Alastair Hall

ECON61001: Econometric Methods

Semester 1

Study Group Questions # 1. Review of Linear Algebra: Practice Questions

Please work on these questions in your study groups and submit your answers to Gradescope following the instructions in the *Weekly Assignments* folder on BB.

Please enter the details of the group in this table:

Group name	Matrix
Student ID 1	11335127
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Student ID 5	11465531

1. Assess whether each of the following matrices is nonsingular and provide a justification for your conclusion in each case:

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & -3 \\ 2 & 2 & -6 \\ 3 & 1 & -9 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{bmatrix}, \qquad C = B'B, \qquad D = BB'.$$

A nonsingular matrix is one that has non-zero determinant.

$$A = \begin{bmatrix} 1 & 3 & -3 \\ 2 & 2 & -6 \\ 3 & 1 & -9 \end{bmatrix} \quad \mathrm{Det}(A) = 1 \det\begin{bmatrix} 2 & -6 \\ 1 & -9 \end{bmatrix} - 3 \det\begin{bmatrix} 2 & -6 \\ 3 & -9 \end{bmatrix} + (-3)\det\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

$$= 1(-18+6) - 3(-18+18) - 3(2-6)$$

$$= 0$$

$\Rightarrow$ A is singular.

$$B = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{bmatrix}$$

A non square matrix can not be nonsingular.

$$C = B'B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1+4+9 & 3+4+3 \\ 3+4+3 & 9+4+1 \end{bmatrix} = \begin{bmatrix} 14 & 10 \\ 10 & 14 \end{bmatrix}$$

$$\Rightarrow \det(C) = 14 \cdot 14 - 10 \cdot 10 = 96 \neq 0$$

$\Rightarrow$ C is nonsingular.

$$D = B \cdot B' = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+9 & 2+6 & 3+3 \\ 2+6 & 4+4 & 6+2 \\ 3+3 & 6+2 & 9+1 \end{bmatrix} = \begin{bmatrix} 10 & 8 & 6 \\ 8 & 8 & 8 \\ 6 & 8 & 10 \end{bmatrix}$$

$$\Rightarrow \det(D) = 10(80-64) - 8(80-48) + 6(64-48)$$

$$= 0$$

$\Rightarrow$ D is singular.

2. Let

$$\mathbf{A} = \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}.$$

Calculate $det(\mathbf{A})$ and $tr(\mathbf{A})$. Assess whether $\mathbf{A}$ is positive definite and provide a justification for your conclusion.

* $\det(A) = a_{11}a_{22} - a_{21}a_{12} = 8$
* $tr(A) = a_{11} + a_{22} = 7$.

* let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$; $x_1 \neq 0, x_2 \neq 0$

$\Rightarrow$ Quadratic form of A : $Q(x) = x' A x$

$$Q(x) = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= 4x_1^2 + 4x_1x_2 + 3x_2^2$$

$$= (2x_1 + x_2)^2 + 2x_2^2$$

$\Rightarrow Q(x) > 0 \quad \forall x_1, x_2 \neq 0$

$\Rightarrow$ A is positive definite.

3. Consider two matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

Calculate $det(\mathbf{A})$ and $tr(\mathbf{A})$, and $det(\mathbf{B})$ and $tr(\mathbf{B})$. Assess whether $\mathbf{A}$ and $\mathbf{B}$ are positive definite matrices and provide a justification for your conclusion in each case.

* $det(A) = a_{11} \cdot a_{22} \cdot a_{33} = 1 \cdot 2 \cdot 1 = 2$

  $tr(A) = a_{11} + a_{22} + a_{33} = 1 + 2 + 1 = 4$

* $det(B) = b_{11} \cdot b_{22} \cdot b_{33} = 10 \cdot (-2) \cdot (-2) = 40$

  $tr(B) = b_{11} + b_{22} + b_{33} = 10 - 2 - 2 = 6.$

* As both A and B are diagonal matrices

  $\Rightarrow$ Eigenvalues of A : $\lambda_A = \{1, 2, 1\}$

  Eigenvalues of B : $\lambda_B = \{10, -2, -2\}.$

As $\lambda_{Ai} > 0 \ \forall i = 1, 2, 3 \Rightarrow$ A is positive definite.

On the other hand, as λ_{Bi} show both signs, B is indefinite