MANIPAL UNIVERSITY JAIPUR
(University under Section 2(f) of the UGC Act)

B.TECH FIRST YEAR

ACADEMIC YEAR: 2020-2021

COURSE NAME: BASIC MECHANICAL ENGINEERING

COURSE CODE : MA 2101

LECTURE SERIES NO : 24 (TWENTY FOUR)

CREDITS : 03

MODE OF DELIVERY : ONLINE (POWER POINT PRESENTATION)

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PROPOSED DATE OF DELIVERY: 14 OCTOBER 2020

MANIPAL UNIVERSITY JAIPUR

VISION

Global Leadership in Higher Education and Human Development

MISSION

- Be the most preferred University for innovative and interdisciplinary learning
- Foster academic, research and professional excellence in all domains
- Transform young minds into competent professionals with good human values

VALUES

Integrity, Transparency, Quality,
Team Work, Execution with Passion, Humane Touch

SESSION OUTCOME

"UNDERSTAND THE PRINCIPLE OF INCLUSION AND EXCLUSION"

ASSIGNMENT

QUIZ

MID TERM EXAMINATION –I, II

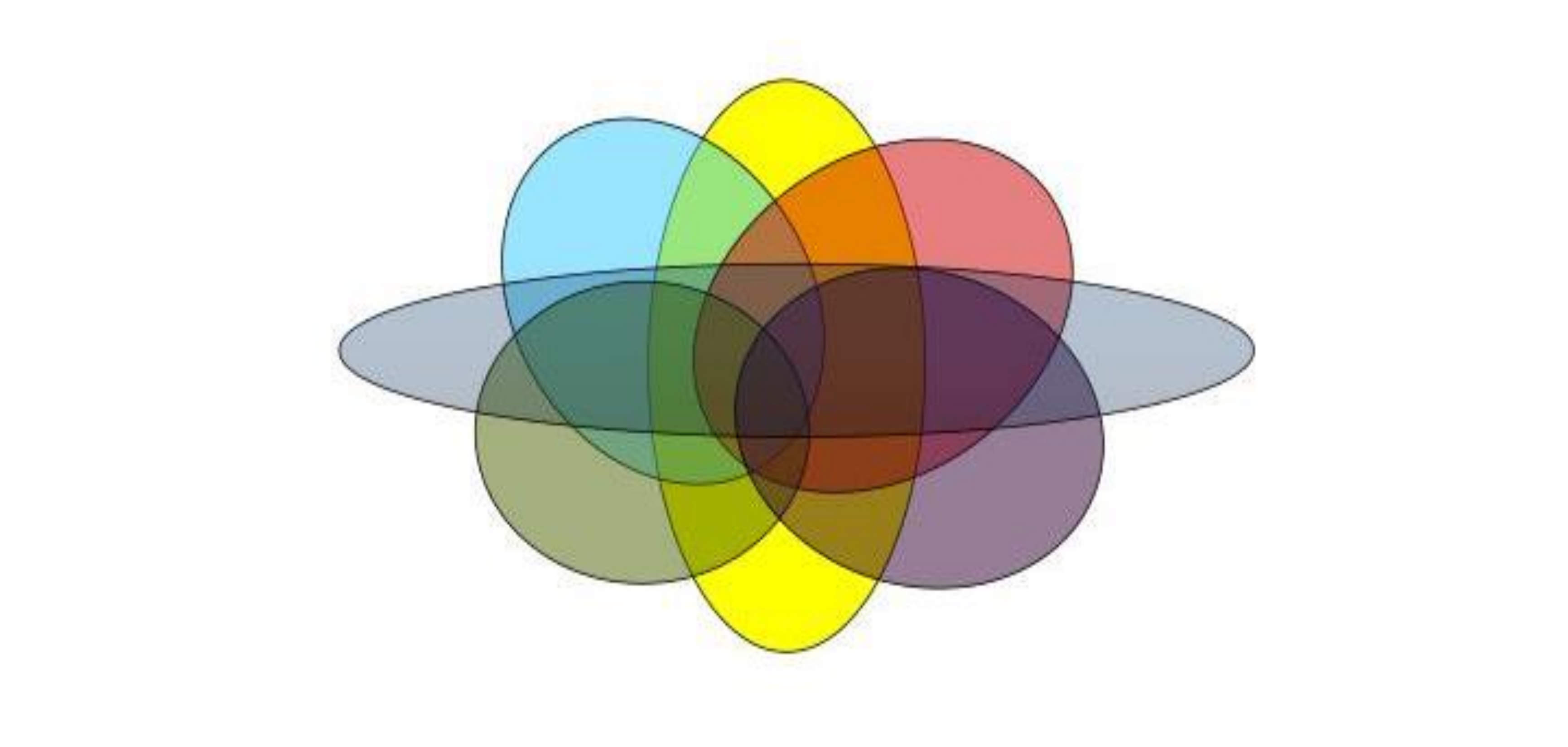
END TERM EXAMINATION

ASSESSMENT CRITERIA'S

PROGRAM OUTCOMES MAPPING WITH CO4

ENGINEERING KNOWLEDGE: APPLY THE KNOWLEDGE OF MATHEMATICS, SCIENCE, ENGINEERING FUNDAMENTALS, AND AN ENGINEERING SPECIALIZATION TO THE SOLUTION OF COMPLEX ENGINEERING PROBLEMS.

PRINCIPLE OF INCLUSION AND EXCLUSION PARTITIONS



If $A \cap B \neq \emptyset$ $then\ sum\ rule$ $|A \cup B| = |A| + |B|$ does not hold. For example if A={a, b, c}, B={c, d, e, f} then $|A| = 3$ and $|B| = 4$ but $|A \cup B|$=6 not 7. the general formula which is true for any finite sets A and B is

$$|A \cup B| = |A| + |B| - |A \cap B| \tag{1}$$

