Problem Set 12

8.7 Including the contribution from strange quarks:

(a) show that $F_2^{ep}(x)$ can be written

$$F_2^{ep}(x) = \frac{4}{9}\left[u(x) + \bar{u}(x)\right] + \frac{1}{9}\left[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)\right],$$

where $s(x)$ and $\bar{s}(x)$ are the strange quark–parton distribution functions of the proton.

(b) Find the corresponding expression for $F_2^{en}(x)$ and show that

$$\int_0^1 \frac{\left[F_2^{ep}(x) - F_2^{en}(x)\right]}{x}\mathrm{d}x \approx \frac{1}{3} + \frac{2}{3}\int_0^1 \left[\bar{u}(x) - \bar{d}(x)\right]\mathrm{d}x,$$

and interpret the measured value of 0.24 ± 0.03.

a)

Equation (8.24) of Thomson gives the expression for the structure function $F_2^{\mathrm{ep}}(x)$:

$$F_2^{\mathrm{ep}}(x) = x\sum_i Q_i^2 q_i^{\mathrm{p}} \tag{1}$$

$$\tag{2}$$

Equation (8.25) expands this with the contributions of the up-, down-, anti-up-, and anti-down parton distribution functions.

$$\begin{aligned} F_2^{\mathrm{ep}}(x) &= x\sum_i Q_i^2 q_i^{\mathrm{p}} && (3)\\ &\approx x\left(\frac{4}{9}u^{\mathrm{p}}(x) + \frac{1}{9}d^{\mathrm{p}}(x) + \frac{4}{9}u^{-\mathrm{p}}(x) + \frac{1}{9}d^{-\mathrm{p}}(x)\right) && (4)\end{aligned}$$

this may be rewritten in a more simplified way such that:

$$\begin{aligned} u^{\mathrm{p}}(x) \equiv u(x) \quad &\text{and} \quad d^{\mathrm{p}}(x) \equiv d(x) && (5)\\ u^{-\mathrm{p}}(x) \equiv \bar{u}(x) \quad &\text{and} \quad d^{-\mathrm{p}}(x) \equiv \bar{d}(x) && (6)\\ &&& (7)\end{aligned}$$

giving us:

$$F_2^{\mathrm{ep}}(x) \approx x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)\right) \tag{8}$$

However, this does not include the contribution of strange quarks, which has $Q = -1/3$ from Table 1.1. Adding their contributions, this becomes:

$$F_2^{\mathrm{ep}}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x) + \frac{1}{9}s(x) + \frac{1}{9}\bar{s}(x)\right) \tag{9}$$

$$\boxed{F_2^{\mathrm{ep}}(x) = \frac{4}{9}x\left[u(x) + \bar{u}(x)\right] + \frac{1}{9}x\left[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)\right]} \tag{10}$$

b)

To get the corresponding $F_2^{\mathrm{en}}(x)$, we note that from the isopin symmetry, we have:

$$\begin{aligned} d^{\mathrm{n}}(x) = u^{\mathrm{p}}(x) \equiv u(x) \quad &\text{and} \quad u^{\mathrm{n}}(x) = d^{\mathrm{p}}(x) \equiv d(x) && (11) \\ d^{-\mathrm{n}}(x) = u^{-\mathrm{p}}(x) \equiv \bar{u}(x) \quad &\text{and} \quad u^{-\mathrm{n}}(x) = d^{-\mathrm{p}}(x) \equiv \bar{d}(x) && (12) \\ & && (13) \end{aligned}$$

thus the expression for $F_2^{\mathrm{en}}(x)$ becomes:

$$\begin{aligned} F_2^{\mathrm{en}}(x) &= x \sum_i Q_i^2 q_i^{\mathrm{p}} && (14) \\ &= x\left(\frac{4}{9}u^{\mathrm{n}}(x) + \frac{1}{9}d^{\mathrm{n}}(x) + \frac{4}{9}u^{-\mathrm{n}}(x) + \frac{1}{9}d^{-\mathrm{n}}(x) + \frac{1}{9}s(x) + \frac{1}{9}\bar{s}(x)\right) && (15) \\ &= x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}\bar{u}(x) + \frac{1}{9}s(x) + \frac{1}{9}\bar{s}(x)\right) && (16) \\ & && (17) \end{aligned}$$

$$\boxed{F_2^{\mathrm{en}}(x) = \frac{4}{9}x\left[d(x) + \bar{d}(x)\right] + \frac{1}{9}x\left[u(x) + \bar{u}(x) + s(x) + \bar{s}(x)\right]} \qquad (18)$$

