Midterms

I swear upon my honor that I have not given nor received any unauthorized help on this exam and that all the work below are my own.

DESABELLE, Olyn D.

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1. **SEMF** [50 pts.]

   For this item, we will refer to Eq. (2.48) of Martin & Shaw's textbook (3rd Ed.).

   (a) For a fixed mass number A, find the proton number Z for the most stable nucleus. *Hint*: The most stable nucleus will have the highest binding energy. [10 pts.]

Equation (2.48) of Martin & Shaw gives the mass deficit $\Delta M(Z, A)$:

$$\Delta M(Z,A) \equiv M(Z,A) - Z(M_p + m_e) - NM_n \tag{2.48}$$

we note that the first term of the RHS of Equation (2.48) is given by Equation (2.49)

$$\mathrm{M(Z,A)} = \sum_{i=0}^{5} f_i(\mathrm{Z,A}) \tag{2.49}$$

where each $f_i(\mathrm{Z,A})$ is given by Equations (2.50-2.55):

$$
\begin{align}
f_0(\mathrm{Z,A}) &= \mathrm{Z}(M_p + m_e) + (\mathrm{A} - \mathrm{Z})M_n \tag{2.50}\\
f_1(\mathrm{Z,A}) &= -a_v A \tag{2.51}\\
f_2(\mathrm{Z,A}) &= a_s A^{2/3} \tag{2.52}\\
f_3(\mathrm{Z,A}) &= a_c \frac{Z(Z-1)}{A^{1/3}} \tag{2.53}\\
f_4(\mathrm{Z,A}) &= a_a \frac{(Z-A/2)^2}{A} \tag{2.54}\\
f_5(\mathrm{Z,A}) &= -f(A) \quad \text{if both } Z, N \text{ are even}\\
&= +f(A) \quad \text{if both } Z, N \text{ are odd} \tag{2.55}\\
&= 0 \quad \text{if either one of } Z, N \text{ is odd and the other is even}
\end{align}
$$

$$a_v = 15.56, \quad a_s = 17.23, \quad a_c = 0.697, \quad a_a = 93.14 \quad a_p = 12$$

where $N = A - Z$, $f(A) = a_p A^{-1/2}$, and the a_i coefficients are in units of MeV$/c^2$. We may plug these into Equation (2.48), from which we may get the binding energy B from:

$$B = -\Delta M c^2 \tag{1}$$

rewriting the conditions for f_5 in terms of A and Z, we thus have:

$$B = \begin{cases} -a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(Z-A/2)^2}{A} + a_p A^{-1/2} & \text{for even } A, \text{ odd Z} \\ -a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(Z-A/2)^2}{A} - a_p A^{-1/2} & \text{for even } A, \text{ even Z} \\ -a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(Z-A/2)^2}{A} & \text{for odd } A \end{cases} \tag{2}$$

to get the highest binding energy B for the most stable nucleus given A, we get its derivative with respect to Z:

$$\frac{dB}{dZ} = a_c \frac{2Z-1}{A^{1/3}} + a_a \frac{2Z-A}{A} \tag{3}$$

at the maxima, $\frac{dB}{dZ} = 0$ and thus we have the relation:

