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Technical Section

AN ELABORATE AMBIGUITY DETECTION METHOD FOR CONSTRUCTING ISOSURFACES WITHIN TETRAHEDRAL MESHES

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Abstract—The algorithm for constructing isosurfaces within tetrahedral meshes has been considered as one approach to solving the ambiguity problem in the marching cubes method and has attracted extensive attention. In this paper, it is pointed out that the ambiguity problem still exists even if isosurfaces are generated through tetrahedral meshes. Then, based on the assumption that the function value distribution along the edges of a cube is linear, the criterion for testing the intersection between an isosurface and a tetrahedron edge is given, followed by the intersection points calculation method. The connection of intersection points in tetrahedra to construct polygons and the triangularization of polygons are discussed in detail. A comparison between the marching cubes method, the existing marching tetrahedra method and the new marching tetrahedra method is presented. It is shown that the isosurfaces generated by our method are independent of the subdivision modes of the tetrahedra from cubes. Finally, two isosurface images generated by our method are shown.

1. INTRODUCTION

Approximating isosurfaces with intermediate geometric primitives is an efficient method for displaying data sets in 3-D space. This popular algorithm for constructing isosurfaces now includes the marching cubes and marching tetrahedra methods, hereafter referred to as the MC method and the MT method, respectively.

The MC method was proposed first by Lorenson and Cline in 1987 [1]. On the assumption that the distribution functions are linear along all edges of cubes, it constructs isosurfaces within a cube according to the function value distributions at its vertices. Because of the simplicity and efficiency, the MC method has been widely adopted. Nevertheless, there exist two limitations: first, the isosurfaces generated by the MC method are merely an approximate representation of actual surfaces which satisfy equation $f(x, y, z) = C_0$, here C_0 is a given constant. Second, there exists an ambiguity for the correct connection among equivalued points within a cube. In recent years, researchers have proposed many improved methods to solve these problems [2–4].

Unlike the MC method, the MT method constructs isosurfaces within tetrahedra generated by subdivision from cubes [5, 6]. There are a few reasons for subdivision of cubes into tetrahedra. First, the tetrahedra are the simplest polyhedra and any other polyhedra can be easily transformed into this form by subdivision, which has a strong application background. Second, a more immediate attempt is to avoid the ambiguity problem which exists in the MC method [6–8].

Most MT methods are designed to partition a cube into five tetrahedra (Fig. 1). According to the distribution of function values of vertices, a tetrahedron may be divided into three configurations (see Fig. 2). If two configurations may be converted into each other by taking contrary signs of all vertices or exchanging the signs of two vertices, these two configurations are considered to be equivalent. In Fig. 2, a vertex with a + sign has function value greater than threshold C_0, while one with a − sign has a function value equal to or less than C_0. If two endpoints of a tetrahedral edge take the same sign, it is assumed that the isosurface with threshold C_0 does not intersect this edge. Otherwise, there exists an

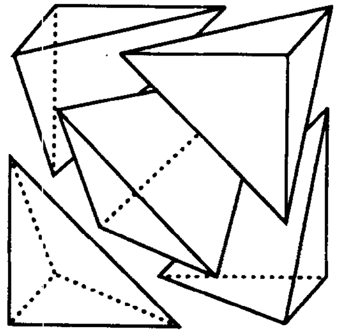

Fig. 1.

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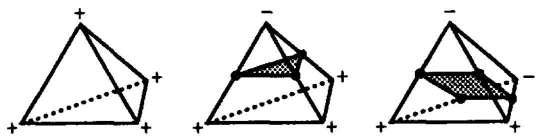


Fig. 2.

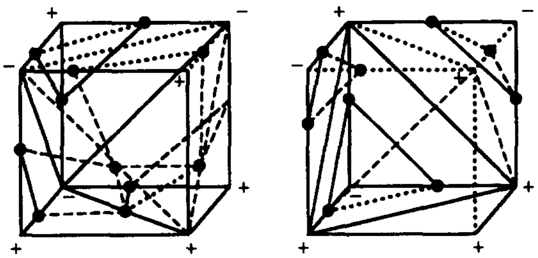


Fig. 3.

intersection point, which is connected with other ones to form a polygon for approximating the isosurface patch.

Although each polygon generated within a tetrahedron is uniquely determined by function values of vertices, there exist two absolutely different modes to subdivide a cube into five tetrahedra. The different subdivision will result in different polygon configurations. This means isosurface construction depends on the subdivision modes of cubes. Figure 3 illustrates this. In order to connect each polygon smoothly, *i.e.* not to generate flaws among polygons, adjacent cubes must be subdivided in mirror-like directions shown in Fig. 4. Thus, the construction of isosurfaces within the whole data sets is dependent on the initial subdivision direction. In other words, the existing MT method cannot solve the ambiguity problem. Unfortunately, so far, most of the published papers discussing the MT method [5, 6, 8, 9] have not touched upon this problem. In ref. [10] the authors pointed out that the orientation of the diagonal affects the connectivity of the vertices and the topological correctness is not provided, but they did not show an approach to solve it throughout.

Based on mathematical strictness and completeness, we first point out, for the assumption that function values vary linearly along each edge of cubes, that the function variations along the diagonal lines of cube's faces are quadratic rather than linear. We cannot simply make the assumption that the isosurfaces do not intersect an edge if its two endpoints have the same sign. Then, we present a criterion for testing the existence of intersection points and the method for calculating them, and discusses the connection rules of equivalued points and triangularization of polygons in detail and experimental results are shown. Finally, a comparison of the new marching tetrahedra method presented in this paper (NMT method), the traditional MC method and the existing MT method is given.

