UNIVERSITY OF TECHNOLOGY - VNUHCM Faculty of Applied Science	Midterm Exam	Semester/Acad. year	2	2020-2021
		Date		May 9th, 2021
	Course title	Probability and Statistics		
	Course ID	MT2013/MT2001		
	Duration	50 minutes	Question sheet code	2025

Instructions to students:

- You are allowed to use your OWN materials and calculator. Total available score: 10.
- At the beginning of the working time, you MUST fill in your full name and student ID on this question sheet. There are 18 questions on 2 pages. Do not round between steps.

Student's full name:	Invigilator 1: .....
Student Id:	Invigilator 2: .....

1. In a course of Statistics, students are allowed to take an exam three times. Suppose that students have a chance of 28% to pass the exam on the first try. If they fail this time, they have a probability of 36% passing the exam on the second try. In case of failing at these two tries, they still have a probability of 58% to pass on the third one. Find the probability that a randomly chosen student needs exactly three times to pass this course.

   (A) 0.1441. (B) 0.4383. (C) The other anwsers are wrong. (D) 0.2673. (E) 0.6003.

2. For ordinary exams, universities in German use a 5-point scale to evaluate their student. The below table summarizes the Calculus scores (X) and the Linear Algebra scores (Y) of students in a certain German university (rounded to be integers). Students will fail at a course if their score is at least 4. Find the probability that a student in this university will fail at Linear Algebra course if he/she has passed the Calculus course.

X	Y				
	1	2	3	4	5
1	0.0662	0.0850	0.0222	0.0968	0.0862
2	0.0775	0.0231	0.0559	0.0233	0.0949
3	0.0747	0.0074	0.0543	0.0195	0.0168
4	0.0319	0.0850	0.0179	0.0051	0.0078
5	0.0410	0.0053	0.0006	0.0006	0.0010

   (A) 0.5592. (B) 0.6625. (C) 0.4792. (D) The other answers are wrong. (E) 0.4199..

3. A university has investigated that 95% of their students graduate from their bachelor's program, but only 12% of them, who have earned the bachelor's degree, will apply for the master's program. Suppose that the decision of applications is independent between students. Choosing 12 students in this university randomly, find the probability that at least 3 of them will apply for the master's program.

   (A) 0.149. (B) 0.359. (C) 0.263. (D) 0.4413. (E) The other answers are wrong.

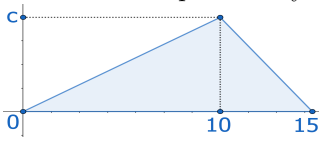
4. Here are the percentages of cotton in material used to manufacture men's shirts: 35.6, 32.1, 39.5, 39.2, 30.6, 39.1, 32.8, 33.7, 38.8, 31.4. Calculate the median of these data.

   (A) The other answers are wrong. (B) 34.75. (C) 34.65. (D) 35.05. (E) 35.15.

5. Of the registered voters in a certain community, 59.2 percent are women and 40.8 percent are men. In the last local election, 67.9 percent of the registered female voters and 62.2 percent of the registered male voters actually voted. If a registered voter from this community is randomly chosen, what is the probability that this person is a woman who voted in the last election?

   (A) 0.7192. (B) 0.6653. (C) The other answers are wrong. (D) 0.613. (E) 0.692.

6. Suppose that the self-study time (hour/week) of a certain student is a random variable with the probability density function $f(x)$ where its graph is given in the figure ($f(x) = 0,\ \forall x \notin (0, 15)$). What is the probability that this student has a self-study time between 10 and 15?



   (A) 0.4794. (B) 0.2174. (C) 0.3579. (D) 0.3333. (E) The other anwsers are wrong.

7. The lifespan of a certain battery type is a random variable with a mean of 11 months and a standard deviation of 4.6 months. Suppose that 380 batteries of this type were randomly selected. Estimate the probability that the average lifespan is more than 11.5 months.

   (A) 0.3073. (B) 0.3119. (C) 0.1175. (D) 0.0171. (E) The other answers are wrong.

8. In a certain town, there is a unique bus line to the airport. The buses depart from the station every 12 minutes and the first depart is at 7 am every day. Suppose that a traveler arrives at the station between 7 am and 7:54 am with the arrival time following a uniform distribution. What is the probability that he/she has missed at least 4 departs?

(A) 0.4075. (B) 0.1301. (C) 0.3333. (D) 0.2176. (E) The other answers are wrong.

