MANIPAL UNIVERSITY
JAIPUR
(University under Section 2(f) of the UGC Act)


## B.TECH SECOND YEAR

ACADEMIC YEAR: 2020-2021

# COURSE NAME: ENGINEERING MATHEMATICS-III

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MANIPAL UNIVERSITY
JAIPUR

**VISION**

Global Leadership in Higher Education and Human Development

**MISSION**

- Be the most preferred University for innovative and interdisciplinary learning
- Foster academic, research and professional excellence in all domains
- Transform young minds into competent professionals with good human values

**VALUES**

Integrity, Transparency, Quality,
Team Work, Execution with Passion, Humane Touch

# SESSION OUTCOME

"KNOWLEDGE OF RULES OF INFERENCES TO FIND VALIDITY OF ARGUMENTS"

ASSIGNMENT

QUIZ

MID TERM EXAMINATION –I & II
END TERM EXAMINATION

# ASSESSMENT CRITERIA'S

# PROGRAM OUTCOMES MAPPING WITH CO5

[PO1]

**ENGINEERING KNOWLEDGE: APPLY THE KNOWLEDGE OF MATHEMATICS, SCIENCE, ENGINEERING FUNDAMENTALS, AND AN ENGINEERING SPECIALIZATION TO THE SOLUTION OF COMPLEX ENGINEERING PROBLEMS.**

# Rules of Inferences

# Consider the Socrates Example

- We have the two premises:
  - “All men are mortal.”
  - “Socrates is a man.”
- And the conclusion:
  - “Socrates is mortal.”
- How do we get the conclusion from the premises?

# The Argument

- We can express the premises (above the line) and the conclusion (below the line) in predicate logic as an argument:

$$\forall x(Man(x) \rightarrow Mortal(x))$$

$$Man(Socrates)$$

$$\therefore \quad Mortal(Socrates)$$

- We will see shortly that this is a valid argument.

# Rules of Inference for Propositional Logic: Modus Ponens

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

**Corresponding Tautology:**

$(p \wedge (p \to q)) \to q$

**Example:**

Let $p$ be "It is snowing."
Let $q$ be "I will study discrete math."

"If it is snowing, then I will study discrete math."
"It is snowing."

"Therefore , I will study discrete math."

**Example:**

"If it snows today, then we will go skiing"
The hypothesis " it snows today," is true, then by modus ponens, "We will go skiing." is true.

# Modus Tollens

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

**Corresponding Tautology:**

$(\neg q \wedge (p \to q)) \to \neg p$

**Example**:

Let $p$ be "it is snowing."

Let $q$ be "I will study discrete math."

"If it is snowing, then I will study discrete math."

"I will not study discrete math."

"Therefore , it is not snowing."

# Hypothetical Syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

**Corresponding Tautology:**
$((p \to q) \land (q \to r)) \to (p \to r)$

**Example**:
Let $p$ be "it snows."
Let $q$ be "I will study discrete math."
Let $r$ be "I will get an A."

"If it snows, then I will study discrete math."
"If I study discrete math, I will get an A."

"Therefore , If it snows, I will get an A."

# Disjunctive Syllogism

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

**Corresponding Tautology:**
$(\neg p \wedge (p \vee q)) \rightarrow q$

**Example**:
Let $p$ be "I will study discrete math."
Let $q$ be "I will study English literature."

"I will study discrete math or I will study English literature."
"I will not study discrete math."

"Therefore , I will study English literature."

# Addition

$$\frac{p}{\therefore p \vee q}$$

**Corresponding Tautology:**

$p \rightarrow (p \vee q)$

**Example**:
Let $p$ be "I will study discrete math."
Let $q$ be "I will visit Las Vegas."

"I will study discrete math."

"Therefore, I will study discrete math or I will visit Las Vegas."

# Simplification

$$\frac{p \wedge q}{\therefore q}$$

**Corresponding Tautology:**

$(p \wedge q) \rightarrow p$

**Example**:

Let $p$ be "I will study discrete math."

Let $q$ be "I will study English literature."

"I will study discrete math and English literature"

"Therefore, I will study discrete math."

# Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

**Corresponding Tautology:**

$((p) \wedge (q)) \rightarrow (p \wedge q)$

# Resolution

$$\begin{array}{l} \neg p \lor r \\ p \lor q \\ \hline \therefore q \lor r \end{array}$$

**Corresponding Tautology:**
$((\neg p \lor r) \land (p \lor q)) \rightarrow (q \lor r)$

**Example**:
Let $p$ be “I will study discrete math.”
Let $r$ be “I will study English literature.”
Let q be “I will study databases.”

