Reflection and transmission of light by a rough surface, including results for surface-plasmon effects

E. Kretschmann and E. Kröger

Institut für Angewandte Physik der Universität Hamburg, D2000 Hamburg 36, Jungiusstrasse 11, Germany
(Received 5 February 1974)

A second-order theory is developed for the reflection and transmission of light by a slightly rough surface. The general results are discussed, especially for a rough interface between vacuum and a free-electron-like metal, where surface-plasma excitation can occur. The influence of the damping of the electron gas and of the shape of the roughness is discussed. Some different previous theories for reflection by a rough surface are shown to be limiting cases of the present theory.

Index Headings: Reflection; Scattering; Surface-guided waves.

It is well known that reflectances and transmittances at a plane interface between two dielectrics are changed by surface roughness. If the surface is rough, there are diffusely scattered waves in all directions (incoherent fields). Moreover, the amplitudes of specularly reflected and directly transmitted light (the spatially coherent fields) are changed by roughness. These phenomena have been used in different ways to determine the surface roughness[1–5] or to determine surface-plasmon effects.[6–8]

However, previous theories fail to describe properly all of the scattering effects. Scalar theories[9] work quite well for roughness scales long compared with the wavelength of the light. They do not allow for the possibility of surface-plasmon generation. In other theories, only the fraction of the incident light is calculated that is absorbed by surface plasmons or which is scattered diffusely.[6] This is not exactly the deficit of the reflection, as has been assumed.

In this work, we calculate directly the first nonvanishing contribution of the change of the directly reflected and transmitted coherent fields, which is of second order in the surface-roughness amplitude. We use a method that we introduced in a previous paper.[10] It is an approximate approach that starts from the exact boundary conditions at the rough surface, similar to the methods in Refs. 11 and 12. The effects of diffuse scattering of the incident light as photons and surface plasmons as well as backscattering of photons and plasmons into the spatially coherent fields are included. The scalar theory[9] and the special results[6] for surface-plasmon excitation are shown to be limiting cases of our results.

I. THE ROUGH SURFACE

We consider two dielectrics ϵ_1, ϵ_2, separated by an interface $z - S(\vec{x}) = 0$, where $\vec{x} = (x, y)$ and x, y, z are rectangular coordinates (see Fig. 1). A vector normal to the interface is

$$\vec{n} = (\partial S/\partial x,\ \partial S/\partial y,\ -1)\ . \tag{1}$$

We assume that $S(\vec{x})$ is a statistical disturbance of the plane interface $z = 0$, with mean value

$$\langle S\rangle_{av} = \frac{1}{F}\int_F d^2\vec{x}\, S(\vec{x}) = 0\ ,$$

where F is a sufficiently large area. The amplitude $S(\vec{x})$ shall be assumed small compared with the wavelength of the electromagnetic fields involved. We denote the Fourier transform of $S(\vec{x})$ by

$$s(\vec{k}) = \frac{1}{(2\pi)^2}\int d^2\vec{x}\, S(\vec{x})\, e^{-i\vec{k}\vec{x}},\quad \vec{k} = (k_x, k_y). \tag{2}$$

For the following calculations, we need the mean square

$$\langle S^2\rangle_{av} = \frac{1}{F}\cdot \int_F d^2\vec{x}\cdot [S(\vec{x})]^2\ , \tag{3}$$

the normalized correlation function

$$G(\vec{x}) = \frac{1}{S^2 F}\cdot \int_F d^2\vec{x}'\cdot S(\vec{x}')\cdot S(\vec{x}'+\vec{x})\ ,$$

and its Fourier transform

$$g(\vec{k}) = (2\pi)^2 \frac{1}{\langle S^2\rangle F}\,|s(\vec{k})|^2\ . \tag{4}$$

In our calculations, we have to take statistical averages of an ensemble of surfaces with the correlation function (4),

$$\langle f(\vec{k}')\rangle_{av} = \left\langle \int d^2\vec{k}\cdot d^2\vec{k}_0\cdot s(\vec{k}'-\vec{k})\cdot s(\vec{k}-\vec{k}_0)\cdot a(\vec{k})\cdot b(\vec{k}_0)\right\rangle_{av} \tag{5}$$

with functions $a(\vec{k})$ and $b(\vec{k})$, which may be arbitrary but independent of surface roughness. Assuming that the averages are independent with translation, we get

$$\langle f(\vec{k}')\rangle_{av} = \langle S^2\rangle_{av}\cdot b(\vec{k}')\cdot \int d^2\vec{k}\cdot g(\vec{k}'-\vec{k})\cdot a(\vec{k})\ . \tag{6}$$

