Question 1

Show that in the regression model $log(y_i) = \alpha + \beta \ log(x_i) + \epsilon_i$, the elasticity of y with respect to x is equal to β (that is, does not depend on the values of x_i and y_i).

Answer

According to the definition of the elasticity, (noted in lecture slides)

$$\text{Elasticity} = \frac{dy/y}{dx/x} = \frac{dy}{dx} \times \frac{x}{y}$$

By exponentiating both sides of the model

$$\begin{aligned} y_i &= e^{(\alpha+\epsilon_i)} \times e^{(\beta \times log(x_i))} \\ &= e^{(\alpha+\epsilon_i)} \times x_i^{\beta} \end{aligned}$$

Now we can calculate the derivative of y with respect to x

$$\frac{dy_i}{dx_i} = e^{\alpha+\epsilon_i} \times \beta \times x_i^{\beta-1} = \beta \times \frac{y_i}{x_i}$$

By substituting the derivative of y with respect to x into the elasticity equation

$$\text{Elasticity} = \frac{dy}{dx} \times \frac{x}{y} = \beta \ \frac{\cancel{y}}{\cancel{x}} \times \frac{\cancel{x}}{\cancel{y}} = \beta$$

Question 2

Determine the elasticity of y with respect to x in the model $y_i = \alpha + \beta\ log(x_i) + \epsilon_i$.

Answer

The process is the same as in the previous question.

First we calculate the derivative of y with respect to x

$$\frac{dy_i}{dx_i} = \beta \times \frac{1}{x_i}$$

Now the elasticity is

$$\text{Elasticity} = \frac{dy}{dx} \times \frac{x}{y} = \beta \times \frac{\cancel{x}}{y} \times \frac{1}{\cancel{x}} = \beta \times \frac{1}{y}$$

Question 3

Determine the elasticity of y with respect to x in the model $log(y_i) = \alpha + \beta\ x_i + \epsilon_i$.

Answer

Exponentiate the model

$$y_i = e^{(\alpha+\epsilon_i)} \times e^{(\beta x_i)}$$

By taking the derivative of y with respect to x

$$\frac{dy_i}{dx_i} = e^{\alpha+\epsilon_i} \times \beta \times e^{(\beta x_i)} = \beta \times y_i$$

Elasticity would be

$$\text{Elasticity} = \frac{dy}{dx} \times \frac{x}{y} = \beta \times y \times \frac{x}{y} = \beta \times x_i$$