“I will not study discrete math or I will study English literature.”
“I will study  discrete math or I will study databases.”

“Therefore, I will study databases or I will English literature.”

$$T \rightarrow (M \vee E)$$
$$S \rightarrow \neg E$$
$$T \wedge S$$
$$\therefore M$$

**Example**:
If today is Tuesday, I have a test in mathematics or Economics. If my Economics Professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics Professor is sick. Therefore I have a test in Mathematics.

Let $T$ denote "Today is Tuesday."
$M$ denote "I have a test in Mathematics."
$E$ denote "I have a test in Economics."
$S$ denote "My Economics Professor is sick."

# Using the Rules of Inference to Build Valid Arguments

- A *valid argument* is a sequence of statements. Each statement is either a premise or follows from previous statements by rules of inference. The last statement is called conclusion.
- A valid argument takes the following form:

$$\begin{array}{l} S_1 \\ S_2 \\ \cdot \\ \cdot \\ \cdot \\ S_n \\ \\ \therefore C \end{array}$$

# Valid Arguments

**Example 1:**

- With these hypotheses:

  "It is not sunny this afternoon and it is colder than yesterday."

  "We will go swimming only if it is sunny."

  "If we do not go swimming, then we will take a canoe trip."

  "If we take a canoe trip, then we will be home by sunset."

- Using the inference rules, construct a valid argument for the conclusion:

  "We will be home by sunset."

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# Valid Arguments

**Solution**:

1. Choose propositional variables:

   *p* : "It is sunny this afternoon." *r* : "We will go swimming." *t* : "We will be home by sunset."

   *q* : "It is colder than yesterday." *s* : "We will take a canoe trip."

2. Translation into propositional logic:

$$\text{Hypotheses: } \neg p \wedge q,\ r \rightarrow p,\ \neg r \rightarrow s,\ s \rightarrow t$$
$$\text{Conclusion: } t$$

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# Valid Arguments

3. Construct the Valid Argument

| **Step** | **Reason** |
|---|---|
| 1. $\neg p \wedge q$ | Premise |
| 2. $\neg p$ | Simplification using (1) |
| 3. $r \rightarrow p$ | Premise |
| 4. $\neg r$ | Modus tollens using (2) and (3) |
| 5. $\neg r \rightarrow s$ | Premise |
| 6. $s$ | Modus ponens using (4) and (5) |
| 7. $s \rightarrow t$ | Premise |
| 8. $t$ | Modus ponens using (6) and (7) |

# Valid Arguments

**Example 2**: From the single proposition

$$p \wedge (p \rightarrow q)$$

Show that $q$ is a conclusion.

**Solution**:

| **Step** | **Reason** |
|---|---|
| 1. $p \wedge (p \rightarrow q)$ | Premise |
| 2. $p$ | Conjunction using (1) |
| 3. $p \rightarrow q$ | Conjunction using (1) |
| 4. $q$ | Modus Ponens using (2) and (3) |

# Handling Quantified Statements

- Valid arguments for quantified statements are a sequence of statements. Each statement is either a premise or follows from previous statements by rules of inference which include:
  - Rules of Inference for Propositional Logic
  - Rules of Inference for Quantified Statements
- The rules of inference for quantified statements are introduced in the next several slides.

# Rules of Inference for Quantified Statements Universal Instantiation (UI)

$$\frac{\forall x P(x)}{\therefore P(c)}$$

**Example**:

Our domain consists of all dogs and Fido is a dog.

"All dogs are cuddly."

"Therefore, Fido is cuddly."

# Universal Generalization (UG)

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$$

Used often implicitly in Mathematical Proofs.

# Existential Instantiation (EI)

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$$

**Example**:

"There is someone who got an A in the course."
"Let's call her *a* and say that *a* got an A"

# Existential Generalization (EG)

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$$

**Example**:

“Michelle got an A in the class.”
“Therefore,  someone got an A in the class.”

# Fallacies

**Some fallacies, arise in incorrect arguments, resemble rules of inference, but are based on contingencies rather than tautologies.**

**Example**: If you do every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics. Therefore, you did every problem in this book.

Is this argument valid?

