University of Zurich

Least Squares method

Annapaola de Cosa

PHYS451 - Experimental Particle Physics

Exercise class 6

25th October 2016

Least Squares method

Contrary to ML method, the least squares method is independent on the *pdf* of observations

- **Given N measurements yi and a model f(x, θ) describing the dependence of y on the variable x**
  - y = f(x, θ)
  - θ is our set of unknown parameters
- **The sum of square deviations of observed y_{obs}^i from expected y_{exp}^i is then minimised**
  - $$\chi^2(\theta) = \sum_{i=1}^{N} (y_i - f(x_i;\theta))^2$$
  - Where y_{obs}^i indicates the entries in the *i*-th bin and y_{exp}^i is the expected number of entries

If the theory is in good agreement with the data, then yobs and yep do not differ by much and the value S will be small

Goodness of fit

- **The expected distribution of the minimum χ^2 value is known and goes like:**
  - $$P(\chi^2; n) = \frac{2^{-n/2}}{\Gamma(n/2)} \chi^{n-2} e^{-\chi^2/2}$$
  - where n is the number of degrees of freedom of the problem (number of measurements N minus the number of fit parameters m).
- **The p-value, defined as the probability of having a χ^2 greater or equal to the value obtained at the fit minimum**
  - Small values for the p-value could indicate a poor description of the theoretical model, once best parameters are found
  - So, the χ^2 is used a goodness-of-fit tool
- **ML estimator does not provide a measurement of the goodness-of-fit**
  - Although in some cases it can be obtained by performing the ration of the likelihood functions evaluated in two hypotheses (Wilks' theorem).

Exercise 1 - Linear regression with Root

Consider the following set of measurements and perform a linear regression:

- x = [1., 2., 3., 4., 5.]
- y = [1.49, 1.58, 1.67, 1.80, 1.908]

- **Graph the set of measurements**
- **Fit the graph using a linear function defined as T2F**
  - $y = a + bx$
  - Which are the best estimates of the two parameters?

Exercise 2 - Linear regression with LS

Consider the following set of measurements and perform a linear regression:

- x = [1., 2., 3., 4., 5.]
- y = [1.49, 1.58, 1.67, 1.80, 1.908]

- **Implement the least squares method to fit this set of measurements**
  - Define the function χ^2
  - Convert this function in a root T2F function, depending on the two parameters a and b
  - Get the values of the two parameters a and b that correspond to the minimum of the function Chi2
    - These will be the values of a and b parameters better fitting data

- **Compare these values with those gotten from step 1**

Exercise 3 - Angular distribution

- **Consider the exercise from previous class (Exercise class 5)**
  - Perform the fit using the least square method instead of the ML one:
    - Compute the χ^2 function
    - Perform a scan of the parameter α
    - Find the value of parameter α that minimises the function χ^2