



**MANIPAL UNIVERSITY
JAIPUR**

(University under Section 2(f) of the UGC Act)



B.TECH SECOND YEAR

ACADEMIC YEAR: 2020-2021



COURSE NAME: ENGINEERING MATHEMATICS-III

COURSE CODE : MA 2101

LECTURE SERIES NO : 37-38 (THIRTY SEVEN – THIRTY EIGHT)

CREDITS : 3

MODE OF DELIVERY : ONLINE (POWER POINT PRESENTATION)

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PROPOSED DATE OF DELIVERY: 19 OCTOBER 2020



**MANIPAL UNIVERSITY
JAIPUR**

VISION

Global Leadership in Higher Education and Human Development

MISSION

- Be the most preferred University for innovative and interdisciplinary learning
- Foster academic, research and professional excellence in all domains
- Transform young minds into competent professionals with good human values

VALUES

Integrity, Transparency, Quality,
Team Work, Execution with Passion, Humane Touch

SESSION OUTCOME

“ TO UNDERSTAND THE CONCEPT
OF ODE AND THEIR APPLICATIONS
AND SOLVE THE PROBLEM”

ASSIGNMENT

QUIZ

MID TERM EXAMINATION –I & II

END TERM EXAMINATION

ASSESSMENT CRITERIA'S

PROGRAM OUTCOMES MAPPING WITH CO1

**ENGINEERING KNOWLEDGE: APPLY THE KNOWLEDGE
OF MATHEMATICS, SCIENCE, ENGINEERING
FUNDAMENTALS, AND AN ENGINEERING
SPECIALIZATION TO THE SOLUTION OF COMPLEX
ENGINEERING PROBLEMS.**

Boolean Algebra

- Boolean algebra provides the operations and the rules for working with the set $\{0, 1\}$.
- These are the rules that underlie **electronic circuits**, and the methods we will discuss are fundamental to **VLSI design**.
- We are going to focus on three operations:
 - Boolean complementation,
 - Boolean sum and
 - Boolean product

Boolean Operations

The **complement** is denoted by a bar. It is defined by

$$0 = 1 \text{ and } 1 = 0.$$

The **Boolean sum**, denoted by + or by OR, has the following values:

$$\begin{aligned} 1 + 1 &= 1, \\ 1 + 0 &= 1, \\ 0 + 1 &= 1, 0 + 0 = 0 \end{aligned}$$

The **Boolean product**, denoted by \times or by AND, has the following values:

$$\begin{aligned} 1 \times 1 &= 1, 1 \times 0 = 0, \\ 0 \times 1 &= 0, \\ 0 \times 0 &= 0 \end{aligned}$$

Examples:

- 1) Find the values of $1.0 + (0 + \bar{1}) + \bar{0}.0$
- 2) Show that $(1.1) + [(0 . \bar{1}) + 0] = 1$
- 3) Find the values of $(\bar{1} . \bar{0}) + (1 . \bar{0})$

Note:

- ➡ The complement, Boolean sum and Boolean product correspond to the logic operators \sim , \vee and \wedge respectively, where 0 corresponds to F (False) and 1 corresponds to T (True)
- ➡ Equalities in Boolean algebra can be considered as equivalences of compound propositions.

Translate the following into logical equivalence:

1) $1.0 + (\overline{0 + 1}) = 0$

2) $(1.1) + [\overline{(0 . 1)} + 0] = 1$

Translate the logical equivalences into Boolean algebra:

1) $(T \wedge T) \vee [\sim(F \wedge T) \vee F] \equiv T$

2) $(T \vee F) \wedge (\sim F) \equiv F$

Boolean Expressions & Boolean Functions

- ♣ Let $B = \{0, 1\}$, then $B^n = \{(x_1, x_2, \dots, x_n) / x_i \in B \text{ for } i = 1 \text{ to } n\}$ is the set of all possible n -tuples of 0's and 1's.
- ♣ **Boolean variable:** The variable x is called a Boolean variable if it assumes values only from B . i.e. if its only possible values are 0 and 1.
- ♣ **Boolean function of degree n :** A function $F: B^n \rightarrow B$, i.e. $F(x_1, x_2, \dots, x_n) = x$, is called a Boolean function of degree n .

