MANIPAL UNIVERSITY JAIPUR
(University under Section 2(f) of the UGC Act)

INSPIRED BY LIFE

FIRST STATE PRIVATE UNIVERSITY IN RAJASTHAN — ACCREDITED WITH GRADE A+ NAAC — 3.28 SCORE

B.TECH SECOND YEAR

ACADEMIC YEAR: 2020-2021

COURSE NAME: ENGINEERING MATHEMATICS-III

COURSE CODE : MA 2101

LECTURE SERIES NO : 35 (THIRTY FIVE)

CREDITS : 3

MODE OF DELIVERY : ONLINE (POWER POINT PRESENTATION)

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MANIPAL UNIVERSITY JAIPUR

VISION

Global Leadership in Higher Education and Human Development

MISSION

- Be the most preferred University for innovative and interdisciplinary learning
- Foster academic, research and professional excellence in all domains
- Transform young minds into competent professionals with good human values

VALUES

Integrity, Transparency, Quality,
Team Work, Execution with Passion, Humane Touch

SESSION OUTCOME

" TO UNDERSTAND THE CONCEPT OF ODE AND THEIR APPLICATIONS AND SOLVE THE PROBLEM"

ASSESSMENT CRITERIA'S

- ASSIGNMENT
- QUIZ
- MID TERM EXAMINATION –I & II
- END TERM EXAMINATION

PROGRAM OUTCOMES MAPPING WITH CO1

[PO1]

ENGINEERING KNOWLEDGE: APPLY THE KNOWLEDGE OF MATHEMATICS, SCIENCE, ENGINEERING FUNDAMENTALS, AND AN ENGINEERING SPECIALIZATION TO THE SOLUTION OF COMPLEX ENGINEERING PROBLEMS.

Algebraic Structures

- Algebraic systems Examples and general properties
- Semi groups
- Monoids
- Groups
- **Subgroups, Normal Subgroups**

Subgroups

- **Def.** A nonempty subset H of a group (G, *) is a subgroup of G, if (H, *) is a group.

Note: For any group {G, *}, {e, * } and (G, *) are trivial subgroups.

- **Ex.** G = {1, -1, i, -i } is a group w.r.t multiplication.

  H_1 = { 1, -1 } is a subgroup of G .

  H_2 = { 1 } is a trivial subgroup of G.

- **Ex.** (Z , +) and (Q , +) are subgroups of the group (R +).
- **Theorem:** A nonempty subset H of a group (G, *) is a subgroup of G iff
- i) $a * b \in H \quad \forall\, a, b \in H$
- ii) $a^{-1} \in H \quad \forall\, a \in H$

Subgroup

A non-empty subset H of a group $(G, *)$ is said to be subgroup of G, if $(H, *)$ is itself a group.

e.g., [{1,-1}, .] is a subgroup of [{1,-1,i,-i} .]

Criteria for a Subset to be a Subgroup

A non-empty subset H of a group G is a subgroup of G if and only if

(i) $a, b \in H \Rightarrow ab \in H$

(ii) $a \in H \Rightarrow a^{-1} \in H$,

where a^{-1} is the inverse of $a \in G$

Theorem

- **Theorem:** A necessary and sufficient condition for a nonempty subset H of a group (G, *) to be a subgroup is that
  $a \in H,\ b \in H \Rightarrow a * b^{-1} \in H.$
- **Proof:** **Case1:** Let (G, *) be a group and H is a subgroup of G
  Let $a,b \in H \Rightarrow b^{-1} \in H$ (since H is is a group)
  $\Rightarrow a * b^{-1} \in H.$ (By closure property in H)
- **Case2:** Let H be a nonempty set of a group (G, *).
  Let $a * b^{-1} \in H \quad \forall\ a, b \in H$
- Now, $a * a^{-1} \in H$ (Taking $b = a$)
  $\Rightarrow e \in H$ i.e., identity exists in H.
- Now, $e \in H,\ a \in H \Rightarrow e * a^{-1} \in H$
  $\Rightarrow a^{-1} \in H$

Contd.,

- $\therefore$ Each element of H has inverse in H.

