ECON60332 Financial Econometrics

Coursework
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This coursework contains the description of the task each group must perform. Note, your answers must be reported following the **template** available on the Blackboard. Reports which do not follow the template receive the mark of zero. Your report, excluding Appendix, should not exceed 5 pages.

This coursework is a group work, where only one answer per group is allowed. Every participant in the group receives the same mark for this coursework. If some group participants did not contribute to the report, indicate this in the appropriate field of the template, in which case the participant receives the mark of zero.

Make sure that you **save your reports as pdf files and submit the single pdf file**. When submitting your reports on Gradescope make sure you appropriately indicate pages where the answer to a specific question is located. For the theoretical questions both typed and scanned answers are allowed, provided that they are neatly written and *clearly* readable.

Good luck!

Theoretical exercise (30P)

Consider the following process for daily log-returns r_t:

$$r_t = c + \varepsilon_t \tag{1}$$

$$\varepsilon_t = \sqrt{\sigma_t^2} z_t \tag{2}$$

where $z_t \overset{i.i.d.}{\sim} \mathcal{N}(0,1)$ Assume that r_t is stationary and its kurtosis is finite.
If your group number is even, i.e. **2, 4, 6, 8 or 10**, then assume that the dynamics of the conditional variance is given by:

$$\sigma_t^2 = \omega + \alpha \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \gamma \right)^2 + \beta \sigma_{t-1}^2 \tag{3}$$

If your group number is odd, i.e. **1, 3, 5, 7, 9 or 11**, then assume that the dynamics of the conditional variance is given by:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 \tag{4}$$

a) Compare the model given in equation (3) or (4) depending on your group number with a simple GARCH(1,1) model. What are the similarities and what are the differences between the two approaches? 4 P

b) Assuming that the process ε_t is stationary, compute its unconditional mean and variance $\mathbb{E}[\varepsilon_t]$ and $\mathbb{V}[\varepsilon_t]$. 2 P

c) Derive $cov[\sigma_{t+1}^2, \varepsilon_t | \mathcal{F}_{t-1}]$, where $\mathcal{F}_t$ is the information set available up to time t. Comment on the ability of the model to capture leverage effect. 8 P

d) Compute the 3-step ahead forecast $\mathbb{E}[\sigma_{t+3}^2 | \mathcal{F}_t]$ of the conditional variance process, based on the model given in equation (3) or (4) depending on your group number. 10 P

e) Derive the News Impact Curve of the model given in equation (3) or (4) depending on your group number. Interpret your findings. 6 P

Practical exercise: predictive power of GARCH models (70P)

Consider the log-return data of a financial asset provided in the file `group_<i>.csv` to estimate and forecast different conditional variance models. The data series you should use depends on the **group number**, e.g. if your group is 6, you should take the file `group_6.csv` and so on. For numerical reasons, the returns are multliplied by a 100, e.g. 3.16 instad of 0.0316.
Use the first 250 observations as an in-sample (estimation) period and the last 250 observations as an out-of-sample (forecasting) period.
For the numerical answers report 2 digits after the decimal point. Use the function `round(x,2)` to avoid mistakes.

a) Investigate the statistical properties of the **in-sample** log-returns (r_t) and the squared in-sample log-returns (r_t^2). In particular:

i) Report the descriptive statistics for r_t.

ii) Test if the mean of r_t is significantly different from zero.

iii) Test if the skewness of r_t is significantly different from zero.

iv) Test if the kurtosis of r_t is significantly different from three.

v) Test r_t for normality using Jarque-Bera test.

vi) Test if there is significant autocorrelation in both r_t and r_t^2 series using the LBQ test for 21 lags.

vii) Plot the r_t, its SACF and SPACF.

Give an economic interpretation of the reported properties. 15 P

b) Assume that the conditional mean of the return series is constant: $r_t = c + \varepsilon_t$, $\varepsilon_t = \sigma_t z_t$. Define the residual series as $e_t = r_t - \hat{c}$. Use the e_t series to estimate the following conditional variance models:

i) GARCH (1,1) with both $z_t \sim \mathcal{N}(0,1)$ and $z_t \sim t_v$, where t_v denotes a Student-t distribution with ν degrees of freedom.

ii) GJR-GARCH (1,1) with both $z_t \sim \mathcal{N}(0,1)$ and $z_t \sim t_v$.

where the degrees of freedom of t_v **have to be estimated** together with the other parameters. Report the estimated parameters and provide an economic discussion of the values and the significance of the estimated parameters. 21 P

c) Plot a NIC of all four estimated models on the same plot, using the values of the parameters you have estimated in b). For the value of σ_e^2 select the unconditional variance from the GARCH(1,1) with the normally distributed z_t. Give an economic interpretation of the plot. 6 P

d) Find which model provides the best fit for the conditional variance by analysing standardised residuals. For each model conduct an LBQ test for 21 lags, report the test statistic and the p-value. Discuss which model provides the best fit to the in-sample daily log returns. 11 P

e) For the **out-of-sample** period, produce 1-step ahead forecasts with all the models from b). For this, use a rolling window estimation scheme, i.e. the number of observations used for estimation $T = 250$ remains constant. Evaluate the forecast accuracy with the help of the root mean squared forecast error (RMSFE) and also apply Diebold-Mariano test with the GARCH(1,1) model as a benchmark, testing the null hypothesis that the forecast produced by the benchmark model is better than the candidate (use $g(x) = x^2$ as a loss function and `alternative = "less"`). As a proxy for the true variance use the squared return series (r_t^2). Which model is the best in predicting the conditional variances? Give an economic interpretation of your findings. 12 P

f) Discuss the differences in the forecasting performance of the models you have used in terms of the trade-off between model misspecification and estimation noise. 5 P