Inequality and a Repeated Joint Project

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Topic:

Influence of share inequality on the scope for cooperation sustainability $\rightarrow \delta$

Novelty:

Renegotiation-proof and coalition-proof solution for a prisoners' dilemma with n players and continuous strategies

Results:

- in a specific model, negative effect of inequality on cooperation, efficiency
- coexistence of one cooperating and one deviating coalition
- repetition of the game and outside options enlarge the scope for redistribution

Repeated PD : Renegotiation proofness

	C	D
C	1,1	-1,2
D	2,-1	0,0

Nash Reversion

$$(1-\delta)\pi^D + \delta\pi^N < \pi^C \Rightarrow \delta > \frac{\pi^D - \pi^C}{\pi^D - \pi^N} \Rightarrow \delta > \frac{1}{2}$$

Problem : both players suffer from the punishment $\Rightarrow$ *incentives to renegotiate.*

Renegotiation Proof Solution

$$\begin{array}{lcl}
\delta_{XA} > \frac{\pi_D - \pi_C}{\pi_C - \pi_P} & \Rightarrow & \delta > \frac{1}{2} \\
\delta_{XP} > \frac{\pi^{P*} - \pi^P}{\pi^C - \pi^P} & \Rightarrow & \delta > \frac{1}{2} \\
\pi^{1/P*} < \pi^{1/P} & \rightarrow & 1 < 2 \\
\pi^{1/P} \geq \pi^C & \rightarrow & 2 > 1
\end{array}$$

Outline

- There exist many examples of voluntarily provided joint projects in the real world:
  collective action problems in management of environmental resources (forests, fisheries, irrigation schemes), financial lobbying, defense alliances, etc.
- In static games, the first-best optimum is known not to be sustainable as nothing prevents agents from deviating.
  $\Rightarrow$ If such a game is infinitely repeated, one can expect new (1^{st} best) solutions to be sustainable.
- If there exists inequality between agents, Olson (1965) argued that this kind of projects work best when one single agent gets all the shares (concentration of all the incentives).
- Olson's result depends on unstated assumptions:
  - Nash behaviour
  - perfect substitutability of efforts
  - identical marginal costs
  - single interactions

Literature review:

- Effect of inequality on efficiency is ambiguous (depending on the cost function) $\rightarrow$ Khwaja (2006), Barnerjee et al. (2001, 2006).
- Tarui (2007), influence of inequality of productivity, access to markets and credit in a dynamic intergenerational game of common property resource use.
- Bardhan & Singh (2005), influence of wealth inequality on cooperation sustained by Nash reversion in an infinitely repeated game.
- Itaya & Yamada (2003), influence of income inequality on a repeated game of private provision of public goods (2 players).
- Vasconcelos (2005), tacit collusion in quantity-setting supergames with asymmetric costs.

- Joint project : voluntary contributions
- Same discount factor for all agents
- Output divided according to some given vector of shares (for instance: wealth inequality):
  $\boldsymbol{\lambda} \equiv [\lambda_1, \lambda_2, \ldots, \lambda_n]$ with $\sum_i \lambda_i = 1$
- Collective action individual payoff: $\pi_i = \lambda_i \sum_{j \in n} e_j - \frac{e_i^\gamma}{\gamma \lambda_i}$, with $\gamma \geq 2$
- Cooperation level of effort: $e_i^C = \lambda_i^{\frac{1}{\gamma-1}}$

  Deviation level of effort: $e_i^N = \lambda_i^{\frac{2}{\gamma-1}}$

  $\Rightarrow e_i^C > e_i^N$ (underprovision of the 'public good')

Repeated Joint Production with Shares

Lemma

The agent who benefits most from deviating, relatively to one's share, is always the one with the lowest share.

Relatively, the poorest player is the one having the most to win.
$\rightarrow$ 'Exploitation by the poor'

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- If $\pi_i^{C^*} < \pi_i^C$, deviating from the cooperation effort is not profitable.
  For deviating to be interesting: $\pi_i^{C^*} > \pi_i^C$.
  If $\lambda_i = n^{-1}$,
  - always true if $n_D = 1$.
  - if $\gamma \geq 2$, true if $n_D < \frac{n+1}{2}$.
  - n_D/n decreases with n

$\rightarrow$ if too many deviations, the small surplus is to be divided among too many for deviating to remain profitable.