Here to find $|A \cup B|$ we included (added) $|A|\ and\ |B|$ and we excluded (subtracted) $|A \cap B|$. Formula (1) is a special case of the inclusion-exclusion principle. Before stating the general inclusion-exclusion principle for n sets we give it for three sets.

Theorem: For any finite set A, B,C we have

$$|\boldsymbol{A} \cup \boldsymbol{B} \cup \boldsymbol{C}| = |\boldsymbol{A}| + |\boldsymbol{B}| + |\boldsymbol{C}| - |\boldsymbol{A} \cap \boldsymbol{B}| - |\boldsymbol{B} \cap \boldsymbol{C}| - |\boldsymbol{A} \cap \boldsymbol{C}| + |\boldsymbol{A} \cap \boldsymbol{B} \cap \boldsymbol{C}|$$

Theorem: (The inclusion-exclusion principle) Let $A_1, A_2, \dots\dots\dots\dots\dots A_n$ be n finite sets. Then

$$|A_1 \cup A_2 \cup A_3 \cup \cdots \dots\dots\dots\dots \cup A_n| = \sum_{i=1}^{n} |A_i| - \sum_{1 \le i < j < k \le n} |A_i \cap A_j| + \sum_{1 \le i < j < k \le n} |A_i \cap A_j \cap A_k|$$

$$\dots\dots.\dots..+(-1)^{n-1}|A_1 \cap A_2 \cap A_3 \dots\dots..\cap A_n|.$$

Alternative form of the inclusion-exclusion principle

Let S be a finite set and $A_1, A_2, \dots\dots\dots\dots\dots A_n$ be subsets of S. Let A'_i **denote the complement of** A_i **then**

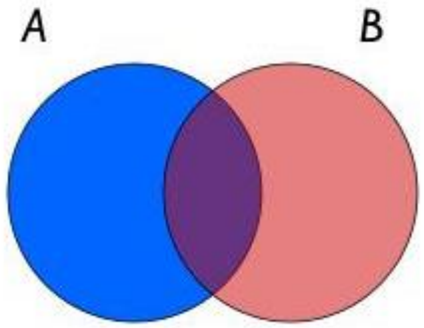
Theorem: If $A_1, A_2, \dots\dots\dots\dots\dots A_n$ are subsets of a finite set S then

$$|A'_1 \cap A'_2 \cap A'_3 \cap \cdots \dots\dots\dots\dots \cap A'_n| = |S| - \sum_{i=1}^{n} |A_i| + \sum_{1 \le i < j < k \le n} |A_i \cap A_j|$$

Inclusion-Exclusion (2 sets)

For two arbitrary sets *A* and *B*

$$|A \cup B| = |A| + |B| - |A \cap B|$$



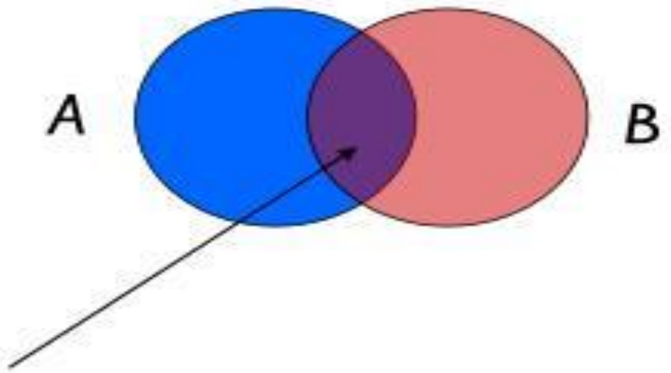
Inclusion-Exclusion (2 sets)

Let S be the set of integers from 1 through 1000 that are multiples of 3 or multiples of 5.