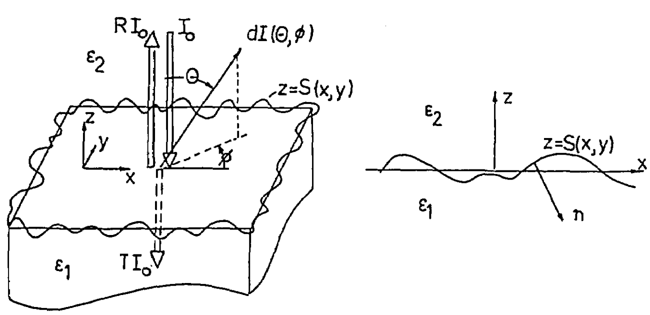

FIG. 1. Geometry of the rough surface.

II. FUNDAMENTAL EQUATIONS FOR SCATTERING THEORY

We start with the usual boundary conditions for the tangential components of the fields at the interface

$$z=S(\vec{x}),\quad \Delta\vec{E}\times\vec{n}=0,\quad \Delta\vec{H}\times\vec{n}=0\ . \tag{7}$$

Δf means $f_2(x,y,z)-f_1(x,y,z)$, where the subscripts 1 and 2 indicate functions in the dielectrics ϵ_1 and ϵ_2, respectively. From Eqs. (1) and (7) we have for $z=S(\vec{x})$

$$\begin{aligned}\Delta E_x(S)&=-\Delta E_z(S)\cdot\frac{\partial S}{\partial x}\ ,\quad \Delta E_y(S)=-\Delta E_z(S)\cdot\frac{\partial S}{\partial y}\ ,\\ \Delta H_x(S)&=-\Delta H_z(S)\cdot\frac{\partial S}{\partial x}\ ,\quad \Delta H_y(S)=-\Delta H_z(S)\cdot\frac{\partial S}{\partial y}\ .\end{aligned}\tag{8}$$

In this and in subsequent equations, an x,y dependence is understood but not written explicitly.

As has been pointed out in Ref. 10, we assume that the fields within both dielectrics can be continued continuously from $z=0$, so that Maxwell's equations are satisfied. Then Eqs. (8) can be transformed into new ones, in which the differences of the fields at $z=0$ appear on the left-hand sides (see Ref. 10, p. 14). Suppressing a time dependence $e^{-i\omega t}$, we get

$$\begin{aligned}\Delta E_x(0)&=-\frac{\partial}{\partial x}\Delta\int_0^S E_z(z)\,dz-i\frac{\omega}{c}\Delta\int_0^S H_y(z)\,dz\ ,\\ \Delta E_y(0)&=-\frac{\partial}{\partial y}\Delta\int_0^S E_z(z)\,dz+i\frac{\omega}{c}\Delta\int_0^S H_x(z)\,dz\ ,\\ \Delta H_x(0)&=-\frac{\partial}{\partial x}\Delta\int_0^S H_z(z)\,dz+i\frac{\omega}{c}\Delta\int_0^S D_y(z)\,dz\ ,\\ \Delta H_y(0)&=-\frac{\partial}{\partial y}\Delta\int_0^S H_z(z)\,dz-i\frac{\omega}{c}\Delta\int_0^S D_x(z)\,dz\ .\end{aligned}\tag{9}$$

From these equations we shall calculate the fields by a perturbation approach in the amplitude of the roughness.

III. CALCULATION OF THE SCATTERED FIELDS

We want to treat the special case in which a plane light wave is incident from the dielectric ϵ_2 perpendicular to the mean interface $z=0$. The electric-field vector is assumed to be parallel to the y axis. We shall treat, successively, the first- and second-order fields. Because $\langle S\rangle=0$, the first-order fields are spatially incoherent and their average amplitude vanishes. Thus, they do not change the specularly reflected and the direct transmitted fields. For the second-order fields, however, there remains a finite average contribution, which changes the reflectance and transmittance. Thus, in general, we have to calculate fields up to second order if we want to calculate the changes of reflectance and transmittance due to roughness. Only for a special case, discussed in Sec. IV A, can we find the change of reflectance from the irradiance scattered into first-order fields by energy-conservation considerations.

