MANIPAL UNIVERSITY JAIPUR

(University under Section 2(f) of the UGC Act)

B.TECH FIRST YEAR

ACADEMIC YEAR: 2020-2021

COURSE NAME: BASIC MECHANICAL ENGINEERING

COURSE CODE : MA 2101

LECTURE SERIES NO : 26 (TWENTY SIX)

CREDITS : 03

MODE OF DELIVERY : ONLINE (POWER POINT PRESENTATION)

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MANIPAL UNIVERSITY JAIPUR

VISION

Global Leadership in Higher Education and Human Development

MISSION

- Be the most preferred University for innovative and interdisciplinary learning
- Foster academic, research and professional excellence in all domains
- Transform young minds into competent professionals with good human values

VALUES

Integrity, Transparency, Quality,
Team Work, Execution with Passion, Humane Touch

SESSION OUTCOME

“UNDERSTAND THE CONCEPT

OF

GENERATING FUNCTION”

ASSIGNMENT

QUIZ

MID TERM EXAMINATION –I, II

END TERM EXAMINATION

ASSESSMENT CRITERIA'S

PROGRAM OUTCOMES MAPPING WITH CO4

ENGINEERING KNOWLEDGE: APPLY THE KNOWLEDGE OF MATHEMATICS, SCIENCE, ENGINEERING FUNDAMENTALS, AND AN ENGINEERING SPECIALIZATION TO THE SOLUTION OF COMPLEX ENGINEERING PROBLEMS.

GENERATING FUNCTION FOR THE SEQUENCE OF REAL NUMBERS

If $(a_0, a_1, a_2, \ldots\ldots\ a_r, \ldots)$ is a sequence of real or complex numbers, then the power series given by

$$\mathrm{A(z)} = \sum_{r=0}^{\infty} a_r z^r\ a_0, +a_1 z + \ a_2 z^2 \ldots\ldots\ a_n z^n + \ldots\ldots\ldots\ \ldots$$

Is called the Generating function for the given sequence.

Multiplying a generating function by a constant

=> scales every term in the associated sequence by the same constant.

$$< 1, 0, 1, 0, \ldots > \quad \leftrightarrow \quad 1 + 0x + 1x^2 + 0x^3 + \ldots \quad = \quad \frac{1}{1 - x^2}$$

Multiply the generating function by 2 gives

$$\frac{2}{1 - x^2} \quad = \quad 2 + 2x^2 + 2x^4 + 2x^6 + \ldots$$

which generates the sequence:

$$< 2, 0, 2, 0, \ldots >$$

Addition

Adding generating functions corresponds to adding sequences term by term.

$$< 1, 1, 1, 1, 1, 1, \ldots > \quad \leftrightarrow \quad \frac{1}{1-x}$$

$$< 1, -1, 1, -1, 1, -1, \ldots > \quad \leftrightarrow \quad \frac{1}{1+x}$$

$$< 2, 0, 2, 0, 2, 0, \ldots > \quad \leftrightarrow \quad \frac{1}{1-x} + \frac{1}{1+x}$$

The same result as in the previous slide.

$$= \frac{2}{1-x^2}$$

Right Shift

$$< 1, 1, 1, 1, \ldots > \quad \leftrightarrow \quad \frac{1}{1-x}$$

How to generate the sequence $<\underbrace{0, 0, \ldots, 0}_{\text{k zeros}}, 1, 1, 1, 1, 1 \ldots>$?

$$< \underbrace{0, 0, \ldots, 0}_{\text{k zeros}}, 1, 1, 1, 1, \ldots > \quad \leftrightarrow \quad x^k + x^{k+1} + x^{k+2} + \ldots$$

$$= x^k (1 + x + x^2 + \ldots)$$

$$= \frac{x^k}{1-x}$$

Adding k zeros ⇔ multiplying x^k on the generating function.

Differentiation

How to generate the sequence <1, 2, 3, 4, 5, …>?