**Solution**: Let $p$ be the proposition "you did every problem in this book." Let $q$ be the proposition "You learned discrete mathematics." **Then this argument is of the form: if $p \rightarrow q$ and $q$, then $p$.**

# Using Rules of Inference

**Example 1**: Using the rules of inference, construct a valid argument to show that

"John Smith has two legs"

is a consequence of the premises:

"Every man has two legs." "John Smith is a man."

**Solution**: Let $M(x)$ denote "$x$ is a man" and $L(x)$ " $x$ has two legs" and let John Smith be a member of the domain.

**Valid Argument**:

| **Step** | **Reason** |
|---|---|
| 1. $\forall x(M(x) \rightarrow L(x))$ | Premise |
| 2. $M(J) \rightarrow L(J)$ | UI from (1) |
| 3. $M(J)$ | Premise |
| 4. $L(J)$ | Modus Ponens using (2) and (3) |

# Using Rules of Inference

**Example 2**: Use the rules of inference to construct a valid argument showing that the conclusion

"Someone who passed the first exam has not read the book."

follows from the premises

"A student in this class has not read the book."

"Everyone in this class passed the first exam."

**Solution**: Let $C(x)$ denote "$x$ is in this class," $B(x)$ denote "$x$ has read the book," and $P(x)$ denote "$x$ passed the first exam."

First we translate the premises and conclusion into symbolic form.

$$\begin{array}{l} \exists x(C(x) \land \neg B(x)) \\ \forall x(C(x) \rightarrow P(x)) \\ \hline \therefore\ \exists x(P(x) \land \neg B(x)) \end{array}$$

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# Using Rules of Inference

**Example 3**: Show that the premises "Everyone in this discrete mathematics class has taken a course in computer science" and "Marla is a student in this class" imply the conclusion "Marla has taken a course in computer science."

**Solution**: Let $D(x)$ denote "$x$ is in this discrete mathematics class" and $C(x)$ "$x$ has taken a course in computer science." The premise are $\forall x(D(x) \rightarrow C(x))$ and $D(\text{Marla})$ and the conclusion is $C(\text{Marla})$.

| Step | Reason |
|---|---|
| 1. $\forall x(D(x) \rightarrow C(x))$ | Premise |
| 2. $D(\text{Marla}) \rightarrow C(\text{Marla})$ | Universal instantiation from (1) |
| 3. $D(\text{Marla})$ | Premise |
| 4. $C(\text{Marla})$ | Modus ponens from (2) and (3) |

$$
\begin{array}{rl}
 & \exists x(C(x) \land \neg B(x)) \\
 & \forall x(C(x) \to P(x)) \\
\hline
\therefore & \exists x(P(x) \land \neg B(x))
\end{array}
$$

**Valid Argument**:

| **Step** | **Reason** |
|---|---|
| 1. $\exists x(C(x) \land \neg B(x))$ | Premise |
| 2. $C(a) \land \neg B(a)$ | EI from (1) |
| 3. $C(a)$ | Simplification from (2) |
| 4. $\forall x(C(x) \to P(x))$ | Premise |
| 5. $C(a) \to P(a)$ | UI from (4) |
| 6. $P(a)$ | MP from (3) and (5) |
| 7. $\neg B(a)$ | Simplification from (2) |
| 8. $P(a) \land \neg B(a)$ | Conj from (6) and (7) |
| 9. $\exists x(P(x) \land \neg B(x))$ | EG from (8) |

$$\frac{\begin{array}{c}\forall x(Man(x) \rightarrow Mortal(x)) \\ Man(Socrates)\end{array}}{\therefore \quad Mortal(Socrates)}$$

**Valid Argument**

| **Step** | **Reason** |
|---|---|
| 1. $\forall x(Man(x) \rightarrow Mortal(x))$ | Premise |
| 2. $Man(Socrates) \rightarrow Mortal(Socrates)$ | UI from (4) |
| 3. $Man(Socrates)$ | Premise |
| 4. $Mortal(Socrates)$ | MP from (2) and (3) |

Universal Modus Ponens combines universal instantiation and modus ponens into one rule.

$$\begin{array}{l} \forall x(P(x) \rightarrow Q(x)) \\ P(a), \text{where } a \text{ is a particular} \\ \qquad \text{element in the domain} \\ \hline \therefore Q(a) \end{array}$$

This rule could be used in the Socrates example.

Thanks for your attention!!