Finals

I swear upon my honor that I have not given nor received any unauthorized help on this exam and that all the work below are my own.

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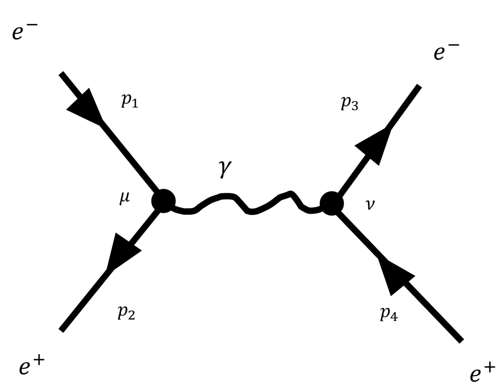
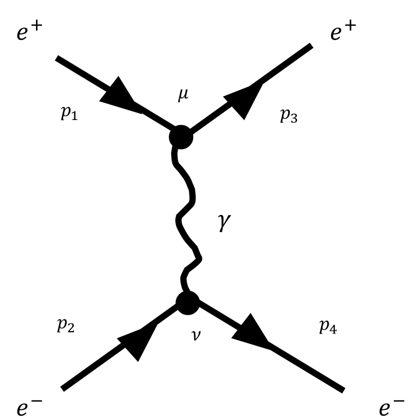
1 Baa Baa Black Sheep [50 pts.]

Consider the process: $e^+e^- \to e^+e^-$.

(a) Draw the lowest-order Feynman diagram/s for this process. [10 pts.]

There are two lowest-order Feynamn diagrams for this process. One is the pair annihilation $\to$ pair creation represented by an s-channel diagram (left) and the other is the scattering represented by a t-channel diagram (right):





(b) Does this process have the same number of lowest-order Feynman diagram as the annihilation process $e^+e^- \to \mu^+\mu^-$ (which only has *one*) considered in class? Why or why not? [5 pts.]

The process $e^+e^- \to e^+e^-$ does not have the same number of lowest-order Feynman diagram as the $e^+e^- \to \mu^+\mu^-$ process since the former has 2 while the latter only has 1. This is because in the lowest-order the former can undergo either pair annihilation then pair creation or scattering, while the latter can only undergo pair annihilation.

We note that the Feynman rules for QED note the following contributions to the matrix element $\mathcal{M}$. We have the following contributions:

$$\begin{aligned}
\text{initial state particle } (e^-) &: u(p) \\
\text{initial state antiparticle } (e^+) &: \bar{v}(p) \\
\text{final state particle } (e^-) &: \bar{u}(p) \\
\text{final state antiparticle } (e^+) &: v(p) \\
\text{interaction vertex factor } &: ie\gamma^\mu \\
\text{photon propagator } &: -\frac{ig_{\mu\nu}}{q^2}
\end{aligned}$$

(c) Use the Feynman rules for QED to write down the corresponding matrix element/s. [10 pts.]

s-channel annihilation + creation

$$-i\mathcal{M} = [\bar{v}(p_2)ie\gamma^\mu u(p_1)]\left[\frac{-ig_{\mu\nu}}{q^2}\right][\bar{u}(p_3)ie\gamma^\nu v(p_4)] \tag{1.1}$$

$$\mathcal{M} = -[\bar{v}(p_2)e\gamma^\mu u(p_1)]\left[\frac{g_{\mu\nu}}{q^2}\right][\bar{u}(p_3)e\gamma^\nu v(p_4)] \tag{1.2}$$

$$\mathcal{M} = \frac{-e^2}{q^2}[\bar{v}(p_2)\gamma^\mu u(p_1)]g_{\mu\nu}[\bar{u}(p_3)\gamma^\nu v(p_4)] \tag{1.3}$$

$$\tag{1.4}$$

$$\boxed{\mathcal{M} = \frac{-e^2}{q^2}[\bar{v}(p_2)\gamma^\mu u(p_1)][\bar{u}(p_3)\gamma_\nu v(p_4)]} \tag{1.5}$$

t-channel scattering

$$-i\mathcal{M} = [\bar{v}(p_3)ie\gamma^\mu v(p_1)]\left[\frac{-ig_{\mu\nu}}{q^2}\right][\bar{u}(p_4)ie\gamma^\nu u(p_2)] \tag{1.6}$$

$$\mathcal{M} = -[\bar{v}(p_3)e\gamma^\mu v(p_1)]\left[\frac{g_{\mu\nu}}{q^2}\right][\bar{u}(p_4)e\gamma^\nu u(p_2)] \tag{1.7}$$

$$\mathcal{M} = \frac{-e^2}{q^2}[\bar{v}(p_3)\gamma^\mu v(p_1)]g_{\mu\nu}[\bar{u}(p_4)\gamma^\nu u(p_2)] \tag{1.8}$$

$$\tag{1.9}$$

$$\boxed{\mathcal{M} = \frac{-e^2}{q^2}[\bar{v}(p_3)\gamma^\mu v(p_1)][\bar{u}(p_4)\gamma_\nu u(p_2)]} \tag{1.10}$$