2. ISOSURFACE EQUATION AND INTERSECTION CURVES BETWEEN ISOSURFACES AND CUBES' FACES

We suppose that the 3-D data sets are uniform and the function values vary linearly along cubes' edges. The distribution function over a cube may be derived as follows. Given the cube $ABCDA_1B_1C_1D_1$ and an arbitrary point $P(x, y, z)$ within it. The intersection points between the cube and the line passing through point P and parallel to Y-axis are denoted by P_1 and P_2, respectively. The projected points from P_1 and P_2 along the X-axis onto the cube are denoted by P_{11} and P_{12}, P_{21} and P_{22}, shown in Fig. 5. The function value of P can be expressed in the following form:

$$f(P) = \frac{|P - P_1|}{|P_2 - P_1|}\,(f(P_2) - (f(P_1)) + f(P_1), \quad (1)$$

where $f(P_1)$ and $f(P_2)$ represent function values of P_1 and P_2. Following the same reasoning, $f(P_1)$ and $f(P_2)$ can be interpolated linearly with corresponding function values of four points P_{11} and P_{12}, P_{21} and P_{22}. Thus, Eq. (1) can be rewritten as follows.

$$f(P) = a_0 + a_1x + a_2y + a_3z + a_4xy$$
$$+ a_5yz + a_6zx + a_7xyz, \quad (2)$$

where a_i ($i = 0, 1, \ldots, 7$) can be obtained by solving the simultaneous linear equations, which can be constructed by substituting coordinates and function values of eight vertices for variables x, y, z and $f(P)$ in Eq. (2). If the cube is not degenerated, the equations always have an unique solution. Thus, the isosurface corresponding to threshold C_0 may be expressed by the following equation:

$$f(P) = C_0. \quad (3)$$

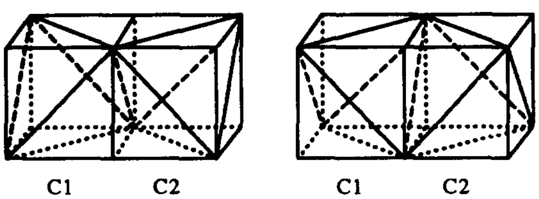


Fig. 4.

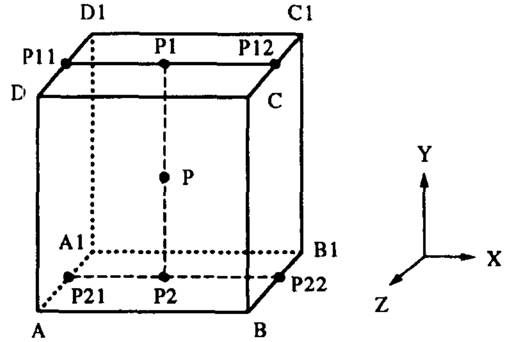
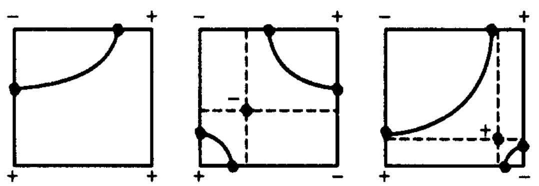


Fig. 5.



(a) (b) (c)

Fig. 6.

Without loss of generality, let the equation of the plane in which a cube's face is located represented by $Z = Z_0$. Substitute it into Eq. (3), we have

$$b_0 + b_1 x + b_2 y + b_3 xy = C_0, \tag{4}$$

where $b_0 = a_0 + a_3 z_0$, $b_1 = a_1 + a_6 z_0$, $b_2 = a_2 + a_5 z_0$, $b_3 = a_4 + a_7 z_0$. Obviously, Eq. (4) represents a hyperbola.

As shown in Fig. 6, if one branch of the hyperbola intersects the cube's face, it is not an ambiguous case in the MC method (Fig. 6a). If two branches of the hyperbola and its asymptotes intersect the cube's face, it is an ambiguous case in the MC method [Fig. 6(b) and (c)]. Nielson presented a method to solve the ambiguity problem [3].

In the MT method, we must analyse the function variation along an edge of a tetrahedron. The edges of a tetrahedron may be classified into two categories: the original edges of the cube and the diagonals of the cube's faces. Following the above assumption, the variation is linear along the original edges of the cube. However, the function distribution along a diagonal line of a cube's face can be derived by combining the diagonal equation with Eq. (4). Taking diagonal AC in Fig. 5 as an example, the equation of line segment AC can be written as

$$\begin{cases} x = x_a + t(x_c - x_a) \\ y = y_a + t(y_c - y_a) \end{cases} \quad (0 \le t \le 1), \tag{5}$$

where (x_a, y_a) and (x_c, y_c) are the coordinates of points A and C, respectively. Substituting Eq. (5) into (4), we find the variation along AC is quadratic, which may be given by

$$f(t) = c_0 + c_1 t + c_2 t^2 \quad (0 \le t \le 1).$$

Here, c_0, c_1, and c_2 are represented by $a_i (i = 0, 1, \ldots, 7)$ and hence by function values at vertices. Therefore, whether the isosurface with the value of threshold C_0 passes AC is dependent on whether the equation

$$c_2 t^2 + c_1 t + c_0 - C_0 = 0 \tag{6}$$

has solutions within the interval [0, 1].

When $c_0 \neq 0$ and $c_1^2 - 4c_2(c_0 - C_0) > 0$, the equation has two different solutions, t_1 and t_2. If values of both t_1 and t_2 fall in the interval [0, 1], there exist two intersection points between the isosurface and the segment AC. Such a case takes place when the two endpoints of the segment take the same sign as shown in Fig. 7a, 7b or 7c.

If we neglect these two intersection points owing to the two endpoints of a diagonal having the same sign, Fig. 7d and 7e will be constructed instead of Fig. 7b and 7c, respectively. It leads to incorrect and inaccurate connection of equivalued points. As shown in Fig. 1, there are 30 edges in total in the five tetrahedra subdivided from the cube and 18 of them are diagonals. It is a nontrivial problem in the MT method.