A. First-order scattered fields

From Eqs. (9) and $E_x^{(0)}=H_y^{(0)}=0$ we obtain, to first order of S,

$$\Delta E_x^{(1)}(0)=\Delta E_y^{(1)}(0)=\Delta H_y^{(1)}(0)=0\ ,$$

$$\Delta H_x^{(1)}(0)=i\cdot\frac{\omega}{c}\cdot S\circ D_y^{(0)}(0)\ . \tag{10}$$

The superscripts indicate the order of approximation. The equivalent Fourier-transformed system with respect to the x,y coordinates is (by use of the convolution theorem)

$$\begin{aligned}&\Delta e_x^{(1)}(\vec{k},0)=\Delta e_y^{(1)}(\vec{k},0)=\Delta h_y^{(1)}(\vec{k},0)=0\ ,\\ &\Delta h_x^{(1)}(\vec{k},0)=i\cdot\frac{\omega}{c}\int d^2\vec{k}_0\cdot s(\vec{k}-\vec{k}_0)\cdot d_y^{(0)}(\vec{k}_0,0)\ .\end{aligned}\tag{11}$$

By the boundary conditions Eqs. (11) at $z=0$, the Maxwell equations, and appropriate boundary conditions for $|z|\to\infty$, the fields are determined uniquely. The resulting amplitudes for the p-polarized (or transverse magnetic) and s-polarized (or transverse electric) fields are

$$\begin{aligned}h_{2p}^{(1)}(\vec{k})&=-\frac{\epsilon_2k_1}{\epsilon_1k_2+\epsilon_2k_1}\Delta h_x^{(1)}(\vec{k})\sin\phi\ ,\\ h_{1p}^{(1)}(\vec{k})&=-\frac{\epsilon_1k_2}{\epsilon_2k_1}h_{2p}^{(1)}(\vec{k})\ ,\\ e_{2s}^{(1)}(\vec{k})&=-\frac{(\omega/c)}{k_1+k_2}\Delta h_x^{(1)}(\vec{k})\cos\phi\ ,\\ e_{1s}^{(1)}(\vec{k})&=e_{2s}^{(1)}(\vec{k})\ ,\end{aligned}\tag{12}$$

where

$$k_n=[\epsilon_n(\omega/c)^2-k^2]^{1/2};\quad \mathrm{Im}(k_n)>0;$$
$$k=|\vec{k}|;\quad \phi=\arccos(k_x/k)\ .$$

k_n (with $n=1,\ 2$) are the z components of the wave vectors in the dielectrics ϵ_n. ϕ denotes the angle between the positive x axis and $\vec{k}$. Note that there are poles of Eqs. (12) with maximum amplitudes for $h_{ip}^{(1)}$ ($i=1,2$), owing to excitation of surface plasmons $\epsilon_2k_1+\epsilon_1k_2=0$, or, equivalently,

$$k=k_{sp}(\omega)=\left(\frac{\omega}{c}\right)\left(\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}\right)^{1/2}\ . \tag{13}$$

The radiant flux scattered into the lower and upper half-space can be calculated by integration of the z component of the Poynting vector just below and above the plane $z=0$ on the xy planes defined by $z=\pm\delta\to 0$

$$\begin{aligned}I_{sc}&=\frac{c}{8\pi}\mathrm{Re}\int d^2\vec{x}((\vec{E}_2^{(1)}\times\vec{H}_2^{(1)*})_z+(\vec{E}_1^{(1)}\times\vec{H}_1^{(1)*})_z)\\ &=\frac{c}{8\pi}(2\pi)^2\,\mathrm{Re}\int d^2\vec{k}\left(\frac{k_2}{\epsilon_2}|h_{2p}^{(1)}|^2+k_2|e_{2s}^{(1)}|^2\right.\\ &\qquad\left.+\frac{k_1}{\epsilon_1}|h_{1p}^{(1)}|^2+k_1|e_{1s}^{(1)}|^2\right)\ .\end{aligned}\tag{14}$$

Putting the incident irradiance $I_0=(c/8\pi)\sqrt{\epsilon_2}\cdot F\circ|E^{(0)}|^2$, we obtain from Eqs. (12) and (14)

$$\frac{I_{sc}}{I_0}=4\sqrt{\epsilon_2}\,|\sqrt{\epsilon_2}-\sqrt{\epsilon_1}|^2\left(\frac{\omega}{c}\right)^2\langle S^2\rangle_{av}\,\mathrm{Re}(Q) \tag{15}$$

with the abbreviations

$$Q=\int d^2\vec{k}g(\vec{k})w'(\vec{k})\ ,$$

$$w'(\vec{k})=w_p(k)\sin^2\phi+w_s(k)\cos^2\phi\ , \tag{16}$$

$$w_p(k)=\frac{k_1k_2(c/\omega)}{\epsilon_1k_2+\epsilon_2k_1}\ ,\quad w_s(k)=\frac{(\omega/c)}{k_2+k_1}\ .$$

