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http://www.physik.uzh.ch/en/teaching/PHY233/HS2016.html

Exercise 1: Runge rule (30 Pts.)

Prove that the Simpson's formula can be obtained by application of the Runge rule to the trapezoidal formula. How does it change the order of accuracy of the method?

Exercise 2: Richardson extrapolation method (20 Pts.)

a) Write a program to determine the integral $\int_0^1 \frac{1}{1+sin(x^2)}dx$ numerically for a given grid of values by a trapezoidal rule, determine the integral with a higher precision using the Richardson extrapolation method, and compare the result with the answer obtained by the Simpson's method.

b) Optional: Propose (and implement) another method of getting the integral with a higher precision for a given grid of values by the trapezoidal rule (**hint:** Euler–Maclaurin). (additionally 20 points)

x	0.	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1
f(x)	1.0	0.984616	0.941213	0.877068	0.801665	0.724235	0.652187	0.590671	0.543044

Exercise 3: Runge-Romberg method (50 Pts.)

Write a program to determine the integral $\int_0^1 \frac{\sqrt{(1-x^2)}}{1.1-x}dx$ numerically for a given grid of values by a trapezoidal rule using the Runge-Romberg algorithm. Why is it not correct to use Richardson and Euler–Maclaurin methods in this case?

x	0.	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1
f(x)	0.909091	1.007266	1.139113	1.278655	1.443376	1.643421	1.889822	2.151657	0.00

Maximum number of points for mandatory tasks on 16.11 : 100
Maximum possible number of points for tasks on 16.11 : 120