Let A be the set of integers from 1 to 1000 that are multiples of 3.

Let B be the set of integers from 1 to 1000 that are multiples of 5.

It is clear that S is the union of A and B,
but notice that A and B are not disjoint.



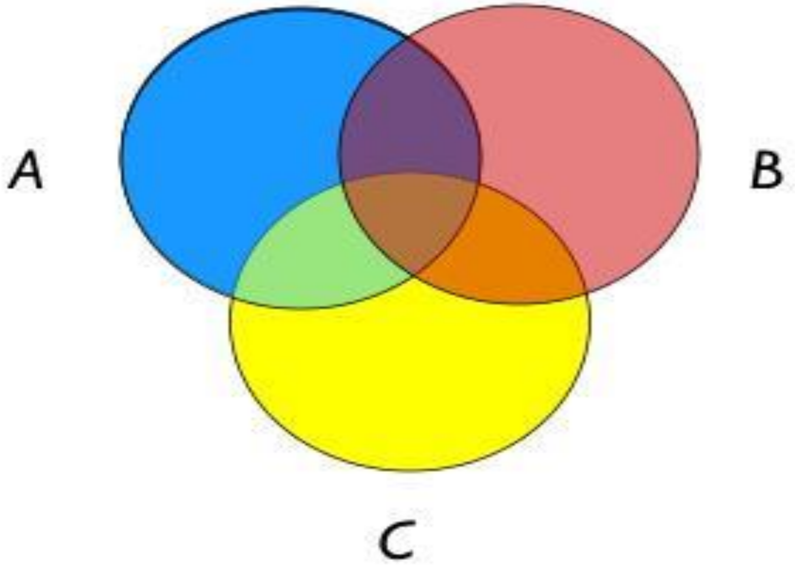
$|A| = 1000/3 = 333$ $\qquad$ $|B| = 1000/5 = 200$

$A \cap B$ is the set of integers that are multiples of 15, and so $|A \cap B| = 1000/15 = 66$

So, by the inclusion-exclusion principle, we have $|S| = |A| + |B| - |A \cap B| = 467$.

Inclusion-Exclusion (3 sets)

$$
\boxed{
\begin{aligned}
|A \cup B \cup C| = |A| + |B| + |C| \\
- |A \cap B| - |A \cap C| - |B \cap C| \\
+ |A \cap B \cap C|
\end{aligned}
}
$$



Inclusion-Exclusion (3 sets)

From a total of 50 students:

$|A| \longrightarrow$ 30 know Java

$|B| \longrightarrow$ 18 know C++

$|C| \longrightarrow$ 26 know C#

$|A \cap B| \longrightarrow$ 9 know both Java and C++

$|A \cap C| \longrightarrow$ 16 know both Java and C#

$|B \cap C| \longrightarrow$ 8 know both C++ and C#

$|A \cup B \cup C| \longrightarrow$ 47 know at least one language.

How many know none?

How many know all?

$|A \cap B \cap C|$

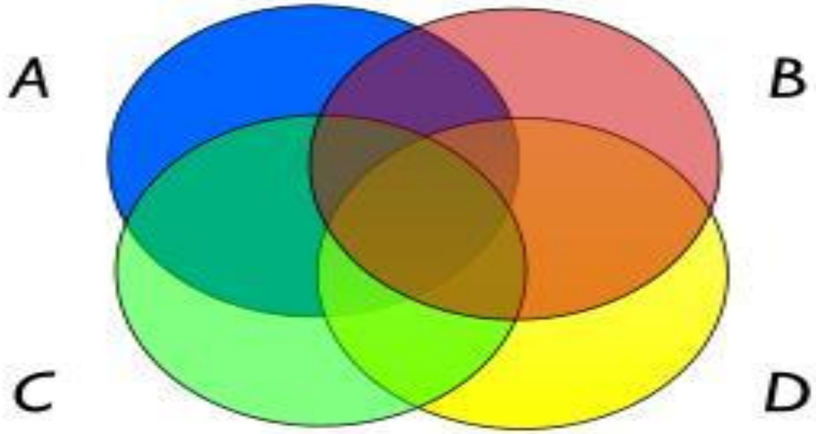
$$\boxed{|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|}$$

$$47 = 30 + 18 + 26 - 9 - 16 - 8 + |A \cap B \cap C|$$

$$|A \cap B \cap C| = 6$$

Inclusion-Exclusion (4 sets)

