Lecture: Thu, 1:00-3:35 PM
Classroom: E202
Office Hours: by appointment

Course Summary: Introduction to stochastic differential equations with emphasis on both theoretical and practical aspects. Topics include stochastic integration and differentiation, theoretical and numerical methods for stochastic differential equations, links between stochastic and partial differential equations, and their financial applications. Prerequisites include an understanding of and proficiency in undergraduate calculus and probability. Some knowledge of numerical methods and programming skills would be helpful, but are not required.

Brief Description: This course is designed to provide the fundamentals of both the theoretical and applied aspects of stochastic differential equations with their applications. We will start with the basic foundations (probability theory, stochastic processes, Brownian motion) that form the basis of stochastic differentiation and integration. An important goal of this course is to introduce students, from the perspective of both mathematically trained engineers and potential financial practitioners, to the theoretical foundations, advanced modeling and analysis techniques used in natural and social science.

Recommended Prerequisites:

Elementary *differential equations* and *Probability* are recommended prerequisites. Students enrolling in this course are expected to have an understanding of and skills in the following topics

- Calculus: differentiation and integration (both single-valued and multi-valued), Riemann–Stieltjes integral, Taylor expansions, Fubini's theorem;
- Probability and Statistics: random variable, expectation, variation; stochastic process; probability spaces, probability distribution functions;
- Primary of differential equations: intuitive understanding of ODEs; separation of variables;

Text Book: There is no official textbook for this course. The lectures will be based on a collection of notes from the instructor and the students are expected to take and study their class notes. The following books are friendly to the beginning students

- Stochastic Differential Equations, An Introduction with Applications (2010) by B. Øksendal
- Mathematics of Financial Markets (2010) by R. J. Elliott and P. E. Kopp
- Stochastic Calculus for Finance, Volume II: Continuous-Time Models (2004) by S. E. Shreve
- Options, Futures, and Other Derivatives (2017) by J. Hull

I will update the list and add more references as the course progresses if needed.

Course Assessment:

The course grades will be given as follows:

- 20% Homework (individual): *There will be 10 homework in total and I will drop the 2 lowest grades towards the final HW grade. Homework will be collected and graded on an individual basis, discussing with classmates is allowed and, indeed, strongly encouraged. Copying other's work is strictly NOT acceptable, resulting in a forfeit of all the HW grades (see Academic Honesty below).*

- 40% Mid-term Exam (individual): *There will be one mid-term closed-book examination. Tentative Date:*

- 40% Final Project and Presentation (group): *20% for the professional paper based on a research topic related to this work (with the instructor's agreement), and another 20% for its professional presentation. The project will be assessed as a joint work, but the members will be evaluated individually for the presentation. Missing the final exam will NOT be accepted. It is the student's job to check the test information.*

Class Attendance & Academic Honesty: This course is structured as a self-organized, continuous learning process, and your class performance will contribute to your overall semester evaluation. Regular attendance at lectures is strongly recommended and encouraged, except in cases of extreme circumstances. It is essential that each student familiarizes themselves with the University's policies on academic integrity and adheres to these standards rigorously. While discussing homework problems with the instructor and classmates is permitted (and encouraged), this course maintains a zero-tolerance policy towards copying homework solutions. Any violations of the University's policies will be reported to the appropriate authorities without hesitation. Additionally, any misconduct related to the course guidelines will result in a downgrade of your final evaluation for this course.

Lecture Topics

- Review of the basic probability and statistics
- Brownian motion and extensions
- Convergence of stochastic process
- Itô calculus; Stratonovich calculus
- Euler–Maruyama method; Milshtein method
- Komogorov's equations: Feymann–Kac formula; Fokker–Planck equation;
- Infinitesimal generator; Dynkin's formula
- Optimal stopping; stopping time
- Dynamic programming principle
- Stochastic control and optimal control
- Financial applications

Detailed Schedule (Tentative)

Week 1: **Brief Overview of Notions in Prob & Stat**: probability space; probability measures; conditional probabilities and independence; random variables; probability distributions; statistics (e.g., expectation; variance); important probabilistic methods; continuous-time processes, Markov processes and martingales;

Week 2: **Brownian Motion and Convergence of Random Variables**: Brownian motion, quadratic variation; extensions of Brownian motion; different convergence fashions and their relations;

Week 3: **Stochastic Integration**: Itô integral, Stratonovich integral; their connections;

Week 4: **Itô Calculus (I)**: properties of Itô integrals; Itô calculus rules; stochastic integration methods;

Week 5: **Itô Calculus (II)**: properties of Itô integrals; Itô calculus rules; stochastic integration methods;

Week 6: **Important SDEs**: Ornstein–Uhlenbeck process; Langevin's equation; McKean–Vlasov process;

Week 7: **Numerical Methods for SDEs**: Monte–Carlo simulations; Euler–Maruyama method; Milshtein method;

Week 8: **Probabilistic Representation of solutions of PDEs**: Feymann–Kac formula; Fokker–Planck equation; Kolmogorov's backward equation;

Week 9: **Optimal Stopping**: Infinitesimal generator; stopping time; Dynkin's formula

Week 10: **Optimal control and Dynamic Programming Principle**: HJB equation; variational inequality;

Week 11: **Financial Derivatives and Pricing Models**: Pricing and hedging via the Black-Scholes, the Cox–Ross–Rubinstein and the Vasicek models; Monte–Carlo methods for derivative pricing;

Week 12: **Financial Options**: European Options; European put-call parity; the American call-option;

Week 13: **Interest Rate Models**: notions of interest rate and yield-curve; LIBOR and SOFR swap rates; IR models: the Vasicek, the Black–Derman–Toy, the Cox–Ingersoll–Ross, the Ho–Lee and the Hull–White models; the affine term-structure and inversion of the yield-curve;

Week 14: **Stochastic Volatility Models**: notions of stochastic volatility; volatility estimation; volatility index (VIX) and its financial derivatives; the Black–Scholes, the Cox–Ross–Rubinstein, the Heston and the CEV model; affine volatility model;

Week 15: research-oriented topics to be determined

Week 16: research-oriented topics to be determined

Week 17: $\boxed{\text{Final Presentation}}$