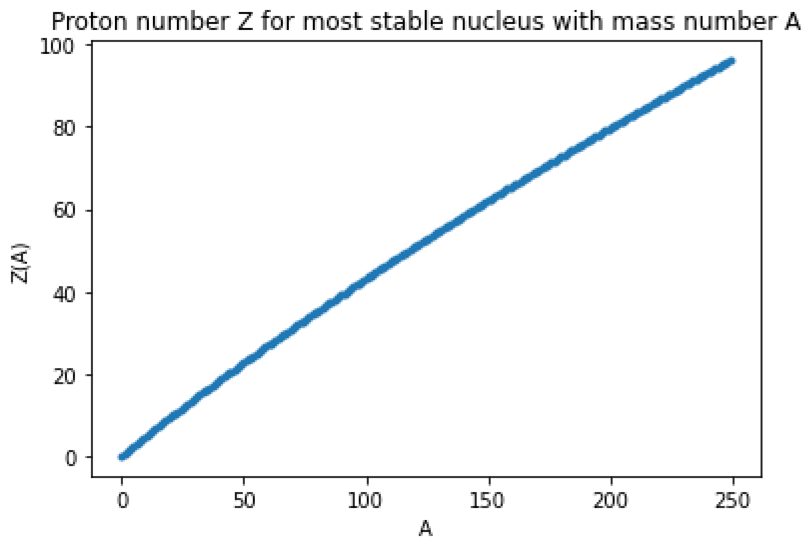
$$0 = a_c \frac{2Z-1}{A^{1/3}} + a_a \frac{2Z-A}{A} \tag{4}$$

$$A^{2/3} a_c (2Z-1) + a_a (2Z-A) = 0 \tag{5}$$

$$\boxed{Z = \frac{A^{2/3} a_c + A a_a}{2a_a + 2A^{2/3} a_c}} \tag{6}$$

(b) Plot the resulting function above (note that your answer in (a) will be Z as a function of A). [10 pts.]

Plotting Equation 6, we get:



It resembles the plot for stable nuclei in Figure 2.13 from Martin & Shaw, noting that $N = A - Z$:

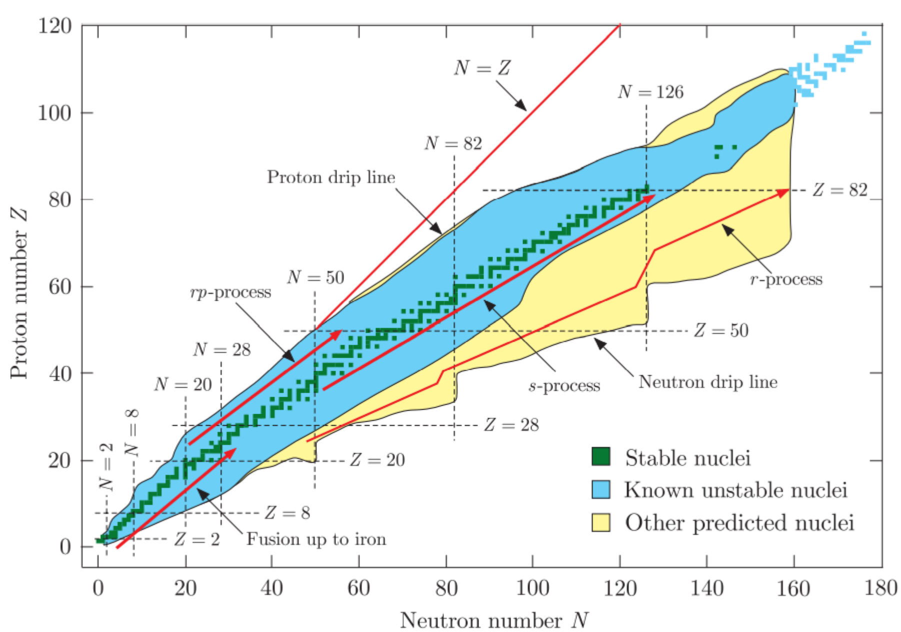


Figure 2.13 The distribution of nuclei. The green squares are the stable nuclei occurring in nature. Known, but unstable, nuclei lie within the blue area, and other predicted nuclei (usually very unstable) lie within the outer yellow area. See text for details. Specific values shown for N and Z are the magic numbers, mentioned below. Source: Bazin (2012), reprinted and adapted by permission from Macmillan Publishers Ltd: copyright 2012.

(c) Show that for a fixed mass number A, $M(Z,A)$ has an extremum Z value. [10 pts.]

