Problem Set 4

Physics 180

Olyn D. Desabelle

8.9 Three nuclei A, B, C are radioactive isotopes of the same element and all decay via α-emission. The half-lives $t_{1/2}$ and ranges r of the alpha particles for A and B are:

$$\mathrm{A}\,(t_{1/2} = 10^3\,\mathrm{yr}, r = 3\,\mathrm{cm}), \quad \mathrm{B}\,(t_{1/2} = 10^2\,\mathrm{days}, r = 4\,\mathrm{cm}).$$

If the range of the α particles from the decay of C is $r = 6\,\mathrm{cm}$, estimate its half-life.

We will be using the Geiger–Nuttall relation to get the half-life of the C isotope:

$$\log_{10}\lambda = c + d\log_{10} r \tag{1}$$

where $\lambda = \frac{\ln 2}{t_{1/2}}$. We begin with finding the values of c and d using the given for the A and B isotopes with:

$$\log_{10}\left(\frac{\ln 2}{t_{1/2}}\right) = c + d\log_{10} r \tag{2}$$

converting them to similar units, we have the system of equations:

$$\log_{10}\left(\frac{\ln 2}{525\ 948\ 766\ \text{min}}\right) = c + d\log_{10}(3\ \text{cm}) \tag{3}$$

$$\log_{10}\left(\frac{\ln 2}{144\ 000\ \text{min}}\right) = c + d\log_{10}(4\ \text{cm}) \tag{4}$$

$$\tag{5}$$

we can solve these using substitution:

$$c = \log_{10}\left(\frac{\ln 2}{525\ 948\ 766\ \text{min}}\right) - d\log_{10}(3\ \text{cm}) \tag{6}$$

$$\log_{10}\left(\frac{\ln 2}{144\ 000\ \text{min}}\right) = \log_{10}\left(\frac{\ln 2}{525\ 948\ 766\ \text{min}}\right) - d\log_{10}(3\ \text{cm}) + d\log_{10}(4\ \text{cm}) \tag{7}$$

$$d = \left[\log_{10}\left(\frac{\ln 2}{525\ 948\ 766\ \text{min}}\right) - \log_{10}\left(\frac{\ln 2}{144\ 000\ \text{min}}\right)\right]\left[\frac{1}{\log_{10}(3\ \text{cm}) - \log_{10}(4\ \text{cm})}\right] \tag{8}$$

from here we get the following values:

$$c = -22.49 \tag{9}$$
$$d = 28.51 \tag{10}$$

getting the half-life for the C isotope:

$$10^{\log_{10}(\ln 2) - c - d\log_{10} r} = t_{1/2} \tag{11}$$
$$10^{\log_{10}(\ln 2) + 22.49 - 28.51\log_{10}(6)} = t_{1/2} \tag{12}$$

$$\boxed{t_{1/2} = 1.37 \text{ min}} \tag{13}$$

8.15 Use the Weisskopf formulas (8.88a) and (8.88b) to calculate the radiative width Γ_γ(E3) expressed in a form analogous to (8.89).

The emission rate $T_{fi}^{E,M}$ is given by:

$$T_{fi}^{E,M} = \frac{1}{4\pi\epsilon_0} \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{1}{\hbar} \left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} B_{fi}^{E,M}(L) \tag{14}$$

in this case, we only need $B_{fi}^{E,M}(L) = B^E(L)$ for the E3 multipole transition. The reduced transition probability for electric radiation ($B^E(L)$) is given by:

$$B^E(L) = \frac{e^2}{4\pi} \left(\frac{3}{L+3}\right)^2 (R_0)^{2L} A^{2L/3} \tag{15}$$

(16)

For E3 multipole transition ($L = 3$), we then have $B^E(L)$ as:

$$B^E(L=3) = \frac{e^2}{4\pi} \left(\frac{1}{2}\right)^2 (R_0)^6 A^2 \tag{17}$$

$$B^E(3) = \frac{e^2}{16\pi} R_0^6 A^2 \tag{18}$$

using this for $T_{fi}^{E,M}$:

$$T^E = \frac{1}{4\pi\epsilon_0} \frac{8\pi(4)}{3[(7)!!]^2} \frac{1}{\hbar} \left(\frac{E_\gamma}{\hbar c}\right)^7 B^E(3) \tag{19}$$

$$T^E = \frac{1}{\epsilon_0} \frac{8}{3[(7)!!]^2} \frac{1}{\hbar} \left(\frac{E_\gamma}{\hbar c}\right)^7 B^E(3) \tag{20}$$

we note that for the double factorial for odd integers, we have:

$$(2n+1)!! = \frac{(2n+1)!}{2^n n!} \tag{21}$$

$$(7)!! = \frac{(7)!}{2^3(3!)} = 105 \tag{22}$$

(23)

going back to the emission rate, we have:

$$\begin{aligned}
T^E &= \frac{1}{\epsilon_0}\frac{8}{3[105]^2}\frac{1}{\hbar}\left(\frac{E_\gamma}{\hbar c}\right)^7\left(\frac{e^2}{16\pi}R_0^6 A^2\right) && (24)\\
&= \frac{e^2 R_0^6}{6\pi[105]^2\epsilon_0\hbar^8 c^7}E_\gamma^7 A^2 && (25)\\
&= \frac{e^2(1.21\ \text{fm})^6}{6\pi[105]^2(55.263\ e^2\text{GeV}^{-1}\text{fm}^{-1})(6.582\times 10^{25}\ \text{GeV s})^8(3\times 10^{23}\ \text{fm/s})^7}E_\gamma^7 A^2 && (26)\\
& && (27)
\end{aligned}$$

to get the radiative width Γ, we use:

$$\begin{aligned}
\Gamma &= \hbar T^E && (28)\\
&= \frac{e^2(1.21\ \text{fm})^6}{6\pi[105]^2(55.263\ e^2\text{GeV}^{-1}\text{fm}^{-1})(6.582\times 10^{25}\ \text{GeV s})^7(3\times 10^{23}\ \text{fm/s})^7}E_\gamma^7 A^2 && (29)\\
& && (30)
\end{aligned}$$

$$\boxed{\Gamma = (2.346\times 10^{16})E_\gamma^7 A^2\ \text{eV}} \qquad (31)$$