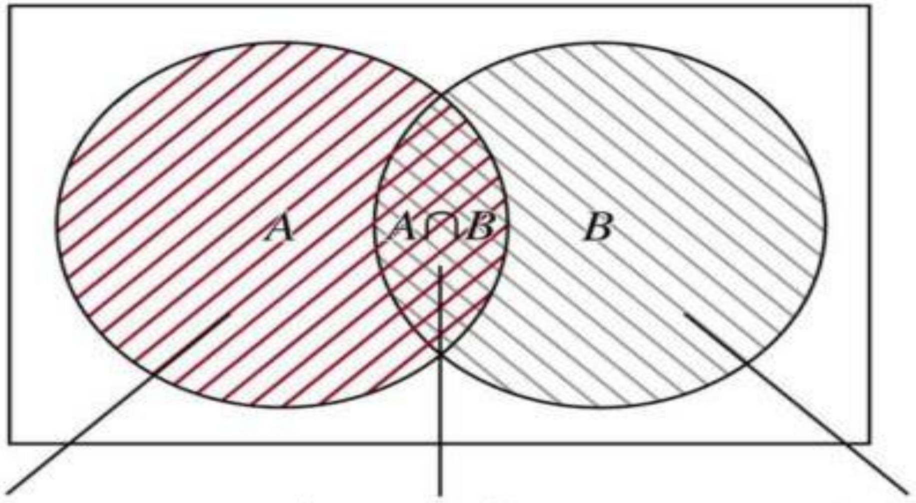
$$
\begin{aligned}
|A \cup B \cup C \cup D| = {} & |A| + |B| + |C| + |D| \\
& - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| \\
& + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| \\
& - |A \cap B \cap C \cap D|
\end{aligned}
$$



- Example 1: In a discrete mathematics class every student is a major in *computer science* or *mathematics*, or both.
- The number of students having computer science as a major (possibly along with mathematics) is 25;
- the number of students having mathematics as a major (possibly along with computer science) is 13;
- and the number of students majoring in both computer science and mathematics is 8.

- How many students are in this class?

$$|A \cup B| = |A| + |B| - |A \cap B| = 25 + 13 - 8 = 30$$



$|A| = 25$ $\quad$ $|A \cap B| = 8$ $\quad$ $|B| = 13$

FIGURE 1 The Set of Students in a Discrete Mathematics Class.

- Example 2: How many positive integers not exceeding 1000 are divisible by 7 or 11?