Nash Reversion

$$(1-\delta_N)\pi_i^{C^*} + \delta_N \pi_i^N < \pi_i^C \Rightarrow \delta_N > \frac{\pi_i^{C^*} - \pi_i^C}{\pi_i^{C^*} - \pi_i^N}$$

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- $\delta_N < 1 \Rightarrow$ the bigger the difference $\pi_i^C - \pi_i^N$, the smaller δ_N and therefore the easier cooperation can be sustained.

$$\lambda_i = n^{-1} \Rightarrow \pi_i^C > \pi_i^N$$

Proposition

Introducing inequality among agents renders the condition to sustain cooperation with Nash reversion more difficult to fulfill.

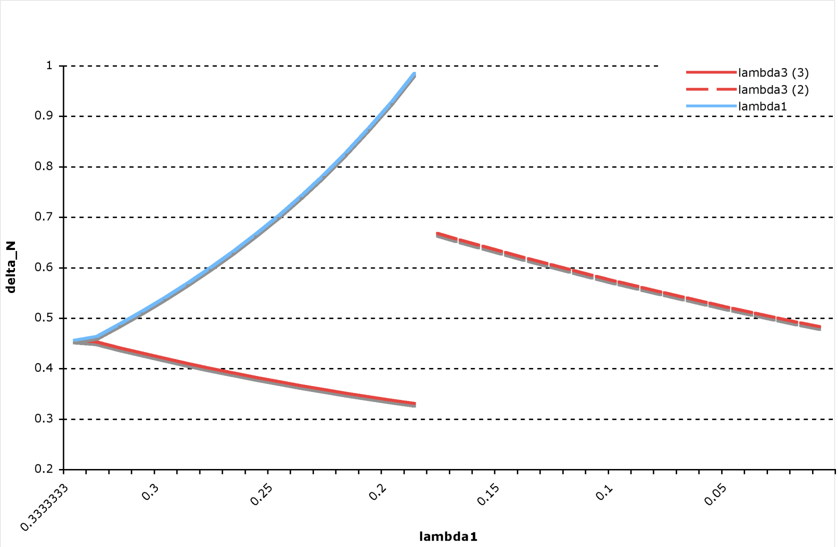

Figure: Influence of the λ_1 distribution on δ_N when $\gamma = 3$ and $n = 3$

A Renegotiation-Proof and Coalition-Proof Equilibrium

→ define R and P: efforts when punishing and being punished

Punishment scheme:

As soon as a coalition deviates, the cooperative agents play a retaliation level of effort R until the deviators have undergone their punishment P.

A Renegotiation-Proof and Coalition-Proof Equilibrium

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Length of punishment:

Trade-off between *ex ante* discouraging deviations and *ex post* encouraging deviations from the punishment (latter effect is dominating).

Lemma

Aiming at the smallest discount factor compatible with a renegotiation-proof and coalition-proof punishment limits the length of the punishment phase to one period.

5 conditions for the equilibrium to be renegotiation-proof and coalition-proof:

2 conditions concerning the deviators

- Ex ante, the punishment must be such that deviations are deterred. ($\rightarrow \delta_{XA}$)
- Ex post, the punishment must be such that deviations from the punishment are deterred. ($\rightarrow \delta_{XP}$)

2 conditions concerning the cooperators

- The payoff of the punishers must be greater when conforming than when deviating from punishing and then conforming, i.e. $\pi_i^{1/P} > \pi_i^{1/P^*}$. $\rightarrow$ gives $R \equiv e_i^N$.
- For the equilibrium to be coalition-proof, no deviation should be credible, i.e. $\pi_i^{1/P^*} < \pi_i^{1/P^{**}}$.

  As no deviation of punishers is credible when the punished conform to their punishment, a fortiori, it is also the case when some punished refuse to undergo the penalty.

1 condition for renegotiation-proofness

- The payoff from punishing a deviating coalition must be greater or equal than the payoff from generalized cooperation, i.e. $\pi_i^{1/P} \geq \pi_i^C$. $\rightarrow$ gives P.

We want to get $\underline{\delta} \equiv \min \max(\delta_{XA}, \delta_{XP})$
Comparing δ_{XA} and δ_{XP} is comparing $\pi^{C^*} - \pi^C$ and $(\leq)\pi^N - \pi^P$.