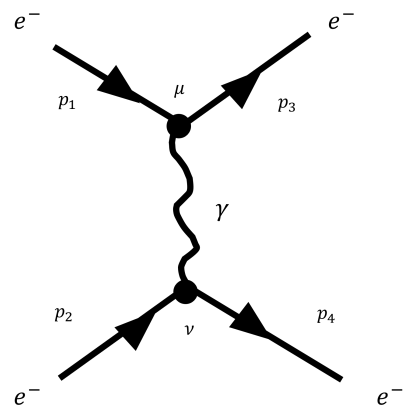
(d) In the *relativistic* limit (i.e., the masses of e^+ and e^- can be neglected), calculate the spin-averaged matrix element. Your final answer must be written in terms of the Mandelstam variables. [25 pts.]

2 Look At Me Roll [50 pts.]

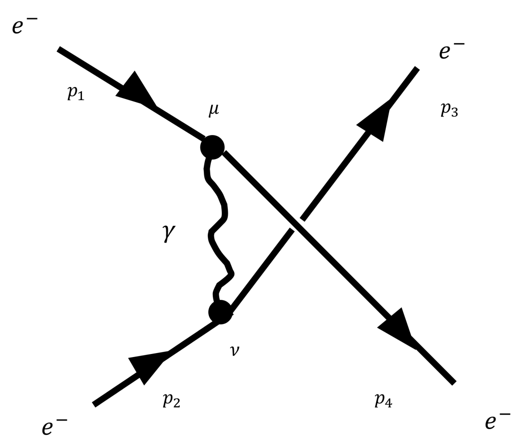
Consider the process: $e^- e^- \to e^- e^-$.

(a) Draw the lowest-order t-channel and u-channel Feynman diagrams for this process. [10 pts.]

The lowest-order t-channel Feynman diagram for the process is given by:



The lowest-order u-channel Feynman diagram for the process is given by:



(b) Use the Feynman rules for QED to write down the corresponding matrix elements. [10 pts.]

t-channel

For the t-channel Feynman diagram, the matrix element $\mathcal{M}$ is given by:

$$-i\mathcal{M} = [\bar{u}(p_3)ie\gamma^{\mu}u(p_1)]\left[\frac{-ig_{\mu\nu}}{q^2}\right][\bar{u}(p_4)ie\gamma^{\nu}u(p_2)] \tag{2.1}$$

$$\mathcal{M} = -[\bar{u}(p_3)e\gamma^{\mu}u(p_1)]\left[\frac{g_{\mu\nu}}{q^2}\right][\bar{u}(p_4)e\gamma^{\nu}u(p_2)] \tag{2.2}$$

$$\mathcal{M} = \frac{-e^2}{q^2}[\bar{u}(p_3)\gamma^{\mu}u(p_1)]g_{\mu\nu}[\bar{u}(p_4)\gamma^{\nu}u(p_2)] \tag{2.3}$$

$$\tag{2.4}$$

$$\boxed{\mathcal{M} = \frac{-e^2}{q^2}[\bar{u}(p_3)\gamma^{\mu}u(p_1)][\bar{u}(p_4)\gamma_{\nu}u(p_2)]} \tag{2.5}$$

u-channel

For the u-channel Feynman diagram, the matrix element $\mathcal{M}$ is given by:

$$-i\mathcal{M} = [\bar{u}(p_3)ie\gamma^{\mu}u(p_2)]\left[\frac{-ig_{\mu\nu}}{q^2}\right][\bar{u}(p_4)ie\gamma^{\nu}u(p_1)] \tag{2.6}$$

$$\mathcal{M} = -[\bar{u}(p_3)e\gamma^{\mu}u(p_2)]\left[\frac{g_{\mu\nu}}{q^2}\right][\bar{u}(p_4)e\gamma^{\nu}u(p_1)] \tag{2.7}$$

$$\mathcal{M} = \frac{-e^2}{q^2}[\bar{u}(p_3)\gamma^{\mu}u(p_2)]g_{\mu\nu}[\bar{u}(p_4)\gamma^{\nu}u(p_1)] \tag{2.8}$$

$$\tag{2.9}$$

$$\boxed{\mathcal{M} = \frac{-e^2}{q^2}[\bar{u}(p_3)\gamma^{\mu}u(p_2)][\bar{u}(p_4)\gamma_{\nu}u(p_1)]} \tag{2.10}$$

(c) In the *non-relativistic* limit (i.e., electron mass is *not* neglected), calculate the spin-averaged matrix element. Your final answer must be written in terms of the Mandelstam variables. [30 pts.]

We note that the spin-averaged matrix element is given by:

t-channel

For the t-channel Feynman diagram,

u-channel

For the u-channel Feynman diagram,