1.4 Markov Chain Monte Carlo Posterior Simulation

In many important Bayesian models, it is not possible to carry out posterior integrals analytically. Alternatively, there are many numerical quadrature integration methods to approximate $p(\boldsymbol{y})$ such as trapezoids, Simpson integration formula, Gauss Hermite quadrature and more (for a brief introduction, see for example Süli and Mayers 2003). Such methods usually work well when $\boldsymbol{\theta}$ is low dimensional because then the construction of the grid of points to integrate over can be reasonably distributed over the parameter space $\boldsymbol{\Theta}$. However, in moderate dimension the construction of such a grid reaches a prohibitive computational cost.

In these cases, either optimization or simulation based methods are used. Regarding optimization algorithms, maximum a posteriori (MAP), Estimation-Maximization (EM) (Dempster et al., 1977) and Variational Inference (VI) (Blei et al., 2017) are among the most popular. An issue with optimization approaches is the way uncertainty is treated: posterior inference under optimization methods is typically limited to point estimates. In case of VI, we can use the variational posterior $q(\boldsymbol{\theta})$ as an approximation for the true posterior $p(\boldsymbol{\theta} \mid \boldsymbol{y})$ and report uncertainty in using q. However q could be a poor approximation for p if the space of variational distributions is too restricted, e.g. under the mean field assumption (independence of the components of $\boldsymbol{\theta}$). See Yin and Zhou (2018) for an expansion on the commonly used analytic variational distribution family that produces accurate variational approximations in a broad range of scenarios.