The given integral may now be expressed as:

$$\begin{aligned} \int_0^1 \frac{\left[F_2^{\mathrm{ep}}(x) - F_2^{\mathrm{en}}(x)\right]}{x}\,\mathrm{d}x &= \int_0^1 \left(\frac{4}{9}\left[u(x) + \bar{u}(x)\right] + \frac{1}{9}\left[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)\right]\right) && (19) \\ &\quad - \left(\frac{4}{9}\left[d(x) + \bar{d}(x)\right] + \frac{1}{9}\left[u(x) + \bar{u}(x) + s(x) + \bar{s}(x)\right]\right)\,\mathrm{d}x && (20) \\ \int_0^1 \frac{\left[F_2^{\mathrm{ep}}(x) - F_2^{\mathrm{en}}(x)\right]}{x}\,\mathrm{d}x &= \int_0^1 \frac{1}{3}\left[u(x) + \bar{u}(x) - d(x) - \bar{d}(x)\right]\,\mathrm{d}x && (21) \end{aligned}$$

the PDFs may be decomposed into contributions from valence quarks and sea quarks:

$$\begin{aligned} u(x) = u_{\mathrm{V}}(x) + u_{\mathrm{S}}(x) \quad \text{and} \quad d(x) = d_{\mathrm{V}}(x) + d_{\mathrm{S}}(x) && (22) \\ && (23) \end{aligned}$$

giving us:

$$\int_0^1 \frac{[F_2^{\mathrm{ep}}(x) - F_2^{\mathrm{en}}(x)]}{x}\,\mathrm{d}x = \frac{1}{3}[u_{\mathrm{V}}(x) + u_{\mathrm{S}}(x) + \bar{u}(x) - d_{\mathrm{V}}(x) - d_{\mathrm{S}}(x) - \bar{d}(x)]\,\mathrm{d}x \tag{24}$$

we note that the valence quark PDFs are normalized such that:

$$\int_0^1 u_{\mathrm{V}}(x)\,\mathrm{d}x = 2 \quad \text{and} \quad \int_0^1 d_{\mathrm{V}}(x)\,\mathrm{d}x = 1 \tag{25}$$

which leaves us with:

$$\begin{aligned}\int_0^1 \frac{[F_2^{\mathrm{ep}}(x) - F_2^{\mathrm{en}}(x)]}{x}\,\mathrm{d}x &= \frac{1}{3}[2 + u_{\mathrm{S}}(x) + \bar{u}(x) - 1 - d_{\mathrm{S}}(x) - \bar{d}(x)]\,\mathrm{d}x && (26)\\ &= \frac{1}{3} + \frac{1}{3}[u_{\mathrm{S}}(x) + \bar{u}(x) - d_{\mathrm{S}}(x) - \bar{d}(x)]\,\mathrm{d}x && (27)\\ & && (28)\end{aligned}$$

expecting the sea PDFs for up- and down- quarks are approximately the same, then we have:

$$u_{\mathrm{S}}(x) = \bar{u}_{\mathrm{S}} \approx d_{\mathrm{S}}(x) = \bar{d}_{\mathrm{S}}(x) \tag{29}$$

we may write $u_{\mathrm{S}}(x) \to \bar{u}_{\mathrm{S}}$ and $d_{\mathrm{S}}(x) \to \bar{d}_{\mathrm{S}}(x)$, giving us:

$$\begin{aligned}\int_0^1 \frac{[F_2^{\mathrm{ep}}(x) - F_2^{\mathrm{en}}(x)]}{x}\,\mathrm{d}x &= \frac{1}{3} + \frac{1}{3}[2\bar{u}(x) - 2\bar{d}(x)]\,\mathrm{d}x && (30)\\ & && (31)\end{aligned}$$

$$\boxed{\int_0^1 \frac{[F_2^{\mathrm{ep}}(x) - F_2^{\mathrm{en}}(x)]}{x}\,\mathrm{d}x = \frac{1}{3} + \frac{2}{3}[\bar{u}(x) - \bar{d}(x)]\,\mathrm{d}x} \tag{32}$$