

# Introduction to PDEs, Fall 2024

## Homework 12 due Jan 2

Name: \_\_\_\_\_

1. Find the Green's function  $G^*(\mathbf{x}; \mathbf{x}_0)$  over  $\Omega := \{(x, y) \in (0, \infty) \times (0, \infty)\}$  being the first quadrant. Verify that it satisfies both conditions of a Green's function.
2. (for advanced students only) Find the Greens function for  $\Omega = B_0(R)$  being the disk centered at the origin with radius  $R$ . Hint: method 1) choose the point dual to  $\mathbf{x}_0$  to be  $\mathbf{x}_0^* = \frac{R^2}{|\mathbf{x}_0|^2} \mathbf{x}_0$ ; method 2) use the facts that the Laplacian in the polar coordinates is

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2},$$

and  $|\mathbf{x} - \mathbf{x}_0| = r^2 + r_0^2 - 2rr_0 \cos(\theta - \theta_0)$ , where  $(r, \theta)$  and  $(r_0, \theta_0)$  are the polar coordinates of  $\mathbf{x}$  and  $\mathbf{x}_0$ .