We recall that $M(Z,A)$ is given by:

$$M(Z,A) = \mathrm{Z}(M_p + m_e) + (\mathrm{A} - \mathrm{Z})M_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(Z - A/2)^2}{A} + f_5 \tag{7}$$

to get its extrema Z value, we set $\frac{dM}{dZ} = 0$:

$$\frac{dM}{dZ} = M_p + m_e - M_n + a_c \frac{2Z-1}{A^{1/3}} + a_a \frac{2Z - A}{A} = 0 \tag{8}$$

$$\boxed{Z = \frac{A^{2/3} a_c + A a_a + M_p + m_e - M_n}{2a_a + 2A^{2/3} a_c}} \tag{9}$$

(d) Is this extremum a minimum or a maximum? [5 pts.]

To find out whether it is a maxima or minima, we get the second derivative with respect to Z:

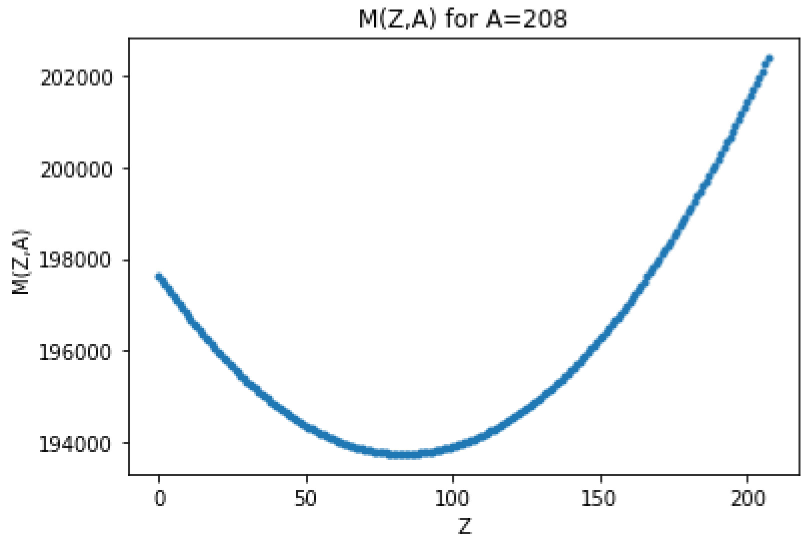
$$\frac{d^2M}{dZ^2} = \frac{d}{dZ}\left[M_p + m_e - M_n + a_c\frac{2Z-1}{A^{1/3}} + a_a\frac{2Z-A}{A}\right] \tag{10}$$

$$\frac{d^2M}{dZ^2} = a_c\frac{2}{A^{1/3}} + a_a\frac{2}{A} > 0 \tag{11}$$

Since the second derivative will always be positive (A is always positive, and the a_i coeffiecients are positive as well), then the extremum is a minimum.

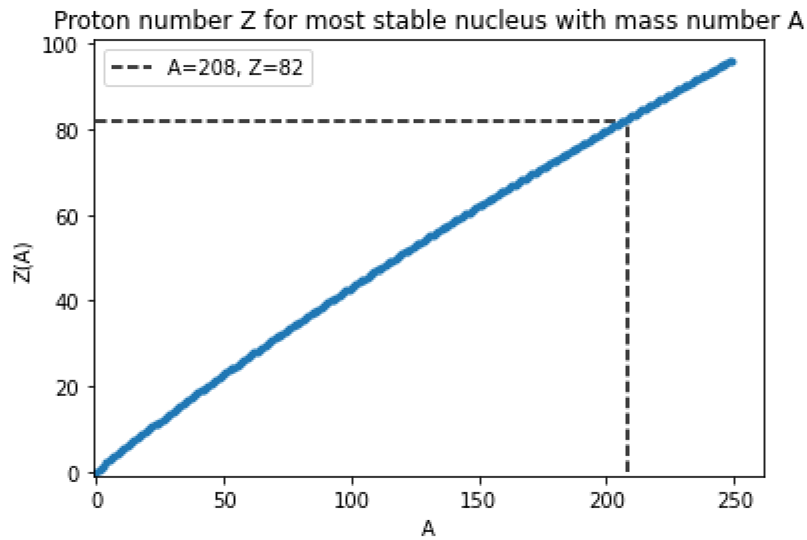
(e) Plot $M(Z, A)$ as a function of Z for a fixed mass number $A = 208$. [10 pts.]

