MANIPAL UNIVERSITY
JAIPUR
(University under Section 2(f) of the UGC Act)

B.TECH FIRST YEAR

ACADEMIC YEAR: 2020-2021

COURSE NAME: BASIC MECHANICAL ENGINEERING

COURSE CODE : MA 2101

LECTURE SERIES NO : 25 (TWENTY FIVE)

CREDITS : 03

MODE OF DELIVERY : ONLINE (POWER POINT PRESENTATION)

FACULTY : DR. ANAMIKA JAIN

EMAIL-ID : anamika.jain@Jaipur.manipal.edu

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MANIPAL UNIVERSITY
JAIPUR

VISION

Global Leadership in Higher Education and Human Development

MISSION

- Be the most preferred University for innovative and interdisciplinary learning
- Foster academic, research and professional excellence in all domains
- Transform young minds into competent professionals with good human values

VALUES

Integrity, Transparency, Quality,
Team Work, Execution with Passion, Humane Touch

SESSION OUTCOME

"UNDERSTAND THE CONCEPT

OF

GENERATING FUNCTION"

ASSESSMENT CRITERIA'S

- ASSIGNMENT
- QUIZ
- MID TERM EXAMINATION –I, II
- END TERM EXAMINATION

PROGRAM OUTCOMES MAPPING WITH CO4

ENGINEERING KNOWLEDGE: APPLY THE KNOWLEDGE OF MATHEMATICS, SCIENCE, ENGINEERING FUNDAMENTALS, AND AN ENGINEERING SPECIALIZATION TO THE SOLUTION OF COMPLEX ENGINEERING PROBLEMS.

DEFINITION OF GENERATING FUNCTION

Generating function provide an alternative way for representing and studying numeric functions and for solving recurrence relations as well as combinatorial problems.

Let $(a_0, a_1, a_2, \ldots.. \; a_r, \ldots)$ be a numeric function a. Then the infinite series in terms of a parameter z,

$$\mathrm{A(z)} = a_0, +a_1 z + \; a_2 z^2 \ldots.. \; a_r z^r + \ldots\ldots. \qquad (1)$$

Is called generating function of the numeric function a.

or

If $(a_0, a_1, a_2, \ldots.. \; a_r, \ldots)$ is a sequence of real or complex numbers, then the power series given by

$$\mathrm{A(z)} = \; \sum_{r=0}^{\infty} a_r z^r \; a_0, +a_1 z + \; a_2 z^2 \ldots.. \; a_n z^n + \ldots\ldots. \, ..$$

Is called the Generating function for the given sequence.

Example: the generating function of the numeric function $(1, 2^1, 2^2, 2^3, \ldots\ldots)$ is

$$1+2z + 2^2z^2 \ldots.. 2^r z^r + \ldots\ldots \qquad (2)$$

Since the series (2) can be written as

$$\frac{1}{1-2z}$$

Therefore the numeric function $(1, 2^1, 2^2, 2^3, \ldots\ldots)$ can be represented as

$$\frac{1}{1-2z}$$

Geometric Series

$$G_n ::= 1 + x + x^2 + \cdots + x^{n-1} + x^n + \cdots$$

What is the closed form expression of G_n?

$$G_n ::= 1 + \cancel{x} + \cancel{x^2} + \cancel{\cdots} + \cancel{x^{n-1}} + \cancel{x^n} \cancel{+ \cdots}$$

$$xG_n = \cancel{x} + \cancel{x^2} + \cancel{x^3} + \cdots + \cancel{x^n} + \cancel{x^{n+1}} + \cdots$$

$$G_n - xG_n = 1$$

$$G_n = \frac{1 - x^{n+1}}{1 - x}$$

More Examples

$$< 1, 1, 1, 1, \ldots > \quad \leftrightarrow \quad 1 + 1x + 1x^2 + 1x^3 + \ldots \quad = \quad \frac{1}{1-x}$$

$$< 1, -1, 1, -1, \ldots > \quad \leftrightarrow \quad 1 - 1x + 1x^2 - 1x^3 + \ldots \quad = \quad \frac{1}{1+x}$$

$$< 1, a, a^2, a^3, \ldots > \quad \leftrightarrow \quad 1 + ax + a^2x^2 + a^3x^3 + \ldots \quad = \quad \frac{1}{1-ax}$$

$$< 1, 0, 1, 0, \ldots > \quad \leftrightarrow \quad 1 + 0x + 1x^2 + 0x^3 + \ldots \quad = \quad \frac{1}{1-x^2}$$

These are all closed form generating functions.

Example

1. For the sequence {1, 2, 3, 4, ...} the generating function

$$G(x) = 1 + 2x + 3x^2 + \cdots = \frac{1}{(1-x)^2}$$

2. Sequence: $\{n_{C_0}, n_{C_1}, n_{C_2}, \dots, n_{C_n}, 0,0,0, \dots\}$

$$G(x) = n_{C_0} + n_{C_1}x + n_{C_2}x^2 + n_{C_3}x^3 + \cdots + n_{C_n} + 0 + 0 + 0 \dots$$

$$= (1+x)^n + 0 + 0 + \cdots$$

$$G(x) = (1+x)^n$$

Theorem: Let a, b and c be numeric functions and let A(z), B(z) and C(z) be respectively the generating functions of a, b and c then

(i) If $b_r = \alpha a_r$, for some constant α, then B(z) =α A(z)

(ii) If $c_r = a_r + b_r$, then C(z) = A(z)+B(z)

(iii) If c is the convolution of a and b; i.e. c = a*b, then C(z) = A(z)B(z)

(iv) If $b_r = \alpha^r a_r$, where is a constant, then B(z) = A(αz)

Theorem: Let A(z) be the generating function of the numeric function a= $(a_0, a_1, a_2, \ldots\ldots a_r, \ldots)$. Then $\frac{1}{(1-z)}$ A(z) is the generating function of the numeric function b which is accumulated sum of a.

Example **:** *Find the generating function of the following series* $1, -1, 1, -1, 1, -1, \ldots$.

Solution: The given series can be directed as $a_0 = 1,\ a_1 = -1,\ a_2 = 1,\ a_3 = -1 \ldots$.

The required generating function is given by

$$A(z) = \sum_{r=0}^{\infty} a_r z^r = a_0 + a_1 z + a_2 z^2 + \ldots = 1 - z + z^2 - z^3 + \ldots = \frac{1}{1+z} = (1+z)^{-1}$$

[sum of infinite G.P. series $S_\infty = \frac{a}{1-$

Example **:** *Find the generating function of the following series* $1, 1, 1, 1, 1, 1, 1$.

Solution: The given series is directed as

$$a_0 = 1,\ a_1 = 1,\ a_2 = 1,\ a_3 = 1,\ a_4 = 1,\ a_5 = 1,\ a_6 = 1.$$

The required generating function is given by

$$a(z) = \sum_{r=0}^{\infty} a_r z^r = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 + a_5 z^5 + a_6 z^6 = 1 + z + z^2 + z^3 + z^4 + z^5 + z^6 = \frac{1-z^7}{1-z}$$


thank you!