As we assume $g(\vec{k})$ to be independent of the angle ϕ, the integration over ϕ is possible and we get

$$Q=\int dk\,kg(k)\pi w(k)\ ,\qquad w(k)=w_p(k)+w_s(k)\ . \tag{17}$$

B. Average second-order fields

As in the case of the zero-order fields the average second-order fields propagate along the positive or negative z axis. Thus they do not depend on x and y, and we get from Eqs. (9) by omitting terms containing the derivatives $\partial/\partial x$, $\partial/\partial y$

$$\begin{aligned}\Delta E_x^{(2)}&=0\ ,\\ \Delta E_y^{(2)}&=i\frac{\omega}{c}\left(S\Delta H_x^{(1)}+\tfrac{1}{2}S^2\Delta\frac{\partial}{\partial z}H_x^{(0)}\right)=-\tfrac{1}{2}\left(\frac{\omega}{c}\right)^2S^2\Delta D_y^{(0)}\ ,\\ \Delta H_x^{(2)}&=i\frac{\omega}{c}\left(S\Delta D_y^{(1)}+\tfrac{1}{2}S^2\Delta\frac{\partial}{\partial z}D_y^{(0)}\right)\ ,\\ \Delta H_y^{(2)}&=-i\frac{\omega}{c}S\Delta D_x^{(1)}\ .\end{aligned} \tag{18}$$

Now we use Eq. (6) and

$$\Delta d_y^{(1)}(\vec{k})=-(\epsilon_2-\epsilon_1)w'(\vec{k})\frac{c}{\omega}\Delta h_x^{(1)}(\vec{k})\ ,$$

which we can obtain from Eq. (12), and get for the averages of the Fourier-transformed fields

$$\begin{aligned}\langle\Delta e_x^{(2)}\rangle_{\rm av}&=\langle\Delta h_y^{(2)}\rangle_{\rm av}=0\ ,\\ \langle\Delta e_y^{(2)}\rangle_{\rm av}&=-\tfrac{1}{2}(\omega/c)^2\langle S^2\rangle\,\Delta d_y^{(0)}\ ,\\ \langle\Delta h_x^{(2)}\rangle_{\rm av}&=\tfrac{1}{2}(\omega/c)^2\langle S^2\rangle\{\sqrt{\epsilon_1}+2(\epsilon_2-\epsilon_1)\cdot Q\}\,\Delta d_y^{(0)}\ ,\end{aligned} \tag{19}$$

where Q is the integral defined by Eq. (16). From the discontinuities of Eqs. (19), we get the average second-order fields, in the same way we got the fields of Eqs. (12). We obtain the result that the reflected and transmitted zero-order amplitudes of the smooth surface

$$r_0=\frac{\sqrt{\epsilon_2}-\sqrt{\epsilon_1}}{\sqrt{\epsilon_2}+\sqrt{\epsilon_1}}\ ,\quad t_0=\frac{2\sqrt{\epsilon_2}}{\sqrt{\epsilon_2}+\sqrt{\epsilon_1}}$$

have to be changed to

$$\begin{aligned}r&=r_0[1-2(\omega/c)^2\langle S^2\rangle\sqrt{\epsilon_2}\{\sqrt{\epsilon_1}+(\epsilon_2-\epsilon_1)Q\}]\ ,\\ t&=t_0[1+(\omega/c)^2\langle S^2\rangle(\sqrt{\epsilon_2}-\sqrt{\epsilon_1})^2\{\tfrac{1}{2}-(\sqrt{\epsilon_1}+\sqrt{\epsilon_2})Q\}]\ ,\end{aligned} \tag{20}$$

which is correct up to the second order of the roughness amplitude for the reflected and transmitted electric-field amplitudes. The reflection coefficient r, Eq. (20) was calculated by Rice[13] in a slightly unusual notation without discussion. For the relative changes of the intensities, we get up to the second order for S, with

$$\begin{aligned}&\frac{\Delta R}{R_0}=\frac{(|r|^2-|r_0|^2)}{|r_0|^2}\ ,\quad \frac{\Delta T}{T_0}=\frac{(|t|^2-|t_0|^2)}{|t_0|^2}\ ,\\ &\frac{\Delta R}{R_0}=-4\left(\frac{\omega}{c}\right)^2\langle S^2\rangle{\rm Re}[\sqrt{\epsilon_2}\{\sqrt{\epsilon_1}+(\epsilon_2-\epsilon_1)Q\}]\ ,\\ &\frac{\Delta T}{T_0}=\left(\frac{\omega}{c}\right)^2\langle S^2\rangle{\rm Re}[(\sqrt{\epsilon_2}-\sqrt{\epsilon_1})^2\cdot\{1-2(\sqrt{\epsilon_1}+\sqrt{\epsilon_2})Q\}]\ .\end{aligned} \tag{21}$$

IV. DISCUSSION OF RESULTS

First, we shall apply Eqs. (15) and (21) to some special problems that have been discussed in previous works.

