Nuclear phenomenology

Nuclides

Nuclides are typically written as:

$$^{\mathrm{A}}_{\mathrm{Z}}\mathrm{Y}$$

where we note that:

$$A \text{ is the } \textbf{mass/nucleon number} \text{ (\# of nucleons)} \tag{1}$$
$$Z \text{ is the } \textbf{proton/atomic number} \text{ (\# of protons)} \tag{2}$$
$$N \text{ is the } \textbf{neutron number} \text{ (\# of neutrons)} \tag{3}$$
$$\tag{4}$$

with $\mathbf{A} = \mathbf{Z} + \mathbf{N}$. We also note that isotopes with: same A = isobars; same Z = isotopes; same N = isotones. Some elements have multiple isotopes, with different stabilty and abundance.

Nuclear shapes and sizes

Nuclei may be treated as static charge distributions with normalization:

$$\int f(\mathbf{r})\mathrm{d}^3\mathbf{r} = Ze \tag{5}$$

where e is the electron charge. Under the Born approximation, the cross-section $\frac{d\sigma}{d\Omega}$ is given by:

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{Z^2\alpha^2(\hbar c)^2}{4\beta^4 E^2 \sin^4(\theta/2)} \tag{6}$$

including the electron spin, the Mott cross-section is given by:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \left(\frac{d\sigma}{d\Omega}\right)_0 [1 - \beta^2 \sin^2(\theta/2)] \tag{7}$$

in the nonrelativistic limit with no spin dependence, the Rutherford cross-section is given by:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \frac{(\hbar c)^2(\alpha Z)^2}{4m^2v^4 \sin^4(\theta/2)} \tag{8}$$

Given $\mathbf{q} = \mathbf{p} - \mathbf{p}'$ where $\mathbf{p}$ and $\mathbf{p}'$ are initial and final electron momenta, the form factor $F(\mathbf{q}^2)$ (Fourier transform of charge distribution) is given by:

$$F(\mathbf{q}^2) = \frac{1}{Ze}\int e^{i\mathbf{q}\cdot\mathbf{r}/\hbar} f(\mathbf{r})\,\mathrm{d}^3\mathbf{r} \tag{9}$$

$$F(\mathbf{q}^2) = \frac{4\pi\hbar}{Zeq}\int_0^\infty r\rho(r)\sin\left(\frac{qr}{\hbar}\right)\,\mathrm{d}r \tag{10}$$

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α emissions

Radioactive decay