9. The life in hours of a 75-watt light bulb is known to be normally distributed with $\sigma = 20$ hours. A random sample of 10 bulbs has a mean life of 1020 hours. Find a 98% two-sided confidence interval on the mean life.

(A) [978, 1008]. (B) [1025, 1055]. (C) [970, 1000]. (D) [1045, 1075]. (E) [1005, 1035].

10. Suppose that a bakery will get a profit of \$15 for each sold cake. If the demand in a certain day is given as below. Find the average and the standard deviation profit of this bakery in the given day.

Number of cakes	0.0000	1.0000	2.0000	3.0000	4.0000
Probability	0.0273	0.1581	0.1596	0.3303	0.3247

(A) $\mu = \$41.505$ and $\sigma = \$16.6381$. (B) $\mu = \$38.2871$ and $\sigma = \$19.5659$. (C) The other answers are wrong.
(D) $\mu = \$41.505$ and $\sigma = \$17.1559$. (E) $\mu = \$45.6573$ and $\sigma = \$17.1559$.

11. Suppose that the number of a certain item purchased in a fashion website folllows a Poisson distribution with a mean of 4 items per hour. If the prior information shows that there are at least 2 items sold between 7pm to 9pm, find the probability that there are exactly 10 items sold in this period.

(A) 0.2852. (B) 0.0996. (C) The other answers are wrong. (D) 0.447. (E) 0.0721.

12. Two random variables $X \sim N(9, 0.2^2)$ and $Y \sim N(8, 0.27^2)$. Find the expected value and variance of $Z = 3X - 4Y$.

(A) $E(Z) = -5$; $Var(Z) = 1.5264$. (B) $E(Z) = -5$; $Var(Z) = 0.8064$. (C) The other answers are wrong.
(D) $E(Z) = -5$; $Var(Z) = 0.4116$. (E) $E(Z) = -47$; $Var(Z) = 1.5264$.

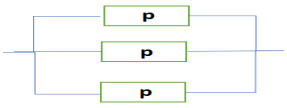
13. A college professor never finishes his lecture before the end of the hour and always finishes his lectures within 5 min after the hour. Let X be the time that elapses between the end of the hour and the end of the lecture and suppose the pdf of X is $f(x) = \begin{cases} 3x^2/5^3, & \text{if } 0 < x < 5 \\ 0, & \text{otherwise} \end{cases}$. Find the median of X.

(A) The other answers are wrong. (B) 3.8141. (C) 3.2882. (D) 3.9685. (E) 4.574.

14. In a certain assembly plant, five machines, B1, B2, B3, B4, and B5, make 18.6%, 69.6%, 8.4%, 0.4%, and 3%, respectively, of the products. It is known from past experience that 4.4%, 10.3%, 7%, 13.6%, and 2.9% of the products made by each machine, respectively, are defective. A finished product is randomly selected and it turns out to be defective. Which machine most likely procduces this item?

(A) B5. (B) B4. (C) B3. (D) B1. (E) B2.

15. The following circuit operates if and only if there is a path of functional devices from left to right. The probabilities that each device functions are as shown where $p = 0.82$. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



(A) 0.7253. (B) 0.9803. (C) The other answers are wrong. (D) 0.8553. (E) 0.9942.

16. The age when smokers first start from previous studies is normally distributed with a population standard deviation of 1.5 years old. A sample of 25 smokers found that their mean starting age was 14 years old. Find a 98% lower (one-sided) confidence bound of the mean.

(A) 13.598. (B) 13.577. (C) 13.385. (D) 13.556. (E) 13.616.

17. The following table shows the joint distribution for an exam where students have to choose one of two questions. The passing score is 3 points or more. What is the probability that a randomly selected student will pass the exam?

	5	4	3	2	1
Question 1	0.0443	0.0713	0.0340	0.1032	0.1369
Question 2	0.1045	0.0978	0.0015	0.1322	0.2743

(A) 0.5012. (B) 0.1107. (C) The other answers are wrong. (D) 0.6747. (E) 0.1827.

18. A civil engineer is analyzing the compressive strength of concrete, which is normally distributed with $\sigma^2 = 1435$ psi^2. It is desired to estimate the compressive strength with an error that is less than 10 psi at 92% confidence. What is the smallest sample size required?

(A) 29. (B) 20. (C) 35. (D) 33. (E) 37.