The generating function is $1 + 2x + 3x^2 + 4x^3 + \ldots$

How to obtain a closed form of this function?

$$\frac{d}{dx}(1 + x + x^2 + x^3 + \ldots) \quad \leftrightarrow \quad \frac{d}{dx}\left(\frac{1}{1-x}\right)$$

$$1 + 2x + 3x^2 + 4x^3 + \ldots \quad \leftrightarrow \quad \frac{1}{(1-x)^2}$$

$$< 1, 2, 3, 4, \ldots > \quad \leftrightarrow \quad \frac{1}{(1-x)^2}$$

We found a generating function for the sequence <1,2,3,…> of positive integers!

More Differentiation

How to generate the sequence <1, 4, 9, 16, 25, …>?

$$\frac{d}{dx}(1 + 2x + 3x^2 + 4x^3 + \ldots)$$

$$= 2 + 6x + 12x^2 + 20x^3 + \ldots$$

$$\leftrightarrow \quad < 2, 6, 12, 20, \ldots >$$

Nice idea. But not what we want.

More Differentiation

How to generate the sequence <1, 4, 9, 16, 25, …>?

$$1 + 2x + 3x^2 + 4x^3 + \ldots \quad \leftrightarrow \quad \frac{1}{(1-x)^2}$$

$$0 + x + 2x^2 + 3x^3 + 4x^4 + \ldots \quad \leftrightarrow \quad \frac{x}{(1-x)^2}$$

$$\frac{d}{dx}(0 + x + 2x^2 + 3x^3 + 4x^4 + \ldots) \quad \leftrightarrow \quad \frac{d}{dx}\frac{x}{(1-x)^2}$$

$$= 1 + 4x + 9x^2 + 16x^3 + 25x^4 + \ldots \quad \leftrightarrow \quad \frac{1+x}{(1-x)^3}$$

$$\boxed{< 1, 4, 9, 16, 25, \ldots > \quad \leftrightarrow \frac{1+x}{(1-x)^3}}$$

The following table represents some sequences and their generating functions.

	Sequence	**Generating Function**
1	1	$\frac{1}{1-z}$
2	$(-1)^n$	$\frac{1}{1+z}$
3	a^n	$\frac{1}{1-az}$
4	$(-a)^n$	$\frac{1}{1+az}$
5	$n+1$	$\frac{1}{1-(z)^2}$
6	n	$\frac{1}{(1-z)^2}$
7	n^2	$\frac{z(1+2)}{(1-z)^3}$
8	na^n	$\frac{az}{(1-az)^2}$

Theorem: Let a, b and c be numeric functions and let A(z), B(z) and C(z) be respectively the generating functions of a, b and c then

(i) If $b_r = \alpha a_r$, for some constant α, then B(z) =α A(z)

(ii) If $c_r = a_r + b_r$, then C(z) = A(z)+B(z)

(iii) If c is the convolution of a and b; i.e. c = a*b, then C(z) = A(z)B(z)

(iv) If $b_r = \alpha^r a_r$, where is a constant, then B(z) = A(αz)

Theorem: Let A(z) be the generating function of the numeric function a= $(a_0, a_1, a_2, \ldots.. a_r, \ldots)$. Then $\frac{1}{(1-z)}$ A(z) is the generating function of the numeric function b which is accumulated sum of a.

Example **:** *Find the generating function of the following series* $1, -1, 1, -1, 1, -1, \ldots$.

Solution: The given series can be directed as $a_0 = 1,\ a_1 = -1,\ a_2 = 1,\ a_3 = -1 \ldots$.

The required generating function is given by

$$A(z) = \sum_{r=0}^{\infty} a_r z^r = a_0 + a_1 z + a_2 z^2 + \ldots = 1 - z + z^2 - z^3 + \ldots = \frac{1}{1+z} = (1+z)^{-1}$$

[sum of infinite G.P. series $S_\infty = \frac{a}{1-}$

Example **:** *Find the generating function of the following series* $1, 1, 1, 1, 1, 1, 1$.

Solution: The given series is directed as

$$a_0 = 1, a_1 = 1, a_2 = 1, a_3 = 1, a_4 = 1, a_5 = 1, a_6 = 1.$$

The required generating function is given by

$$a(z) = \sum_{r=0}^{\infty} a_r z^r = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 + a_5 z^5 + a_6 z^6 = 1 + z + z^2 + z^3 + z^4 + z^5 + z^6 = \frac{1-z^7}{1-z}$$


thank you!