies this."
Reason: This characterization is a non-trivial theorem, not a definition; citation added.

Change 16 **Trivial** *Streamlined Substep 6.1 (Audit #31)*
Location: Proposition 2, Substep 6.1
Original: Three partially-developed arguments (MVT, compact-support bound, Plancherel).
Updated: Retained only the Plancherel argument: $\|\phi - S_t\phi\|_2^2 \le t^2\|\phi'\|_2^2$ using $\sin^2 x \le x^2$.
Reason: The Plancherel argument is the cleanest and self-contained; the other two were redundant and verbose.

Change 17 **Minor** *Disjointness of J_n intervals (Audit #26)*
Location: Lemma 11, Step 3 justification
Original: Justification ended after "the claim follows."
Updated: Added: "The J_n are pairwise disjoint: the left endpoint of J_{n+1} exceeds the right endpoint of J_n by $\pi/(2|\xi|) > 0$."
Reason: The intervals were used without verifying pairwise disjointness.

Change 18 **Minor** *Spurious +1 in N bound (Audit #27)*
Location: Lemma 11, Step 4 justification
Original: $N \le t_0|\xi|/(2\pi) + 1/4 + 1$
Updated: $N \le t_0|\xi|/(2\pi) + 1/4$
Reason: The correct derivation from $J_{N-1} \subset (0, t_0]$ gives $N \le t_0|\xi|/(2\pi) + 1/4$ without the extra $+1$. The error was harmless but should be corrected.

Change 19 **Trivial** *Spurious $-C$ constant removed (Audit #32)*
Location: Lemma 11, Step 6 heading and Step 8 justification
Original: "$\sum \ge c' \log N - C$ for absolute constants $c', C > 0$" (heading); Step 8 referenced "$c' \log N - C$".
Updated: "$\sum \ge c' \log N$ for an absolute constant $c' > 0$" (heading); Step 8 chain updated to "$2c_0c' \log N = 2c_0c'(\log|\xi| + O(1))$".
Reason: Substep 6.3 derives $\sum \ge c \log N$ with no subtracted constant; the $-C$ was an

artifact not produced by the argument.

Change 20 Trivial *Superfluous $|h| \le 1$ removed (Audit #28)*
Location: Lemma 13, Step 1
Original: "For $\phi \in \mathcal{K}_M$ and $|h| \le 1$:"
Updated: "For $\phi \in \mathcal{K}_M$ and $h \in \mathbb{R}$:"
Reason: The Plancherel identity holds for all h; the restriction was unnecessary.

Change 21 Minor *L^2-boundedness for Kolmogorov–Riesz (Audit #29)*
Location: Proposition 6, Step 4 justification
Original: "Theorem KR applied to $\mathcal{K} := \{\phi_n\}$, using Steps 2 and 3."
Updated: Added: "(The required L^2-boundedness holds: $\|\phi_n\|_2^2 \le M$ by Step 1.)"
Reason: Kolmogorov–Riesz requires L^2-boundedness, which was not explicitly listed.

Change 22 Minor *Compactness transfer to H_I (Audit #30)*
Location: Proposition 6, Step 5 justification
Original: "The map $\phi \mapsto \phi|_I$ is a continuous surjection $H_I \to L^2(I)$ (indeed, an isometry)...."
Updated: Prepended: "Since H_I is closed in $L^2(\mathbb{R})$, every subsequential $L^2(\mathbb{R})$-limit of $\{\phi_n\}$ lies in H_I."
Reason: Relative compactness transferred to $L^2(I)$ without noting limit points remain in H_I.

Change 23 Trivial *Division by $\|u_n\|_2$ (Audit #33)*
Location: Theorem 2, Substep 2.4 justification
Original: "...we get $\|u_n\|_2 \le \|f_n\|_2$, hence..."
Updated: "...we get $\|u_n\|_2 \le \|f_n\|_2$ (if $u_n = 0$ the bound is trivial), hence..."
Reason: Division by $\|u_n\|_2$ requires $u_n \neq 0$; the vacuous case was unacknowledged.