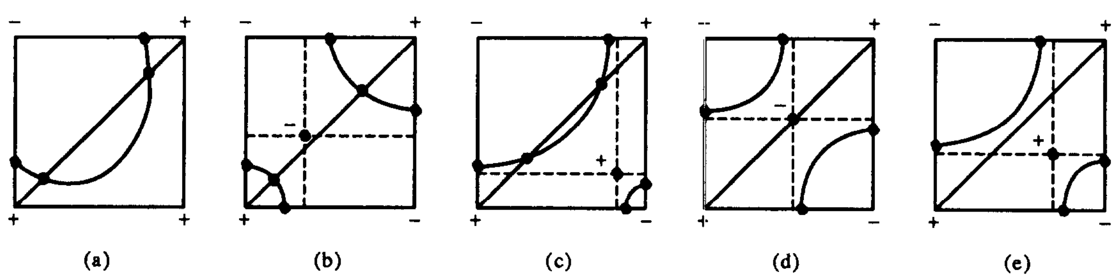
3. CONSTRUCTING ISOSURFACES WITHIN TETRAHEDRA

The construction of isosurfaces can be divided into three steps: calculation of the intersection points between the isosurface and the edges of tetrahedra; connection of the intersection points to form polygons; triangularization of polygons.

Calculation of intersection points

Apparently, values of the intersection points between the isosurface and the edges of tetrahedra will satisfy Eq. (3). Thus the pseudo program to evaluate them may be written as follows.

```
If E1E2 is an original edge of a cube's face
{
  if the function values of E1 and E2 take contrary
      signs
    calculate the intersection point by linear
      interpolation and output it;
  else
    no intersection point exist;
}
else
```



(a) (b) (c) (d) (e)

Fig. 7. The cases of intersection between the isosurface and the diagonal of the tetrahedron.

{
if the function values of E_1 and E_2 take the same sign
and the discriminant of Eq. (6) is not negative
solve Eq. (6) and output the intersection points which lie in between the two endpoints;
else if the function values of E_1 and E_2 take contrary signs
evaluate the two intersection points and take one of them which lies in between the two endpoints and output, discard the other one.
}

Connection of intersection points and construction of polygons

The intersection points between isosurfaces and the edges of triangles are called equivalued points. After the equivalued points lying on tetrahedral edges have been found, we should establish how to connect these points in order to construct polygons. First, we consider the connection of equivalued points on a tetrahedra face. In order to avoid the generation of a flaw on the common face of two adjacent tetrahedra, the intersection curve on the common face should be determined uniquely by the property of the common face itself, rather than be affected by other vertices of the tetrahedra. Therefore, the connecting lines among equivalued points cannot coincide with any edge of the triangle and cannot intersect with each other.

For a given triangle, there are six cases about the sign configuration of the function values of vertices and the distribution of the equivalued points (shown in Fig. 8).

Case 1 does not have any intersection point. It indicates that the isosurfaces do not intersect the triangle. In case 5, connecting I_1 and I_2, we have a contour passing through points I_1 and I_2.

Case 2 indicates that there exists a contour intersecting one edge of the triangle at two points which does not have any intersection point on other edges. The segment of the contour which connects points I_1 and I_2 will fall in the triangle, and it is required to find an equivalued point O so that segments I_1O and OI_2 approximate the contour. This point is referred as an additional equivalued point. The calculation of point O can be performed as follows. If point A is the triangle vertex opposite to the triangle edge passing through I_1 and I_2, and point M is the middle point of I_1I_2 (see Fig. 8–2), then, point M takes a negative sign while point A takes a positive sign. Thus, line segment AM must pass the isosurface, and the intersection point lies within the triangle. Although the coordinates of point O can be calculated by solving simultaneous equations, a simple method is to approximate the equivalued point by linear interpolation of the two endpoints. We have

$$O = M + \frac{C_0 - f(M)}{f(A) - f(M)}\,(A - M),$$

where C_0 is the threshold, $f(A)$ and $f(M)$ are the function values of points A and M.

Obviously, the connection of equivalued points in cases 2 and 5 can be determined uniquely, but for cases 3, 4 and 6, there exist several possible connection modes. So we need more delicate methods to deal with these cases. For convenience, both cases 2 and 5 are called the exclusive connection mode of contour, and cases 3, 4 and 6 the selective connection mode of contour.

In case 3, two possible connection modes exist (Fig. 8–3), we take $P = \frac{1}{4}\sum_{i=1}^{4} I_i$. If the sign of function value of P is opposite to those of triangle vertices, we follow the connection mode in Fig. 8–3(a); otherwise,

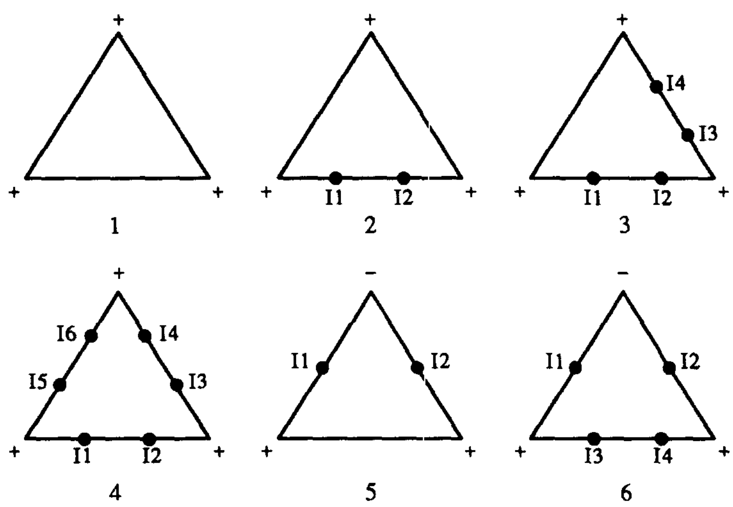


Fig. 8. The sign configuration of the function values of vertices and the distribution of equivalued points in a triangle. $I_i (i = 0, 1, \ldots, 5)$ are the equivalued points.