A. Scattered radiation and excitation of surface plasmons at a rough vacuum-metal interface without damping

Consider the case $\epsilon_2=1$ and $\epsilon_1=\epsilon$, ${\rm Re}(\epsilon)<0$, ${\rm Im}(\epsilon)\to 0$. Then the integral ${\rm Re}(Q)$ can be split into two parts. The first part is from $0<k<\omega/c$ and represents the light that is scattered into the vacuum by the surface roughness. If we put

$$d^2\vec{k}=k\cdot dk\cdot d\phi=(\omega/c)^2\sin\theta\cos\theta\,d\theta\,d\phi=(\omega/c)^2\cos\theta\,d\Omega\ ,$$

where θ and ϕ are the polar angles of observation of the scattered light, indicated in Fig. 1, we obtain per steradian $d\Omega$ from Eq. (14)[14]

$$\left(\frac{dI}{I_0\,d\Omega}\right)_{\rm light}=4\left(\frac{\omega}{c}\right)^4g\left(\frac{\omega}{c}\sin\theta\right)|\sqrt{\epsilon}-1|^2\cos^2\theta\langle S^2\rangle\times\left\{\frac{|\sin^2\theta-\epsilon|\sin^2\phi}{|\epsilon\cos\theta+(\epsilon-\sin^2\theta)^{1/2}|^2}+\frac{\cos^2\phi}{|\cos\theta+(\epsilon-\sin^2\theta)^{1/2}|^2}\right\}\ . \tag{22}$$

The term in $\sin^2\phi$ is the p-polarized fraction and the term in $\cos^2\phi$ is the s-polarized fraction of the scattered light.

A second part for ${\rm Re}(Q)$ is obtained if $k=k_{\rm sp}(\omega)$, see Eq. (13). This is a δ-function contribution and tells us how much energy is scattered into the nonradiative surface plasmon. We get from Eq. (15)

$$\left(\frac{I}{I_0}\right)_{\rm sp}=(2\pi)^2\left(\frac{\omega}{c}\right)^4\langle S^2\rangle\frac{\epsilon^2}{(-\epsilon-1)^{5/2}}g(k_{\rm sp})\ . \tag{23}$$

Equations (22) and (23) agree with those of Ref. 7. An equation similar to Eq. (22) has been obtained also in Ref. 8.

If we evaluate $\Delta R/R_0$ for the optical constants, specialized at the beginning of this section, we find from Eq. (21)

$$\frac{\Delta R}{R_0}=\Delta R=-\int d\Omega\left(\frac{dI}{I_0\,d\Omega}\right)_{\rm light}-\left(\frac{I}{I_0}\right)_{\rm sp}\ . \tag{24}$$

Thus we find that the decrease of the power of the specularly reflected light is equal to the power of the scattered light plus the power scattered into surface plasmons. This is a special result, which is no longer true if there is damping within the metal. Numerical evaluations of the quantities $(I/I_0)_{\rm sp}$ from Eq. (23) and ΔR [Eq. (24)] for a free-electron gas are presented as special cases of more-general results in Sec. IV C and shown in Fig. 5.

The evaluation of $\Delta T/T_0$ is a little more complicated, because a Cauchy integral around $k=k_{\rm sp}(\omega)$ is involved.

B. The long-wavelength limit of surface roughness: The scalar theory

For the long-wavelength limit of surface roughness, we approximate the correlation function $g(\vec{k})$ by the δ

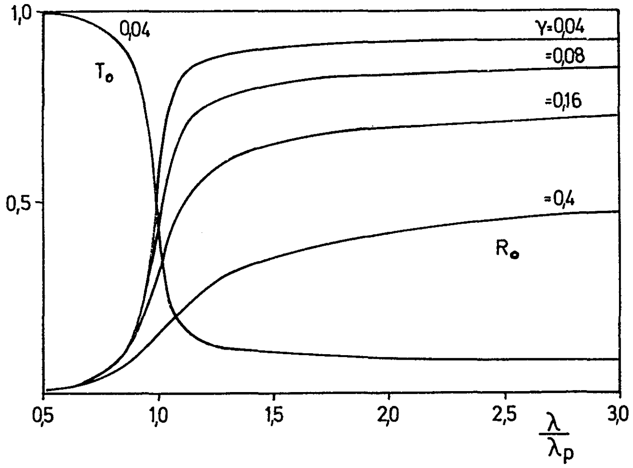

FIG. 2. Reflectance R_0 and transmittance T_0 at the smooth interface vacuum–metal (free-electron-like) for different damping constants γ.