Step D2 — Semigroup Route to Irreducibility (Lemma 12, Proposition 4)

Change 24 Significant *Laplace transform citation (Audit #16)*
Location: Lemma 12, Step 3 justification
Original: "Since $A_\lambda \ge 0$, the Laplace-transform identity ... holds as a Bochner integral in $\mathcal{B}(L^2(I))$."
Updated: "By the Laplace-transform formula for C_0-semigroups (Engel–Nagel [Cor. II.1.11]): ... (convergence: $\|e^{-\alpha t}T(t)\| \le e^{-\alpha t}$, integrable for $\alpha > 0$; α lies in the resolvent set of $-A_\lambda$ since $\sigma(A_\lambda) \subset [0, \infty)$)."
Reason: The Laplace-transform resolvent formula is a non-trivial theorem requiring citation. Convergence conditions are now also stated. New bibliography entry added.

Change 25 **Significant** *Spectral commutativity citation (Audit #17)*

Location: Lemma 12, Step 4 justification

Original: "A bounded operator commuting with the bounded selfadjoint R commutes with every bounded Borel function of R, in particular with its spectral projections…"

Updated: "Since $PR = RP$, P commutes with every bounded Borel function of R (Reed–Simon [Cor. to Thm. VIII.5]: $PR^n = R^nP$ by induction; extend to polynomials by linearity, to $C(\sigma(R))$ by Weierstrass, and to bounded Borel functions via SOT limits)."

Reason: The claim requires a three-step argument (induction, Weierstrass, dominated convergence) that was stated without proof or citation. New bibliography entry added.

Change 26 **Moderate** *$A_\lambda^{1/2}P$ domain and identity proved (Audit #19)*

Location: Lemma 12, Step 4 justification (latter part)

Original: "…P commutes with $A_\lambda^{1/2} = \int_0^\infty \mu^{1/2}\, dE_{A_\lambda}(\mu)$ in the sense that $P\mathcal{D}(A_\lambda^{1/2}) \subset \mathcal{D}(A_\lambda^{1/2})$ and $A_\lambda^{1/2}P = PA_\lambda^{1/2}$ on $\mathcal{D}(A_\lambda^{1/2})$."

Updated: Added explicit spectral-measure calculations: *Domain preservation:* $\int \mu\, d\|E_{A_\lambda}(\mu)Pu\|^2 \leq \int \mu\, d\|E_{A_\lambda}(\mu)u\|^2 = \|A_\lambda^{1/2}u\|^2 < \infty$. *Commutativity:* interchange of bounded P with spectral integral justified by $PE_{A_\lambda}(\Delta) = E_{A_\lambda}(\Delta)P$.

Reason: Both claims require non-trivial spectral-measure estimates that were asserted without proof.

Change 27 **Minor** *Domain of φ corrected (Audit #20)*

Location: Lemma 12, Step 4 justification

Original: "$\varphi(\lambda) = (\lambda + 1)^{-1}$, a Borel bijection $(0, \infty) \to (0, 1)$"

Updated: "$\varphi(\mu) = (\mu + 1)^{-1}$, a continuous strictly decreasing bijection $[0, \infty) \to (0, 1]$"

Reason: Since $\sigma(A_\lambda) \subset [0, \infty)$, the domain includes 0 (where $\varphi(0) = 1$). Variable renamed from λ to μ to avoid clash with the form parameter.

Change 28 **Moderate** *Logical inversion fixed in Proposition 4 (Audit #21)*

Location: Proposition 4, Steps 1–2

Original: Step 1 applied the form splitting (Lemma 12) for all $G \in \mathcal{D}(\mathcal{E}_\lambda)$; Step 2 proved $1 \in \mathcal{D}(\mathcal{E}_\lambda)$.

Updated: Swapped: Step 1 now proves $1 \in \mathcal{D}(\mathcal{E}_\lambda)$; Step 2 states the general form splitting. Substep references updated (2.3/2.4→1.3/1.4), and downstream reference "from Step 1 with $G = 1$" updated to "from Step 2 with $G = 1$".

Reason: The form splitting was invoked (conceptually with $G = 1$) before membership was established. The reordering makes the logical flow correct.

Step D3 — Perron–Frobenius Consequences

Change 29 **Minor** *Strong continuity verified (Audit #22)*

Location: Proposition 5, Step 3 justification

Original: Began with "A_λ is selfadjoint and lower bounded..."