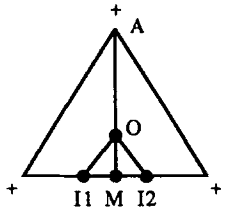


Fig. 8-2. The evaluation of the additional equivalued point.

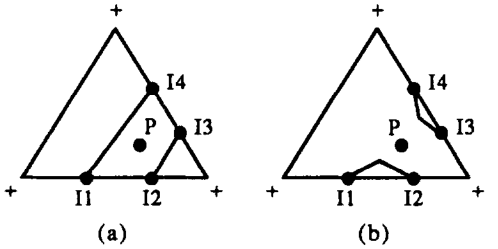


Fig. 8-3.

Table 1.

Sign of P_1	Sign of P_2	Sign of P_3	Connection mode
+	+	+	b
+	+	−	d
+	−	+	c
+	−	−	a
−	+	+	e
−	+	−	a
−	−	+	a
−	−	−	a

the connection mode in Fig. 8–3(b) is reasonable. Similarly, the coordinates of the additional equivalued point can be interpolated with point P and the middle point of two equivalued points.

Case 4 is the most complicated case. It possesses five possible connection modes. Table 1 lists all the processing methods, here $P_1 = \frac{1}{4}(I_1 + I_2 + I_3 + I_4)$, $P_2 = \frac{1}{4}(I_3 + I_4 + I_5 + I_6)$ and $P_3 = \frac{1}{4}(I_5 + I_6 + I_1 + I_2)$. In Table 1, the left three columns show all the possible cases of values taken by points P_1, P_2 and P_3, the rightmost column matches corresponding connection modes. For instance, sign "b" corresponds to Fig. 8–4(b). The location of the additional equivalued points can be derived by interpolation of corresponding middle points of the intersection points and the point taking positive sign among P_1, P_2 and P_3. For example, the additional equivalued point connecting I_5 and I_6 can be calculated by interpolating between point P_3 and the middle point of I_5 and I_6. It is necessary to explain that we force the case "two negative and one positive" in Table 1 corresponding to Fig. 8–4(a) for simplicity. The details will be discussed elsewhere.

In case 6, we take $P = \frac{1}{4}\sum_{i=1}^{4} I_i$. Figure 8–6(a) and 8–6(b) give the two modes of connection corresponding to $f(P)$ taking negative and positive signs, respectively.

The strategy to construct polygon is as follows. First, pick up a connecting line of equivalued points from a triangle of the tetrahedron, and then find out the next line, which starts from the end point of the preceding one from the triangle in which the end point of the preceding line lies. Then, mark the end point and compare it with the first one. If it is not the same as the first, continue to search for its next edge. Otherwise, the polygon has been closed and output this polygon. Then, test whether there exist connecting lines which have not been visited. If so, select a connecting line again and repeat the above steps until all the polygons have been output.

Triangularization of polygons

If we only consider the exclusive connection mode of contour and the selective connection mode without the cases with additional equivalued points, there are 16 polygon configurations shown in Fig. 9. These configurations can be divided into three groups. Case

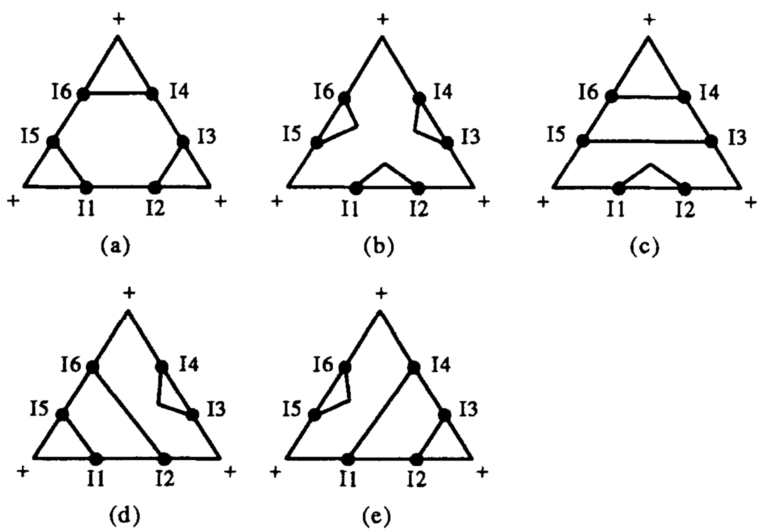


Fig. 8–4.

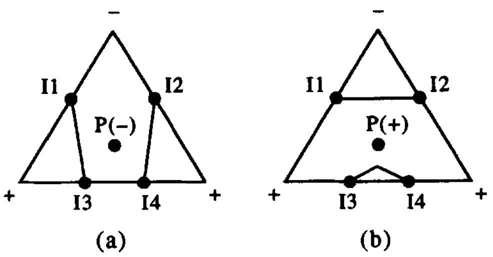


Fig. 8–6.

1 to case 4 are in group 1, case 5 to case 7 in group 2, the others in group 3. All the cases which belong to the same group have the same distribution of signs in vertex function values. The first case in each group (cases 1, 5 and 8) are considered by the existing MT method. However, not all the other cases are involved in it. Both cases 2 and 14 have one additional equivalued point, both cases 9 and 10 have two, case 13 has three and case 11 has four.