- if $P = e_i^C$, $\underline{\delta} = \delta_{XA} = \delta_{XP}$.
- if $P > e_i^C$, we have to minimize P to get $\underline{\delta} \Rightarrow P \equiv \underline{P}$.
- if $P < \underline{P}$, the punishment would not be RPCP.
- if $\underline{P} < e_i^C$, agents could alternate between deviating and being punished.

$$\Rightarrow P \equiv \max(e_i^C, \underline{P})$$

It means that, under an equalitarian distribution:

- $P = e_i^C$, when $n_D \geq \frac{n}{2}$ if $\gamma \geq 2$
- in the other cases, $P = \underline{P}$

The condition to be fulfilled for cooperation to be sustainable is:

$$\pi_i^C > \pi_i^N$$

Proposition

As long as $\delta_{XP} < \delta < 1$, $\lambda_i = n^{-1}$ and the game is infinitely repeated, it is possible to sustain cooperation with a renegotiation-proof and coalition-proof punishment.

Once we introduce inequality, the same rule applies.

Proposition

After introducing inequality, the agents losing from the disequalizing change in the distribution of shares have to be more patient than before to produce the efficient level of effort when the punishment is renegotiation-proof and coalition-proof.

- $$\delta_N \leq \delta_{XP}$$

Redistribution and Cooperation

Coexistence of 2 coalitions

If all know that some are so poor that $\pi_k^N > \pi_k^C$ and therefore cannot afford to cooperate, the rich players are expected not to punish them.

Observation

If a subset of agents do not cooperate because of a low $\underline{\lambda}_k$ (such that $\pi_k^C < \pi_k^N$), a low-producing (putting in the Nash level of effort) and a high-producing (providing the efficient level of effort) coalition can coexist.

2 players with a high enough positive share are needed for the cooperative effort to be produced.

In the presence of 2 coalitions, the total level of effort put in the project is $\sum_i e_i = \sum_{j \in n_C} \lambda_j^{\frac{1}{\gamma-1}} + \sum_{k \in n_D} \lambda_k^{\frac{2}{\gamma-1}}$.

Increasing inequality,

- $D \to D \rightsquigarrow C$, $\triangle^+$ total level of effort
- $D \to C$, depends on γ ($\geq 3 : \triangle^-$,$\exists \tilde{\gamma} \in [2, 3[: \triangle = 0$, $< \tilde{\gamma} : \triangle^+$)
- $D \to D$, depends on γ ($< 3 : \triangle^+$, $= 3 : \triangle = 0$,$> 3 : \triangle^-$)

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A disequalizing redistribution can therefore increase or decrease the total level of efforts put in for the project, depending on the value of γ and the status of the concerned players. $\rightsquigarrow$ U shape

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Diminishing inequality

There are cases where, following their best interest, the rich players would benefit from redistributing part of their share to the poor so that the latter can afford to cooperate.

Outside Option

Nash Reversion

$$(1-\delta)\pi_i^{C^*} + \delta\pi_i^N < \pi_i^C \Rightarrow \delta > \frac{\pi_i^{C^*} - \pi_i^C}{\pi_i^{C^*} - \pi_i^N}$$

$$(1-\delta)\pi_i^{C^*} + \delta\pi_i^S < \pi_i^C \Rightarrow \delta > \frac{\pi_i^{C^*} - \pi_i^C}{\pi_i^{C^*} - \pi_i^S}$$

$$\Rightarrow \pi_i^C > \max(\pi_i^N, \pi_i^S)$$

RPCP

Besides conditions on δ_{XA} and δ_{XP},

$$(1-\delta)\pi_i^P + \delta\pi_i^C > \pi_i^S \Rightarrow \delta > \frac{\pi_i^S - \pi_i^P}{\pi_i^C - \pi_i^P}$$

$$\Rightarrow \pi_i^C > \max(\pi_i^N, \pi_i^S)$$

Summary Table

$$\boldsymbol{\pi_i^{C^*}} \quad > \quad \boldsymbol{\pi_i^{C}} \quad > \quad \boldsymbol{\pi_i^{N}}$$

$$\begin{array}{cccc}
\uparrow & \uparrow & \uparrow & \uparrow \\
\pi_i^S & (1-\delta)\pi_i^{C^*} + \delta\pi_i^S & \pi_i^C & \pi_i^C \\
 & & \text{if } \delta_{N\to S} < \delta & \text{if } \delta_N < \delta
\end{array}$$