Plotting Equation 7 for $A = 208$, we have:

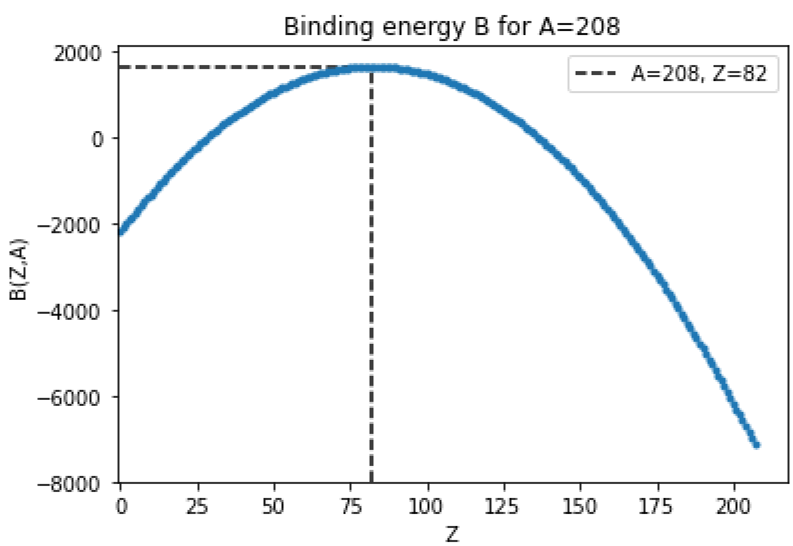


(f) From your results, approximate the number of protons Z in the most stable nucleus of A = 208. In reality, this is ^{208}Pb with $Z = 82$. How close are your predictions from plots (b) and (e)? [5 pts.]

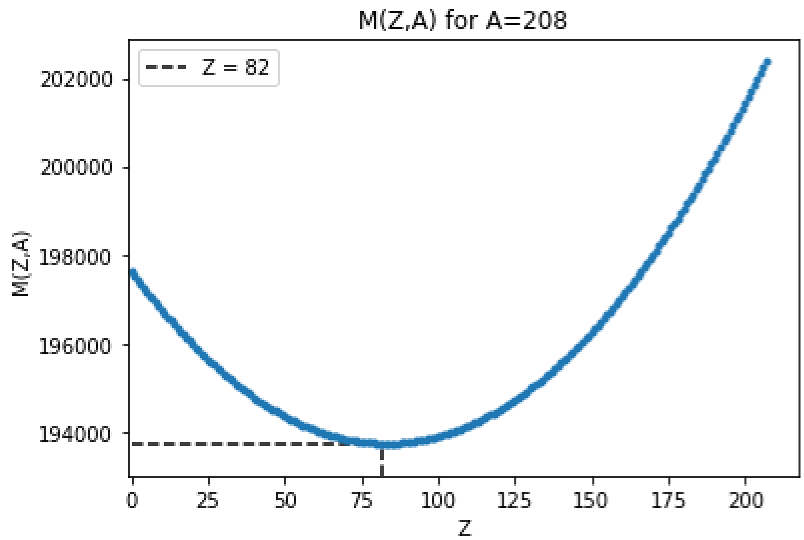
In the following plots, $Z = 82$ and/or $A = 208$ is presented in the dashed lines:



In the figure above, it shows that $Z = 82$ and $A = 208$ intersect along the plot, meaning it is a stable pair. Given that it is the most stable nucleus, this also shows the highest binding energy when plotted:



Highlighting it at the plot of $M(Z, A)$:



In the figure above, $Z = 82$ lies at/close to the minima of the plot. This makes sense, as in the equation for binding energy, the highest binding energy (which gives the most stable nucleus) may be obtained for lower values of $M(Z, A)$.