Usually, the algorithm generates at most four polygons within a tetrahedron and each polygon has at most eight vertices. In general, the polygons generated by connecting lines are nonplanar, which is not convenient to be further processed. If a polygon has more than three vertices, we prefer to make it triangularization. The principle of triangularization is as follows. Two vertices of a polygon in the same face of the tetrahedron cannot be connected with each other in order to prevent the connection from producing a singularity on that face. From the evaluation and connection of the equivalued points, it is apparent that the connecting lines between equivalued points in a face are determined entirely by the three vertices of a triangle. Since the generation of polygons within a tetrahedron is based on the faces of the tetrahedron, then the connection of the equivalued points on the common face of two adjacent tetrahedra is uniform, which guarantees a smooth connection of polygons on the common faces.

For a polygon with additional equivalued points, the triangularization algorithm only needs to connect any equivalued point with other vertices of the polygon to form $n-2$ triangles (where n is the number of vertices in the polygon). For other polygons, the algorithm connects two consecutive edges by a new edge to form a triangle. This triangle is then removed from the polygon and proceeds to the next one until $n-2$ triangles have been formed. But case 7 is an exception, no matter how the vertices are connected, there is eventually a generated triangle falling on the face. This corresponds to a subdivision case of a cube (see Fig. 10).

If less accuracy is acceptable, we take the centroid of the tetrahedron and connect it with all the vertices of the polygon. A more delicate method may be designed as follows. In the tetrahedron, we take one edge with two endpoints having a positive sign and the other edge with two endpoints having a negative sign. Then, two middle points of the equivalued points from each of these two edges are picked up.

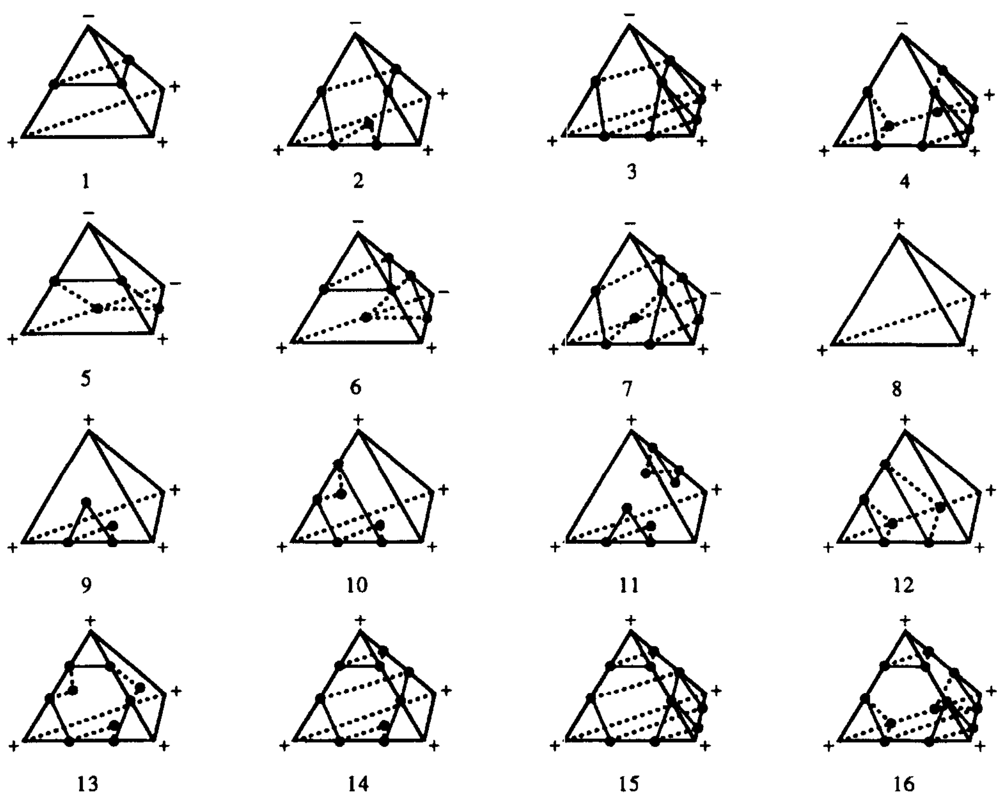


Fig. 9. Sixteen polygon configurations within a tetrahedron (not considering selective connection modes with additional equivalued points).

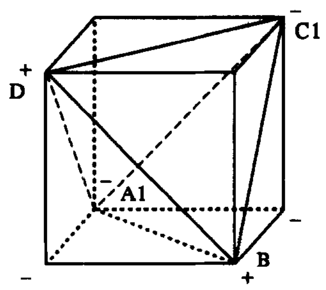


Fig. 10. Tetrahedron DBC_1A_1 corresponds to case 7 in Fig. 9.

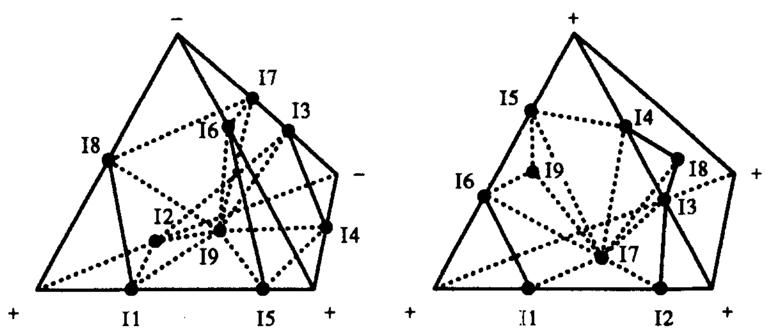
Thus, these two middle points have opposite signs. We calculate an additional equivalued point by interpolation of these two endpoints to substitute for the centroid. Figure 11 gives two examples of triangularization corresponding to cases 7 and 13.

If the cases with additional equivalued points in the selective connection mode of contour are considered, then there are 59 polygon configurations, shown in the appendix, rather than 16 polygon configurations in Fig. 9.

