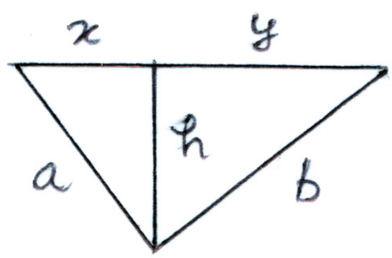
# JAVÍTÓKULCS

## I. feladat

a)




$h^2 = a^2 - x^2$ 1 p $x = \sqrt{h^2 + a^2}$

$h^2 = b^2 - y^2$ $y = \sqrt{b^2 - h^2}$

$x + y = c$

$$a^2 - h^2 + b^2 - h^2 + 2 \cdot \sqrt{(a^2 - h^2) \cdot (b^2 - h^2)} = c^2$$

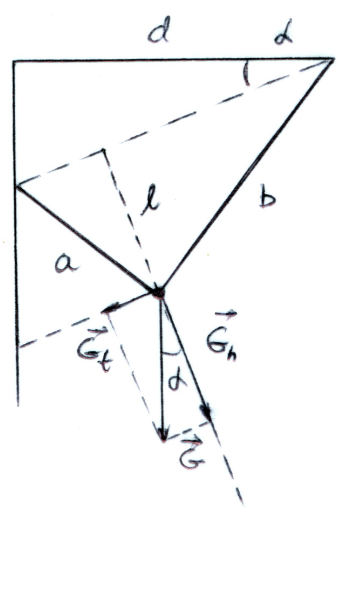
$$(a^2 - 2h^2 + b^2 - c^2) = 4(a^2 - h^2)(b^2 - h^2)$$

$h^2 = 3600 \quad \rightarrow \quad h = 60$ cm 2 p

$T = 2\pi\sqrt{\frac{l}{g}}$, ahol l = h 1 p

$$T = 2\pi\sqrt{\frac{0{,}6}{0{,}8}} = 1{,}553\, s$$

*b)* 1 p



$ab = cl \quad \rightarrow \quad l = (ab)/c$ 1 p

$\cos\alpha = d/c \quad \rightarrow \quad d = \cos\alpha$

$d = (ab)/l \cos\alpha$ 1 p

$T = 2\pi\sqrt{\frac{l}{gn}} = 2\pi\sqrt{\frac{l}{g \cdot \cos(\alpha)}} \qquad l = \frac{T^2}{4 \cdot \pi^2} g\cos(\alpha)$ 1 p

$d = \frac{ab\, 4 \cdot \pi^2}{T^2 \cdot g} = 1{,}00095m \approx 1\ m$ 1 p

## II. feladat

a) $F = k\,(m_x m)/r^2 \quad mg_x = km_x m/r^2 \; g_x = km_x/r^2$ 1 p

$m_x = \rho(4\pi r^3)/3 \quad g_x = (4\pi)/3\ k\,\rho\, r$

$g_0 = (4\pi)/3\ \mathrm{k}\,\rho\, \mathrm{R}_0 \qquad g_x = rg_0/\mathrm{R}_0$ 1 p

$\mathrm{F}_x = mg_x = r(mg_0)/\mathrm{R}_0 \qquad \mathrm{F}_x \sim r \qquad k = mg_0/\mathrm{R}_0$ 1 p

A labda rezgőmozgást végez 0,5 p

$T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{R_0}{g_0}} = 5024\,s$ 1 p

$t = \mathrm{T}/2 = 2512\ s = 41{,}87$ perc 0,5 p

b) A maximális sebességet a Föld középpontjában éri el. 0,5 p

$v_{\max} = \omega \mathrm{R}_0$

$v_{max} = \frac{2\pi}{T} \cdot R_0 = \sqrt{R_0 \cdot g_0} = 8000\frac{m}{s}$ 0,5 p

c) $\mathrm{L} = \Delta \mathrm{E}_C$ 0,5 p $\quad \mathrm{L} = \mathrm{F}_m h$ 0,5 p

$\mathrm{F} = k\,(\mathrm{M}m)/r^2 = \mathrm{K}/r^2 \qquad F_m = \sqrt{F_A \cdot F_B} = \frac{K}{r_A \cdot r_B}$ 0,5 p

$$L = \frac{K}{r_A \cdot r_B} \cdot (r_B - r_A) = K \cdot \left(\frac{1}{r_A} - \frac{1}{r_B}\right) = K \cdot \left(\frac{1}{R} - \frac{1}{R+h}\right)$$ 0,5 p

$$\frac{K}{R} \cdot \left(1 - \frac{1}{1+\frac{h}{R}}\right) = \frac{m \cdot v_0^2}{2} \quad \rightarrow \quad \frac{kM}{R} \cdot \left(1 - \frac{1}{1+\frac{h}{R}}\right) = \frac{g_0 \cdot R}{2}$$ 0,5 p

$$\rightarrow \quad g_0 \cdot R \cdot \left(1 - \frac{1}{1+\frac{h}{R}}\right) = \frac{g_0 \cdot R}{2} \quad \rightarrow \quad \mathrm{h} = \mathrm{R}$$

Rezgőmozgás 2R amplitúdóval 1 p

**III. feladat**

a) Állóhullámok kialakulásának feltétele

$l = k\,\lambda/2$ 0,5 p

$\lambda = v\mathrm{T}$ 0,5 p $v=\sqrt{\dfrac{F}{\mu}}=\sqrt{\dfrac{Fl}{m}}=400\,\dfrac{m}{s}$ 1 p

$\omega = 10\pi$ rad/$s$ $\omega = 2\pi/\mathrm{T}$ $\mathrm{T} = 2\pi/\omega = 0{,}2\ s$ 0,5 p

$\lambda = 80\ m$ $k = 2l/\lambda = 5$ 5 orsópont alakul ki 0,5 p

$x_M = (2k-1)\lambda/4$ $k = 1, 2, \ldots$ $x = l - x_1 = 60\ m$

$(2k-1)\lambda/4 = x$ $k = 1/2(4x/\lambda + 1) = 2$

A rezgésforrástól $x_1 = 140\ m$ maximum alakul ki, melynek amplitúdója $A_1 = 2A = 10\ cm$ 1 p

b) $y=2\,A\cos\left(2\pi\right)\dfrac{2x+\dfrac{\lambda}{2}}{2\lambda}\sin\left(2\pi\right)\left(\dfrac{t}{T}-\dfrac{2l+\dfrac{\lambda}{2}}{2\lambda}\right)$ 2 p

$A' = \left|2\,A\cos\left(2\pi\right)\dfrac{2x+\dfrac{\lambda}{2}}{2\lambda}\right|$ 1 p

$\left|\cos\left(2\pi\,\dfrac{2x+\dfrac{\lambda}{2}}{2\lambda}\right)\right|=\dfrac{1}{2}$ $\cos\left(2\pi\right)\dfrac{2x+\dfrac{\lambda}{2}}{2\lambda}=\dfrac{\pm 1}{2}$ 1 p

A fonal rögzített végéhez eső legközelebbi 2 pont: helyzete meghatározható a

$2\pi\,\dfrac{2\cdot x_1+\dfrac{\lambda}{2}}{2\lambda}=\dfrac{2\pi}{3}$ , illetve $2\pi\,\dfrac{2\cdot x_2+\dfrac{\lambda}{2}}{2\lambda}=\dfrac{4\pi}{3}$ 1 p

feltételekből → $x_1 = 20/3\ cm$ $x_2 = 100/3\ cm$

$\Delta x = x_2 - x_1 = \lambda/3 = 80/3\ cm$ 1 p