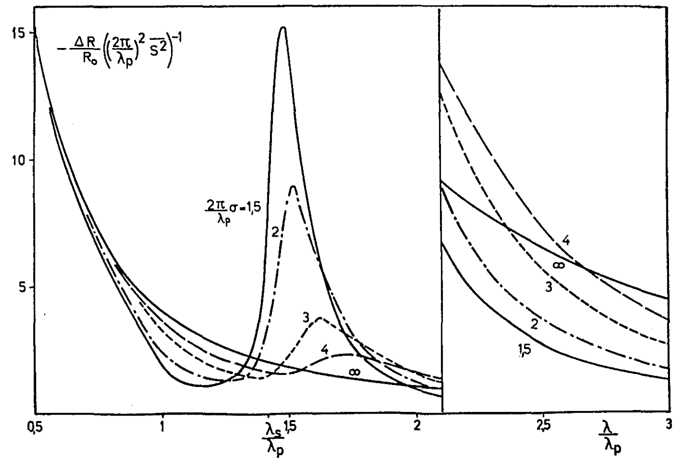

FIG. 4. Relative change of reflectance $-\Delta R/R_0 = -(R-R_0)/R_0$ from a rough-surface vacuum–free-electron-like metal ($\gamma = 0.04$) for various correlation lengths.

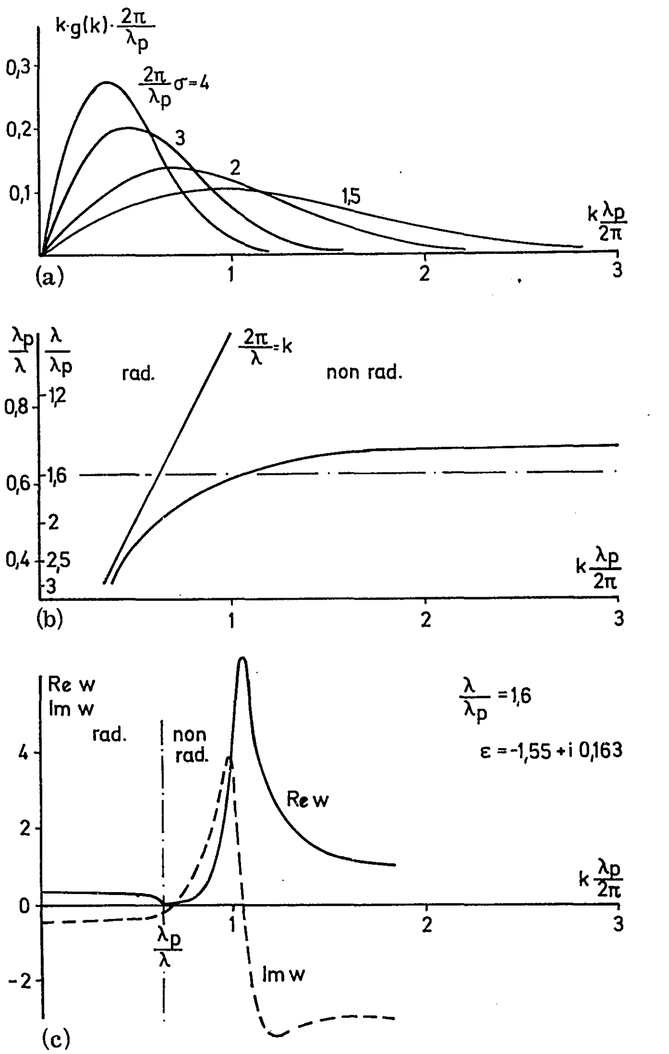

FIG. 3. (a) Surface-roughness function $k \cdot g(k)$ for various correlation lengths. (b) Dispersion curve of surface-plasma waves at a free-electron-like metal; the curve denotes the maximum of $\mathrm{Re}[w(k)]$. (c) Real and imaginary part of $w(k)$ for $\lambda/\lambda_p = 1.6$ and $\gamma = 0.04$.