$$|A \cup B| = |A| + |B| - |A \cap B| = 142 + 90 - 12 = 220$$

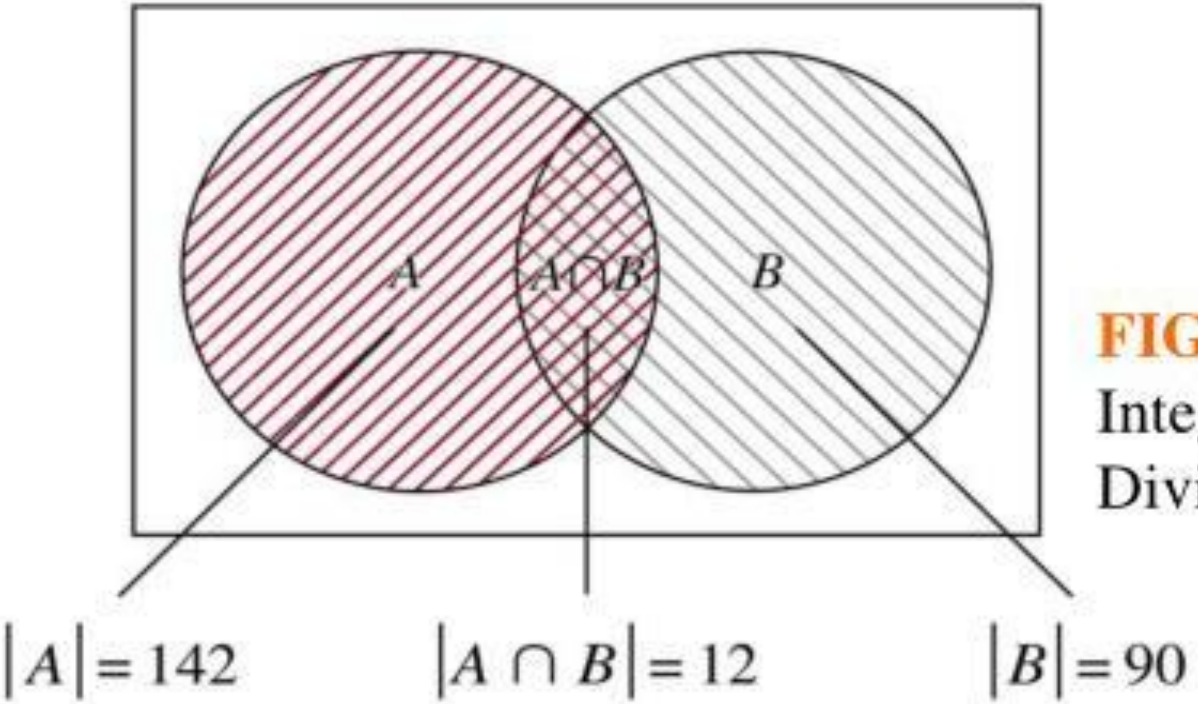
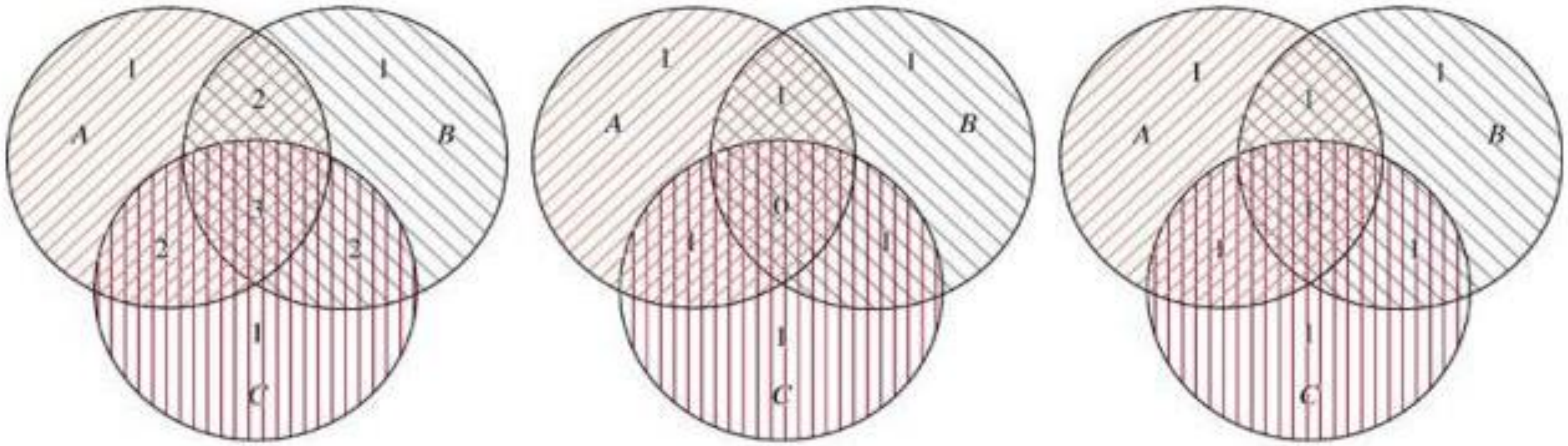


FIGURE 2 The Set of Positive Integers Not Exceeding 1000 Divisible by Either 7 or 11.

- $|A \cup B \cup C| =$

  $|A|+|B|+|C|-|A \cap B|-|A \cap C|-|B \cap C|+|A \cap B \cap C|$



(a) Count of elements by $|A|+|B|+|C|$

(b) Count of elements by $|A|+|B|+|C|-|A \cap B|-|A \cap C|-|B \cap C|$

(c) Count of elements by $|A|+|B|+|C|-|A \cap B|-|A \cap C|-|B \cap C|+|A \cap B \cap C|$

FIGURE 3 Finding a Formula for the Number of Elements in the Union of Three Sets.


thank you!