Physics 185 Midterm Exam
1st Sem AY 2021-2022

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INSTRUCTIONS: Show complete and detailed solutions. Kindly include an honor pledge at the start of your work: "I swear upon my honor that I have not given nor received any unauthorized help on this exam and that all the work below are my own."

1. **SEMF** [50 pts.]

   For this item, we will refer to Eq. (2.48) of Martin & Shaw's textbook (3rd Ed.).

   (a) For a fixed mass number A, find the proton number Z for the most stable nucleus. *Hint*: The most stable nucleus will have the highest binding energy. [10 pts.]

   (b) Plot the resulting function above (note that your answer in (a) will be Z as a function of A). [10 pts.]

   (c) Show that for a fixed mass number A, $M(Z, A)$ has an extremum Z value. [10 pts.]

   (d) Is this extremum a minimum or a maximum? [5 pts.]

   (e) Plot $M(Z, A)$ as a function of Z for a fixed mass number $A = 208$. [10 pts.]

   (f) From your results, approximate the number of protons Z in the most stable nucleus of A = 208. In reality, this is ^{208}Pb with $Z = 82$. How close are your predictions from plots (b) and (e)? [5 pts.]

2. **Dirac and Weyl** [50 pts.]

   In obtaining the spinor solutions, we explicitly used the Dirac-Pauli representation of the γ matrices given in Eq. (4.35) of Thomson's textbook (*Note*: all equation numbers mentioned hereinafter will come from Thomson's textbook). Here we will use the *Weyl* representation whose γ matrices are given by

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$$

   Note that the γ^k are still similar to the Dirac-Pauli representation, the only difference is that for Weyl, γ^0 is not diagonal, explicitly

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

   (a) Show that the γ matrices in Weyl representation still satisfy Eq. (4.33) and Eq. (4.34). [10 pts.]

   (b) Solve the Dirac equation for a particle at rest using the Weyl representation. *Hint*: Have a look at Section 4.6.1 of Thomson's textbook. [15 pts.]

   (c) Show that in Weyl representation, the Dirac equation (for a general free particle not necessarily at rest) still leads to coupled equations similar to Eq. (4.45) and Eq. (4.46). [15 pts.]

   (d) Show that the coupled equations in (c) *decouple* the moment you set $m = 0$. The resulting uncoupled equations are known as *Weyl equations.* [10 pts.]