function $\delta(\vec{k})$. Then we get

$$Q = w'(\vec{k}=0) = \frac{1}{\sqrt{\epsilon_1}+\sqrt{\epsilon_2}} . \tag{25}$$

If we put $\epsilon_2 = 1$ and let $\epsilon_1 = \epsilon$ be an arbitrary dielectric constant, which may have an imaginary part, we have

$$\frac{\Delta R}{R_0} = -4\left(\frac{\omega}{c}\right)^2 \langle S^2 \rangle , \tag{26}$$

$$\frac{\Delta T}{T_0} = -\left(\frac{\omega}{c}\right)^2 \langle S^2 \rangle \mathrm{Re}(1-\sqrt{\epsilon})^2 . \tag{27}$$

Equation (26) is identical with the lowest order of the so-called scalar theory. It has been used for qualitative determinations of the surface-roughness height.[1] It is not possible to describe surface-plasmon generation with this simplified model.

C. Reflectance and transmittance change for a rough surface with finite correlation length of an electron gas with damping

We shall discuss Eqs. (21) for a vacuum–metal interface. The metal is assumed to have the dielectric function of a free-electron gas,

$$\epsilon_1 = 1 - \left(\frac{\lambda}{\lambda_p}\right)^2 \frac{1}{1 + i\gamma(\lambda/\lambda_p)} , \tag{28}$$

where λ_p is the plasma wavelength and γ stands for the plasmon damping (see Fig. 2). For the correlation function, we shall use a gaussian $G(\vec{x}) = \exp(-\vec{x}^2/\sigma^2)$. Then, we get for the Fourier-transformed correlation function

$$g(\vec{k}) = g(k) = \frac{1}{4\pi}\sigma^2 \cdot \exp(-\sigma^2 k^2/4) . \tag{29}$$

A key for understanding the results that we shall get is the form of the integrand of the expression for Q, Eq. (16). In Fig. 3 we give results for the function $k \cdot g(k)$ with different correlation lengths as parameter and for the function $w(k)$ for a certain value of λ. The curve for the maximum value of $\mathrm{Re}\{w(k)\}$ for different values of λ is also shown. This is the well-known sur-

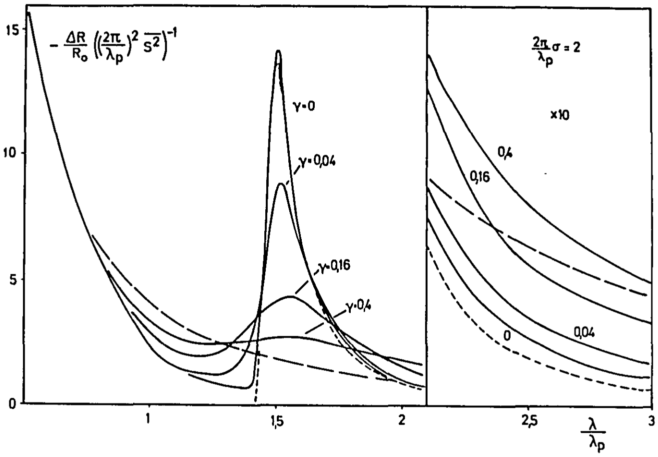

FIG. 5. Relative decrease of reflectance for various damping constants of the metal ($2\pi\sigma/\lambda_p = 2$). The curve for $\gamma = 0$ is equal to ΔR from Eq. (24). Dashed line - - -: $(I/I_0)_{sp}$ from Eq. (23); dashed line — —: scalar limit Eq. (26) (independent of ϵ).

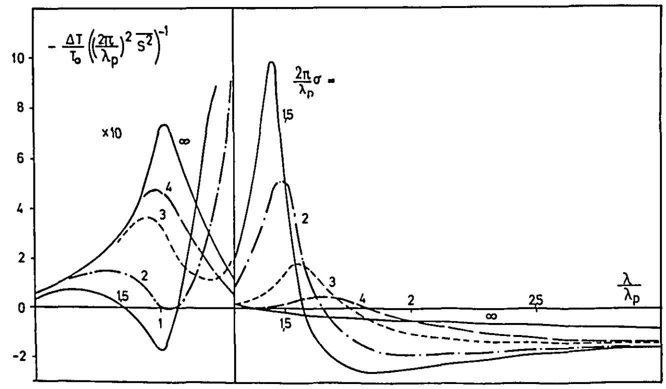

FIG. 6. Relative change of transmittance. Same parameters as in Fig. 4. $\sigma = \infty$ denotes the scalar limit [Eq. (27)].

face-plasmon dispersion. We shall have the maximum values of $\mathrm{Re}(Q)$ at approximately those wavelengths where the maxima of $k \circ g(k)$ and $\mathrm{Re}\{w(k)\}$ coincide. In the interval $0 < k < 2\pi/\lambda$, $w(k)$ contributes mainly to photon scattering. The maximum of $\mathrm{Re}\{w(k)\}$ and the related zero of $\mathrm{Im}\{w(k)\}$ are due to the excitation of surface plasmons. If the damping γ becomes zero, $\mathrm{Re}\{w(k)\}$ for $k > 2\pi/\lambda$ is transformed to a δ function at $k = k_{sp}$. This case is discussed in Sec. IV A.