We may also find that using previous equations with $A = 208$, $Z = 82$ may be almost obtained. Using Equation 6, we obtain $Z = 82.46674080645812$ (meaning it is the proton number for the most stable nucleus). Using Equation 9, we obtain $Z = 82.46341480094144$ (meaning it has the lowest $M(Z, A)$).

My python code for the plots and computations are available at
https://colab.research.google.com/drive/1lrSsf7ZJqOBmzF7LExekYsIC9AZipV4T?usp=sharing

2. **Dirac and Weyl** [50 pts.]

In obtaining the spinor solutions, we explicitly used the Dirac-Pauli representation of the γ matrices given in Eq. (4.35) of Thomson's textbook (*Note*: all equation numbers mentioned hereinafter will come from Thomson's textbook). Here we will use the *Weyl* representation whose γ matrices are given by

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$$

Note that the γ^k are still similar to the Dirac-Pauli representation, the only difference is that for Weyl, γ^0 is not diagonal, explicitly

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(a) Show that the γ matrices in Weyl representation still satisfy Eq. (4.33) and Eq. (4.34). [10 pts.]

Equation 4.33

Equation 4.33 reads:

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \tag{4.33}$$

this shows the identity relations and anticommutation relation:

$$(\gamma^0)^2 = I \tag{12}$$
$$(\gamma^k)^2 = -I \tag{13}$$
$$\gamma^\mu \gamma^\nu = -\gamma^\nu \gamma^\mu \quad \text{for } \mu \neq \nu \tag{14}$$

$(\gamma^0)^2$

$$(\gamma^0)^2 \stackrel{?}{=} I \tag{15}$$
$$(\gamma^0)^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{16}$$
$$(\gamma^0)^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{17}$$

$$\boxed{(\gamma^0)^2 = I} \tag{18}$$

$\underline{\underline{\gamma^\mu\gamma^\nu = -\gamma^\nu\gamma^\mu}}$

$\mu = 0,\ \nu = 1$

$$\gamma^0\gamma^1 \stackrel{?}{=} -\gamma^1\gamma^0 \tag{19}$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \stackrel{?}{=} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{20}$$

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{21}$$

$$\tag{22}$$

$$\boxed{\gamma^0\gamma^1 = -\gamma^1\gamma^0} \tag{23}$$

$\mu = 0,\ \nu = 2$

$$\gamma^0\gamma^2 \stackrel{?}{=} -\gamma^2\gamma^0 \tag{24}$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \stackrel{?}{=} - \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{25}$$

$$\begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \tag{26}$$

$$\tag{27}$$

$$\boxed{\gamma^0\gamma^2 = -\gamma^2\gamma^0} \tag{28}$$

$\mu = 0,\ \nu = 3$

$$\gamma^0\gamma^3 \stackrel{?}{=} -\gamma^3\gamma^0 \tag{29}$$

$$\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix} \stackrel{?}{=} -\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix}\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix} \tag{30}$$

$$\begin{pmatrix}-1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix} = \begin{pmatrix}-1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix} \tag{31}$$

$$\tag{32}$$

$$\boxed{\gamma^0\gamma^3 = -\gamma^3\gamma^0} \tag{33}$$

Equation 4.34

Equation 4.34 reads:

$$\gamma^{0\dagger} = \gamma^0 \quad \text{and} \quad \gamma^{k\dagger} = -\gamma^k \tag{4.34}$$

since the Weyl representation only differs for γ^0, we only verify the first equality in the equation:

$$\gamma^{0\dagger} \stackrel{?}{=} \gamma^0 \tag{34}$$

$$\gamma^{0\dagger} = \overline{(\gamma^0)^T} \tag{35}$$

to get $\gamma^{0\dagger}$, we get the complex conjugate of the transpose of γ^0:

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{36}$$

$$(\gamma^0)^T = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}^T \tag{37}$$

$$(\gamma^0)^T = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{38}$$

$$\tag{39}$$

since all elements do not have a complex part, we then have:

$$\overline{(\gamma^0)^T} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \gamma^{0\dagger} \tag{40}$$

$$\tag{41}$$

The obtained matrix for $\gamma^{0\dagger}$ is the same as the given matrix for γ^0, thus we have shown that the Weyl representation still satisfies $\gamma^{0\dagger} = \gamma^0$ of Eq.(4.34) [and since the Dirac-Pauli representation and Weyl representation have the same γ^k matrices for $k \neq 0$, the other γ^k matrices of the Weyl representation also satisfy Eq.(4.34)].

(b) Solve the Dirac equation for a particle at rest using the Weyl representation. *Hint*: Have a look at Section 4.6.1 of Thomson's textbook. [15 pts.]

Following 4.6.1 of Thomson, the free-particle wavefunction for a particle at rest ($\mathbf{p} = \mathbf{0}$) is given by:

$$\psi = u(E, 0)e^{-iEt} \tag{42}$$

$$E\gamma^0 u = mu \tag{43}$$

using the new γ^0 from the Weyl representation, we have:

$$E\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} = m\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} \tag{44}$$

$$\tag{45}$$

solving for its eigenvalues λ, we have:

$$\begin{vmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ 1 & 0 & -\lambda & 0 \\ 0 & 1 & 0 & -\lambda \end{vmatrix} = 0 \tag{46}$$

$$\lambda = 1, 1, -1, -1 \tag{47}$$

solving for the corresponding eigenvectors, we have:

$\underline{\lambda = 1}$

$$\begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = 0 \tag{48}$$

$$-a + c = 0 \tag{49}$$

$$-b - d = 0 \tag{50}$$

$$a - c = 0 \tag{51}$$

$$b - d = 0 \tag{52}$$

$$\text{eigenvectors} : \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \tag{53}$$

$\underline{\lambda = -1}$

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = 0 \tag{54}$$

$$a + c = 0 \tag{55}$$

$$b + d = 0 \tag{56}$$

$$a + c = 0 \tag{57}$$

$$b + d = 0 \tag{58}$$

$$\text{eigenvectors} : \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \tag{59}$$

The first two eigenvectors have positive energy eigenvalues ($E = +m$), whilst the latter two have negative energy eigenvalues($E = -m$). Along with normalization constant N, we thus have:

$$u_1 = N \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \; u_2 = N \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \; u_3 = N \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \; u_4 = N \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \tag{60}$$

$$\boxed{\psi_1 = N \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{-imt}, \; \psi_2 = N \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} e^{-imt}, \; \psi_3 = N \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{+imt}, \; \psi_4 = N \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} e^{+imt},} \tag{61}$$

(c) Show that in Weyl representation, the Dirac equation (for a general free particle not necessarily at rest) still leads to coupled equations similar to Eq. (4.45) and Eq. (4.46). [15 pts.]

The Dirac equation for a general free particle is given by:

$$(E\gamma^0 - p_x\gamma^1 - p_y\gamma^2 - p_z\gamma^3 - m)u = 0 \tag{62}$$

using the Weyl representation, this becomes:

$$\left[E\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} - \begin{pmatrix} 0 & \sigma\cdot\mathbf{p} \\ -\sigma\cdot\mathbf{p} & 0 \end{pmatrix} - m\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}\right] u = 0 \tag{63}$$

$$\begin{pmatrix} -mI & EI - \sigma\cdot\mathbf{p} \\ EI + \sigma\cdot\mathbf{p} & -mI \end{pmatrix}\begin{pmatrix} u_A \\ u_B \end{pmatrix} = 0 \tag{64}$$

$$\tag{65}$$

for the coupled equations, we have:

