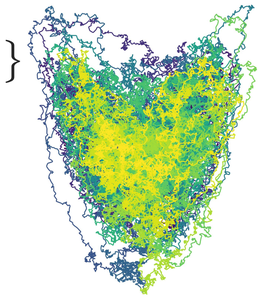
(a1) **Lorenz model:**

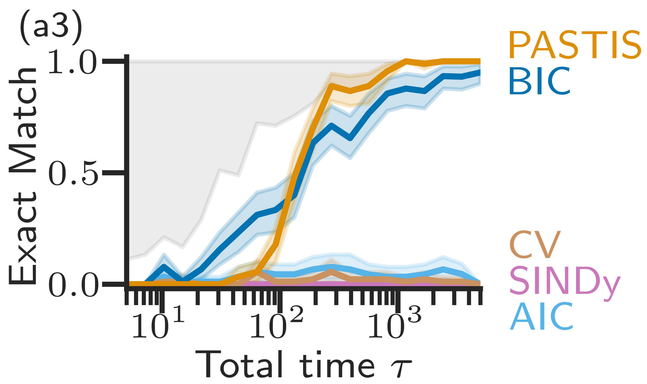
$$\dot{x} = \sigma(z - x) + \sqrt{2D}\xi_x$$
$$\dot{y} = xz - \beta y + \sqrt{2D}\xi_y$$
$$\dot{z} = x(\rho - y) - z + \sqrt{2D}\xi_z$$

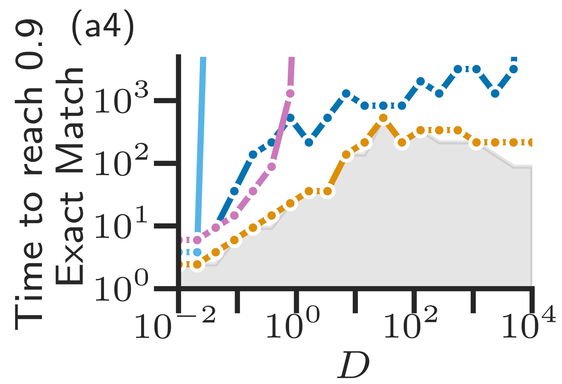
$\mathcal{B}_0 = \{1, x, y, \ldots, yz, z^2\}$

$n_0 = 30, n^* = 7$

(a2) $\tau = 50, D = 100$







(b1) **10-dimensional Ornstein-Uhlenbeck model:**

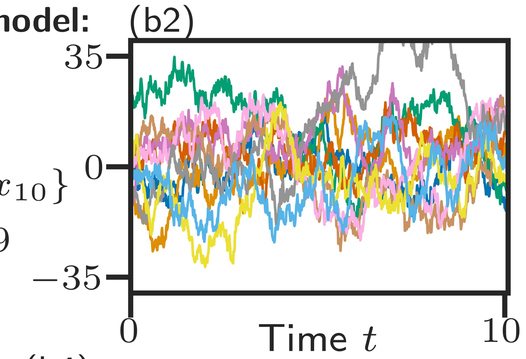
$$\dot{x}_\mu = -\sum_\nu A_{\mu\nu} x_\nu + \sqrt{2D}\xi_\mu$$

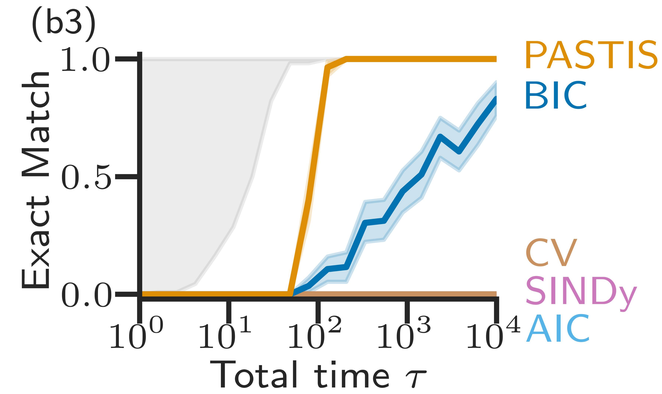
$$\mathbf{A} = \begin{pmatrix} 1 & -1 & \cdot\cdot & 0 \\ \cdot\cdot & 1 & 0 & 0 \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\ 0 & 0 & \cdot\cdot & 1 \end{pmatrix}$$