E.g.

- 1) $F(x, y) = x \cdot \bar{y}$ from the set of ordered pairs of Boolean variables to the set $\{0, 1\}$ is a Boolean function of degree 2 with given values in table:

x	y	\bar{y}	F
1	1	0	0
1	0	1	1
0	1	0	0
0	0	1	0

Examples of Boolean Functions

2) Boolean Function:

$$F = x + \bar{y}z$$

↔ Truth Table

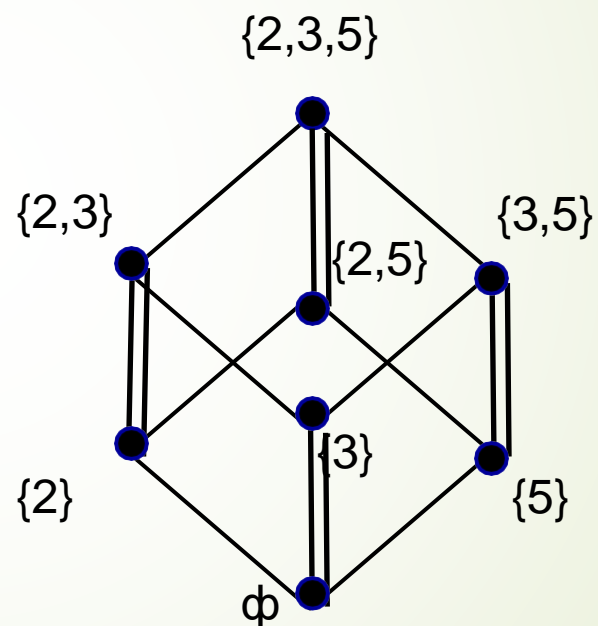
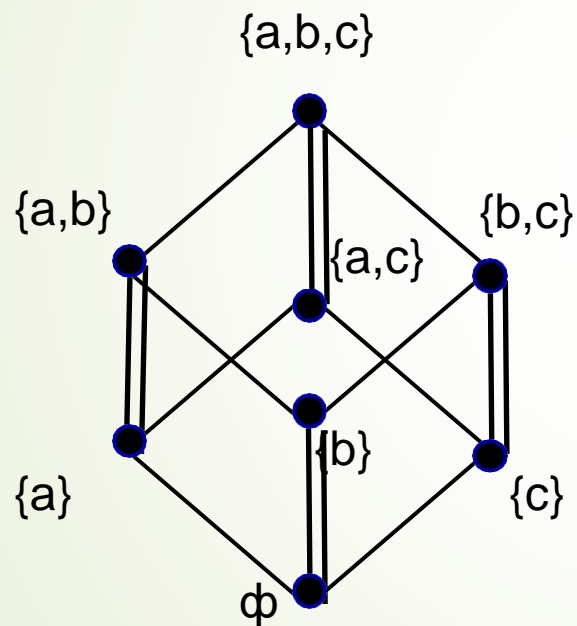
All possible combinations
of input variables

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Finite Boolean Algebra

Example:

$S = \{a, b, c\}$ and $T = \{2, 3, 5\}$. consider the Hasse diagrams of the two lattices $(P(S), \subseteq)$ and $(P(T), \subseteq)$.



Note : the lattice depends only on the number of elements in set, not on the elements.

Finite Boolean Algebras

♣ $(P(S), \subseteq)$

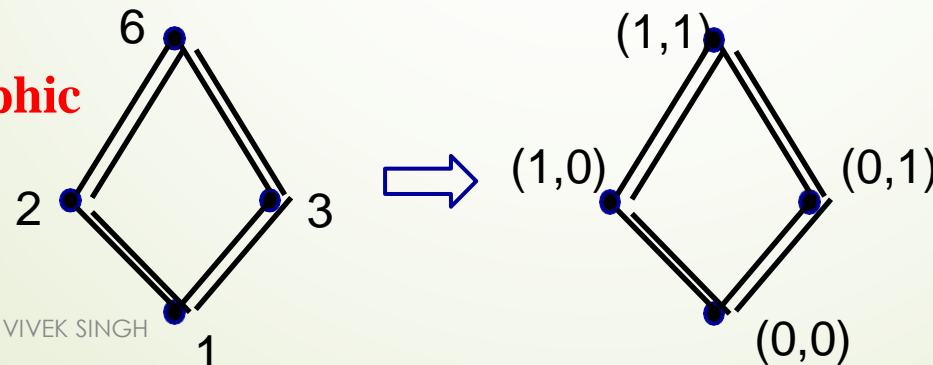
Each x and y in B_n correspond to subsets A and B of S . Then $x \leq y$, $x \wedge y$, $x \vee y$ and x' correspond to $A \subseteq B$, $A \cap B$, $A \cup B$ and A^c . Therefore,