$$-mIu_A + (EI - \sigma\cdot\mathbf{p})u_B = 0 \tag{66}$$

$$(EI + \sigma\cdot\mathbf{p})u_A - mIu_B = 0 \tag{67}$$

$$\tag{68}$$

$$\boxed{\begin{aligned} u_B &= \frac{mu_A}{E - (\sigma\cdot\mathbf{p})} \\ u_A &= \frac{mu_B}{E + (\sigma\cdot\mathbf{p})} \end{aligned}} \tag{69}$$

expanding everything in matrices, we have:

$$\left[\begin{pmatrix} 0 & 0 & E & 0 \\ 0 & 0 & 0 & E \\ E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & p_x \\ 0 & 0 & p_x & 0 \\ 0 & -p_x & 0 & 0 \\ -p_x & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & -ip_y \\ 0 & 0 & ip_y & 0 \\ 0 & ip_y & 0 & 0 \\ -ip_y & 0 & 0 & 0 \end{pmatrix}\right. \tag{70}$$

$$\left. - \begin{pmatrix} 0 & 0 & p_z & 0 \\ 0 & 0 & 0 & -p_z \\ -p_z & 0 & 0 & 0 \\ 0 & p_z & 0 & 0 \end{pmatrix} - \begin{pmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{pmatrix}\right] \begin{pmatrix} u_{A1} \\ u_{A2} \\ u_{B1} \\ u_{B2} \end{pmatrix} = 0 \tag{71}$$

$$\begin{pmatrix} -m & 0 & E - p_z & -p_x + ip_y \\ 0 & -m & -p_x - ip_y & E + p_z \\ E + p_z & p_x - ip_y & -m & 0 \\ p_x + ip_y & E - p_z & 0 & -m \end{pmatrix} \begin{pmatrix} u_{A1} \\ u_{A2} \\ u_{B1} \\ u_{B2} \end{pmatrix} = 0 \tag{72}$$

$$\tag{73}$$

the corresponding equations we have are:

$$-mu_{A1} + (E - p_z)u_{B1} + (-p_x + ip_y)u_{B2} = 0 \tag{74}$$

$$-mu_{A2} + (-p_x - ip_y)u_{B1} + (E + p_z)u_{B2} = 0 \tag{75}$$

$$(E + p_z)u_{A1} + (p_x - ip_y)u_{A2} + (-m)u_{B1} = 0 \tag{76}$$

$$(p_x + ip_y)u_{A1}(E - p_z)u_{A2} + (-m)u_{B2} = 0 \tag{77}$$

we note that at least one of the elements of u_A (u_{A1} and u_{A2}) and u_B (u_{B1} and u_{B2}) are mixed in each equation.

(d) Show that the coupled equations in (c) *decouple* the moment you set $m = 0$. The resulting uncoupled equations are known as *Weyl equations.* [10 pts.]

the coupled equations earlier show that:

$$u_B = \frac{mu_A}{E - (\sigma \cdot \mathbf{p})} \tag{78}$$
$$u_A = \frac{mu_B}{E + (\sigma \cdot \mathbf{p})} \tag{79}$$

$$(E - (\sigma \cdot \mathbf{p}))u_B = mu_A \tag{80}$$
$$(E + (\sigma \cdot \mathbf{p}))u_A = mu_B \tag{81}$$

setting $m = 0$, we get:

$$\boxed{\begin{aligned}[E - (\sigma \cdot \mathbf{p})]u_B = 0 \\ [E + (\sigma \cdot \mathbf{p})]u_A = 0\end{aligned}} \tag{82}$$

the equations have decoupled. We can also see this in the expanded versions of the equation when $m = 0$:

$$(E - p_z)u_{B1} + (-p_x + ip_y)u_{B2} = 0 \tag{83}$$
$$(-p_x - ip_y)u_{B1} + (E + p_z)u_{B2} = 0 \tag{84}$$
$$(E + p_z)u_{A1} + (p_x - ip_y)u_{A2} = 0 \tag{85}$$
$$(p_x + ip_y)u_{A1}(E - p_z)u_{A2} = 0 \tag{86}$$