4. COMPARISON OF THE MC, MT AND NMT METHODS

Considering all the cases of the function value distribution of a face in a cube, a comparison of the MC, MT and NMT methods is shown in Fig. 12. The first column in Fig. 12 represents the correct connection of the intersection between an isosurface and the face, *i.e.* the contour segment. The other columns show the various results generated by the different methods with different subdivision modes. Cases 1–3 correspond to different rows in Fig. 12. The face in each row has the same distribution of function values of vertices. These three methods do not generate any contour in case 1. For case 2, the connection in all three methods are correct in topology, but the results are slightly different. In the MC method and subdivision 1 of MT method, the contour segments are approximated by the connection lines of two equivalued points. In the subdivision 2 of MT method, the intersection point on the diagonal is calculated by linear interpolation, while in the NMT method it is calculated by solving a quadratic equation.

For case 3, a great difference takes place among these three methods. In the MC method, ambiguity



(7) (13)

Fig. 11. Case 7: I_9 is an additional equivalued point. After triangularization of polygon $I_1I_2I_3I_4I_5I_6I_7I_8$, the following triangles are produced: $I_1I_2I_9$, $I_2I_3I_9$, $I_3I_4I_9$, $I_4I_5I_9$, $I_5I_6I_9$, $I_6I_7I_9$, $I_7I_8I_9$, $I_8I_1I_9$. Case 13: I_7, I_8 and I_9 are additional equivalued points. After triangularization of polygon $I_1I_7I_2I_3I_8I_4I_5I_9I_6$, the following triangles are produced: $I_2I_7I_3$, $I_3I_7I_8$, $I_8I_7I_4$, $I_4I_7I_5$, $I_5I_7I_9$, $I_9I_7I_6$, $I_6I_7I_1$.

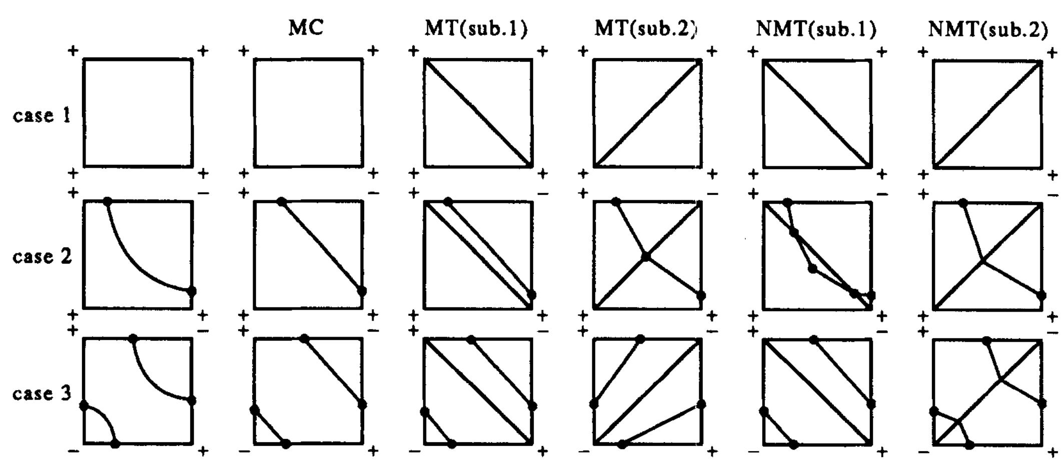


Fig. 12. Comparison of the three different methods: MC, MT and NMT. Each row corresponds to a case of distribution of function values of a face in a cube, and each column shows a result corresponding to the different methods with different subdivision modes.

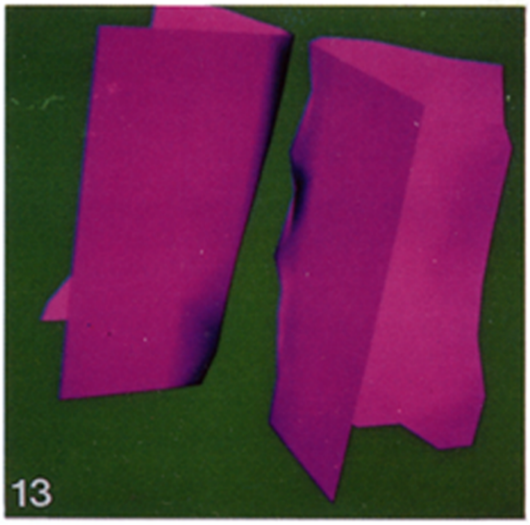


Fig. 13.

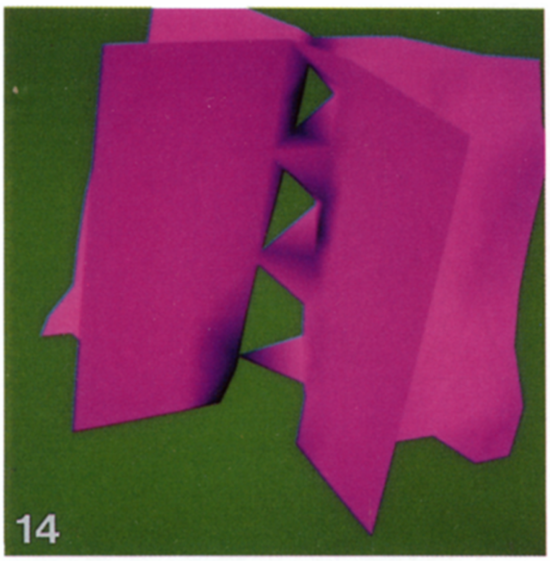


Fig. 14.

exists and the two possible connection modes are shown. In the MT method, different results in connection topology will be generated with respect to different subdivision modes. Only the NMT method can produce correct connections which approximate the actual contour segments no matter how subdivision is given.