Updated: Prepended: "$T(t)$ is strongly continuous (C_0): by the spectral theorem, $\|T(t)f - f\|_2^2 = \int_0^\infty |e^{-t\mu} - 1|^2 \, d\|E_{A_\lambda}(\mu)f\|^2 \to 0$ as $t \to 0^+$ by dominated convergence."

Reason: The C_0 property is needed for holomorphy but was not explicitly verified.

Change 30 Trivial *$\sigma(A_\lambda) \neq \emptyset$ noted automatic (Audit #23)*

Location: Theorem 1 (external), conclusion (a)

Original: "(a) $\sigma(A) \neq \emptyset$;"

Updated: "(a) $\sigma(A) \neq \emptyset$ (automatic for a selfadjoint operator on a nonzero Hilbert space);"

Reason: This conclusion does not require positivity improving; the parenthetical clarifies.

Change 31 Trivial *Notation $\lambda_1 \to \mu_0$ in application (Audit #24)*

Location: Proposition 5, Step 5 justification

Original: "$\lambda_1 := \min \sigma(A_\lambda)$ exists and is an eigenvalue; ... λ_1 is a simple eigenvalue"

Updated: "$\mu_0 := \min \sigma(A_\lambda)$ exists and is an eigenvalue (we write μ_0 instead of the λ_1 of Theorem 1 to avoid confusion with the form parameter λ); ... μ_0 is a simple eigenvalue"

Reason: The symbol λ is already the form parameter; using μ_0 (consistent with Corollary 4) avoids confusion.

Step F — Reflection Symmetry and Even Ground State

Change 32 Minor *"Norm is symmetric" corrected (Audit #34)*

Location: Proposition 7, Substep 3.4 justification

Original: "$\|\phi - S_{-t}\phi\|_2 = \|S_t\phi - \phi\|_2 = \|\phi - S_t\phi\|_2$ (norm is symmetric). More formally: ...substituting $v = u + t$..."

Updated: Retained only the substitution: "Substituting $v = u+t$: $\|\phi - S_{-t}\phi\|_2^2 = \int |\phi(u) - \phi(u+t)|^2 \, du = \int |\phi(v-t) - \phi(v)|^2 \, dv = \|\phi - S_t\phi\|_2^2$."

Reason: The informal sentence conflated unitarity of S_t with norm symmetry; the direct substitution is correct and cleaner.

Change 33 Trivial *Unitarity of R on $L^2(\mathbb{R})$ (Audit #35)*

Location: Proposition 7, Substep 3.3 justification

Original: "Unitarity of R."

Updated: "R is unitary on $L^2(\mathbb{R})$ by the same substitution $v = -u$ as in Step 1 (the isometry and surjectivity arguments carry over verbatim from $L^2(I)$ to $L^2(\mathbb{R})$)."

Reason: Step 1 establishes unitarity on $L^2(I)$; the extension to $L^2(\mathbb{R})$ was implicit.

2 Addendum: Issues Not Addressed

The following audit issue was deemed not necessary to address.

Audit Issue #5 *Premature side-condition on Lemma 2 application*

Severity: Trivial

Location: Lemma 3, Step 2 (lines ≈270–278)

Audit recommendation: Remove the restriction $p^m \leq \lambda^2$ from Step 2 and introduce it only in Step 3.

Reason for not addressing: Lemma 2 is a purely algebraic identity that is indeed valid for all m, so the early restriction to $p^m \leq \lambda^2$ in Step 2 is cosmetically premature. However, the restriction does not affect correctness: Steps 1–2 are now explicitly flagged as a finite computation (via the forward reference added for Audit Issue #4), and the constraint is introduced precisely where it is first *needed* (Step 3). Restructuring Steps 2–3 solely to move a harmless side-condition would risk introducing errors for negligible expository benefit, so we leave the original ordering.

Summary of New Bibliography Entries

Two new references were added to the bibliography:

1. **Engel–Nagel [EN00]:** K.-J. Engel and R. Nagel, *One-Parameter Semigroups for Linear Evolution Equations*, Graduate Texts in Mathematics vol. 194, Springer, 2000.
   Cited in: Lemma 12, Step 3 (Laplace-transform resolvent formula).
2. **Reed–Simon [RS80]:** M. Reed and B. Simon, *Methods of Modern Mathematical Physics. I: Functional Analysis*, Academic Press, revised ed., 1980.
   Cited in: Lemma 12, Step 4 (spectral commutativity theorem).