In Fig. 4, we see a peak of $-\Delta R/R_0$ due to surface-plasmon excitation, which increases with decreasing σ, as expected from Fig. 3. The shift of this peak to greater wavelengths with increasing σ is explained also by Fig. 3. The case $\sigma \to \infty$ is the scalar-theory limit, Eq. (26), for which no surface-plasmon excitation is possible.

For $\lambda \to 0$ all of our calculated $-\Delta R/R_0$ curves are close to the scalar limit, with a typical increase proportional to $(\lambda_p/\lambda)^2$. This quadratic dependence, as well as the change of reflectance at large wavelengths, which is much smaller than the scalar-theory-limit predicts, has been found experimentally.[8]

In Fig. 5, we see that, with increased damping, the scalar theory comes closer to our results. A decrease of the maximum of $-\Delta R/R_0$ by surface-plasmon excitation due to increasing damping is found. The maximum is shifted a little to greater wavelengths. For infinite damping, neither Eq. (23) alone nor the sum of the decrease Eq. (23) and the scalar limit (as has been used in Ref. 3) gives the correct result.

The relative change of transmittance $\Delta T/T_0$ is shown in Fig. 6 for different correlation lengths. Its wavelength dependence is more complicated than $\Delta R/R_0$, because it depends on both the real and imaginary part of $w(k)$, even for vanishing damping, while $\Delta R/R_0$ depends for small values of ϵ_i mainly on $\mathrm{Re}[w(k)]$. We see, that $-\Delta T/T_0$ becomes negative for wavelengths greater than a certain value greater than the surface-plasma limit $\lambda_s = \lambda_p \cdot \sqrt{2}$; this value depends on the correlation length σ. The negative value of $-\Delta T/T_0$ means that the amplitude of the coherent field within the metal becomes greater with the increase of surface roughness. For wavelengths near λ_s, we get a decrease of transmittance. The maximum of this decrease is at slightly smaller wavelengths than the decrease of reflectance. The results of measurements of the transmittance of a rough silver foil are in qualitative agreement with our theoretical results.

ACKNOWLEDGMENT

We are indebted to Professor Dr. H. Raether for discussions on this work.

[1] H. E. Bennett, J. Opt. Soc. Am. 53, 1389 (1963).
[2] J. L. Stanford, H. E. Bennett, J. M. Bennett, E. J. Ashley, and E. T. Arakawa, Bull. Am. Phys. Soc. 13, 989 (1968).
[3] A. Daude, A. Savary, and S. Robin, J. Opt. Soc. Am. 62, 1 (1972).
[4] J. G. Endriz and W. E. Spicer, Phys. Rev. B 4, 4144 (1971).
[5] E. Kretschmann, thesis (Universität Hamburg, 1972).
[6] J. M. Elson, and R. H. Ritchie, Phys. Rev. B 4, 4129 (1971); J. Crowell and R. H. Ritchie, J. Opt. Soc. Am. 60, 794 (1970); E. Kretschmann, Z. Phys. 237, 1 (1970).
[7] J. M. Elson and R. H. Ritchie, Phys. Status Solidi B 62, 461 (1974).
[8] D. Beaglehole and O. Hunderi, Phys. Rev. B 2, 309 (1970).
[9] H. E. Bennett, J. Opt. Soc. Am. 51, 123 (1961); H. Davies, Proc. IEEE 101, 209 (1954); P. Beckmann and A. Spizzichino, *The Scattering of Electromagnetic Waves from Rough Surfaces* (Pergamon, New York, 1973).
[10] E. Kröger and E. Kretschmann, Z. Phys. 237, 1 (1970).
[11] S. O. Rice, Comm. Pure Appl. Math. 4, 351 (1951).
[12] H. J. Juranek, Z. Phys. 233, 324 (1970).
[13] Reference 11, Eq. (7.29).

[14] Equation (22) is valid for complex ϵ also.