Numerical Methods
Exercise Sheet 2

HS 16
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http://www.physik.uzh.ch/en/teaching/PHY233/HS2016.html

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Exercise 1: Lagrange's Interpolation (40 Pts.)
Write an interpolation program for a given data set by Lagrange's method. The input data are represented in a .txt file as:

0 0
1 1
2 4

Algorithm:

1. Open the input data file. (1 point)
2. Identify the number of points on which your interpolation will be based. (1 point)
3. Create a two-dimensional array of the right size. (1 point)
4. Read the data into an array. (1 point)
5. Close the file. (1 point)
6. Identify minimum x_{min} and maximum x_{max} of the arguments value. (2 points)
7. Identify an interpolation's step $\delta x = \frac{(x_{max} - x_{min})}{N}$, where N is a number of points in which interpolation function will be calculated($N \geq 100$ is recommended). (2 points)
8. Open a new file to write results. (1 point)
9. Iterate over k, starting at 0. (1 point)
10. Compute the k^{th} abscissa x_k^{int} of interpolation function. (3 points)
    $x_k^{int} = x_{min} + \delta x \cdot k$
11. If $x_k^{int} > x_{max}$, go to item 16. (2 point)
12. Compute the k^{th} ordinate of the interpolation function $y_k^{int} = L(x_k^{int})$ (15 points)

$$L(x) = \sum_{i=0}^{n} y_i \prod_{j=0, j\neq i}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

13. Write the values (x_k^{int}, y_k^{int}) to file. (1 point)

14. Increase the value of k by one. (2points)
15. Go back to item 10.
16. Close the file with results. (1 point)
17. Plot results. (5 points)
18. Optional: Plot results using programming code. (extra 10 points)

Exercise 2: Newton's interpolation (40 Pts.)

Write an interpolation program for a given data set by Newton's method. Input data are represented in a .txt file as:

0 0
1 1
2 4

Algorithm:

1. Open the input data file. (1 point)
2. Identify the number of points on which the interpolation will be based. (1 point)
3. Create a two-dimensional array of the right size. (1 point)
4. Read the data into a array. (1 point)
5. Close the file. (1 point)
6. Check the uniformity of the grid by comparing $(x_1 - x_0)$ with $(x_{i+1} - x_i)$ for $i = 1, ..., (n-1)$. If uniformity is not achieved, then display a notification and finish the program. (3 points)
7. Identify the grid size h. (2 points)
8. Identify the minimum x_{min} and maximum x_{max} of the arguments value. (2 points)
9. Identify an interpolation's step $\delta x = \frac{(x_{max} - x_{min})}{N}$, where N is a number of points in which interpolation function will be calculated ($N \geq 100$ is recommended). (2 point)
10. Open a new file to write results. (1 point)
11. Iterate over k, starting at 0. (1 point)
12. Compute the k^{th} abscissa x_k^{int} of interpolation function. (2 point)
    $x_k^{int} = x_{min} + \delta x \cdot k$
13. If $x_k^{int} > x_{max}$, go to item 18. (1 point)

14. Compute the k^{th} ordinate of the interpolation function, y_k^{int}. (14 points)

$$N(x) = y_0 + \frac{\Delta^{(1)} y_0}{h}(x - x_0) + \frac{\Delta^{(2)} y_0}{2!h^2}(x - x_0)(x - x_1) + ... + \frac{\Delta^{(n)} y_0}{n!h^n}(x - x_0)(x - x_1)...(x - x_{n-1})$$

, where $\Delta^{(k)} y_i = \sum_{s=0}^{k} (-1)^{s+k} \frac{k!}{s!(k-s)!} y_{i+s}$

15. Write values (x_k^{int}, y_k^{int}) to file. (1 point)
16. Increase the value of k by one. (2 points)
17. Go back to item 12.
18. Close the file with results. (1 point)
19. Plot the results. (3 points)
20. Optional: Plot results using programming code. (extra 10 points)
21. Optional: Using of one program with ability to set the interpolation method (Newton, Lagrange) by command line options. (extra 10 points)

Exercise 3: Cubic splines (20 Pts.)
Students should be able to answer the questions and substantiate their answers:

1. What is the diference between spline and Lagrange's interpolations.
2. Why do we need to use (cubic) spline interpolation.
3. Which function is used for cubic spline interpolation.
4. What is the diference between B-spline interpolation and regular splines.

Exercise 4: Cubic splines: optional (bonus 30 Pts.)
Write an interpolation program for a given data set by cubic spline method (use cubic B-splines).

Maximum number of points for mandatory tasks on 03.10 : 100
Maximum possible number of points for tasks on 03.10 : 160

Maximum number of points for mandatory tasks on 19.10 : 70
Maximum possible number of points for tasks on 19.10 : 130

Maximum number of points for mandatory tasks on 24.10 : 30
Maximum possible number of points for tasks on 24.10 : 90

Maximum number of points for mandatory tasks after 24.10 : 0
Maximum possible number of points for tasks after 24.10 : 60