  Further, $a \in H,\ b \in H \Rightarrow a \in H,\ b^{-1} \in H$

  $\Rightarrow a * (b^{-1})^{-1} \in H.$

  $\Rightarrow a * b \in H.$

  $\therefore$ H is closed w.r.t $*$.

- Finally, Let $a,b,c \in H$

  $\Rightarrow a,b,c \in G$ (since $H \subseteq G$)

  $\Rightarrow (a * b) * c = a * (b * c)$

  $\therefore$ $*$ is associative in H

- **Hence, H is a subgroup of G.**

Ex. Show that the intersection of two sub groups of a group G is again a sub group of G.

- **Proof:** Let $(G, *)$ be a group.
- Let H_1 and H_2 are two subgroups of G.
- Let $a, b \in H_1 \cap H_2$.
- Now, $a, b \in H_1 \Rightarrow a * b^{-1} \in H_1$ (Since, H_1 is a subgroup of G)
- again, $a, b \in H_2 \Rightarrow a * b^{-1} \in H_2$ (Since, H_2 is a subgroup of G)
- $\therefore \; a * b^{-1} \in H_1 \cap H_2$.
- **Hence, $H_1 \cap H_2$ is a subgroup of G .**

Ex. Show that the union of two sub groups of a group G need not be a sub group of G.

Proof: Let G be an additive group of integers.

- Let $H_1 = \{0, \pm 2, \pm 4, \pm 6, \pm 8, \ldots\}$
- and $H_2 = \{0, \pm 3, \pm 6, \pm 9, \pm 12, \ldots\}$
- Here, H_1 and H_2 are groups w.r.t addition.
- Further, H_1 and H_2 are subsets of G.
- $\therefore$ H_1 and H_2 are subgroups of G.
- $H_1 \cup H_2 = \{0, \pm 2, \pm 3, \pm 4, \pm 6, \ldots\}$
- Here, $H_1 \cup H_2$ is not closed w.r.t addition.
- **For ex.** $2, 3 \in G$
- But, $2 + 3 = 5$ and 5 does not belongs to $H_1 \cup H_2$.
- **Hence, $H_1 \cup H_2$ is not a subgroup of G.**

NORMAL SUBGROUPS

Definition:

A subgroup N of a group G is said to be a normal subgroup of G if,

$$gng^{-1} \in N \quad \forall \quad g \in G,\ n \in N$$

Equivalently, if $gNg^{-1} = \{gng^{-1} \mid n \in N\}$, then N is a normal subgroup of G if and only if

$$gNg^{-1} \subset N \quad \forall\, g \in G.$$

Theorem

The subgroup N of a group G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

Proof: Let N be a normal subgroup of G. Then $gNg^{-1} = N \ \forall \ g \in G$ (by theorem 1)

$(gNg^{-1})g = Ng \quad \forall \quad g \in G$

Or $gN(g^{-1}g) = Ng \ \forall \quad g \in G$

$\mathbf{gN = Ng} \ \forall \ g \in G$

i.e., every left coset gN is the right coset Ng.

Conversely, assume that every left coset of a subgroup N of G is the right coset of N in G.

Thus, for $g \in G$, a left coset gN must be a right coset.

$\therefore gN = Nx$ for some $x \in G$.

Now, $e \in N$ $ge = g \in gN$.

$g \in Nx$ (since $gN = Nx$)

Also, $g = eg \in Ng$, a right coset of N in G.

Thus, two right cosets Nx and Ng have common element g.

$N x = Ng$ (since two right cosets are either identical or disjoint.)

$\therefore$ Ng is the unique right coset which is equal to the left coset gN.

$\therefore$ $gN = Ng \ \forall \ g \in G$

$gNg^{-1} = Ngg^{-1} \quad \forall \ g \in G$

$gNg^{-1} = N \ \forall \ g \in G$ (since, $gg^{-1} = e$ and $Ne = N$)

N is a normal subgroup of G.

THANK YOU

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