In the following, we will give a test example which demonstrates the great difference between our new marching tetrahedra method and the existing MT method. The mathematical model designed for the test is

$$F(x, y, z) = (xy - zx - zy + 4)e^{y-2} - e^{x+0.25z-2}$$

The cubic meshes are organized as $10 \times 10 \times 10$ with the grid interval equal to 4. The origin of the

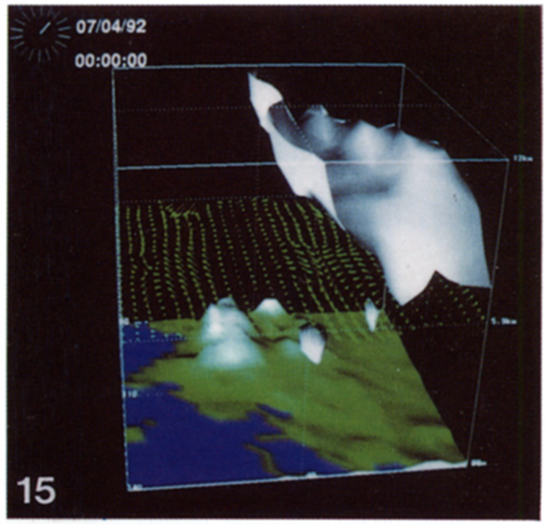


Fig. 15.

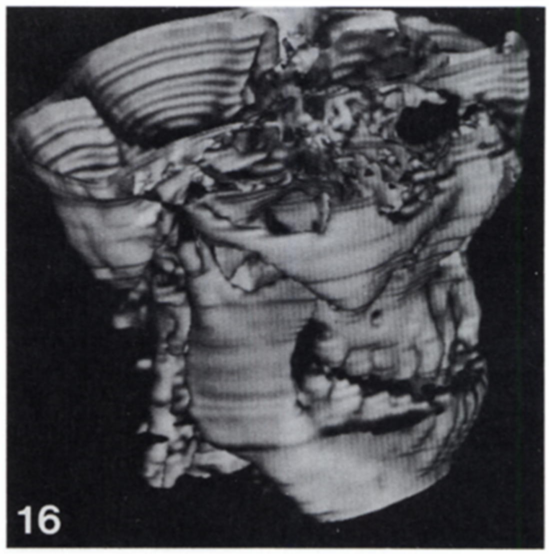


Fig. 16.

coordinates is the centroid of the meshes and the threshold C_0 is equal to zero.

Figure 13 is generated by our algorithm, while Fig. 14 is generated by the existing MT method. These two results have obvious differences. In Fig. 14 two components of the isosurface connect with each other at some grids, which is incorrect.

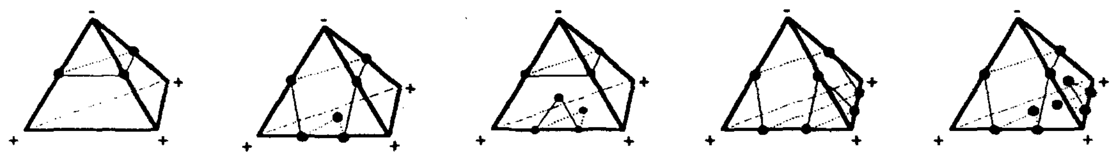
5. IMPLEMENTATION AND RESULTS

The algorithm presented in this paper has been implemented in C language on an SGI IRIS 4D/25 workstation. The triangles generated by our algorithm are displayed by GL hardware. The isosurface images in Figs 15 and 16 are generated by our algorithm. Figure 15 is a vortex isosurface of a meteorological data set over an area on north hemisphere of the earth, including China at certain time. The bottom of this image is a topographic drawing and the middle is the wind vector distribution. In Fig. 16, the skull of a man is shown.

REFERENCES

1. W. E. Lorensen and H. E. Cline, Marching cubes: a high resolution 3D surface construction algorithm. *Computer Graphics* **21**(4), 163–169 (1987).
2. R. S. Gallagher and J. C. Nagtegaal, An efficient 3D visualization technique for finite element models and other coarse volumes. *Computer Graphics* **23**(3), 185–194 (1989).
3. G. Nielson and B. Hamann, The asymptotic decider: resolving the ambiguity in marching cubes. *IEEE Visualization '91*, San Diego, 83–91 (1991).
4. J. Wilhelm and A. V. Gelder, Topological considerations in isosurface generation extended abstract. *Computer Graphics* **24**(5), 79–86 (1990).
5. A. Doi and A. Koide, An efficient method of triangulating equi-valued surface by using tetrahedral cells. *IEICE Trans.* **E74**, 214–224 (1991).
6. B. A. Payne and A. W. Toga, Surface mapping brain function on 3D models. *IEEE CG&A* **10**(5), 33–41 (1990).
7. J. Bloomenthal, Polygonization of implicit surfaces, *CAGD* **5**, 341–355 (1988).
8. C. Giertsen, Volume visualization of sparse irregular meshes. *IEEE CG&A* **12**(2), 40–48 (1992).
9. T. T. Elvins, A survey of algorithms for volume visualizing. *Computer Graphics* **26**(3), 194–201 (1992).
10. P. Ning and J. Bloomenthal, An evaluation of implicit surface tilers. *IEEE CG&A* **13**(6), 33–41 (1993).

