Dirac Equation

Klein-Gordon Equation

The Einstein energy-momentum relationship when expressed in terms of operators becomes:

$$E^2 = \mathbf{p}^2 + m^2$$
$$\hat{E}^2\psi(\mathbf{x},t) = \hat{\mathbf{p}}^2\psi(\mathbf{x},t) + m^2\psi(\mathbf{x},t)$$

with the operators rewritten as $\hat{\mathbf{p}} = -i\nabla$ and $\hat{E} = i\frac{\partial}{\partial t}$, the **Klein-Gordon wave equation** is given by:

$$\frac{\partial^2\psi}{\partial t^2} = \nabla^2\psi - m^2\psi \tag{1}$$

or in Lorentz-invariant form,

$$(\partial^\mu\partial_\mu + m^2)\psi = 0 \tag{2}$$

where

$$\partial^\mu\partial_\mu = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

This has plane wave solutions in the form of:

$$\psi(\mathbf{x},t) = Ne^{i(\mathbf{p}\cdot\mathbf{x} - Et)} \tag{3}$$

The Dirac Equation