Restricted access to the project

$$\boldsymbol{\pi_i^{C^*}} \quad > \quad \boldsymbol{\pi_i^{N}} \quad > \quad \boldsymbol{\pi_i^{C}}$$

$$\begin{array}{cccc}
\uparrow & \uparrow & \uparrow & \uparrow \\
\pi_i^S & (1-\delta)\pi_i^{C^*} + \delta\pi_i^S & (1-\delta)\pi_i^{C^*} + \delta\pi_i^N & (1-\delta)\pi_i^{C^*} + \delta\pi_i^N
\end{array}$$

Conclusion

- Inequality reduces the scope for cooperation, efficiency (through $\triangle^+\delta_N$ or δ_{XP}).
- Possibility of coexistence of a cooperating and deviating coalition.
- Share inequality can have a U-shaped relationship with the aggregate level of effort.
- Repetition of the game and outside options increase the scope for redistribution.

Condition

Ex ante, the punishment must be such that deviations are deterred.

For a return to cooperation after the punishment phase
(and for preventing the deviators from alternating between deviating and being punished):

$$(1-\delta)\pi_i^{C^*} + \delta(1-\delta)\pi_i^P + \delta^2\pi_i^C < \pi_i^C \Rightarrow \delta_{XA} > \frac{\pi_i^{C^*} - \pi_i^C}{\pi_i^C - \pi_i^P}$$

Condition

Ex post, the punishment must be such that deviations from the punishment are deterred.

$$(1-\delta)\pi_i^P + \delta\pi_i^C > (1-\delta)\pi_i^{P^*} + \delta(1-\delta)\pi_i^{P'} + \delta^2\pi_i^C \Rightarrow \delta_{XP} > \frac{\pi_i^{P^*} - \pi_i^P}{\pi_i^C - \pi_i^{P'}}$$

$$\frac{\partial\delta_{XP}}{\partial n_{D^*}} > 0 \Rightarrow \underset{n_{D^*}\in(1,n_D)}{\arg\max}\ \delta_{XP} = n_D$$

Condition

The payoff of the punishers must be greater when conforming than when deviating from punishing and then conforming, i.e. $\pi^{1/P} > \pi^{1/P^*}$.

This condition prevents punishers to skip the punishment phase.

If $\lambda_i = n^{-1}$, we get:

$$\gamma \frac{n_{C^*}}{n}(R - R^*) > (R^\gamma - {R^*}^\gamma)$$

- it must be always true if $n_{C^*} = 1$.
- if it is not true, when $n_{C^*} > 1$, this deviating subcoalition cannot be credible.

Condition

For the equilibrium to be coalition-proof, no deviation should be credible, i.e. $\pi_i^{1/P^*} < \pi_i^{1/P^{**}}$.

If $\lambda_i = n^{-1}$ and $n_{C^{**}} = 1$,

$$\frac{\gamma}{n} < \frac{(1 - n^{\frac{-\gamma}{\gamma-1}})}{(1 - n^{\frac{-1}{\gamma-1}})}$$

We know from those two conditions that, if $R \equiv e_i^N \equiv \lambda_i^{\frac{2}{\gamma-1}}$, no credible coalition of punishers can deviate from giving a punishment.

$$R \equiv e_i^N \Rightarrow \overline{\delta_{XP}} > \frac{\pi_i^N - \pi_i^P}{\pi_i^C - \pi_i^P}$$

As no deviation of punishers is credible when the punished conform to their punishment, a fortiori, it is also the case when some punished refuse to undergo the penalty.

Condition

The payoff from punishing a deviating coalition must be greater or equal than the payoff from generalized cooperation, i.e. $\pi_i^{1/P} \geq \pi_i^C$.

$$\lambda_i = n^{-1} \Rightarrow \underline{P} = n^{\frac{-2}{\gamma-1}}[n^{\frac{\gamma}{\gamma-1}}(1-\gamma^{-1}) - (n_C - \gamma^{-1})]\frac{1}{n_D}$$

$$\lambda_i \neq n^{-1} \Rightarrow$$

$$\underline{P}_j = \frac{\lambda_k}{\sum_{k \in n_D} \lambda_k}\left[\sum_{i \in n} \lambda_i^{\frac{1}{\gamma-1}} - \sum_{j \in n_C} \lambda_j^{\frac{2}{\gamma-1}} - \gamma^{-1}\frac{\sum_{j \in n_C} \lambda_j^{\frac{2-\gamma}{\gamma-1}}(1-\lambda_j^{\frac{\gamma}{\gamma-1}})}{n_C}\right]$$