APPENDIX



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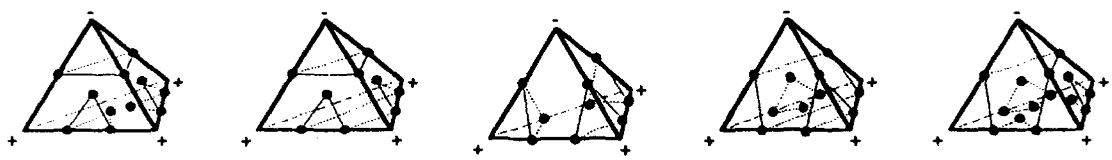

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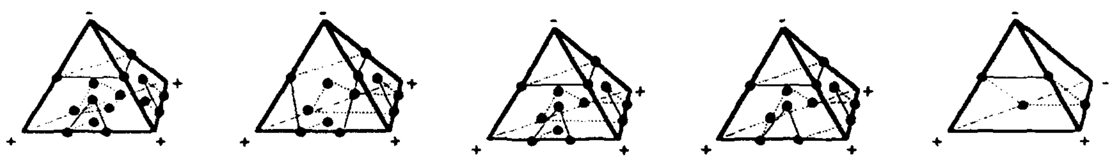

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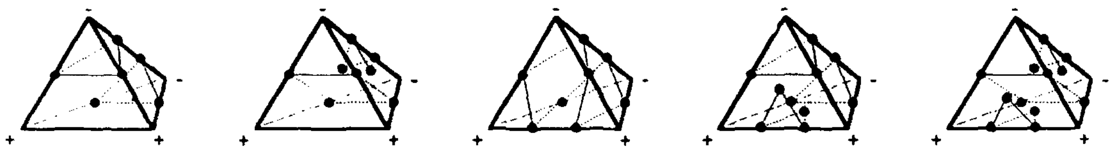

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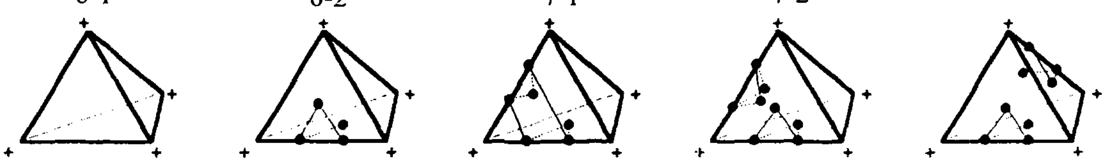

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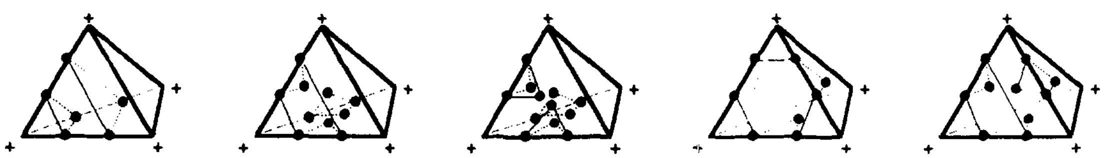

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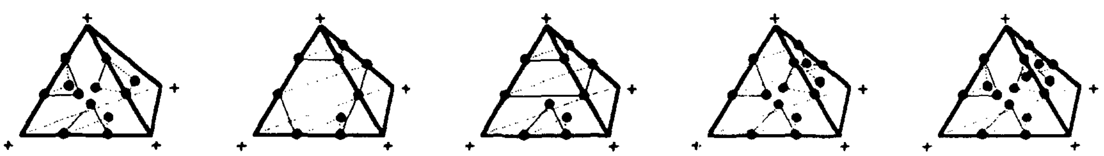

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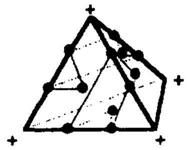
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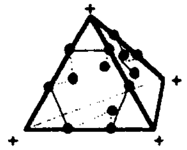
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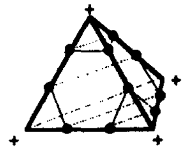
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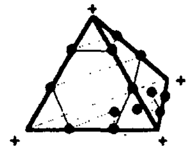
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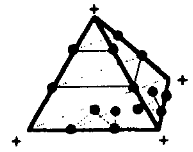
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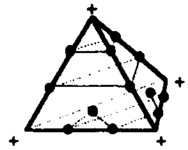
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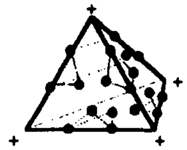
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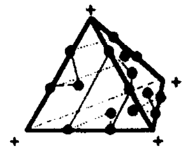
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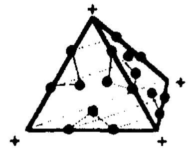
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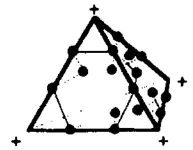
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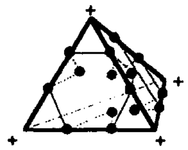
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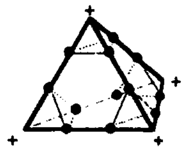
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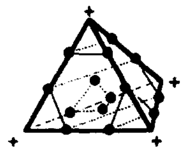
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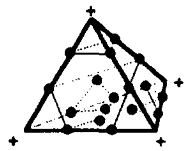
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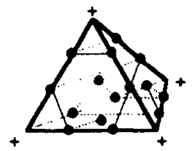
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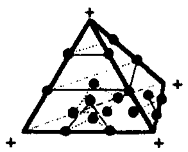
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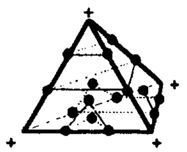
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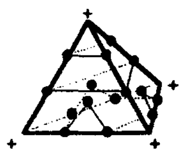
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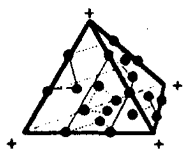
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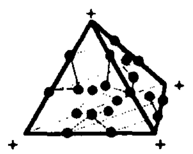
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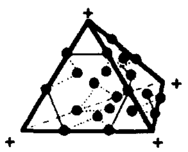
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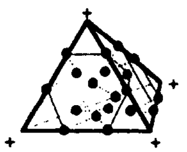
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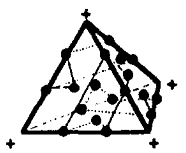
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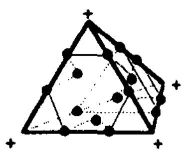
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