Numerical Methods
Exercise Sheet 3

HS 16
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Tasks with cubic splines are transferred to this sheet as they were not performed by anyone on the last seminar.

Exercise 1: Cubic splines (60 Pts.)

a) Students should be able to answer the questions and substantiate their answers: (20 points)

1. What is the difference between spline and Lagrange's interpolations.
2. Why do we need to use a (cubic) spline interpolation.
3. Which function is used for a cubic spline interpolation.
4. What is the difference between a B-spline interpolation and a regular splines.

b) Write an interpolation program for a given data set by a cubic spline method. (40 points)

- Define the cubic interpolation polynomial for the each domain of the $F[x_0, .., x_n]$ function. (see lecture)
- Use the continuity condition of the function, the continuity condition of the first and second derivatives and the free (natural) boundary conditions to find coefficients of the polynomial (feel free to use any other boundary conditions, for instance, the clamped boundary conditions).
- Solve the resulting system of equations to find coefficients.
- Evaluate the interpolation function based on the calculated coefficients.

c) Optional: Plot results. (extra 10 points)

Exercise 2: Bilinear interpolation (40 Pts.)

a) Show how to obtain the 2D formula for a bilinear interpolation (1), in case of a coordinate system in which the four points where function is known are (0, 0), (0, 1), (1, 0), and (1, 1) by a successively linear interpolation in x and in y axis. (20 points)

$$P(x,y) = \begin{pmatrix} 1-x & x \end{pmatrix} \cdot \begin{pmatrix} f(0,0) & f(0,1) \\ f(1,0) & f(1,1) \end{pmatrix} \cdot \begin{pmatrix} 1-y & y \end{pmatrix} \tag{1}$$

b) Show how to obtain the 2D formula for a bilinear interpolation (1), in case of a coordinate system in which the four points where function is known are (0, 0), (0, 1), (1, 0), and (1, 1) by a finding coefficients of: (20 points)

$$P(x, y) = a_0 + a_1 x + a_2 y + a_3 xy$$

Exercise 3: Extrapolation: optional (40 Pts.)

Based on the lecture notes, extrapolate the function $f(x) = \frac{(cos(x)-1)}{sin(x)}$ to x = 0 with $h_k = 2^{-k}$ using interpolating polynomials P_n for $n = (1, .., 4)$.

Maximum number of points for mandatory tasks on 19.10 : 100
Maximum possible number of points for tasks on 19.10 : 150