$(P(S), \subseteq)$ is isomorphic with B_n , where $n=|S|$

♣ Example

Consider the lattice D_6 consisting of all positive integer divisors of 6 under the partial order of divisibility.

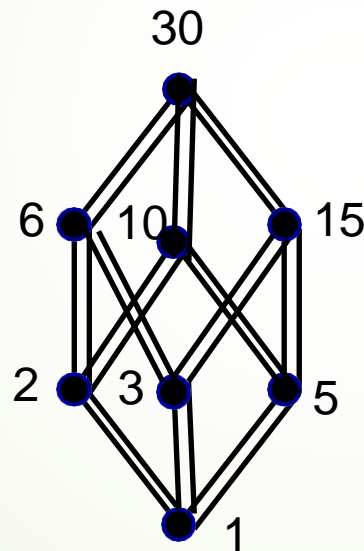
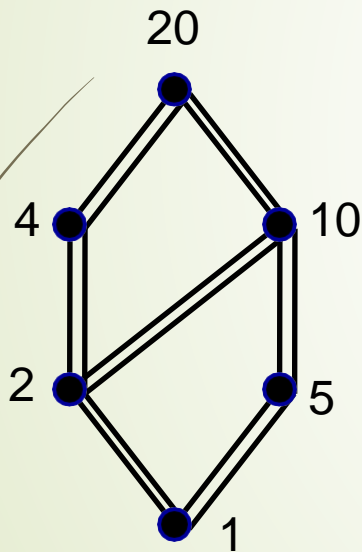
D_6 is isomorphic



Finite Boolean Algebras

♣ Example

Consider the lattices D_{20} and D_{30} of all positive integer divisors of 20 and 30, respectively.

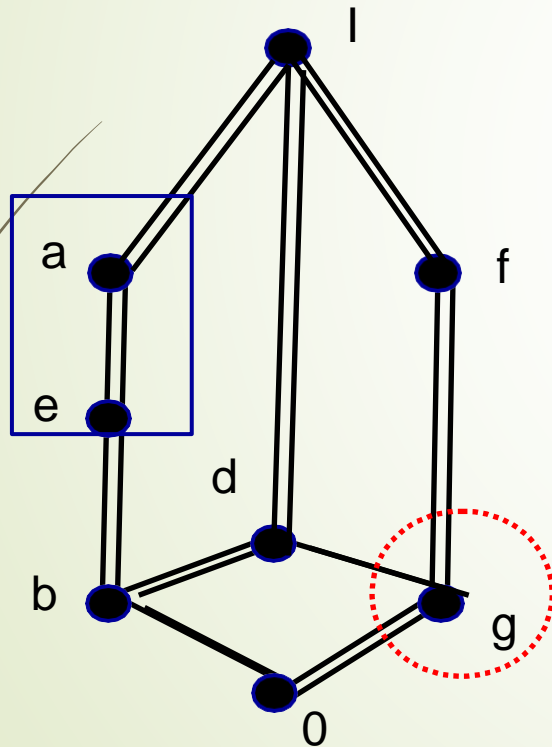


**D_{20} is not a Boolean algebra
(why? 6 is not 2^n)**

D_{30} is a Boolean algebra, $D_{30} \cong B_3$

Finite Boolean Algebras

Example: Show the lattice whose Hasse diagram shown below is not a Boolean algebra.



a and e are both complements of g

Theorem (e.g. properties 1~14) is usually used to show that a lattice L is not a Boolean algebra.

THANK YOU

