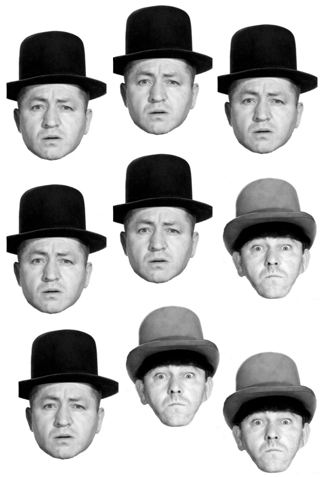
Combinations	Permutations

based on **Order** and **Repetition**

Combinations

Order Doesn't Matter



Repetition **Allowed**

Only **1 Un-Ordered Set** exists for each combination of characters.

Formula:

$$\binom{r+n-1}{r} = \frac{(r+n-1)!}{r!(n-1)!}$$

- **n** = **2** stooges
- **r** = **3** selections
- 4! / 3! x 1!
- 24 / 6 = **4** Combinations

Repetition **Not Allowed**

Very **Few Un-Ordered Set** exists!

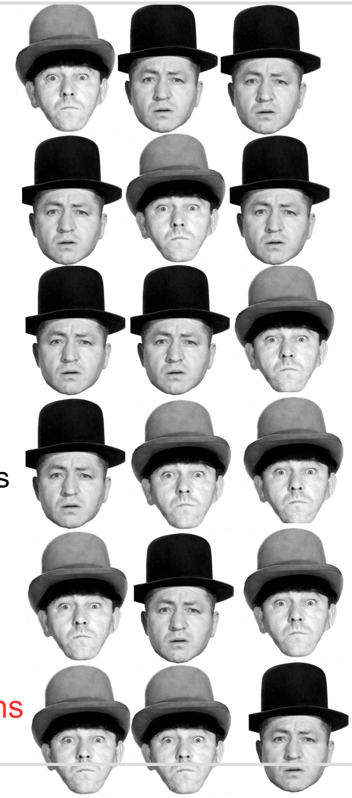
Formula:

$$\frac{n!}{r!(n-r)!} = \binom{n}{r}$$

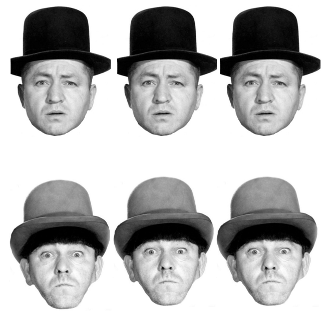
- **n** = **3** stooges
- **r** = **2** selections
- 3! / (2! x 1!)
- 6 / (2 x 1) = **3** Combinations

Permutations

Order Matters



Repetition **Allowed**



Many **Sets** and **Orders** exists

Formula: n^r

- **n** = **2** stooges
- **r** = **3** selections
- 2^3 = **8** Permutations

Repetition **Not Allowed**

Fewer **Sets** and **Orders** exists

$$\frac{n!}{(n-r)!}$$

- **n** = **3** stooges
- **r** = **2** selections
- 3! / 1!
- 6 / 1 = **6** Permutations

Order Doesn’t Matter

2 Types:

1. Repetition **Allowed**
2. Repetition **Not Allowed**

1. Repetition **Allowed**

→○○○→→→
○→→○→→○
○→→→→○○

- **ex**: how many different ways can we arrange arrows and circles?
- Notice that there are always 3 circles (3 scoops of ice cream) and 4 arrows (we need to move 4 times to go from the 1st to 5th container).
- So (being general here) there are **r + (n−1)** positions, and we want to choose **r** of them to have circles.
- This is like saying "we have r + (n−1) pool balls and want to choose r of them". In other words it is now like the pool balls question, but with slightly changed numbers. And we can write it like this:

$$\binom{r+n-1}{r} = \frac{(r+n-1)!}{r!(n-1)!}$$

where ***n*** is the number of things to choose from, and we choose ***r*** of them **repetition** allowed, **order** doesn't matter.

$$\frac{(3+5-1)!}{3!(5-1)!} = \frac{7!}{3!\times4!} = \frac{5040}{6\times24} = 35$$

2. Repetition **Not Allowed**

- **ex**: Let’s say pool balls 1, 2 and 3 are chosen.
- These are the possibilities:

Order does matter: (**Permutation**)	Order doesn't matter (**Combination**)
123	
132	
213	123
231	
312	
321	

So, **permutations have 6 times** as many .

Formula:

Order Matters

2 Types:

1. Repetition **Allowed**
2. Repetition **Not Allowed**

1. Repetition **Allowed**

- **ex**: *Permutation Lock*:
- There are 10 numbers to choose from (0,1,2,3,4,5,6,7,8,9) and we choose 3 of them:
- 10 × 10 × ... (3 times) = 10^3 = 1,000 permutations

n = number of items in the set.

r = times we choose from the set

Formula: n^r

2. Repetition **Not Allowed**

- **ex**: what order can *16 pool balls* be in?
- After choosing the “14 Ball" we can't choose it again.
- So, our first choice has 16 possibilities
  - our next choice has 15 possibilities,
  - then 14, 13, 12, 11, ... etc.

16 × 15 × 14 × 13 × ... = 20,922,789,888,000

- But maybe we don't **want to choose them all, just 3 of them**, and that is then:

16 × 15 × 14 = 3,360

n! **r!(n − r)!** $$\frac{n!}{r!(n-r)!} = \binom{n}{r}$$ It is often called "**n choose r**" (such as "**16 choose 3**") And is also known as the Binomial Coefficient.	**n** = number of items in the set each time **!** = times we choose from the set **Factorial:** **3!** = 16 × 15 × 14 = 3,360 **Formula**: $$\frac{n!}{(n-r)!}$$ ***P*(16,3)** = 3,360 Example Our "order of 3 out of 16 pool balls example" is: $$\frac{\mathbf{16!}}{\mathbf{(16\text{-}3)!}} = \frac{\mathbf{16!}}{\mathbf{13!}} = \frac{\mathbf{20{,}922{,}789{,}888{,}000}}{\mathbf{6{,}227{,}020{,}800}} = \mathbf{3{,}360}$$ (which is just the same as: **16 × 15 × 14 = 3,360**)