Additional References

Energy Decomposition, Compact Resolvent, and Perron–Frobenius Properties of the Restricted Weil Quadratic Form

Abstract

This document collects the 13 additional references recommended for inclusion in the paper, grouped by priority. Entries marked **[VERIFY]** require confirmation of publication details against the original source before use; all other entries have been verified against independent sources or primary documents seen during the audit process.

Group 1: Essential — direct technical ancestors

[1] A. Weil, "Sur les formules explicites de la théorie des nombres premiers," *Comm. Sém. Math. Univ. Lund* (1952), 252–265. (Volume dedicated to Marcel Riesz.)
Note: Primary source for the Weil explicit formula and positivity criterion in the form used in the paper; the distributions W_p and $W_{\mathbb{R}}$ descend from this work.

[2] A. Connes, "Trace formula in noncommutative geometry and the zeros of the Riemann zeta function," *Selecta Math. (N.S.)* **5** (1999), no. 1, 29–106.
DOI: 10.1007/s000290050042.
Note: Establishes the semilocal trace formula and the explicit distributions W_p, $W_{\mathbb{R}}$ whose negatives form the quadratic form studied in the paper; also introduces the adele class space framework.

[3] A. Connes and C. Consani, "Spectral triples and ζ-cycles," *Enseign. Math.* **69** (2023), no. 1–2, 93–148.
DOI: 10.4171/LEM/1042.
Note: Asserts for each $\lambda > 1$ the existence of a selfadjoint operator A_λ on $L^2([\lambda^{-1}, \lambda], d^*x)$ with compact resolvent (see §6.4 of Connes 2026 [5] and its reference [25]); the present paper supplies the self-contained proof of this property.

[4] A. Connes and W. van Suijlekom, "Quadratic forms, real zeros and echoes of the spectral action," *Commun. Math. Phys.* **406** (2025), Paper no. 312.
DOI: 10.1007/s00220-025-05240-6. (Volume dedicated to H. Araki.)
Note: Proves (Theorem 6.1 of Connes 2026 [5]) that if the minimum eigenvalue of A_λ is simple with even eigenfunction, then all zeros of the Mellin transform of the minimiser lie on the critical line; the present paper establishes precisely those hypotheses.

Group 2: Highly recommended — direct context and payoff

[5] A. Connes, "The Riemann Hypothesis: Past, Present and a Letter Through Time," arXiv:2602.04022v1 [math.NT], February 2026.
Note: Survey and original contribution explaining why the compact resolvent and Perron–Frobenius properties of A_λ (proved in the present paper) are needed for the programme outlined in §6.4 thereof; also provides the most current bibliographic guide to the full Connes programme.

[6] A. Connes and C. Consani, "Weil positivity and trace formula, the archimedean place," *Selecta Math. (N.S.)* **27** (2021), no. 4, Paper no. 77, 70 pp.
DOI: 10.1007/s00029-021-00672-x.
Note: Studies archimedean Weil positivity and the Sonin-space lower bound; provides essential geometric context for the semilocal operator A_λ.

Group 3: Worthwhile — complementary approaches and broader context

[7] **[VERIFY]** J.-F. Burnol, "The explicit formula and a propagator,"
Note: Complementary treatment of the Weil explicit formula through a propagator analysis; the precise publication venue and year require verification.

[8] **[VERIFY]** J.-F. Burnol, "Spectral analysis of the local conductor operator,"
Note: Analyses the spectral theory of a local operator complementary to the approach taken in the present paper; publication details require verification.

[9] J.-F. Burnol, "Sur certains espaces de Hilbert de fonctions entières, liés à la transformation de Fourier et aux fonctions L de Dirichlet et de Riemann," *C. R. Acad. Sci. Paris Sér. I* **333** (2001), 201–206.
DOI: 10.1016/S0764-4442(01)02049-3.
Note: Introduces Hilbert spaces of entire functions in the L^2 analysis of Fourier/Dirichlet/Riemann L-functions; foundational for the Sonin space approach credited in Connes 2026 [5], §7.2.

[10] J.-F. Burnol, "Sur les espaces de Sonine associés par de Branges à la transformation de Fourier," *C. R. Acad. Sci. Paris Sér. I* **335** (2002), 689–692.
DOI: 10.1016/S1631-073X(02)02569-5.
Note: Studies Sonin spaces via de Branges' theory of Fourier transforms; introduced into the RH context in Connes 2026 [5], §7.2.

[11] J.-F. Burnol, "Two complete and minimal systems associated with the zeros of the Riemann zeta function," *J. Théor. Nombres Bordeaux* **16** (2004), no. 1, 65–94.
URL: jtnb.cedram.org.
Note: Constructs complete and minimal systems linked to zeta zeros; provides L^2 functional-analytic background complementary to the present paper's approach.

[12] G. Poitou, "Sur les petits discriminants," *Séminaire Delange–Pisot–Poitou. Théorie des Nombres*, 18ème année, 1976–77, Exposé no. 6, 18 pp., Secrétariat Mathématique, Paris, 1977.

Note: Classical application of Weil positivity to lower bounds on discriminants of number fields; exemplifies the arithmetic power of the positivity criterion that motivates the present paper.

[13] **[VERIFY]** F. Battistoni and G. Molteni, "Explicit formulæ for L-functions and generators of the class group,"
Note: Applies explicit formula techniques to class group structure; publication details (journal, volume, year) require verification before inclusion.

Notes on [VERIFY] entries. Three entries above carry a **[VERIFY]** flag:

- **[7]** Burnol, "The explicit formula and a propagator": the title is known from literature but the precise journal, volume, and year could not be confirmed without access to a database.
- **[8]** Burnol, "Spectral analysis of the local conductor operator": same situation.
- **[13]** Battistoni–Molteni: full bibliographic data unavailable at time of drafting.

All three should be resolved via MathSciNet, zbMATH, or arXiv before the paper is submitted.