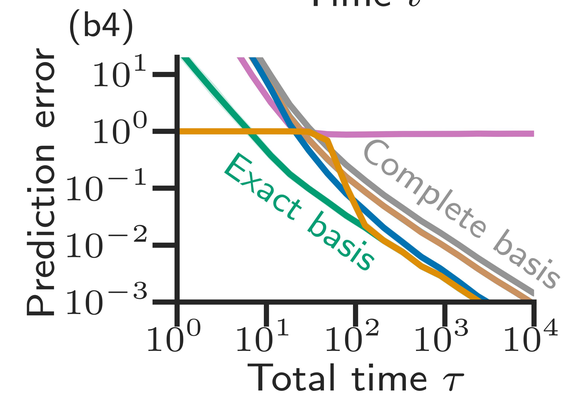
10 %off-diagonal terms are nonzero

$\mathcal{B}_0 = \{1, x_1, \ldots, x_{10}\}$

$n_0 = 110, n^* = 19$





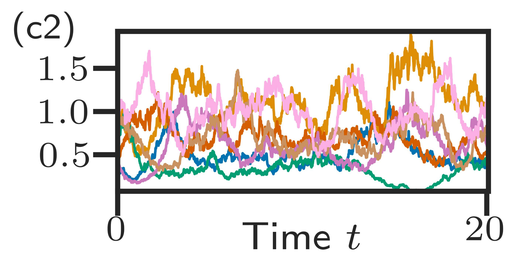


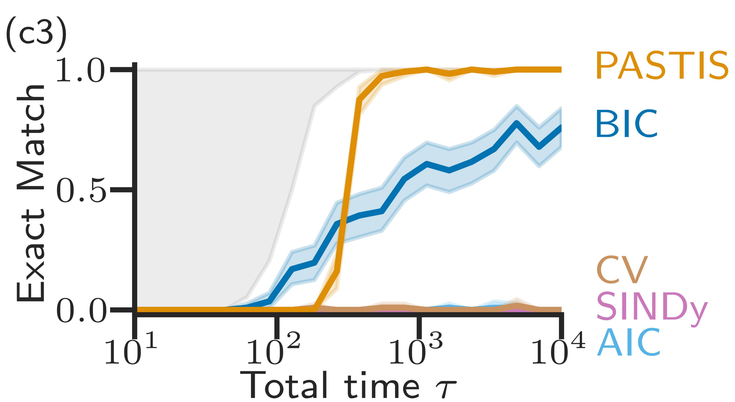
(c1) **Lotka-Volterra model:**

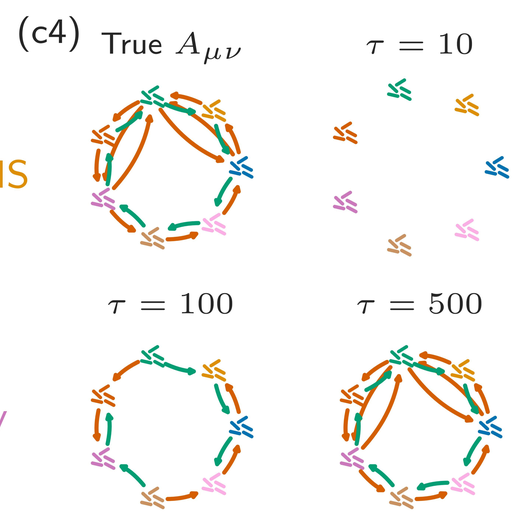
$$\frac{\mathrm{d}x_\mu}{\mathrm{d}t} = x_\mu(r_\mu + \textstyle\sum_\nu A_{\mu\nu}x_\nu + \sqrt{2D}\xi_\mu)$$

$$\mathcal{B}_0 = \left\{x_\mu, x_\mu x_1, \ldots, x_\mu^2, \ldots, x_\mu x_7\right\}$$

$n_0 = 56, \quad n^* = 32$

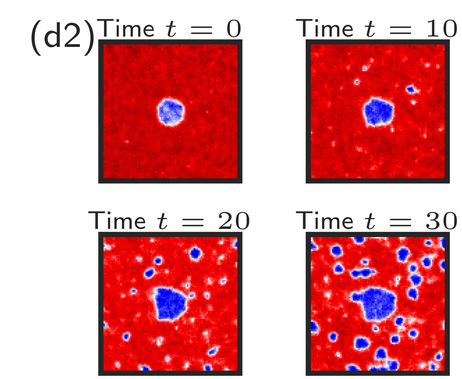
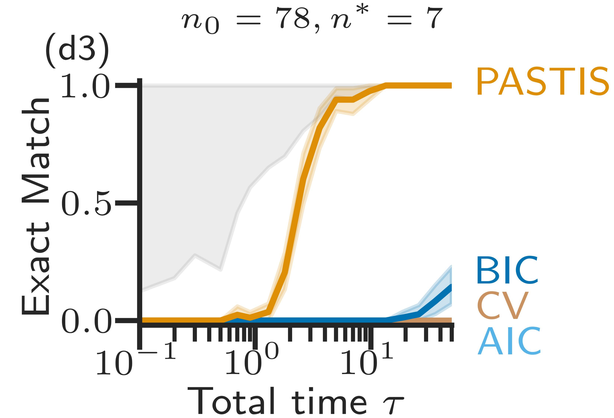
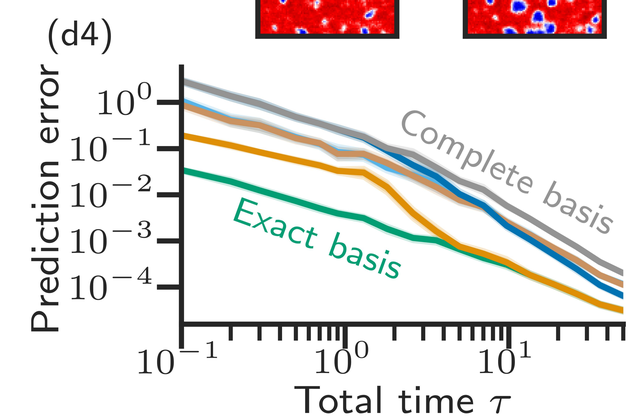






(d1) **Gray-Scott model :**

$$\partial_t u = D_u \nabla^2 u - uv^2 + F(1-u) + \sqrt{2D}\xi_u$$
$$\partial_t v = D_v \nabla^2 v + uv^2 - (F+k)v + \sqrt{2D}\xi_v$$

$$\mathcal{B}_0 = \{1, u, v, uv, u^2, \ldots, uv^3, v^4, \partial_x u, \ldots, u\partial_x v, \ldots, \partial_x v \partial_y v, \nabla^2 u, \nabla^2 v\}$$

$n_0 = 78, n^* = 7$